

Physics
Teach Yourself Series
Topic 1: Projectile motion (Units 2 & 3)

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What you should know

As it appears in Unit 3

Investigate and analyse the motion of projectiles near Earth's surface including a qualitative description of the effects of air resistance

Projectile motion

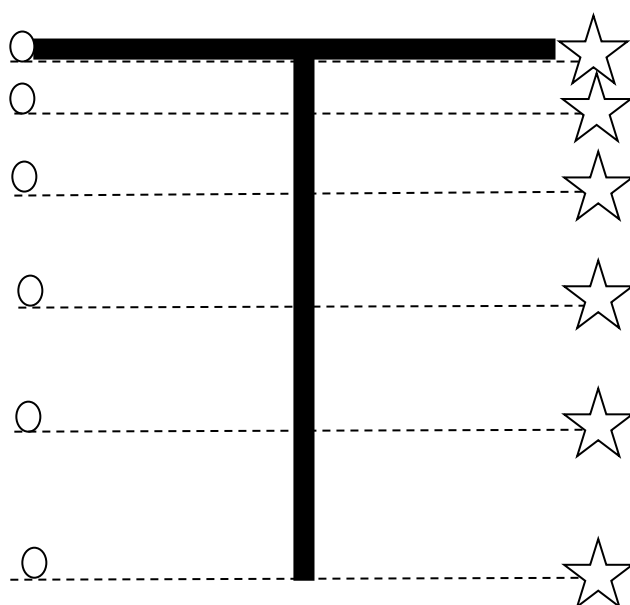
As it appears in Unit 3

A projectile is an object that has been thrown or launched into the air. Projectile motion refers to the motion of any object that is projected into the air.

Galileo was the first person to talk about more than one factor affecting the motion of a projectile. The force that acts vertically is the projectile's weight (i.e. the force due to Earth's gravitational attraction), and this causes the object to accelerate towards the earth at 9.8 m s^{-2} .

But while the object is falling downwards, the projectile is also moving horizontally at the same time. Galileo was able to prove that the path taken by any projectile (except those dropped straight downwards or that have a power source) is parabolic.

In the absence of air resistance, all objects fall with the same uniform acceleration. Therefore, objects dropped from the same point will hit the ground at the same time as shown in the diagram below.



The intervals between the lines represent how far the objects have fallen in a set period of time. The distance between the lines increases as the objects descend indicating that they are speeding up. However, the horizontal displacement of the objects has not changed.

Falling downwards

As it appears in Unit 2

If an object is released above the ground, the only forces acting on it are gravity and air resistance.

When answering questions based on this concept, a couple of assumptions are usually made:

- 1) The accepted figure for acceleration due to gravity is 9.8 m s^{-2} .
- 2) Unless told otherwise, the upwards force exerted on a falling object due to air resistance is negligible compared to the downwards gravitational force.

The following equations for straight line motion apply:

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Where:

s = displacement (m)

u = initial velocity (m s^{-1})

v = final velocity (m s^{-1})

a = acceleration (m s^{-2})

t = time (s)

Sample question

A person is standing on a lookout 300 m above ground level and they drop their camera over the edge. Calculate the following:

- i. How long will it take for the camera to hit the ground?

$$s = 300 \text{ m}$$

$$a = 9.8 \text{ m s}^{-2}$$

$$u = 0 \text{ m s}^{-1}$$

$$s = ut + \frac{1}{2}at^2$$

$$300 = 0t + \frac{1}{2} \times 9.8 \times t^2$$

$$t^2 = \frac{300}{4.9}$$

$$t^2 = 61.22$$

$$t = 7.82 \text{ s}$$

- ii. Calculate how far the camera will have travelled after each whole second.

$$\text{Use } s = ut + \frac{1}{2}at^2$$

$$\text{At } t = 1$$

$$s = \frac{1}{2} \times 9.8 \times 1^2$$

$$s = 4.9 \text{ m}$$

$$\text{At } t = 2$$

$$s = \frac{1}{2} \times 9.8 \times 2^2$$

$$s = 19.6 \text{ m}$$

$$\text{At } t = 3$$

$$s = \frac{1}{2} \times 9.8 \times 3^2$$

$$s = 44.1 \text{ m}$$

$$\text{At } t = 4$$

$$s = \frac{1}{2} \times 9.8 \times 4^2$$

$$s = 78.4 \text{ m}$$

$$\text{At } t = 5$$

$$s = \frac{1}{2} \times 9.8 \times 5^2$$

$$s = 122.5 \text{ m}$$

$$\text{At } t = 6$$

$$s = \frac{1}{2} \times 9.8 \times 6^2$$

$$s = 176.4 \text{ m}$$

$$\text{At } t = 7$$

$$s = \frac{1}{2} \times 9.8 \times 7^2$$

$$s = 240.1 \text{ m}$$

- iii. Explain the pattern of the results?

It can be seen that the distance covered during each time interval of one second increases, indicating the fact that vertical velocity is changing.

INTERVAL	DISTANCE TRAVELLED
1	4.9
2	14.7
3	24.5
4	34.3
5	44.1
6	53.9
7	63.7

Review Questions

A person jumps off a cliff into a pool below. A friend who is timing the jump notes that the splash occurs 2.5 seconds after they jump.

1. What is the distance between the top of the cliff and the pool below?

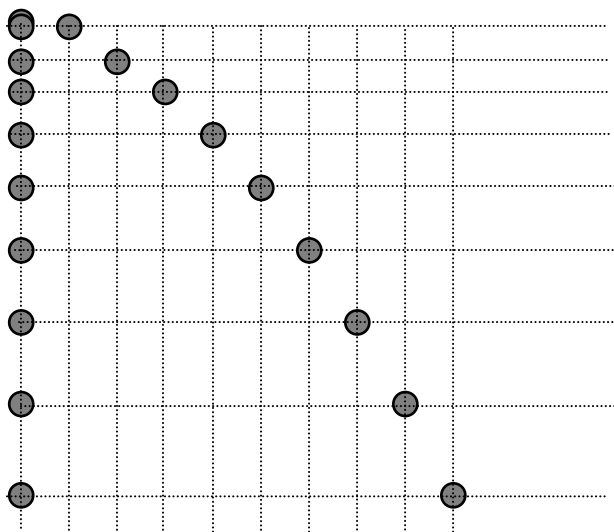
2. What was the person's velocity when they hit the pool?

Changing horizontal and vertical velocity

As it appears in Unit 3

Imagine two objects falling. The first is allowed to fall from rest; the second is given an initial horizontal velocity. The vertical motion determines the time to fall so the magnitude of the horizontal velocity will not have any effect on the downward acceleration or the time it takes for the object to reach the ground.

The diagram below shows that the vertical velocity is increasing, but the horizontal velocity is the same. This means there is no horizontal acceleration because there is no net force acting horizontally.



The diagram also shows the characteristic parabolic shape due to the constant horizontal velocity and the changing vertical velocity.

The final piece of information obtained from this diagram is that two objects: one dropped vertically and the other given an initial horizontal velocity will hit the ground at the same time.

When solving this type of problem, it is necessary to keep the vertical and horizontal components separate.

The vertical component is often used to calculate the time of flight.

Once the total time has been established, it can then be used to calculate the horizontal distance (range) travelled by the object.

Sample question

Several people have been caught in a rip. A rescue helicopter travelling at a speed of 20 m s^{-1} drops a safety raft into the ocean. Assuming that the helicopter is 50 m above the water, calculate the following:

- i. The time it will take the safety raft to hit the water.
- ii. The range of the safety raft.

Firstly, work out the vertical and horizontal components of the motion.

Vertical component	Horizontal component
$u = 0 \text{ m s}^{-1}$	$u = 15 \text{ m s}^{-1}$
$a = 9.8 \text{ m s}^{-2}$	$a = 0 \text{ m s}^{-2}$
$t = ?$	$t = ?$
$s = 50 \text{ m}$	$s = ?$

- i. This answer is obtained using the vertical component information.

$$s = ut + \frac{1}{2}at^2$$
$$50 = 0 + 4.9t^2$$
$$50 = 4.9t^2$$
$$t^2 = 10.2$$
$$t = 3.19 \text{ s}$$

- ii. This answer is obtained using the time already calculated and the horizontal component information.

$$s = ut + \frac{1}{2}at^2$$
$$s = (15 \times 3.19) + 0$$
$$s = 47.85 \text{ m}$$

Sometimes it is necessary to treat the question as a quadratic equation.

Sample problem (for the mathematically inclined)

A stone is thrown from the top of a 60 m building. If its initial velocity is 20 m s^{-1} down, how long will it take for the stone to reach the ground?

$$s = 60 \text{ m}$$
$$u = 20 \text{ m s}^{-1}$$
$$a = 9.8 \text{ m s}^{-2}$$
$$t = ?$$
$$s = ut + \frac{1}{2}at^2$$

$$60 = 20t + \frac{1}{2} \times 9.8 \times t^2$$

$$60 = 20t + 4.9t^2$$

Since we have to work out both t and t^2 , it is not possible to solve for t while the equation is in this format. The equation needs to be altered into a quadratic format and then solved using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The standard format for a quadratic equation is $ax^2 + bx + c = 0$

So in this case the equation now reads $4.9t^2 + 20t - 60 = 0$

So $a = 4.9$, $b = 20$ and $c = -60$

$$t = \frac{-20 \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-20 \pm \sqrt{20^2 - 4 \times 4.9 \times -60}}{2 \times 4.9}$$

$$t = \frac{-20 + 39.7}{9.8} \text{ or } t = \frac{-20 - 39.7}{9.8}$$

$$t = 2.01 \text{ s or } t = -6.09 \text{ s}$$

The only sensible answer is 2.01 s

Therefore it would take 2.01 seconds for the stone to reach the ground.

Review Question

3. A ball is thrown downwards with a velocity of 10 m s^{-1} from an unknown height. If it hits the ground 3 seconds later, determine the height that it is thrown from.

Straight up and down

As it appears in Unit 2

In some cases, a projectile is launched straight upwards.

Once the projectile is launched and in the air, the only force acting on it is gravity, and so, it will be accelerating downwards. The projectile will slow down until it comes to a halt, then it will move downwards with an increasing speed.

If air resistance is ignored, the projectile will return to the original position with the same speed at which it was projected.

An example of this would be a cricket player throwing the ball straight upwards. The sequence of events is:

- 1) The ball moves upwards with an initial velocity.
- 2) The ball moves with a downwards acceleration due to gravity.
- 3) Eventually the upwards velocity decreases to zero and the ball is instantaneously at rest in the air.
- 4) The ball then descends, continuing to experience a downwards acceleration due to gravity.

Sample question

A cricket ball is thrown upwards with a velocity of 20 m s^{-1} . Prove that the speed with which it was thrown upwards and its speed on returning is the same.

Starting with the upwards motion

$$u = 20 \text{ m s}^{-1}$$

$$v = 0 \text{ m s}^{-1} \text{ (because the ball will come to rest at the top of its displacement)}$$

$$a = -9.8 \text{ m s}^{-2}$$

$$d = ?$$

$$t = ? \text{ (but is irrelevant in this case)}$$

$$s = \frac{v^2 - u^2}{2a}$$

$$s = \frac{0^2 - 20^2}{2 \times -9.8} = 20.41 \text{ m}$$

Now the downwards motion

$$u = 0 \text{ m s}^{-1}$$

$$v = ? \text{ (but should be } 20 \text{ m s}^{-1}\text{)}$$

$$a = 9.8 \text{ m s}^{-2}$$

$$s = 20 \text{ m}$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times 9.8 \times 20.41$$

$$v^2 = 400$$

$$v = 20 \text{ m s}^{-1}$$

which is the same as the initial speed.

Sample question

A child throws a ball straight up into the air with a velocity of 5 m s^{-1} . Assuming they will keep their hand stationary, how much time passes before they catch the ball?

This type of problem can be approached in 2 ways.

Method 1: The entire motion

In this case:

$$u = 5 \text{ m s}^{-1}$$

$$a = -9.8 \text{ m s}^{-2} \text{ (acceleration due to gravity)}$$

The final velocity v must equal -5 m s^{-1} because the ball returns to the starting position with an equal and opposite magnitude to the initial velocity.

$$v = u + at$$

$$-5 = 5 + (-9.8 \times t)$$

$$-5 = 5 - 9.8t$$

$$t = 1.02 \text{ s}$$

Method 2: Half of the motion

In this case:

$$u = 5 \text{ m s}^{-1}$$

$$a = -9.8 \text{ m s}^{-2}$$

The final velocity of the ball must be zero because the ball comes to rest at the top of its path.

$$v = u + at$$

$$0 = 5 + (-9.8t)$$

$$t = 0.51 \text{ s}$$

It takes 0.51 s for the ball to reach the top of its path and it takes an equal amount of time to return to the original position, therefore 1.02 s would have passed.

Either method will produce the same answer, but it is generally best to use the first method if possible. When solving problems where a single equation involves both upwards and downwards motion, it is preferred to call the upwards direction positive.

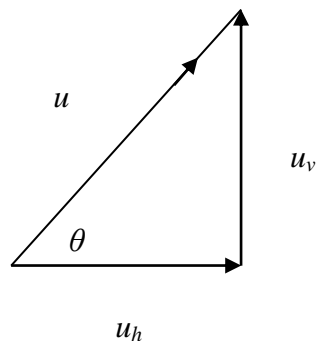
Review Questions

- A bullet is fired straight up into the air with an initial velocity of 70 m s^{-1} . Calculate the maximum height it will reach.
- A cricket player throws a ball straight upwards. If the ball takes 6 seconds to return to the original position (the position it was thrown from), Calculate the velocity of the ball as it reaches the original position.

Projectiles at an angle: Velocity components

As it appears in Unit 3

As in previous examples, the velocity of a projectile can be split into the horizontal and vertical components.



u = initial velocity

θ = the angle of projection

u_h = initial horizontal velocity component.

u_v = initial vertical velocity component.

If a projectile is fired with a velocity u at the angle θ , then the following information can be obtained from the diagram by the use of trigonometry.

Working out the horizontal velocity component

$$\cos \theta = \frac{u_h}{u}$$

$$\text{therefore } u \cos \theta = u_h$$

Working out the vertical velocity component

$$\sin \theta = \frac{u_v}{u}$$

$$\text{therefore } u \sin \theta = u_v$$

Sample question

A projectile is launched with a velocity of 80 m s^{-1} at an angle of 30° to the horizontal. Establish the initial vertical and horizontal velocity components.

Horizontal velocity

$$u = 80 \text{ m s}^{-1}$$

$$u \cos \theta = u_h$$

$$u_h = 80 \cos 30^\circ$$

$$u_h = 69.3 \text{ m s}^{-1}$$

Vertical velocity

$$u \sin \theta = u_v$$

$$80 \sin 30^\circ = u_v$$

$$u_v = 40 \text{ m s}^{-1}$$

Review Question

6. A person hits a tennis ball at an angle of 50° to the horizontal with a velocity of 20 m s^{-1} . Establish the initial horizontal and vertical velocity components.

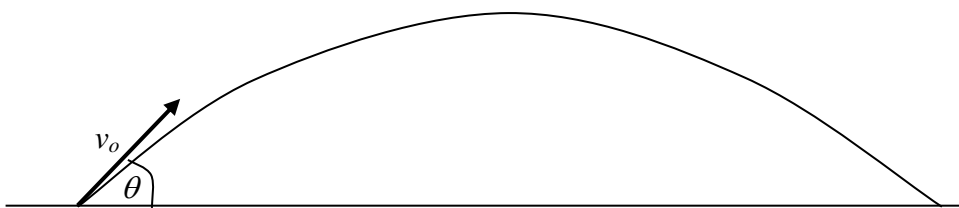
Projectiles at an angle: total time of flight

As it appears in Unit 3

Consider the diagram below.

$$\text{Initial vertical velocity } (u) = \text{velocity} \times \sin \theta$$

$$\text{Final vertical velocity } (v) = - \text{velocity} \times \sin \theta$$



As with vertical motion, air resistance will be ignored. Hence, the projectile will return to ground with the same speed at which it was projected.

This information can be used to work out the time of flight.

There are 2 common ways of working out the time of flight as shown in the following example.

Sample question

A soccer ball is kicked with an initial velocity of 12 m s^{-1} at an angle of 30° . How long will the ball remain in flight.

One method is to use the formula;

$$v = u + at$$

Take the upwards direction as positive as this equation will involve upward and downward velocity components.

$$u_v = 12 \sin 30^\circ = 6$$

$$v_v = -u_v = -6$$

$$t = ?$$

$$a = -9.8 \text{ m s}^{-2}$$

$$t = \frac{-6 - 6}{-9.8}$$

$$t = 1.22 \text{ s}$$

A second way of working out the answer is to use the “time of flight” formula.

$$t_f = \frac{2v \sin \theta}{g}$$

$$t_f = \frac{2 \times 12 \sin 30^\circ}{9.8}$$

$$t_f = 1.22 \text{ s}$$

Review Questions

7. An archer fires an arrow with an initial speed of 40 m s^{-1} at an angle of 35° . How long will the arrow be in flight?

- 8.** An aeroplane is at an altitude of 500 m and moving horizontally at 160 m s^{-1} . A small object is dropped from it. Find:
- i.** The vertical component of the objects initial velocity immediately after being dropped.
 - ii.** The vertical component of its initial displacement.
 - iii.** The vertical component of its acceleration.
 - iv.** The horizontal component of the objects initial velocity immediately after being dropped.
 - v.** The horizontal component of the objects final velocity.
 - vi.** The horizontal component of the objects acceleration.
 - vii.** If the object is in the air for 10 seconds, what is the horizontal component of its displacement?

Projectiles at an angle: maximum height

As it appears in Unit 3

The maximum height of a projectile's flight is calculated using the formula $v^2 = u^2 + 2as$ with $v = 0$ and $a = g$ in the downward direction.

The equation can be rearranged to:

$$s = \frac{u^2}{2g}$$

Sample question

A stunt man is launched from cannon with a velocity of 35 m s^{-1} at an angle of 40° to the ground, what is the maximum height that he will reach?

$$u = 35 \sin 40^\circ$$

$$g = 9.8 \text{ m s}^{-2}$$

$$s = \frac{(35 \sin 40^\circ)^2}{2 \times 9.8}$$

$$s = 25.82 \text{ m}$$

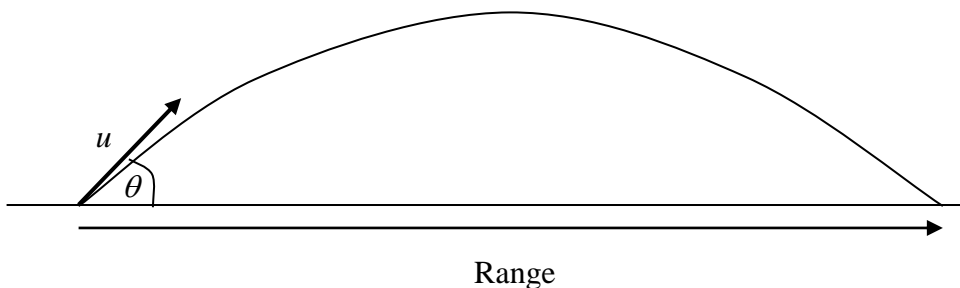
Review Question

9. An object is thrown into the air at an angle of 35° with a velocity of 30 m s^{-1} . Calculate the maximum height it reaches.

Projectiles at an angle: range

As it appears in Unit 3

The range of an object is the total horizontal distance travelled. It is dependent on the horizontal velocity and the angle of projection (but only if the object is launched from ground level).



$\text{Range} = t \times u_h \quad \Rightarrow \quad \text{Range} = ut\cos\theta$
(though in order to use this method the time of flight must be calculated first.)

If t is not given, it can be calculated as $t = \frac{2u\sin\theta}{g}$

t = time of flight

u = the initial velocity of the object

u_h = horizontal velocity

θ = the angle of projection

g = gravitational acceleration

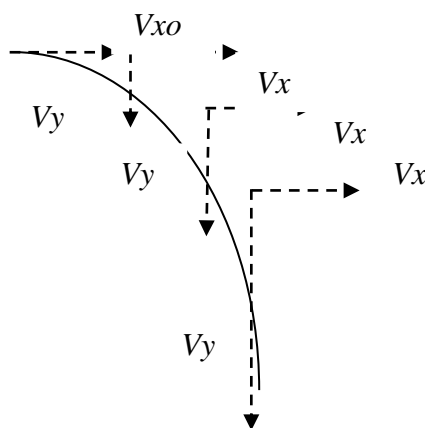
Review Question

10. A stone is thrown, from ground level, at an angle of 40° with a velocity of 20 m s^{-1} . Determine its range.

Objects thrown horizontally

As it appears in Unit 3

Once an object is projected horizontally, it has a downward acceleration due to gravity. Therefore the vertical velocity (V_y) is continually increasing. Horizontal velocity (V_x) remains constant and is equal to V_{xo} . The two vectors V_x and V_y are used (via Pythagoras theorem) to find the velocity at each point on the path as shown in the diagram below.



Sample question

A person throws a stone horizontally from the top of a cliff. The stone has an initial horizontal velocity of 20 m s^{-1} .

- i. If the stone takes 6 seconds to reach the ground, how high is the cliff?
Considering vertical motion

$$s = ut + \frac{1}{2} at^2$$

$$s = 0 + \frac{1}{2} \times 9.8 \times 6^2$$

$$s = 176.4 \text{ m}$$

Therefore the cliff is 176.4 m high.

- ii. What is the actual speed of the stone 2.5 seconds after being thrown?

Vertical velocity

$$v_y = u_y + at$$

$$v_y = 0 + 9.8 \times 2.5$$

$$v_y = 24.5 \text{ m s}^{-1}$$

Horizontal velocity $v_x = 20 \text{ m s}^{-1}$

$$v = \sqrt{24.5^2 + 20^2}$$

$$v = 31.6 \text{ m s}^{-1}$$

Review Questions

A ball is projected horizontally from the top of a table. It hits the floor at a point 2.9 m away. The surface of the table is 1.5 m above the floor.

11. How long does it take for the ball to hit the floor?

12. At what speed did the ball leave the table?

13. What is the speed of the ball just before it hits the floor?

14. What angle does the direction of travel make with the floor just before it hits?

- 15.** A car drives up a ramp which makes an angle of 30° with the horizontal. If the initial velocity is 100 km h^{-1} , what is the horizontal component of the car's initial velocity?
- 16.** What is the vertical component of the initial velocity?
- 17.** Assuming there is no driving force acting on the car, determine the maximum height reached by the car before it fully stops.
- 18.** The driver wishes to cross a gap of 100 m. Will they make it?

Solutions to Review Questions

1. $s = ut + \frac{1}{2}at^2$

$$u = 0 \text{ m s}^{-1}$$

$$a = 9.8 \text{ m s}^{-2}$$

$$t = 2.5 \text{ s}$$

$$s = 0 + \frac{1}{2} \times 9.8 \times 2.5^2$$

$$s = 30.63 \text{ m}$$

2. $u = 0 \text{ m s}^{-1}$
 $a = 9.8 \text{ m s}^{-2}$
 $s = 30.63 \text{ m}$

$$v = u + at$$

$$v = 0 + 9.8 \times 2.5 = 24.5 \text{ m s}^{-1}$$

OR

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times 9.8 \times 30.63$$

$$v^2 = 600.35$$

$$v = 24.5 \text{ m s}^{-1}$$

3. $u = 10 \text{ m s}^{-1}$
 $t = 3.0 \text{ s}$
 $a = 9.8 \text{ m s}^{-2}$
 $s = ut + \frac{1}{2}at^2$

$$s = 10 \times 3 + \frac{1}{2} \times 9.8 \times 3^2$$

$$s = 74.1 \text{ m}$$

4. $a = -9.8 \text{ m s}^{-2}$
 $u = 70 \text{ m s}^{-1}$
 $v = 0 \text{ m s}^{-1}$

$$s = \frac{v^2 - u^2}{2a}$$

$$s = \frac{0^2 - 70^2}{2 \times -9.8} = 250 \text{ m}$$

5.

Since time of the whole trip is 6 seconds

And since air resistance is ignored

Then the time for the ball to reach maximum height = $\frac{\text{total time of trip}}{2} = \frac{6}{2} = 3 \text{ s}$

Given that the velocity of the ball at maximum height = 0 m s^{-1}

Then the givens can be re-written as follows:

$u = 0 \text{ m s}^{-1}$
 $a = g = 9.8 \text{ m s}^{-2}$
 $t = 3 \text{ s}$
 $v = ?$

Solution

$v = u + at$
 $v = 0 + 9.8 \times 3 = 29.4 \text{ m s}^{-1}$

Therefore the ball will return to the original position at a velocity of 29.4 m s^{-1}

6. $u = 20 \text{ m s}^{-1}$
 $\theta = 50^\circ$

Horizontal velocity

$u_h = u \cos \theta$
 $u_h = 20 \cos 50^\circ$
 $u_h = 12.9 \text{ m s}^{-1}$

Vertical velocity

$u_v = u \sin \theta$
 $u_v = 20 \sin 50^\circ$
 $u_v = 15.3 \text{ m s}^{-1}$

7. $v = 40 \text{ m s}^{-1}$
 $\theta = 35^\circ$

$$t_f = \frac{2 v \sin \theta}{g}$$

$$t_f = \frac{2 \times 40 \sin 35^\circ}{9.8}$$

$$t_f = 4.68 \text{ s}$$

8. The answers are as follows

i. 0 m s^{-1}

ii. 500 m

iii. 9.8 m s^{-2} downwards

iv. 160 m s^{-1}

v. 160 m s^{-1}

vi. 0 m s^{-2}

vii. $160 \times 10 = 1600 \text{ m}$

9. $u = 30 \sin 35^\circ$

$$s = \frac{u^2}{2g}$$

$$s = \frac{(30 \sin 35^\circ)^2}{19.6} = 15.1 \text{ m}$$

10. $u = 20 \text{ m s}^{-1}$
 $\theta = 40^\circ$

$$\text{Range} = ut \cos \theta$$

$$t_f = \frac{2 v \sin \theta}{g}$$

$$t_f = \frac{2 \times 20 \sin 40^\circ}{9.8} = 2.62 \text{ s}$$

$$\text{Range} = 20 \times 2.62 \cos 40^\circ$$

$$\text{Range} = 40.14 \text{ m}$$

11. Vertically

$$s = -1.5$$

$$u = 0 \text{ m s}^{-1}$$

$$a = -9.8 \text{ m s}^{-2}$$

$$s = ut + \frac{1}{2}at^2$$

$$-1.5 = 0 + \frac{1}{2} \times -9.8 \times t^2$$

$$4.9t^2 = 1.5$$

$$t = 0.55 \text{ s}$$

12. Horizontally

$$s = 2.9 \text{ m}$$

$$t = 0.55 \text{ s}$$

$$u = \frac{s}{t}$$

$$u = \frac{2.9}{0.55}$$

$$u = 5.27 \text{ m s}^{-1}$$

13. Vertically

$$u = 0 \text{ m s}^{-1}$$

$$a = -9.8 \text{ m s}^{-2}$$

$$s = -1.5 \text{ m}$$

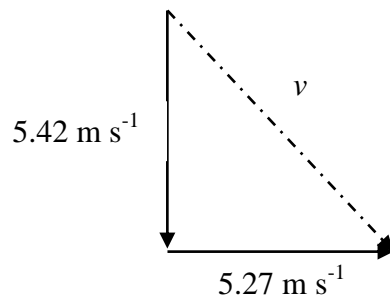
$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times -9.8 \times -1.5$$

$$v^2 = 29.4$$

$$v = 5.42 \text{ m s}^{-1}$$

Horizontal velocity is the same as the initial velocity.



$$v^2 = \sqrt{5.42^2 + 5.27^2}$$

$$v = 7.56 \text{ m s}^{-1}$$

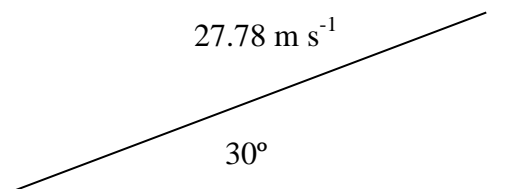
14. This answer needs to be based on the same diagram as shown for question 13.

$$\tan \theta = \frac{5.42}{5.27}$$

$$\text{So } \theta = 45.8^\circ$$

15. The diagram would look like this:

$$(100 \text{ km h}^{-1} = 27.78 \text{ m s}^{-1})$$



$$u_h = 27.78 \cos 30^\circ$$

$$u_h = 24.06 \text{ m s}^{-1}$$

16. $u_v = 27.78 \sin 30^\circ$

$$u_v = 13.89 \text{ m s}^{-1}$$

17. Alongside the ramp

$$u = 27.78 \text{ m s}^{-1}$$

$$v = 0 \text{ m s}^{-1}$$

$$a = -9.8 \times \sin 30 = -4.9 \text{ m s}^{-2}$$

$$s = \frac{v^2 - u^2}{2a}$$

$$s = \frac{0^2 - 27.78^2}{2 \times -4.9} = 78.74 \text{ m}$$

$$\text{To find the height from ground level, } \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{height}}{\text{distance on the ramp}}$$

$$\text{Hence, the height} = 78.74 \sin 30^\circ = 39.37 \text{ m}$$

18. First work out the time of flight.

$$u = 13.89 \text{ m s}^{-1}$$

$$v = -13.89 \text{ m s}^{-1}$$

$$a = -9.8 \text{ m s}^{-2}$$

$$t = \frac{v - u}{a}$$

$$t = \frac{-13.89 - 13.89}{-9.8} = 2.83 \text{ s}$$

$$s_h = u_h \times t$$

$$s_h = 24.06 \times 2.83$$

$$s_h = 68.1 \text{ m}$$

The car driver will not be able to cross the gap.



Physics

Teach Yourself Series

Topic 2: Newton's Laws, Momentum and Work (Units 2 & 3)

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What you should know

As it appears in Unit 3

- Investigate and apply theoretically and practically Newton's three laws of motion in situations where two or more coplanar forces act along a straight line and in two dimensions
- Investigate and apply theoretically and practically the laws of energy and momentum conservation in isolated systems in one dimension.
- Investigate and analyse theoretically and practically impulse in an isolated system for collisions between objects moving in a straight line: $F\Delta t = m\Delta v$
- Investigate and apply theoretically and practically the concept of work done by a constant force using:
 - work done = constant force . distance moved in direction of net force
 - work done = area under force-distance graph
- Analyse transformations of energy between kinetic energy, strain potential energy, gravitational potential energy and energy dissipated to the environment (considered as a combination of heat, sound and deformation of material):
- kinetic energy at low speeds: $E_k = \frac{1}{2}mv^2$; elastic and inelastic collisions with reference to conservation of kinetic energy
- strain potential energy: area under force-distance graph including ideal springs obeying Hooke's Law: $E_s = \frac{1}{2}k\Delta x^2$
- gravitational potential energy: $E_g = mg\Delta h$ or from area under a force-distance graph and area under a field-distance graph multiplied by mass

Newton's Laws of Motion

As it appears in Unit 3

Newton's first law of motion

This is known as the law of inertia: an object continues in its state of rest or uniform motion unless made to change by an unbalanced (a non-zero net) force.

The net force (F_{net}) is the vector sum of forces acting on an object.

Newton's second law of motion

This law relates to the behaviour of objects for which the existing forces are not balanced. The acceleration of an object is dependent on the net force acting on the object and on the mass of the object.

Acceleration depends directly on the net force acting on the object and inversely on its mass. When net force increases so does acceleration and when mass increases acceleration decreases.

The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object.

This law can be expressed as the following equation:

$$F_{net} = \frac{\Delta p}{\Delta t}$$

Since $p = mv$, this equation can also be expressed as

$$F_{net} = \frac{m\Delta v}{\Delta t}$$

And finally as

$$F_{net} = ma$$

Where

F = force in N (or kg m s^{-2})

m = mass in kg

a = acceleration in m s^{-2}

Other forms of the final equation:

$$a = \frac{F}{m}$$

$$m = \frac{F}{a}$$

Newton's third law of motion

When an object applies a force to another object, the second object applies an equal and opposite force to the first object. This law is often paraphrased as for every action there is a reaction that is equal in magnitude and opposite in direction.

Steps for solving questions

As it appears in Unit 3

In order to reduce the chance of making oversights it is a good idea to use the following steps.

Sample question

A 2 kg object is subjected to a net force of 12 N. Determine its acceleration.

Step 1: Identify what you are being asked to establish and all information that will assist you to solve the problem.

$$a = ?$$

$$m = 2 \text{ kg}$$

$$F = 12 \text{ N}$$

Step 2: Write down the formula of the equation you need to solve the problem.

$$a = \frac{F}{m}$$

Step 3: Solve the equation by substituting the information into the equation.

$$a = \frac{12}{2}$$
$$a = 6 \text{ m s}^{-2}$$

Other things to consider

- Check your work and make sure that all numbers have an SI unit label and that you have the correct SI unit for your answer.
- If your answer is a vector, don't forget to include the direction.
- Make sure that your answer makes sense. It is a good idea to always double check your answers because it is very easy to hit the wrong button on your calculator.

Review Questions

1. Calculate the acceleration of a 4 kg object subjected to a net force of 12 N.
2. An object is accelerating at the rate of 3 m s^{-2} . If the mass of the object is doubled and the net force is tripled, what will be the new acceleration of the object? Explain your answer.

Kinetic energy

As it appears in Unit 3

Kinetic energy is the energy associated with the movement of an object.

The formula for calculating kinetic energy is as follows:

$$E_k = \frac{1}{2}mv^2 \quad \text{or} \quad \frac{mv^2}{2}$$

Where:

E_k = kinetic energy in Joules (J)

m = mass in kilograms (kg)

v = velocity in metres per second (m s^{-1})

Review Question

3. Calculate the kinetic energy of a car with a mass of 1.2 tonne travelling at 60 km h^{-1} .

Momentum

As it appears in Unit 3

The momentum of a body is the product of its mass and velocity.

The equation for calculating the momentum of an object is:

$$p = mv$$

Where

p = momentum in kg m s^{-1} or newton seconds N s

m = mass in kg

v = velocity in m s^{-1}

As a result, momentum is a vector having both a magnitude and a direction.

Sample question

A bowler has a 12 kg bowling ball which travels along a bowling alley at a speed of 7 m s^{-1} . Establish its momentum?

$$m = 12 \text{ kg}$$

$$v = 7 \text{ m s}^{-1}$$

$$p = mv$$

$$p = 12 \times 7$$

$$p = 84 \text{ kg m s}^{-1}$$

Keep in mind that momentum is a vector quantity. Since the bowling ball is travelling forwards, the answer must include this information.

The momentum of the bowling ball is 84 kg m s^{-1} forwards.

Review Question

4. Which of the following has the momentum of greater magnitude; a 600 kg car traveling at 10 km h^{-1} or a 200 kg bike traveling at 30 km h^{-1} ?

Impulse

As it appears in Unit 3

From Newton's second law of motion,

$$F = ma \text{ and since } a = \frac{v-u}{t} \text{ then } F = \frac{mv-mu}{t}$$

$$\therefore F\Delta t = mv - mu$$

When a force is applied to an object, so is an impulse. Increasing the force means that the impulse is also increased; the two are directly proportional which means if the force is doubled so is the impulse.

However, the two are not the same because impulse also depends on how long the force is applied. Increasing the time increases the impulse in exactly the same way as increasing the force; if time is tripled so is the impulse.

Therefore, the impulse exerted on an object depends directly on how much force is applied and how long the force is applied for.

Impulse is the product of force acting and the time of contact

$$I = F\Delta t$$

Impulse is a vector quantity. The unit is the newton-second. (N s)

Impulse can be measured by the change in momentum produced.

Impulse is the change in momentum $I = p_2 - p_1$

Where p_1 = initial momentum

p_2 = final momentum

Sample question

A force of 5.0 N is applied to an object for 6.0 s. Calculate the impulse.

$$I = F\Delta t$$

$$F = 5.0 \text{ N}$$

$$t = 6.0 \text{ s}$$

$$I = 5.0 \times 6.0$$

$$I = 30 \text{ N s}$$

Sample questions

A vehicle of mass 1500 kg and speed of 16 m s^{-1} collides with a pole. It takes 0.05 s to come to rest.

- i. What is the change in momentum of the vehicle?

It is important to remember that momentum is a vector so directions must be assigned. Since the vehicle was initially travelling forwards this direction will be positive and going backwards will be negative.

$$\begin{aligned}m &= 1500 \text{ kg} \\u &= 16 \text{ m s}^{-1} \\v &= 0 \text{ m s}^{-1}\end{aligned}$$

$$\begin{aligned}\Delta p &= mv - mu \\&= 1500 (0 - 16) \\&= -24000 \text{ kg m s}^{-1}\end{aligned}$$

The change in momentum of the vehicle is $24000 \text{ kg m s}^{-1}$ in the opposite direction to the original direction of the vehicle.

- ii. What is the impulse exerted on the vehicle?

The impulse on the vehicle is the same as the change in momentum. Since the change in momentum has been shown to be $-24000 \text{ kg m s}^{-1}$ the impulse on the vehicle has to be $24000 \text{ kg m s}^{-1}$ in the opposite direction to the original direction of the vehicle.

- iii. Calculate the magnitude of the force exerted by the wall on the vehicle.

$$I = F\Delta t$$

$$\begin{aligned}I &= 24000 \text{ N s} \\t &= 0.05 \text{ s}\end{aligned}$$

$$\begin{aligned}F &= \frac{I}{t} \\&= \frac{24000}{0.05} \\&= 480000 \text{ N}\end{aligned}$$

- iv. Assume that, instead of coming to rest, the vehicle rebounds from the wall with a speed of 5 m s^{-1} after having been in contact with the wall for 0.05 s. Calculate the net force exerted on the vehicle by the wall.

To solve this problem it is necessary to use 2 different equations. Firstly you have to work out the impulse and then the impulse can be used to work out the net force.

$$I = m\Delta v$$

$$\begin{aligned}m &= 1500 \text{ kg} \\ \Delta v &= -5 - 16 = -21 \text{ m s}^{-1}\end{aligned}$$

$$\begin{aligned}I &= 1500 \times -21 \\I &= -31500 \text{ N s}\end{aligned}$$

Now use $I = F\Delta t$

$$I = -31500 \text{ N s}$$

$$t = 0.05 \text{ s}$$

$$F = \frac{-31500}{0.05}$$

$$F = -630000 \text{ N}$$

Review Questions

5. An object experiences an impulse of 22 N s for a time period of 2.0 s. What is the force on the object?
6. A ball with a mass of 500 g is thrown at a wall at a velocity of 10 m s^{-1} . The wall applies a force of 4000 N to the ball which rebounds with a velocity of 8 m s^{-1} . Calculate how long the ball was in contact with the wall.

7. A car has a mass of 600 kg and it is travelling in a westerly direction with a velocity of 25 m s^{-1} . It then collides with a brick wall and rebounds with a velocity of 8 m s^{-1} . The car was in contact with the wall for 0.5 s.

i. Determine the initial momentum of the car.

ii. Determine the change in momentum of the car as it is in contact with the wall.

iii. Determine the impulse applied to the car.

iv. Calculate the average force that the car exerts on the wall during the collision.

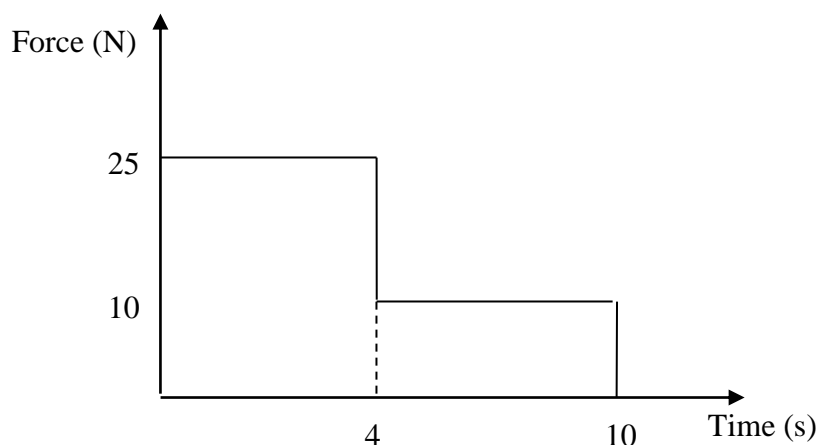
Impulse from a graph

As it appears in Unit 3

Since impulse is dependent on force and time, it can be calculated by using a graph showing force against time.

This is because $F\Delta t = m\Delta v$ for a constant force, and so impulse will always be given by the area under the force-time graph.

This area also measures the change in momentum as the area under the graph = Impulse = Δ momentum.



In this case, the information can be broken up into 2 rectangles as shown by the dotted line.

The impulse is calculated by adding the area of both shapes together.

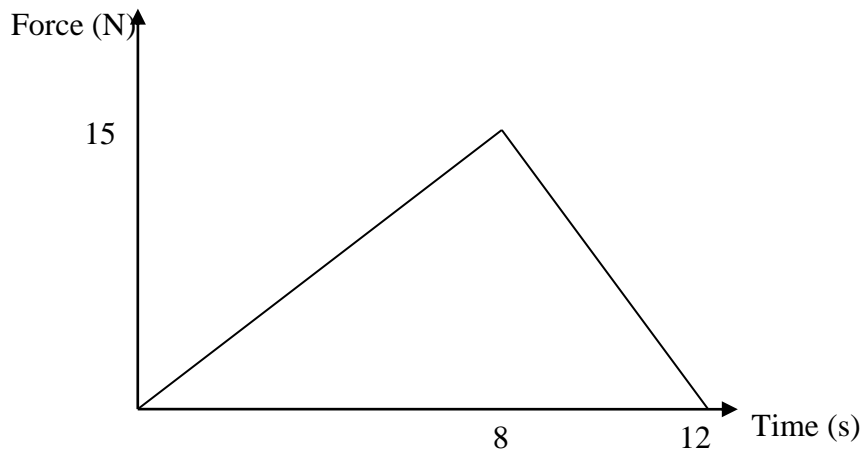
The impulse shown in this graph = $(25 \times 4) + (10 \times 6) = 160 \text{ N s}$

Force time graphs can come in a variety of shapes. If possible they can be broken down into a series of geometrical shapes.

However, the same information can also be obtained by counting the squares if you are provided with a question where the graph provided has gridlines.

Review Question

8. Use the diagram below to calculate the impulse on the object.

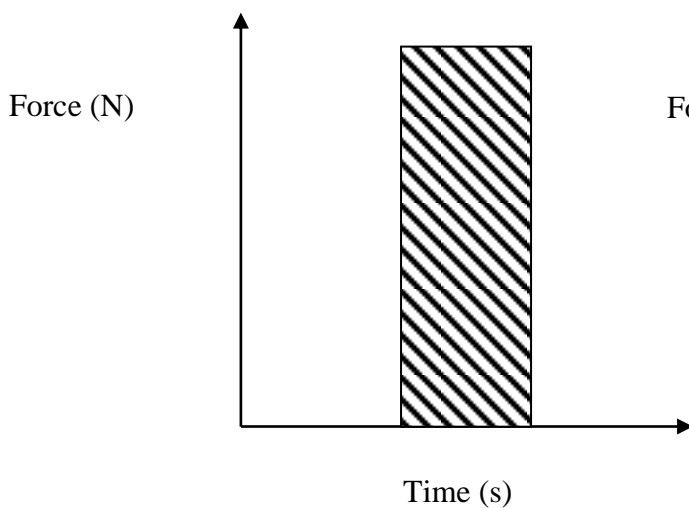


Impulse and collisions

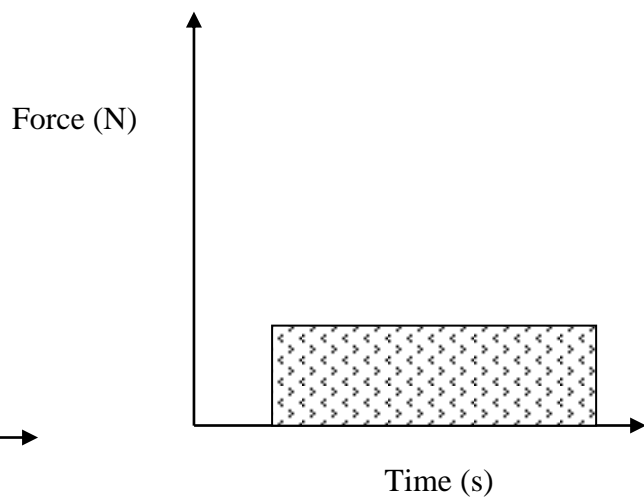
As it appears in Unit 3

Impulse explains how an object behaves when a collision occurs. Common examples include how a ball behaves when kicked or hit with a bat or what happens to a person in a car when an accident occurs. When a car is involved in a collision, we want the impulse to occur over a longer time interval to reduce the forces involved reducing the severity of injuries.

Shorter time interval; larger force



Longer time interval; smaller force



Review Question

9. A car travelling at 60 km h^{-1} collides with a wall. The front of the car crumples on impact and the car comes to a complete halt over a distance of 75 cm. The driver was wearing a seatbelt and comes to rest in the same time and distance as the entire car. However, a passenger in the front seat was not wearing a seat belt and their body travelled forward and struck the dashboard. During the impact with the dashboard, their body moved through a distance of 10 cm. Assume each of the occupants had a mass of 80 kg.

- i. Calculate the impulse on the driver.

- ii. Calculate the average acceleration of the driver during the impact.

- iii. Calculate the impulse on the passenger.

- iv. Calculate the average acceleration of the passenger during the impact.

- v. Explain the purpose of crumple zones in modern cars.

- vi. Use your knowledge of physics to explain why the use of seat belts is supposed to reduce the severity of injury.

Conservation of momentum

As it appears in Unit 3

The total momentum of a system remains constant provided that no external forces act on the system. This has important implications in the study of collisions.

In simple terms, we say that the total momentum before = total momentum after but it is also important to remember that the share of the momentum may change.

If objects bounce off each other and the total kinetic energy is the same (conserved) at the end as it is at the start, then the collision is elastic.

If some kinetic energy is lost, (converted into heat, light sound, deformation energy, etc), then the collision is inelastic. Most collisions tend to be inelastic.

Momentum is always conserved in collisions.

Think about two objects travelling in the same direction. The table below shows the properties of the objects:

PROPERTY	OBJECT 1	OBJECT 2
Mass	M	m
Initial velocity	u_1	u_2
Final velocity	v_1	v_2

Conservation of momentum is given by the following equation.

$$Mu_1 + mu_2 = Mv_1 + mv_2$$

If the collision is elastic then:

Total kinetic energy before = total kinetic energy after

$$\frac{1}{2}Mu_1^2 + \frac{1}{2}mu_2^2 = \frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2$$

Sample question

A car of mass 800 kg travelling at a velocity 10 m s^{-1} hits the back of smaller car of mass 400 kg travelling at a speed of 5 m s^{-1} . As a result of the collision the larger cars velocity decreases to 8 m s^{-1} and the smaller cars velocity increases to 9 m s^{-1} .

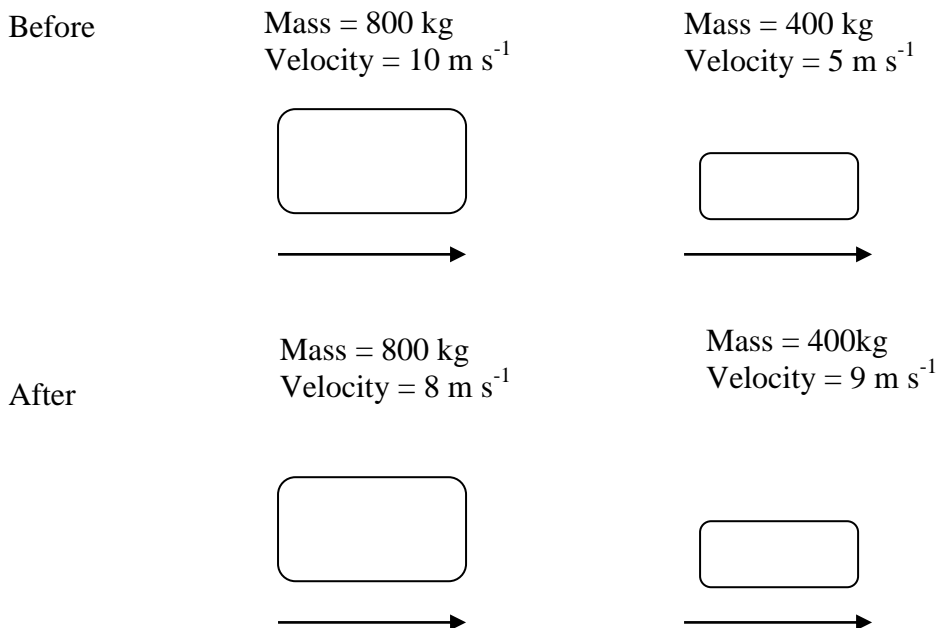
Work out the following:

- i. The total kinetic energy before the collision
- ii. The total kinetic energy after the collision
- iii. Whether the collision is elastic or not

Kinetic energy is a scalar quantity.

Solving the problem

It is a good idea to start out with a diagram showing all of the available information.



Then complete the calculations. Since both vehicles are going in the same direction positive figures are used for both.

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

i. Initial kinetic energy = $(\frac{1}{2} \times 800 \times 10^2) + (\frac{1}{2} \times 400 \times 5^2)$

$$\text{Initial KE} = 40000 + 5000$$

$$\text{Initial KE} = 45000 \text{ J}$$

ii. Final kinetic energy = $(\frac{1}{2} \times 800 \times 8^2) + (\frac{1}{2} \times 400 \times 9^2)$

$$\text{Final KE} = 25600 + 16200$$

$$\text{Final KE} = 41800 \text{ J}$$

To establish whether the collision is elastic or inelastic work out the difference between the initial kinetic energy and the final kinetic energy.

iii. Difference in kinetic energy = $45000 - 41800 = 3200 \text{ J}$

This collision must be inelastic because 3200 J of kinetic energy has been lost.

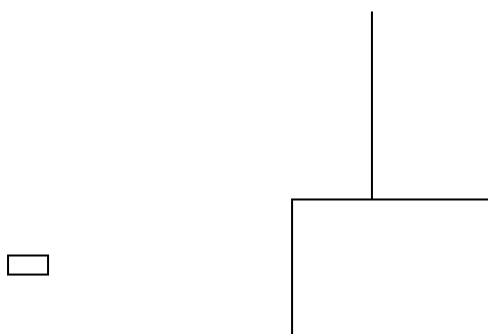
Sample Question

A bullet of mass 50 g is travelling horizontally at 300 m s^{-1} when it strikes a stationary wooden block of mass 10 kg suspended on a string so that it can swing freely. The bullet is embedded in the block.

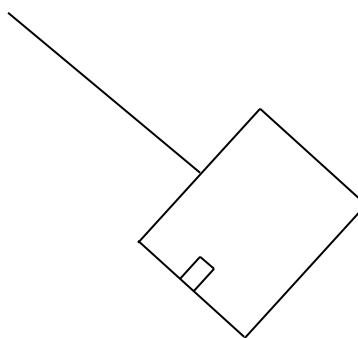
Calculate

- i.** The velocity at which the block begins to swing when the bullet hits it
- ii.** The height the block will rise to, compared to its initial position
- iii.** How much kinetic energy is lost when the bullet hits the block.

Before



After



Solving Problem

- i. Momentum before = momentum after

Initially only the bullet is moving so the block of wood has no momentum.

$$\text{Momentum before} = (0.05 \times 300) + 0 = 15 \text{ kg m s}^{-1}$$

After the collision, the bullet and the block of wood move at the same speed ($v \text{ m s}^{-1}$) and the mass is the combined mass of the bullet and the block.

$$\text{Momentum after} = 10.05v$$

Momentum before = momentum after

$$15 = 10.05v$$

$$v = \frac{15}{10.05}$$

$$v = 1.49 \text{ m s}^{-1}$$

- ii. To work out the height you need to use the principle of conservation of energy.

Kinetic energy after collision = potential energy at maximum height

$$\frac{mv^2}{2} = mgh$$

Mass will cancel out, this leaves

$$\frac{v^2}{2} = gh$$

Which ultimately is rearranged to $h = \frac{v^2}{2g}$

In this case

$$v = 1.49 \text{ m s}^{-1}$$

$$g = 9.8 \text{ m s}^{-2}$$

$$h = \frac{1.49^2}{2 \times 9.8}$$

$$h = 0.11 \text{ m}$$

- iii. To work this out we need to know the kinetic energy of the bullet before the collision and the kinetic energy of the block and the bullet after the collision.

$$KE = \frac{mv^2}{2}$$

$$KE \text{ before collision} = \frac{0.05 \times 300^2}{2} = 2250 \text{ J}$$

$$KE \text{ after collision} = \frac{10.05 \times 1.49^2}{2} = 11 \text{ J}$$

$$\text{Change in KE} = 2250 - 11 = 2239 \text{ J}$$

Review Questions

10. A slinky is placed at the top of the stairs and then set off. As it moves down the stairs, its speed decreases indicating that it is losing kinetic energy. Identify at least 2 types of energy that the lost kinetic energy has converted into.
11. A vehicle of mass 500 kg travelling at 5 m s^{-1} hits another vehicle that has a mass 400 kg and is travelling at 2 m s^{-1} in the same direction. After the collision the velocity of the second vehicle has increased to 4 m s^{-1} .
Assuming the collision is elastic,
- i. Use conservation of energy to find the velocity of the first vehicle after the collision
- ii. Is the collision given in the question realistic?

Collision of objects going in opposite directions

As it appears in Unit 3

In any collision or explosion where there are no external forces acting on the objects undergoing the interactions, the sum of all internal forces is zero since all the forces are equal and opposite.



So the rate of change of momentum of object A is equal and opposite to the rate of change of momentum of object B.

As previously momentum before and after the collision remains the same.

It is important to remember that a sign convention will be essential when solving this type of problem.

Sample question

Two trolleys are approaching each other. Trolley A has a mass of 50 kg and is travelling 2 m s^{-1} east and trolley B has a mass of 80 kg and is travelling 1 m s^{-1} west.

The 2 trolleys collide and trolley B **rebounds** at a speed of 0.5 m s^{-1} .

What is the velocity of trolley A after the collision?

Since the trolleys are going in two different directions, one direction will need to be positive and the other negative. The final velocity of trolley B is to the east so proceed on the assumption that east is positive and west is negative.

Initially
Trolley A
 $m = 50 \text{ kg}$
 $v = 2 \text{ m s}^{-1}$

Finally
Trolley A
 $m = 50 \text{ kg}$
 $v = ?$

Trolley B
 $m = 80 \text{ kg}$
 $v = -1 \text{ m s}^{-1}$

Trolley B
 $m = 80 \text{ kg}$
 $v = 0.5 \text{ m s}^{-1}$

$$(50 \times 2) + (80 \times -1) = (50 \times v) + (80 \times 0.5)$$

$$100 + -80 = 50v + 40$$

$$-20 = 50v$$

$$v = \frac{-20}{50}$$

$$v = -0.4 \text{ m s}^{-1} = 0.4 \text{ m s}^{-1} \text{ west}$$

What if the trolleys couple rather than rebound?

$$\text{Momentum before} = (50 \times 2) + (80 \times -1) = 20$$

$$\text{Momentum after} = (80 + 50) \times v \text{ m s}^{-1}$$

$$20 = 130v$$

$$v = \frac{20}{130}$$

$$v = 0.154 \text{ m s}^{-1} \text{ east}$$

Review Questions

12. Two students are studying collisions by sliding blocks on a frictionless surface.

A block with a mass of 3 kg is slid across the table with a speed of 4 m s^{-1} . It hits a second block with a mass of 2 kg which was at rest. After the collision the 2 kg block has a speed of 2 m s^{-1} . Calculate the final speed of the 3 kg block.

13. A vehicle of mass 500 kg travelling north at 5 ms^{-1} hits another vehicle with mass 400 kg travelling south at 2 m s^{-1} . After the collision, both cars get linked together.

i. Calculate the velocity of the linked cars after the collision.

ii. What is the impulse that the second car gave to the first car in the collision?

Work

As it appears in Unit 2

Work is the amount of mechanical energy transferred to or from an object.

It is calculated as the product of the magnitude of the force and the distance through which an object moves in the direction of the force.

Work is a scalar quantity and it is expressed in Joules. 1 Joule of work is done when a force of 1 Newton causes an object to be displaced by 1 metre in the same direction as the force. This means that 1 Joule also equals 1 Newton metre.

The formula for calculating work is:

$$W = F\Delta x$$

Where:

W = work in Joules (J)

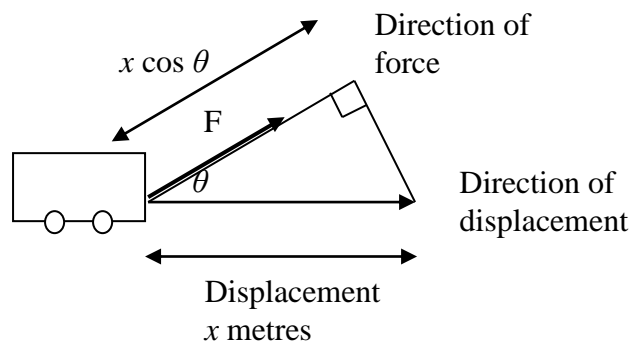
F = Force in Newtons (N)

Δx = displacement distance (m)

Review Questions

14. A force of 120 N is used to lift a box a height of 2 metres. Calculate the work done.

Displacement does not always occur in the same direction as the force applied. A common example involves a cart being pulled by a rope.



In this type of scenario the rope makes a an angle θ to the horizontal.

The equation for work done has to be modified to:

$$W = F\Delta x \cos \theta$$

15. A child is pulling a toy duck on a string with a force of 20 N. The angle of the rope is 35° to the horizontal. If the child pulls the toy for a horizontal distance of 15 m, how much work is done on the toy?

Work in collisions

As it appears in Unit 3

Energy can be transferred from one object to another as the result of work being done.

As shown previously, the work done on an object with a mass m by the net force acting on it is given by the formula

$$W = F_{net}x$$

Since $F_{net} = ma$ the equation can be altered to:

$$W = max$$

However since $v^2 = u^2 + 2ax$ then x can be expressed as $x = \frac{v^2 - u^2}{2a}$

The formula for work can now be altered to $W = \frac{ma(v^2 - u^2)}{2a}$

This can be simplified to:

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Which is the same as the equation for the change in kinetic energy.

So it can be mathematically proven that the work done on an object by the application of a net force is equal to the change in the kinetic energy of the object.

This information can then be used to work out the net force on the object using the formula $W = Fx$

Review Question

16. A car with a mass of 800 kg is travelling along a road at 20 m s^{-1} . It then collides with a fence and comes to a stop over the distance of 50 cm.

- i. How much work was done by the fence in order to stop the car?
- ii. What was the net force acting on the car in the course of it coming to a halt.

Solutions to Review Questions

1. $m = 4 \text{ kg}$
 $F = 12 \text{ N}$

$$a = \frac{F}{m}$$

$$a = \frac{12}{4}$$

$$a = 3 \text{ m s}^{-2}$$

2. The original acceleration is multiplied by 3 and then divided by 2 so will be 4.5 m s^{-2}

3. $v = 60 \text{ km h}^{-1} = 16.67 \text{ m s}^{-1}$
 $m = 1.2 \text{ tonne} = 1200 \text{ kg}$

$$E_k = \frac{mv^2}{2}$$

$$E_k = \frac{1}{2} \times 1200 \times 16.67^2$$

$$E_k = 1.67 \times 10^5 \text{ J}$$

4. For the car
 $m = 600 \text{ kg}$
 $v = 10 \text{ km h}^{-1} = 2.77 \text{ m s}^{-1}$

$$p = mv$$

$$p = 600 \times 2.77$$

$$p = 1666.67 \text{ kg m s}^{-1}$$

For the bike

$$m = 200 \text{ kg}$$

$$v = 30 \text{ km h}^{-1} = 8.33 \text{ m s}^{-1}$$

$$p = 200 \times 8.33$$

$$p = 1666.67 \text{ kg m s}^{-1}$$

Therefore both have the same momentum.

5. $t = 2.0 \text{ s}$
 $I = 22 \text{ N s}$

$$I = F\Delta t$$

$$F = \frac{22}{2}$$

$$F = 11 \text{ N}$$

6. $F = -4000 \text{ N}$
 $m = 0.5 \text{ kg}$

$$p = m\Delta v = m(v_2 - v_1)$$

$$p = 0.5(-8 + 10) = 9 \text{ kg m s}^{-1}$$

$$t = \frac{\Delta p}{F} = \frac{9}{4000} = 0.00225 \text{ s}$$

7. The answers are as follows:

i. $m = 600 \text{ kg}$
 $v = 25 \text{ m s}^{-1}$

$$p_1 = mv$$

$$p_1 = 600 \times 25$$

$$p_1 = 15000 \text{ kg m s}^{-1} \text{ west or } 1.5 \times 10^4 \text{ kg m s}^{-1} \text{ west}$$

ii. Make east positive and west negative

$$\Delta p = p_2 - p_1$$

$$\Delta p = 600 \times 8 - (-15000)$$

$$\Delta p = 19800 \text{ kg m s}^{-1} = 19800 \text{ kg m s}^{-1} \text{ east}$$

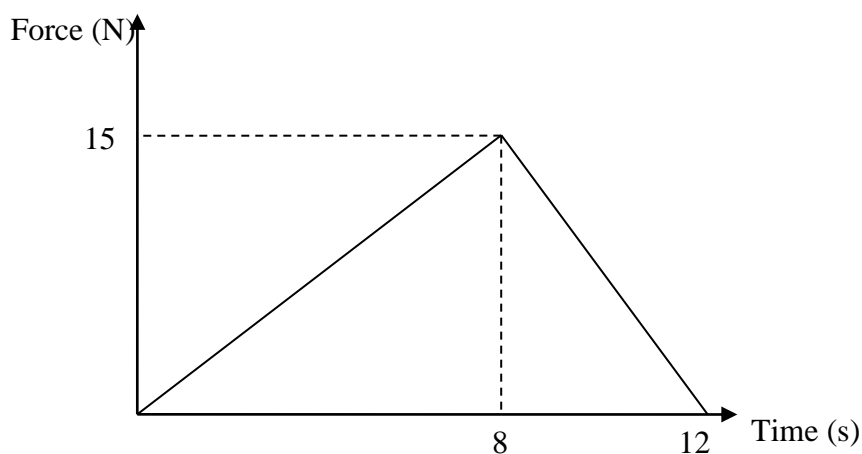
iii. The change in momentum and impulse are equal so impulse = 19800 N s east.

iv. $F = \frac{\Delta p}{\Delta t}$

$$F = \frac{19800}{0.5}$$

$$F = 39600 \text{ N east}$$

8. This shape can be split up into 2 triangles.



The area of a triangle = $\frac{1}{2} bh$

$$I = \frac{1}{2} \times 8 \times 15 + \frac{1}{2} \times 4 \times 15$$

$$I = 90 \text{ N s}$$

9. Assign the original direction as being positive.

i. $m = 80 \text{ kg}$
 $\Delta v = 0 - 16.67 \text{ m s}^{-1}$

$$I = m\Delta v$$
$$I = 80 \times -16.67$$
$$I = -1333.3 \text{ N s or } 1333.3 \text{ N s in the opposite direction.}$$

ii. $x = 0.75 \text{ m}$
 $u = 16.67 \text{ m s}^{-1}$
 $v = 0 \text{ m s}^{-1}$

$$v^2 = u^2 + 2ax$$
$$0 = 16.67^2 + 2 \times a \times 0.75$$
$$a = \frac{-278}{1.5}$$
$$a = -185 \text{ m s}^{-2}$$

iii. $m = 80 \text{ kg}$
 $\Delta v = 0 - 16.67 \text{ m s}^{-1}$

$$I = m\Delta v$$
$$I = 80 \times -16.67$$
$$I = -1333.3 \text{ N s or } 1333.3 \text{ N s in the opposite direction.}$$

iv. $x = 0.10 \text{ m}$
 $u = 16.67 \text{ m s}^{-1}$
 $v = 0 \text{ m s}^{-1}$

$$v^2 = u^2 + 2ax$$
$$0 = 16.67^2 + 2 \times a \times 0.10$$
$$a = \frac{-278}{0.2}$$
$$a = -1389 \text{ m s}^{-2}$$

v. The purpose of the crumple zones is to allow the change in momentum (impulse) to take place over a longer distance and therefore over a longer time interval. This reduces the force applied to the individuals in the car.
This is due to the impulse (change in momentum) being a fixed value therefore if the time of impact increases the amount of force decreases.

vi. If there is no seat belt the person keeps moving forwards with the car rather than stopping. As a result the passenger moves forwards and collides with the dashboard in a smaller interval of time subjecting them to a greater force than if they were wearing a seatbelt. Therefore wearing a seatbelt should decrease the chance of injury occurring or the severity of injury.

10. Sound or heat.

11.

i. $M = 500 \text{ kg}$
 $u_1 = 5 \text{ m s}^{-1}$
 $m = 400 \text{ kg}$
 $u_2 = 2 \text{ m s}^{-1}$
 $v_2 = 4 \text{ m s}^{-1}$

$$\frac{1}{2}Mu_1^2 + \frac{1}{2}mu_2^2 = \frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2$$

$$\left(\frac{1}{2} \times 500 \times 5^2\right) + \left(\frac{1}{2} \times 400 \times 2^2\right) = \left(\frac{1}{2} \times 500 \times v_1^2\right) + \left(\frac{1}{2} \times 400 \times 4^2\right)$$

$$v_1^2 = \frac{3850}{250}$$

$$v_1 = 3.9 \text{ m s}^{-1}$$

ii.

$$\text{Initial momentum} = 500 \times 5 + 400 \times 2 = 3300 \text{ kg m s}^{-1}$$

$$\text{Final momentum} = 500 \times 3.9 + 400 \times 4 = 3550 \text{ kg m s}^{-1}$$

Momentum has not been conserved so the collision is not realistic.

12. Initial $p = (3 \times 4) + 0 = 12$ Final $p = 3v + 2 \times 2$

$$3v + 8 = 12$$

$$4 = 3v$$

$$v = 1.33 \text{ m s}^{-1}$$

13.

i. $m_1 = 500 \text{ kg}$
 $v_1 = 5 \text{ m s}^{-1}$
 $m_2 = 400 \text{ kg}$
 $v_2 = -2 \text{ m s}^{-1}$ (negative as it is going in the opposite direction)
 $m_{12} = 500 + 400 = 900 \text{ kg}$
Due to the law of conservation of momentum

$$m_1v_1 + m_2v_2 = m_{12}v_{12}$$
$$500 \times 5 + 400 \times (-2) = 900 v_{12}$$

$$v_{12} = \frac{2500 - 800}{900} = 1.89 \text{ ms}^{-1} \text{ north}$$

ii. $I = m\Delta v = 500 \times (1.89 - 5)$
 $= -1555 \text{ N s} = 1555 \text{ N s south}$

14. $W = Fx$
 $F = 120 \text{ N}$
 $x = 2 \text{ m}$

$$W = 120 \times 2$$

$$W = 240 \text{ J}$$

15. $W = Fx \cos \theta$
 $F = 20 \text{ N}$
 $x = 15 \text{ m}$
 $\theta = 35^\circ$

$$W = 20 \times 15 \cos 35^\circ$$

$$W = 245.7 \text{ N}$$

16. The answers are as follows:

i. $W = \Delta E_k = \frac{1}{2} mv^2$ (as the final kinetic energy is zero)

$$m = 800 \text{ kg}$$

$$v = 20 \text{ m s}^{-1}$$

$$W = \frac{1}{2} \times 800 \times 20^2$$

$$W = 160000 \text{ J}$$

ii. $W = Fx$
 $W = 160000 \text{ J}$
 $x = 0.5 \text{ m}$

$$F = \frac{W}{x}$$

$$F = \frac{160000}{0.5}$$

$$F = 320000 \text{ N}$$

Physics

Teach Yourself Series

Topic 3: Circular motion (Unit 3)

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What you should know

As it appears in Unit 3

- Investigate and analyse theoretically and practically the uniform circular motion of an object moving in a horizontal plane: $F_{net} = \frac{mv^2}{r}$ including:
 - a vehicle moving around a circular road
 - a vehicle moving around a banked track
 - an object on the end of a string
- Investigate and apply theoretically Newton's second law to circular motion in a vertical plane (forces at the highest and lowest positions only)

Circular motion

As it appears in Unit 3

For linear motion, if there is a non-zero resultant force then acceleration occurs in the direction of the net force resulting in a change in speed.

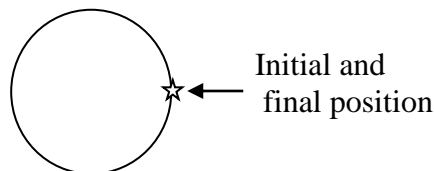
Circular motion differs in that the velocity changes but the speed does not, this is because the net force is acting at right angles to the direction of the motion.

Instantaneous velocity

As it appears in Unit 3

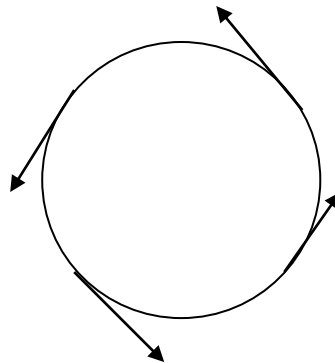
It is important to remember that there is a difference between speed and velocity. Velocity is a vector quantity having both a magnitude and a direction while speed only has a magnitude.

If an object travels in a circular path such as the one shown below, the average velocity of the object will always be zero because the object will always end up in the same place.



However, although the magnitude of the velocity will always be the same the direction will constantly be changing.

The direction of the velocity is indicated by drawing a tangent to the circular pathway as shown below. So, at any instant, the velocity is tangential to the path and is at right angles to the radius and acceleration direction.



The speed of an object can be calculated using the formula:

$$v = \frac{d}{t}$$

Where:

v = speed (m s^{-1})

d = distance (m)

t = time (s)

However, for objects travelling in a circular pathway, it is more convenient to use the period (the time taken to complete a single revolution of a repeated circular motion) and circumference. So;

$$v = \frac{\text{circumference}}{\text{period}}$$

The circumference of a circle = $2\pi r$

Where:

r = the radius of the circle (m)

T = period of the circle (s)

Sample question

A dog is attached to a 10 m long chain which is attached to a post in the ground so that the dog is able to run in circles around the post. It takes the dog 12 seconds to complete one circuit.

Work out the following:

- i. What is the dog's average speed?

$$r = 10 \text{ m}$$

$$T = 12 \text{ s}$$

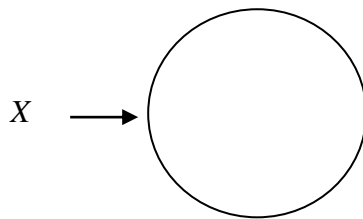
$$v = \frac{2\pi \times 10}{12}$$

$$v = 5.24 \text{ m s}^{-1}$$

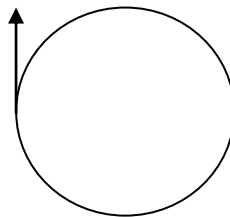
- ii. The dog completes 5 circuits, what is the dog's average velocity?

The dog completes 5 circuits, hence, it is back in its original position, which means that its average velocity is zero.

- iii. Assuming that the dog is travelling clockwise at a constant rate, determine its instantaneous velocity at point X.



When solving this kind of question it helps to indicate the direction of travel on the diagram.



From the information supplied the dog had to be travelling due North at this point in time.

The magnitude of the velocity was previously established as being 5.24 m s^{-1} , so the instantaneous velocity has to be 5.24 m s^{-1} north.

Review Questions

Use the information below to answer the following questions.

A miniature train track travels a circular pathway at a constant speed of 10 m s^{-1} . It takes 196 s for the train to complete 2 circuits of the track.

1. Calculate the radius of the train track.
2. What is the magnitude of the train's instantaneous velocity when it has travelled $\frac{3}{4}$ of the way around the track?
3. What is the average velocity of the train if it completes 2 entire circuits?

Acceleration

As it appears in Unit 3

Although the magnitude of the velocity is constant, the direction of the object is changing. This means that the velocity is changing, and therefore, an object travelling in a circular pathway must be accelerating. Acceleration is the result of a non-zero net force.

If an object is being rotated and is then released, it will leave the circular pathway with the same velocity that it had at the point of release.

Acceleration can be calculated using the formula $F = ma$

and

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

Where:

v = average speed (m s^{-1})

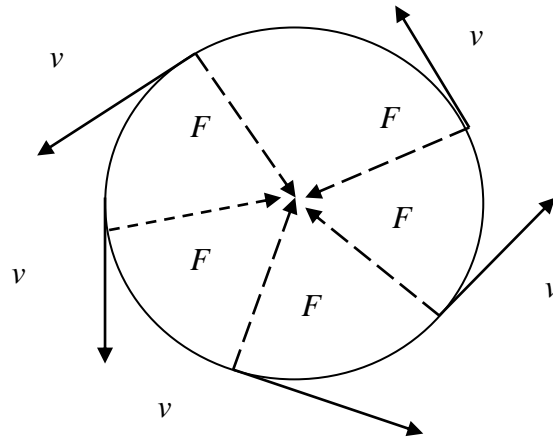
r = radius (m)

T = period (s)

The direction of the force

As it appears in Unit 3

Any object that travels at a constant speed in a circle has a force acting on it at any instant that is centripetal. (centre-seeking or centre-directed) as shown in the diagram below.



Acceleration is determined by working out the change in velocity. This is done by using vector addition.

$$\Delta v = v - u$$

Where:

v = final velocity

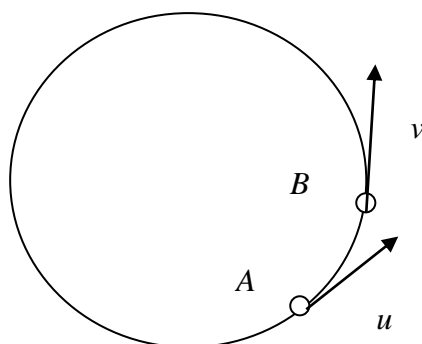
u = initial velocity

So

$$\Delta v = v + -u$$

(note that alternate symbols such as v_2 or v_f may be used instead of v , and v_1 or v_i may be used instead of u)

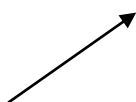
The diagram below shows the original position and direction of instantaneous velocity (point A) as well as the final position and direction of instantaneous velocity (point B)



A vector addition diagram can be constructed using this information.



The arrow for the final velocity is exactly as shown on the diagram

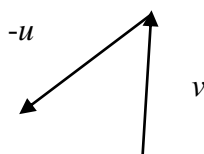


Since the arrow for the initial velocity looks like this

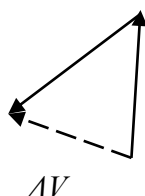


The arrow indicating $-u$ will need to be equal and opposite like this

When the two vectors are added together the result will look like this.



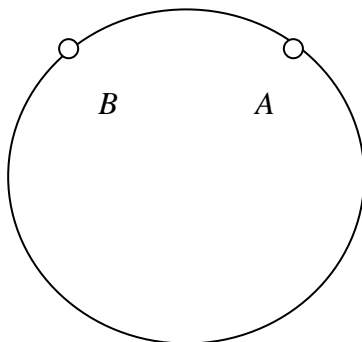
The change in velocity is indicated by the dotted line and as can be seen the acceleration and hence the resultant force (which is in the direction of the acceleration) acts radially inwards. This will always be the case.



This type of diagram can be used to prove that the motion of objects travelling in a circular pathway complies with Newton's 2nd law: the net force acting on an object is in the same direction as the acceleration ($F = ma$)

Review Question

4. Produce a vector addition diagram showing ΔV for an object that moves from point A to point B.



The magnitude of the force

As it appears in Unit 3

The formula for the magnitude of a net force necessary to keep an object in uniform circular motion is:

$$F = ma = \frac{mv^2}{r} = \frac{m4\pi^2r}{T^2}$$

Where:

F = force (N)

m = mass (kg)

v = velocity (m s^{-1})

r = radius (m)

T = period (s)

Remember sometimes it will be necessary to use this formula in conjunction with $F = ma$

Sample question

A car with a mass of 900 kg is driven around a roundabout which has a radius of 4.5 m. The car is travelling at a constant speed of 9.6 m s^{-1} .

Use this information to determine the following:

- i. The magnitude and direction of the acceleration of the car.

$$m = 900 \text{ kg}$$

$$r = 4.5 \text{ m}$$

$$v = 9.6 \text{ m s}^{-1}$$

$$a = \frac{v^2}{r}$$

$$a = \frac{9.6^2}{4.5} = 20.48 \text{ m s}^{-2}$$

The acceleration of an object travelling in a circular pathway is always towards the centre, so the acceleration of the car is 20.48 m s^{-2} towards the centre of the roundabout.

- ii. The time it takes the car to complete one circuit of the roundabout (that is the period of the car).

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 4.5}{9.6} = 2.95 \text{ s}$$

The car takes 2.95 s to complete one circuit of the roundabout.

- iii. The magnitude and direction of the net force on the car at the instant it is travelling anticlockwise in a south easterly direction.

In this case, any of the formulae for net force could be used. You could use the formula $F = ma$ in conjunction with the acceleration calculated in part i. The potential problem with this approach is that you will be using derived information, so if you made a mistake in part i. this will be perpetuated in part ii. Also rounding of an earlier result can affect the final answer.

The calculations below use both of the circular motion formulae, although in this case the one involving the period also uses derived information.

$$F = \frac{mv^2}{r}$$

$$m = 900 \text{ kg}$$

$$r = 4.5 \text{ m}$$

$$v = 9.6 \text{ m s}^{-1}$$

$$F = \frac{900 \times 9.6^2}{4.5} = 18432 \text{ N}$$

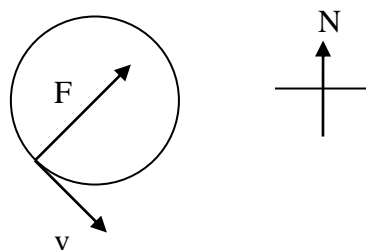
$$F = \frac{m4\pi^2 r}{T^2}$$

$$m = 900 \text{ kg}$$

$$r = 4.5 \text{ m}$$

$$T = 2.95 \text{ s}$$

$$F = \frac{900 \times 4\pi^2 \times 4.5}{2.95^2} = 18373 \text{ N}$$



The net force on the car is 18432 N north-east as it has to be at right-angles to the velocity.

Note the answer using the period is different because of the rounded value used for the period though the values are the same to 3 significant figures.

Review Questions

Use this information to answer the following questions

A boy has a slot car set which includes a toy car weighing 50 g and a circular track with a radius of 50 cm. If the car completes one circuit in 8.1 seconds, calculate the following:

5. The centripetal acceleration of the car.

6. The centripetal force acting on the car.

Cars taking a horizontal curve

As it appears in Unit 3

When a car is going around a bend, the bend may be considered to be an arc of a circle. In order for the car to get around the bend safely the net force must be towards the centre.

If the road is level then the only force towards the centre is the sideways friction. So the sideways friction is the entire magnitude of the net force. This means:

$$\text{Net force} = \text{sideways friction} = \text{centripetal force}$$

The forwards friction between the road and the tyres keeps the car moving forwards.

If the sideways frictional forces are insufficient, then the net force will not be towards the centre and the vehicle will leave the circular path.

As velocity increases the force needed to keep the car moving in a circle greatly increases because the net force is proportional to v^2 .

For a car to stay on the road the following condition must be met:

$$\text{Friction} \geq \frac{mv^2}{r}$$

The maximum speed a car can maintain on a curve is given by:

$$v_{\max} = \sqrt{\frac{\text{friction} \times r}{m}}$$

Sample question

A 900 Kg car is being driven across a corner with a radius of 8 m. If the car's speed is 40 km h⁻¹, what is the magnitude of the force needed to keep the car in its path?

$$r = 9.0 \text{ m}$$

$$m = 900 \text{ kg}$$

$$v = \frac{40}{3.6} = 11.11 \text{ m s}^{-1}$$

In order for the car to not skid off its track, $F_{\text{friction}} = F_{\text{centrepital}}$

$$F_c = \frac{mv^2}{r} = \frac{900 \times 11.11^2}{9} = 12300 \text{ N}$$

Review Questions

Use this information to answer questions 7 and 8.

A cyclist is riding a bike at a constant speed of 12 m s⁻¹ when taking a corner with a radius of 6.5 m. If the centripetal force acting on the cyclist and the bike is 2200 N towards the centre of the corner.

7. Calculate the combined mass of the cyclist and the bike.

8. Determine the sideways frictional force acting on the tyres of the bike if the rider gets around the corner safely.

9. A truck with a mass of 2500 kg travels around a corner with a radius of 20.0 m. The sideways friction is 23000 N. If the truck is travelling at 15 m s⁻¹, will it be able to safely navigate the corner? Show calculations to support your answer.

10. A person is driving around a corner at the maximum speed consistent with safety. Halfway through the corner they hit a patch of loose gravel which greatly reduces the sideways frictional force. Explain what will happen to the car.

Banked roads

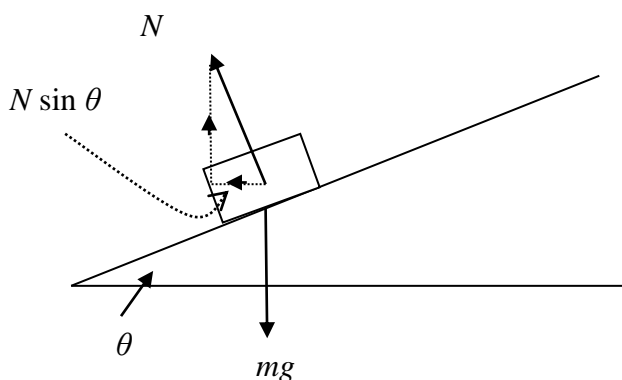
As it appears in Unit 3

In questions relating to banked roads, the assumption is that friction does not exist. If the road is banked at an angle, θ , towards the centre of the circle the horizontal component of the normal reaction $N \sin \theta$ contributes to the centripetal force.

In general, $F_{net} = F_{friction} \cos \theta + N \sin \theta$

However, since friction is assumed to be zero this formula is rearranged to: $F_{net} = N \sin \theta$

This means that banking the road increases the centripetal force, which means that a car can safely travel around the corner at a higher speed.



The forces can be divided into vertical and horizontal components and this information can then be used to calculate the maximum safe velocity on the banked road.

Horizontally $\frac{mv^2}{r} = N \sin \theta$

Vertically $mg = N \cos \theta$

$$\frac{v^2}{g} = \tan \theta \quad \left(\text{as } \tan \theta = \frac{\sin \theta}{\cos \theta} \right)$$

$$v = \sqrt{rg \tan \theta}$$

Review Question

11. The curve of a road has a radius of 500 m. On average cars travel around the curve at a speed of 31.3 m s^{-1} . At what angle should the road be banked for the cars to travel around the curve safely?

Objects attached to strings

As it appears in Unit 3

For an object being spun at the end of a string, the maximum speed of the object is determined using the equation:

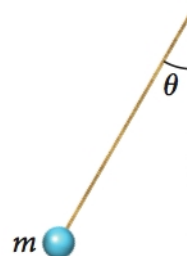
$$v = \sqrt{rg \tan \theta}$$

Where:

r = radius (m)

$g = 9.8 \text{ m s}^{-2}$

θ = angle between the string and the vertical



Questions relating to this type of scenario are usually solved using a combination of trigonometry and the use of appropriate formulae.

You may also be asked to determine the amount of tension in the string. This type of problems is solved by considering vertical and horizontal components.

Review Questions

Use this information to answer the following questions:

A child is whirling a yoyo weighing 50 g in a circular path. The length of the string is 1.7 m and their hand is 0.8 m above the path of the yoyo.

12. Calculate the radius of the circular path taken by the yoyo.

13. Determine the net force acting on the yoyo.

14. Calculate the tension on the string of the yoyo.

Non uniform circular motion

As it appears in Unit 3

Non uniform circular motion refers to conditions where the speed of the object is not constant.

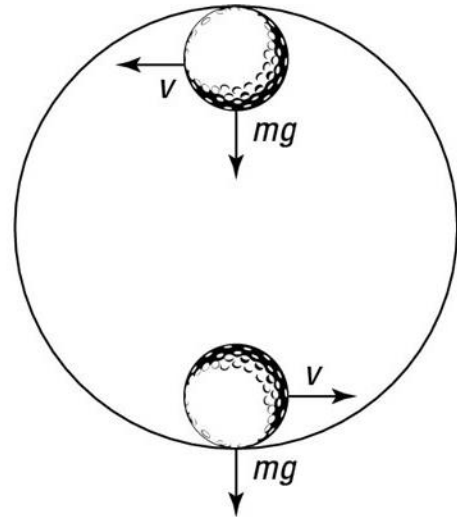
If an object is being spun vertically rather than horizontally, the effect of gravity will cause the object to move more rapidly at the bottom of the circle compared to the top.

If a toy car is travelling in a loop then at the top:

$$N + mg = F_c = \frac{mv^2}{r}$$

At the bottom:

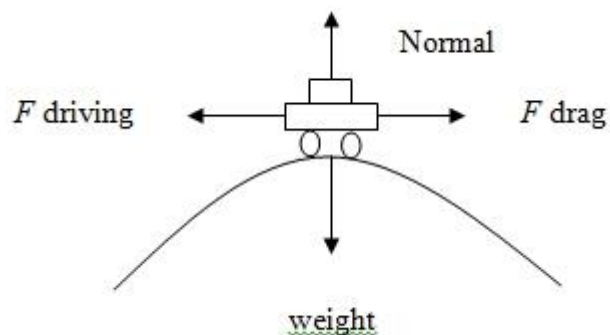
$$N - mg = F_c = \frac{mv^2}{r}$$



If an object is travelling in a vertical loop, it is important to remember that the normal reaction must always be equal to or greater than zero otherwise the object will fall off the loop.

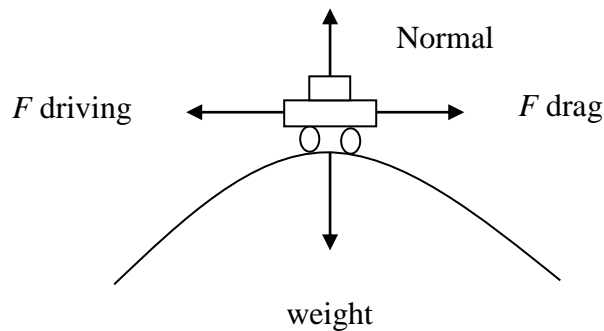
If a toy car is travelling on the top of a loop:

$$mg - N = F_c = \frac{mv^2}{r}$$



Sample questions

A car with a mass of 1000 kg travels over the crest of a hill. Identify all of the forces acting on the car.



The car is travelling fast enough so that as it passes over the crest of the hill it momentarily leaves the ground. Calculate the centripetal force acting on the car at this point.

Since the car leaves the ground the normal reaction force is zero.

$$F_c = F_{net} = W - N$$

Since $N = 0$

$$F_c = mg$$

$$m = 1000 \text{ kg}$$

$$g = 9.8 \text{ m s}^{-2}$$

$$F_c = 1000 \times 9.8$$

$$F_c = 9800 \text{ N}$$

If the radius of the hill crest is 5.0m, calculate the speed of the car at the point where it leaves the road.

$$v = \sqrt{\frac{Fr}{M}}$$

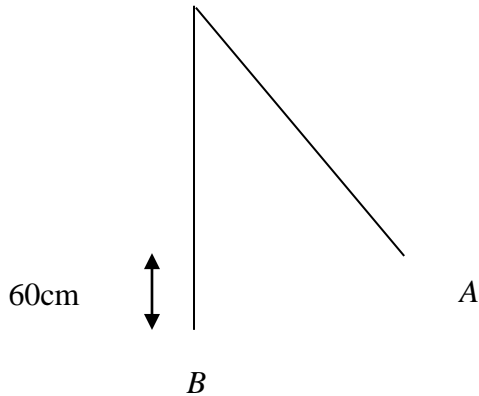
$$v = \sqrt{\frac{9800 \times 5}{1000}}$$

$$v = 7 \text{ m s}^{-1}$$

Sometimes energy conservation needs to be used in order to solve a circular motion question.

Sample question

A child is placed onto a swing and pushed to point A where they are briefly held at rest before being released. The combined mass of the child and the swing seat is 20 kg and the chain of the swing is 2.0 m long.



The vertical distance between point A and point B is 60 cm.

Calculate the speed at which the swing will be travelling at point B.

To solve this problem you need to use:

$$KE_B = PE_A$$

$$\frac{1}{2}mv^2 = mg\Delta h$$

$$v^2 = 2g\Delta h$$

$$v^2 = 2 \times 9.8 \times 0.6$$

$$v = 3.43 \text{ m s}^{-1}$$

Calculate the centripetal force acting on the child at point B.

$$F = \frac{mv^2}{r}$$

$$F = \frac{20 \times 3.43^2}{2}$$

$$F = 118 \text{ N}$$

Review Questions

Use this information to answer the following questions:

A skateboarder with a mass of 60 kg is traveling at a speed of 12 m s^{-1} at the bottom of a circular ramp of diameter 3.5 metres..

- 15.** Calculate the total mechanical energy of the skateboarder.
- 16.** Calculate the centripetal force on the skateboarder when they reach the top of the ramp.
- 17.** Calculate the normal force on the skateboarder at the top of the ramp.

Solutions to Review Questions

1. $r = \frac{vT}{2\pi}$

$$v = 10 \text{ m s}^{-1}$$

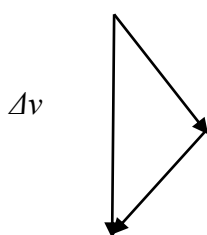
$$T = \frac{196}{2} = 98 \text{ s}$$

$$r = \frac{vT}{2\pi} = \frac{10 \times 98}{2 \times 3.14} = 156 \text{ m}$$

2. The magnitude of the velocity is always the same so it must be 10 m s^{-1} .

3. 0 m s^{-1}

4. The diagram would look like this:



5. $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

$$r = 0.5 \text{ m}$$

$$T = 8.1 \text{ s}$$

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$a = \frac{4\pi^2 \times 0.5}{8.1^2}$$

$$a = 0.3 \text{ m s}^{-2} \text{ towards centre of track}$$

6. $F = ma$

$$m = 0.05 \text{ kg}$$

$$a = 0.3 \text{ m s}^{-2}$$

$$F = 0.05 \times 0.3$$

$$F = 0.015 \text{ N towards the centre of the track.}$$

7. $F = \frac{mv^2}{r}$

$$F = 2200 \text{ kg}$$

$$v = 12 \text{ m s}^{-1}$$

$$r = 6.5 \text{ m}$$

$$m = \frac{Fr}{v^2} = \frac{2200 \times 6.5}{144} = 99.3 \text{ kg}$$

8. The sideways frictional force is also 2200 N towards the centre because the sideways friction and the centripetal force are the same.

9. $F = \frac{mv^2}{r}$

$$m = 2500 \text{ kg}$$

$$v = 15 \text{ m s}^{-1}$$

$$r = 20.0 \text{ m}$$

$$F = \frac{2500 \times 15^2}{20}$$

$$F = 28125 \text{ N}$$

For the truck to navigate the corner safely friction should be greater than centripetal force. Since this is not the case the truck will not navigate the corner safely.

10. Since the sideways frictional force has been decreased the vehicle will leave the road travelling at a straight line from the point where they hit the gravel patch.

11. $\tan \theta = \frac{v^2}{rg}$

$$\theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$$

$$\theta = \tan^{-1} \left(\frac{31.3^2}{500 \times 9.8} \right)$$

$$\theta = 11.31^\circ$$

12. This is solved using Pythagoras' theorem

$$x^2 = 1.7^2 - 0.8^2$$

$$x = 1.5 \text{ m}$$

13. First work out the angle.

$$\cos \theta = \frac{0.8}{1.7}$$

$$\theta = 61.93^\circ$$

Then work out v

$$v = \sqrt{rg \tan \theta}$$

$$r = 1.5 \text{ m}$$

$$g = 9.8 \text{ m s}^{-2}$$

$$\theta = 61.93^\circ$$

$$v = \sqrt{1.5 \times 9.8 \tan 61.93^\circ}$$

$$v = 5.25 \text{ m s}^{-1}$$

$$F = \frac{mv^2}{r}$$

$$m = 0.05 \text{ kg}$$

$$v = 5.25 \text{ m s}^{-1}$$

$$r = 1.5 \text{ m}$$

$$F = \frac{0.05 \times 5.25^2}{1.5}$$

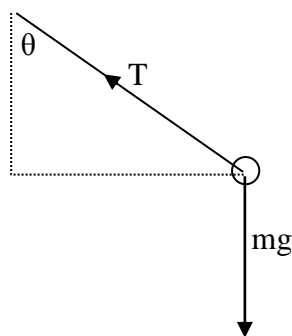
$$F = 0.92 \text{ N centripetally}$$

14. $T \cos \theta = mg$

$$T = \frac{mg}{\cos \theta}$$

$$T = \frac{0.05 \times 9.8}{\cos 61.93^\circ}$$

$$T = 1.04 \text{ N}$$



15. The skateboarder's mechanical energy is

$$\text{K.E} = \frac{mv^2}{2} = \frac{60 \times 12^2}{2} = 4320 \text{ J}$$

16. As the height of the ramp is 3.5m, the P.E. at top = $mgh = 60 \times 3.5 \times 9.8 = 2058 \text{ J}$

Due to the law of conservation of energy, total energy = P.E. + K.E.

Since maximum K.E. = 4320 J (at the bottom of the ramp).

Then $K.E._{\text{top}} = \text{Maximum energy} - \text{PE at top}$
 $= 4320 - 2058 = 2262 \text{ J}$

Hence, velocity at the top of the ramp = $v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 2262}{60}} = 8.68 \text{ m s}^{-1}$

Hence, $F_c = \frac{mv^2}{r} = \frac{60 \times 8.68^2}{3.5} = 1293 \text{ N}$ vertically downwards

17. $F_{\text{net}} = N + W$

$N = F_{\text{net}} - W$

$F_{\text{net}} = 1293 \text{ and } W = 60 \times 9.8 = 588 \text{ N}$

$N = 1293 - 588$

$N = 705 \text{ N}$ downwards

Physics

Teach Yourself Series

Topic 4: Gravity (Units 2 & 3)

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What you need to know

As it appears in Unit 2

- Model the force due to gravity, F_g , as the force of gravity acting at the centre of mass of a body, $F_g = mg$, where g is the gravitational field strength (9.8 N kg⁻¹ near the surface of Earth)

As it appears in Unit 3

- Analyse the use of gravitational fields to accelerate mass, $g = \frac{GM}{r^2}$ and $F_g = \frac{GM_1M_2}{r^2}$;
- Apply the concepts of force due to gravity F_g , and the normal reaction force F_N including satellites in orbit where the orbits are assumed to be uniform and circular
- Model satellite motion (artificial, moon, planet) as uniform circular orbital motion ($a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$);

Gravity, mass and weight

As it appears in Unit 2

Gravity is the force of attraction that exists between any two objects that have mass.

Generally this force is too small to measure unless at least one of the objects has a very large mass.

Mass is the amount of material in a body measured in kilograms (kg). The mass of a body is the same anywhere in the universe.

Weight is the force on an object due to the pull of gravity. The value of the force of gravity on an object will depend on where it is measured, for example in deep space away from any large masses the weight of anything is zero.

The force of gravity is calculated using the following equation:

$$F_g = mg$$

Where:

F_g = force of gravity (N)

m = mass of the body (kg)

g = gravitational field strength (N kg⁻¹)

Note that, at the Earth's surface, the standard value for the Earth's gravitational field is 9.8 N kg⁻¹.

Review Questions

1. A person has a mass of 60 kg. Determine their weight:
 - i. On Earth

 - ii. On the moon where $g = 1.60 \text{ N kg}^{-1}$

2. An astronaut wearing a suit standing on the moon has a weight of 400 N; what would be their mass on Earth?

Gravitational field strength

As it appears in Unit 3

Isaac Newton suggested that it was the force of gravity that keeps the planets in orbit around the Sun.

When the mass of an object exerting the force of gravity increases so does the magnitude of the gravitational field strength. This is written as:

$$g \propto M$$

The relationship between the gravitational field and the distance (r) from the centre of the mass exerting the force of gravity is written as:

$$g \propto \frac{1}{r^2}$$

Newton combined these 2 pieces of information together with the universal gravitational constant to produce the formula for gravitational field strength:

$$g = \frac{GM}{r^2}$$

Where:

g = gravitational field strength (N kg^{-1})

G = gravitational constant = $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

M = the mass of the object exerting the force of gravity (kg)

r = the distance from the centre of object (m)

Sample question

Given the following data:

$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$, mass of the Earth = $6.0 \times 10^{24} \text{ kg}$, radius of the Earth = $6.4 \times 10^6 \text{ m}$.

Calculate the gravitational field strength at the surface of the Earth.

$$g = \frac{GM}{r^2}$$

$$g = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{(6.4 \times 10^6)^2}$$

$$g = 9.8 \text{ N kg}^{-1}$$

Review Questions

3. If the mass of Mars is 6.43×10^{23} kg and its radius is 3.4×10^6 m, calculate the gravitational field strength at the surface of Mars.
4. Calculate the strength of the gravitational field due to an object with a mass of 70 kg at a distance of 6.4×10^6 m.

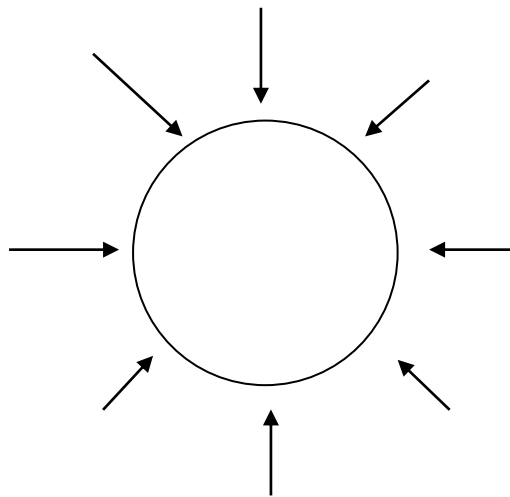
Gravitational fields

As it appears in Unit 3

Gravitational force becomes weaker at large distances. This can be shown in a field line diagram in which the direction of each field line shows the direction or the force of attraction experienced by an object placed there.

The density of the field lines on the diagram indicates the strength of the field; the lines indicating a strong force will be closer together than those of a weaker force.

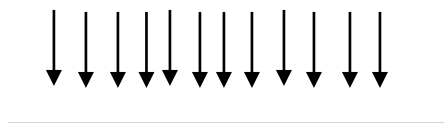
The diagram below models the field lines for the Earth.



As shown on the diagram, the field lines are directed inwards because any object in the Earth's field will experience a force directed towards the centre of the Earth.

The other piece of information shown is that the field lines spread out as the distance from Earth increases which indicates that the field strength is decreasing.

If this diagram was redrawn showing the field lines closer to earth, it would look more like this:



Newton's law of universal gravitation

As it appears in Unit 3

The law of universal gravitation refers to the attraction between objects. Every object in the universe attracts every other object in the universe with a force that has a magnitude that is directly proportional to the product of their masses and inversely proportional to the distance between their centres squared.

It is important to remember that the gravitational field strength at any point in a field is independent of the mass placed there because it is a property of the central object. This means that two objects of different mass placed at the same point in the field will experience the same field strength, but will feel different gravitational forces.

The equation used to calculate the force is

$$F_g = mg = \frac{GMm}{R^2}$$

Where:

F_g = gravitational force on each mass (N)

G = universal gravitational constant = $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

M, m = masses (kg)

R = distance between the centres of the masses M and m (in metres)

If you need to calculate the period of an object in orbit around a central mass the gravitational force can be equated to the centripetal force on the object

$$\frac{GMm}{R^2} = \frac{m4\pi^2 R}{T^2}$$

Where T = the period in seconds

So

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

Review Questions

5. Use your result from question 4 to answer the following question:

Explain why a small object and the Earth exert the same size force on each other despite the fact that the gravitational field of the small object is much weaker than that of the Earth.

6. How would the force between 2 objects change if the distance between them is halved?

7. How would the force between 2 objects change if the distance between them was increased by a factor of 3?

Use the following information to answer questions 8 and 9.

On June 2340, a new satellite was sent to gather information about Neptune.

Given that the mass of Neptune is estimated to be around 102.4×10^{24} kg, the satellite had a mass of 600 kg, and it was orbiting Neptune at a distance of 3×10^7 m from its centre,

8. Calculate the magnitude of Neptune's gravitational field near the satellite.

9. The satellite is travelling at a constant speed. Determine the speed at which it is travelling.

- 10.** The international space station is orbiting Earth. The gravitational field strength in the region of the space station is 88% of that at sea level

Calculate the gravitational field strength experienced on the space station

- 11.** An astronaut on the space station has a mass of 75 kg. Calculate the gravitational force on the astronaut.

- 12.** A satellite of mass 1000 kg was orbiting Mars at a radius of 2.5×10^6 m. If the mass of Mars is given as 6.42×10^{23} kg, calculate the gravitational force on the satellite.

Relationship between the Earth and the Moon

As it appears in Unit 3

The Moon has a roughly circular pathway around the Earth. This means that the concepts covered in circular motion can be applied.

The Moon experiences a centripetal force caused by the effect of the Earth's gravitational field on the Moon.

Therefore $F_g = F_c$

The velocity of the moon can be calculated using the formula:

$$v = \sqrt{\frac{GM}{r}}$$

Where:

G = the universal gravitational constant

M = the mass of the Earth

r = radius of the moons orbit

Review Question

13. If the mass of the Earth is 5.98×10^{24} kg and the radius of the moons orbit is 3.84×10^8 m. Calculate the average velocity of the moon

Satellites

As it appears in Unit 3

If a satellite is in a stable circular orbit, then the only force acting on the satellite is the gravitational force between it and the central body.

The kinetic and gravitational potential energy of a satellite remains constant because the force acting on the satellite is perpendicular to its motion.

The force of gravity holds the satellites in their orbits and causes them to have centripetal acceleration. Just like an object at the end of a string, if gravity was switched off the satellites would leave their orbits at a tangent. This does not happen because the satellites are falling towards the earth at the same rate as the satellite would be moving away from the Earth if there was no gravity.

Since satellites are in a continual state of free-fall, their acceleration will equal the gravitational field strength at that point.

The acceleration of a satellite is calculated using:

$$a = \frac{v^2}{R} \quad \text{However, since } v = \frac{2\pi R}{T}, \quad a = \frac{4\pi^2 R}{T^2}$$

$$\text{Also } a = \frac{GM}{R^2} = g$$

Where v = speed (m s^{-1})

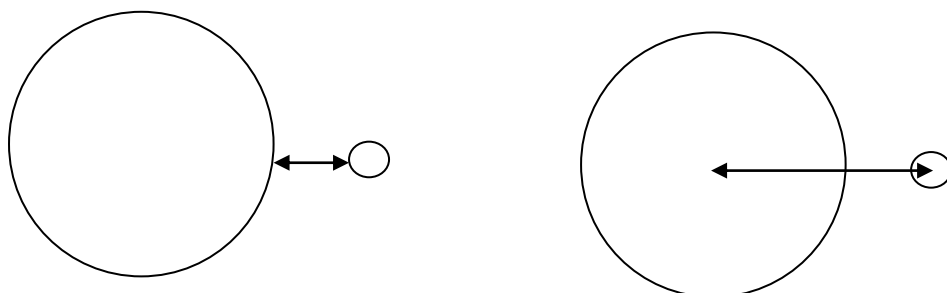
R = radius of orbit (m)

T = period of orbit (s)

M = central mass (kg)

g = gravitational field strength (N kg^{-1}).

It is important to be careful establishing the radius of a satellites orbit because you also need to include the radius of the earth. Often the altitude of the satellite will be given which is measured from the Earth's surface.



For example, if a spaceship has an altitude of 400 km then the radius = $R_E + 400 \text{ km}$

The other area to be careful with is making sure the units are the same. The radius of the Earth is usually given in metres and the altitude of a satellite is generally given in kilometers so you need to convert the altitude to metres e.g. $1\text{ km} = 10^3$ metres so in this example $400\text{ km} = 400 \times 10^3$ metres

Review Questions

14. If the Mir space station orbits the Earth at an altitude of 400 km and the radius of the earth is 6.37×10^6 m, what is the radius of the orbit of the space station?

15. The Mir space station completes 15.7 orbits every 24 hours. Calculate the period of the space station.

16. Calculate the speed of the MIR space station,

17. Draw a diagram showing the direction of any forces on the space station as it orbits the earth.

Apparent weight

As it appears in Unit 3

Your weight feels normal to you if you are travelling at a constant velocity.

If you are accelerating vertically the normal reaction will not equal mg . This alters your perception of your weight causing you to feel heavier or lighter.

When the normal reaction force is less than mg you will feel lighter, and when the normal reaction force is greater than mg you will feel heavier.

If $ma = mg$ then you are in free fall and will feel weightless.

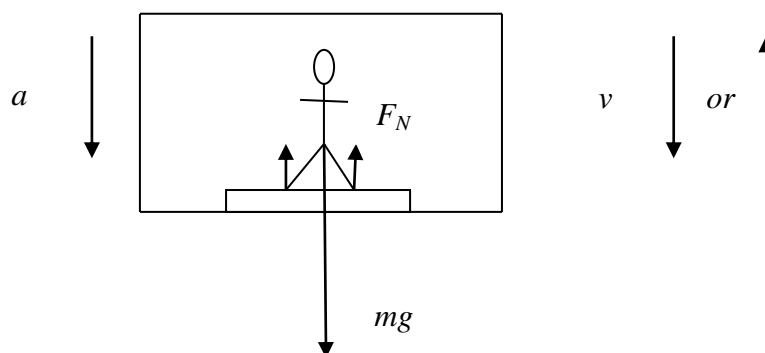
So the apparent weight of a person is equal in magnitude to the normal force supplied by the supporting surface.

For example astronauts experience apparent weightlessness because they are in the same orbit as their spaceship. Both the ship and the astronaut are experiencing centripetal acceleration towards the Earth. The gravitational pull from the Earth curves their trajectory towards the deck, but the deck is falling away from them at the same rate. This means that the deck is not exerting a normal force on the astronauts giving the feeling of apparent weightlessness.

True weightlessness is only possible when the strength of the gravitational field is zero. This condition can only be approximated in deep space where an object is a long distance away from any other objects.

Another example involves a person in an elevator.

Consider that the person in the elevator is standing on a set of scales. The forces on the person are marked on the diagram below.



The normal force comes from the contact between the person and the set of scales, so therefore the scales are measuring F_N not mg .

If the person is accelerating downwards the normal force is not supplying force that is equal to mg so the scales will measure an apparent weight less than mg and as a result the person feels lighter.

Review Questions

- 18.** A spaceship is in a stable orbit 500 km above the Earth's surface. An item with a mass of 5 kg is placed onto a spring balance attached to the inside surface of the spaceship. Predict the weight shown on the spring balance. Provide a reason to justify your answer.

- 19.** Explain what is meant by apparent weightlessness

- 20.** In order to sleep, astronauts need to strap themselves onto a bed attached to the walls of the space station. Predict the magnitude of the force that the bed exerts on the astronaut. Provide a reason to justify your answer.

Solutions to Review Questions

1. The answers are as follows:

i. $W = mg$
 $m = 60 \text{ kg}$
 $g = 9.8 \text{ m s}^{-2}$

$$W = 60 \times 9.8$$

$$W = 588 \text{ N}$$

ii. $m = 60 \text{ kg}$
 $g = 1.60 \text{ N kg}^{-1}$

$$W = 60 \times 1.60$$

$$W = 96 \text{ N}$$

2. $W = 400 \text{ N}$
 $g = 1.60 \text{ N kg}^{-1}$

$$m = \frac{W}{g}$$

$$m = \frac{400}{1.6}$$

$$m = 250 \text{ kg}$$

Their mass on earth will be the same as their mass on the moon.

3. $m = 6.43 \times 10^{23} \text{ kg}$
 $r = 3.40 \times 10^6 \text{ m}$

$$g = \frac{GM}{r^2}$$

$$g = \frac{6.67 \times 10^{-11} \times 6.43 \times 10^{23}}{(3.4 \times 10^6)^2}$$

$$g = 3.7 \text{ N kg}^{-1}$$

4. $m = 70 \text{ kg}$
 $r = 6.4 \times 10^6 \text{ m}$

$$g = \frac{GM}{r^2}$$

$$g = \frac{6.67 \times 10^{-11} \times 700}{(6.4 \times 10^6)^2} = 1.14 \times 10^{-22} \text{ N kg}^{-1}$$

5. This is because the actual force acting on an object in a gravitational field is the product of the field strength and the mass of the object placed there.

For the 70 kg object, the force = $70 \times 9.8 = 686 \text{ N}$

For the earth, the force = $6.0 \times 10^{24} \times 1.14 \times 10^{-22} = 684 \text{ N}$

Allowing for rounding errors, the force on the object in the Earth's gravitational field and the force on the Earth in the object's gravitational field are the same.

6. Since the force is inversely proportional to r^2 the force would be $2^2 = 4$ times the original strength.
7. The force would be $\frac{1}{9}$ of the force in the original scenario.

8.

$$g = \frac{GM}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 102.4 \times 10^{24}}{(3 \times 10^7)^2} = 7.59 \text{ N kg}^{-1}$$

9. Since the gravitational field strength is the same as the centripetal acceleration

$$v = \sqrt{gR} = \sqrt{7.59 \times 3 \times 10^7}$$

$$= 1.5 \times 10^4 \text{ m s}^{-1}$$

10. $9.8 \times 0.88 = 8.6 \text{ N kg}^{-1}$

11. $F_g = mg$

$$F_g = 75 \times 8.6$$

$$F_g = 645 \text{ N}$$

12. $F_g = \frac{GMm}{r^2}$

$$F_g = \frac{6.67 \times 10^{-11} \times 6.42 \times 10^{24} \times 1000}{(2.5 \times 10^6)^2}$$

$$F_g = 6851 \text{ N}$$

13.

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{3.84 \times 10^8}}$$

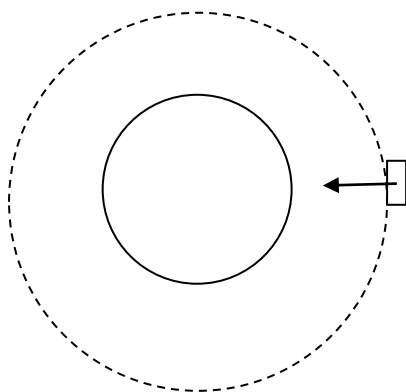
$$v = 1.02 \times 10^3 \text{ m s}^{-1}$$

14. $r = 6.37 \times 10^6 \text{ m} + 400 \times 10^3 \text{ m}$
 $r = 6.77 \times 10^6 \text{ m}$

15. Time = $24 \times 60 \times 60 = 86400 \text{ s}$
 $T = \frac{86400}{15.7}$
 $T = 5503 \text{ s}$

16. $v = \frac{2\pi r}{T}$
 $v = \frac{2\pi \times 6.77 \times 10^6}{5503}$
 $v = 7.73 \times 10^3 \text{ m s}^{-1}$

17. The diagram should look like this as there is only 1 force acting on the space station.



18. The scales should read zero.

Scales such as a spring balance measure the reaction force. The net force = $mg - T$ Since $a = g$ then T must equal zero.

19. Apparent weightlessness is the sensation of feeling as if you have no weight.

It is the normal reaction from the surface that gives us the sensation of having weight. If the normal force is zero then a person experiences apparent weightlessness.

Apparent weightlessness occurs due to the object being in freefall, accelerating at the rate of g therefore due to $\Sigma F = W + N$ and as $\Sigma F = W$, $N = 0$

20. Both the space station and the astronaut are in free fall, so they are accelerating at the same rate therefore the bed does not exert a force on the astronaut.

Physics

Teach Yourself Series

Topic 5: Einstein's special relativity (Unit 3)

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What you should know

As it appears in Unit 3

- Describe Einstein's two postulates for his theory of special relativity that:
 - the laws of physics are the same in all inertial (non-accelerated) frames of reference
 - the speed of light has a constant value for all observers regardless of their motion or the motion of the source
- Compare Einstein's theory of special relativity with the principles of classical physics
- Describe proper time (t_0) as the time interval between two events in a reference frame where the two events occur at the same point in space
- Describe proper length (L_0) as the length that is measured in the frame of reference in which objects are at rest
- Model mathematically time dilation and length contraction at speeds approaching c using the equations: $t = t_0\gamma$ and $L = \frac{L_0}{\gamma}$ where $\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-\frac{1}{2}}$
- Explain why muons can reach Earth even though their half-lives would suggest that they should decay in the outer atmosphere.
- Interpret Einstein's prediction by showing that the total 'mass-energy' of an object is given by: $E_{\text{tot}} = E_k + E_0 = \gamma mc^2$ where $E_0 = mc^2$, and where kinetic energy can be calculated by:
$$E_k = (\gamma - 1)mc^2$$
- Describe how matter is converted to energy by nuclear fusion in the Sun, which leads to its mass decreasing and the emission of electromagnetic radiation.

Einstein's special relativity

When objects have speeds approaching the speed of light, Newton's laws must be modified to describe the motion. Einstein's special relativity no longer assumes that quantities such as length, mass and time are fixed.

Before Einstein

As it appears in Unit 3

Relative motion

Suppose that you are standing stationary at one end of a railway carriage that is D m long and that a friend standing at the other end of the carriage throws a ball to you. Suppose also that it takes T s to reach you. Then the speed of the ball is given by:

$$\text{Ball speed} = \frac{D}{T}$$

But, suppose an observer outside the train sees the train moving with speed V_{train} in the same direction as that of the ball, then the speed of the ball is:

$$\text{Ball speed} = \frac{D}{T} + V_{\text{train}}$$

Absolute zero

Obviously the “observed speed” of the ball depends on the frame of reference in which it is being measured. To find the “true speed” of the ball it would seem reasonable to consider all frames of reference in which the ball is moving. If the train is moving along the Earth’s equator in the same direction as the Earth’s motion due to its spin, then the ball speed is given by:

$$\text{Ball speed} = \frac{D}{T} + V_{\text{train}} + \frac{2\pi R_E}{T_{\text{SPIN}}}$$

Where R_E is the Earth’s radius and T_{SPIN} is the time for the Earth to spin once about its axis. Now taking into account the Earth’s revolution about the Sun the ball speed becomes:

$$\text{Ball speed} = \frac{D}{T} + V_{\text{train}} + \frac{2\pi R_E}{T_{\text{SPIN}}} + \frac{2\pi R_{\text{ORBIT}}}{T_{\text{ORBIT}}}$$

Don’t forget also that the the Solar System is moving through space at

The problem with this logic is what happens when you are in a spacecraft drifting through space. There are no signposts in space, so how do you tell if you are actually moving? And what if you meet another spacecraft travelling towards you? Are you stationary with the other space craft travelling towards you or is it the other way around? It is impossible to tell. Fortunately there is no real problem as all of the above frames of reference are *inertial frames of reference*. An inertial frame of reference is a non-accelerating frame of reference. All of the laws of Physics (e.g. Newtons 3 Laws of Motion) apply in an inertial frame of reference. If you are trying to solve a Physics problem involving things that are moving, then you just choose the most appropriate reference frame then use the usual laws of Physics.

All of the above suggests that there is no Absolute Frame of Reference or Absolute Zero.

Review Question

1. You are standing at one end of a railway carriage which is 10 m long and throw a ball to a friend. This takes exactly 1.0 s. The train is travelling at its speed limit which is 100 km h^{-1} (or 28 m s^{-1}) and the train is travelling around the equator. What is the speed of the ball as observed by a person standing on the Sun? (This observer has an airconditioned fire proof space suit). Note that the radius of the Earth is 6400 km, the radius of the Earth’s orbit about the Sun is 1 500 000 km and the time for 1 orbit about the Sun is 365 days.

Important experiments

As it appears in Unit 3

By the 1800's, many experiments had been performed involving light and experiments performed by Thomas Young in particular provided strong evidence that light is a wave phenomenon (as opposed to particles). The speed of light had been measured in a variety of ways and all agreed that the speed of light was somewhere around $3 \times 10^8 \text{ m s}^{-1}$.

An especially significant result obtained by James Maxwell was the mathematical prediction of the speed of light. He used Faraday's Law and Ampere's Law and some other lesser known laws to establish that the speed of light should travel at $3 \times 10^8 \text{ m s}^{-1}$ and, more importantly, that the speed of light was only affected by the medium through which it travelled and not on the speed of the source or observer.

This gives rise to another question. If water waves need water to travel through and sound waves need air (or some other medium), then what do light waves travel through? A popular explanation in the 1800's was the idea of the *Luminiferous Aether*, a substance through which light could travel.

Two physicists, Michelson and Morley, set out to confirm the existence of the aether. Their experiment used the following logic. Suppose that you were to measure the speed of sound coming from two separate sources, one due North of you and the other due East. Suppose also that there was a strong North wind blowing so that the medium carrying the sound was moving towards you from the North. You would therefore obtain slightly different results which confirm the existence of a sound wave carrying medium.

Michelson and Morely's experiment used light from a distant source which passed through a semisilvered mirror. Half of the light travelled along one path to the observer while the other half travelled along a second path which was perpendicular to the first. Their experiment was not to measure the speed of light but to detect any difference in the speed of light travelling along the two perpendicular paths. Their equipment did not detect the slightest difference in speed no matter where or when they performed their experiment. This refuted the existence of the aether and confirmed Maxwell's prediction that the speed of light only depends on the medium which it is travelling through. It doesn't matter whether you are moving towards or away from a light source or whether the light source is moving towards or away from you, The speed of light is always the same.

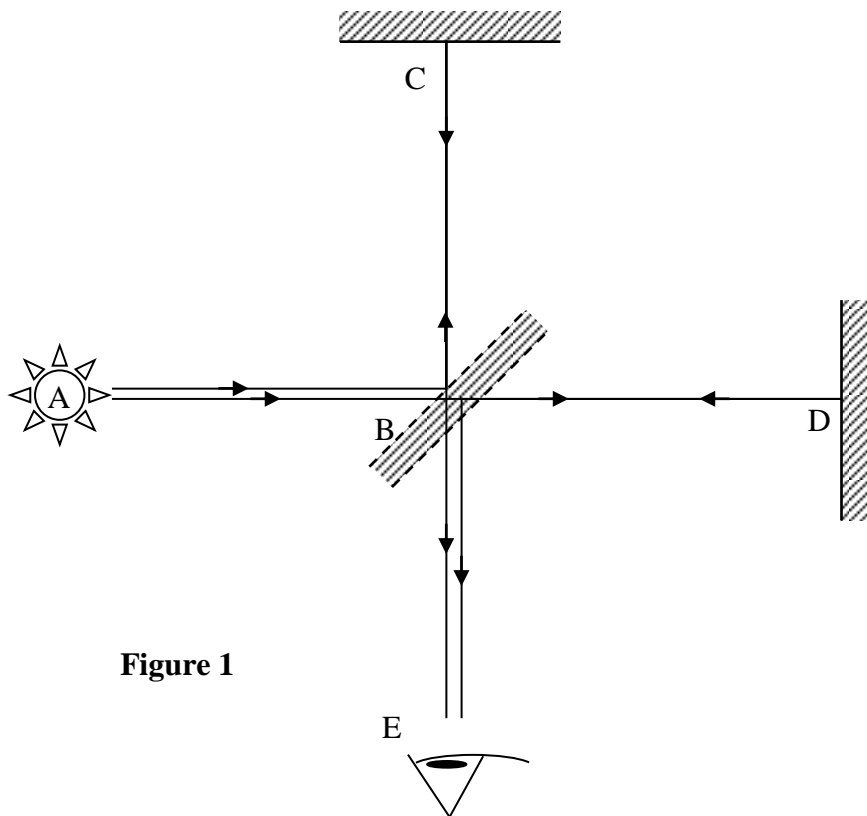


Figure 1

Figure 1 shows a simplified version of Michelson and Morley's apparatus. The path difference $BC - BD$ produces an interference pattern. If light is travelling through the drifting aether then it should be travelling faster along one of the paths.

Rotating the apparatus by 90° should produce a different interference pattern. However this was not the case, so the speed of light is the same in all directions

Review Question

2. The purpose of the Michelson-Morley experiment was to:
 - A. measure the speed of light.
 - B. prove the existence of the aether
 - C. show that light is a wave phenomenon
 - D. prove that light is an electromagnetic wave.

Einstein's interpretation

Einstein's postulates

As it appears in Unit 3

The puzzle concerning the speed of light laid the groundwork for Einstein's special theory of relativity. Two golden rules that had to be obeyed are known as Einstein's two postulates for special relativity. They are:

- The laws of Physics are the same in all inertial (i.e. non-accelerating) frames of reference
- The speed of light is the same for all observers regardless of their motion or the motion of the light source.

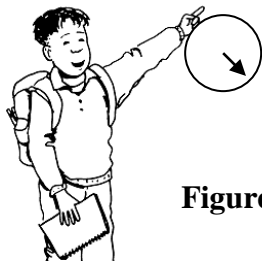
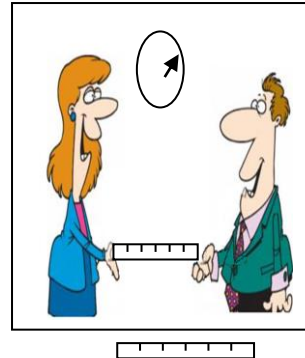
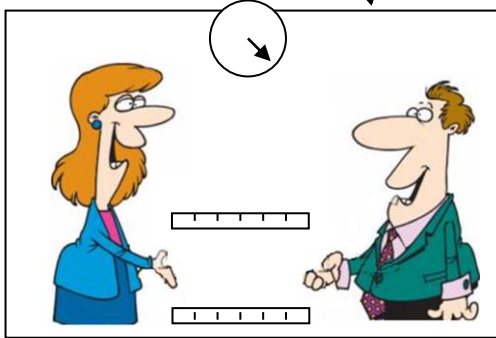
The consequence of this is that measurements of time and distance are no longer constant but depend on the relative speed of the observer. When an object travels at a speed close to the speed of light, lengths in the direction of motion contract and measurements of time expand.

Distance contraction and time dilation

As it appears in Unit 3

In the bus at
normal speed

In the spacecraft
close to the speed
of light



Stationary
observer

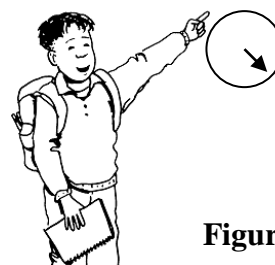


Figure 2a

Figure 2b

Figure 2a shows an observer watching two people on a bus travelling at a regular everyday speed. The experiment has progressed for a short time. Both the passengers in the bus and the stationary observer agree on the distance between the passengers and how long the experiment has taken. The experiment is repeated in Figure 2b but this time the passengers are in a spacecraft travelling close to the speed of light. The observer states that, based on his ruler, the passengers are closer together. The observer and the passengers disagree on the duration of the experiment. The observer says that based on his clock, the experiment took a longer time than the passengers' measurement.

The Lorentz transformations

As it appears in Unit 3

These are equations that can be used to determine the extent of the length contraction or time dilation when observing motion at speeds close to the speed of light.

$$L = \frac{L_0}{\gamma} \quad T = T_0\gamma \quad \text{where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- L_0 is the “proper length or the length of a body in the same reference frame as the observer.
- T_0 is the proper time or the time duration of an event which occurs in the same reference frame as the observer.
- γ is the Lorentz factor or Transformation factor.
- v is the speed of the body relative to the observer
- c is the speed of light

Review Questions

3. Dan and Wen are to trial a space ship and have a careful look around before takeoff. They notice that the seats are comfortably located, each being 120cm apart. They have a tool kit which contains an accurate ruler and a clock. The space ship is launched and quickly reaches a speed of 0.6 c relative to Con, a stationary observer who also has a ruler and accurate clock. They know that on Earth it takes 5 minutes to cook an egg.

a. Determine the Lorentz factor for this experiment.

b. How far apart do the seats appear to be according to Con?

c. Wen cooks an egg while the spaceship is moving. How long does this take according to Con?

d. Dan is in the same spacecraft and performs the same tasks as Wen. How far apart are the seats according to Dan?

e. How long does it take to cook an egg in the moving spacecraft according to Dan?

Mass and special relativity

As it appears in Unit 3

As the speed of a body approaches the speed of light, its mass increases. At the speed of light, mass becomes infinite. This places an upper speed limit on all bodies because no matter how much force is applied, according to Newton's 2nd law i.e. $F = m a$, with an infinite mass, the acceleration must be zero.

The mass of a body is given by: $m = m_0 \gamma$ where m_0 is called the *rest mass*.

The total energy of a body is given by:

$$E_{TOTAL} = E_{KINETIC} + E_{REST} = mc^2$$

Combining the above equations gives:

$$E_K = (\gamma - 1) m_0 c^2$$

The energy required to create a particle of mass m or the energy produced when a particle of mass m is totally destroyed is given by

$$E = m c^2$$

Review Questions

4. In a particle accelerator an electron can reach a speed of $0.6c$. What is its *relativistic mass* (i.e. its mass as observed by a stationary observer) if the rest mass of an electron is 9.109×10^{-31} kg?

5. In the fusion process, a proton of rest mass 1.673×10^{-27} kg and a neutron of rest mass 1.675×10^{-27} kg combine to form a deuterium nucleus of rest mass 3.344×10^{-27} kg. The energy released is:
 - A. 1.2×10^{-21} J
 - B. 3.6×10^{-13} J
 - C. 4.0×10^{-3} J
 - D. 3.6×10^{-14} J

6. A square sheet of paper is moving towards you (face on) at a constant high speed? The paper is
 - A. Rectangular and the same size as when it is stationary
 - B. Square and exactly the same size as when it is stationary
 - C. Square and smaller than when it is stationary
 - D. Rectangular and smaller than when it is stationary

7. Suppose T_1 is the time taken for light to travel a return trip parallel to the Earth's motion and T_2 the time taken for light to travel a return trip of exactly the same distance perpendicular to the Earth's motion. The value of $\frac{T_1}{T_2}$ would be
- A. Slightly less than 1
 - B. Equal to 1
 - C. Slightly greater than 1
 - D. Significantly greater than 1
8. An electron moves with a speed such that its total energy is 15% greater than its rest mass energy. The rest mass energy of an electron is 8.2×10^{-14} J. What is its kinetic energy?
9. What is the Lorentz factor for the situation in question 8 above?
10. What is the speed of the electron relative to the observer?

Solutions to Review Questions

$$\begin{aligned}
 1. \text{ Ballspeed} &= \frac{D}{T} + V_{\text{train}} + \frac{2\pi R_E}{T_{\text{SPIN}}} + \frac{2\pi R_{\text{ORBIT}}}{T_{\text{ORBIT}}} \\
 &= \frac{10}{1.0} + 28 + \frac{2\pi \times 6400 \times 10^3}{24 \times 60 \times 60} + \frac{2\pi \times 1500000 \times 10^3}{365 \times 24 \times 60 \times 60} \\
 &= 10 + 28 + 465 + 299 \\
 &= 802 \text{ m s}^{-1}
 \end{aligned}$$

2. The speed of light was already known. The M&M experiment compared the speed of light along two perpendicular paths to detect the presence of the aether. So the answer is **B**.

$$3. \text{ a. } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.6c)^2}{c^2}}} = 1.25$$

$$\text{b. } L = \frac{L_0}{\gamma} = \frac{1.2}{1.25} = 0.96 \text{ m}$$

$$\text{c. } T = T_0 \gamma = 5 \times 1.25 = 6.25 \text{ min}$$

- d. Dan is in the same reference frame as the spaceship so there is no contraction. Observed distance is 1.2 m

- e. For the same reason above, time will be the same, 5 minutes

4. If $v = 0.6c$, then $\gamma = 1.25$

$$m = m_0 \gamma = 9.109 \times 10^{-31} \times 1.25 = 1.14 \times 10^{-30} \text{ kg}$$

$$\begin{aligned}
 5. \text{ } E &= \Delta m c^2 \\
 &= (1.673 \times 10^{-27} + 1.675 \times 10^{-27} - 3.344 \times 10^{-27}) \times (3 \times 10^8)^2 \\
 &= 3.6 \times 10^{-13}
 \end{aligned}$$

So answer **B** is correct

6. There is contraction in the direction of motion only. So the paper will appear thinner but still appear square with the same side lengths. Answer is **B**.

7. This was the basis of the Michelson-Morely experiment that found that there was no difference. So the answer is **B**.

8. $E_{TOTAL} = E_{KINETIC} + E_{REST} = mc^2$

$$E_{REST} \times \frac{115}{100} = E_{KINETIC} + E_{REST} \times \frac{100}{100}$$

$$E_{KINETIC} = E_{REST} \times \frac{15}{100} = 8.2 \times 10^{-14} \times \frac{15}{100} = 1.23 \times 10^{-14} \text{ J}$$

9. $E_K = (\gamma - 1) m_0 c^2$

$$1.23 \times 10^{-14} = (\gamma - 1) \times 8.2 \times 10^{-14}$$

$$(\gamma - 1) = 0.15$$

$$\gamma = 1.15$$

10. $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2}$$

$$v = 1.48 \times 10^8 \text{ m s}^{-1}$$



Physics

Teach Yourself Series

Topic 6: Electric fields (Unit 3)

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What you should know

As it appears in Unit 3

- Describe electricity using a field model.
- Investigate and compare theoretically and practically electric fields, including directions and shapes of fields, attractive and repulsive fields, and the existence of dipoles and monopoles.
- Investigate and compare theoretically and practically gravitational fields and electrical fields about a point mass or charge (positive or negative) with reference to:
 - the direction of the field
 - the shape of the field
 - the use of the inverse square law to determine the magnitude of the field
 - potential energy changes (qualitative) associated with a point mass or charge moving in the field
- Identify fields as static or changing, and as uniform or non-uniform.
- Analyse the use of an electric field to accelerate a charge, including:
 - electric field and electric force concepts: $E = k \frac{q}{r^2}$ and $F = k \frac{q_1 q_2}{r^2}$
 - potential energy changes in a uniform electric field: $W = qV$, $E = \frac{V}{d}$
 - the magnitude of the force on a charged particle due to a uniform electric field: $F = qE$
- Describe the interaction of two fields, allowing that electric charges, magnetic poles and current carrying conductors can either attract or repel, whereas masses only attract each other

Electric Fields

As it appears in Unit 3

A field in physics is defined as an area where each point is affected by a force. We have already looked at gravitational fields and will later investigate magnetic fields. Within this section we will investigate electric fields.

An electric field surrounds an electric charge. When a charged particle is placed in that region it experiences an electric force that either attracts or repels it.

An electric field is a vector quantity, it has both direction and magnitude.

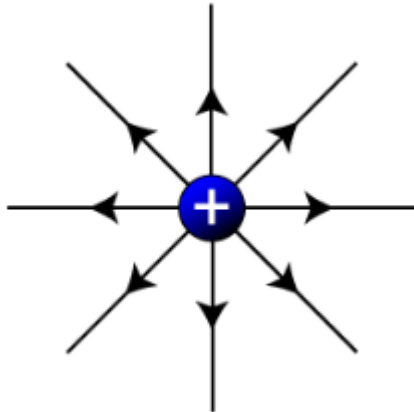
In order to visualize electric fields, as they cannot be seen, we use electric field lines. These electric field lines show the direction of force a positive charge will experience if it was to be placed within the field. The intensity of the field is displayed by how close the lines are together.

Some basic rules when drawing field lines

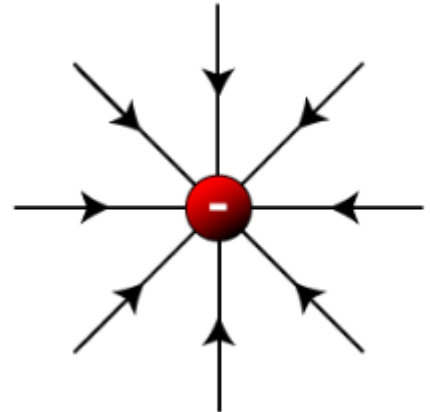
1. Electric field lines point away from positive charges, and toward negative charges.
2. Electric field lines never cross.
3. Electric field lines always intersect conductors at right angles to the surface.
4. Stronger fields have closer lines.
5. Field strength and line density decreases as you move away from the charges.

Drawing electric field lines

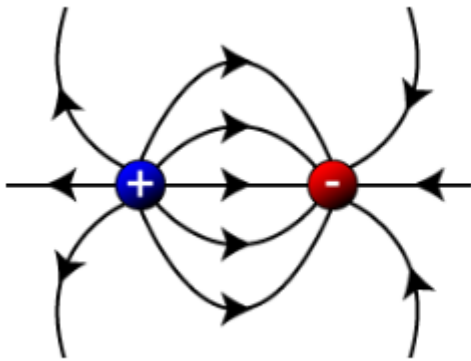
Single positive charge



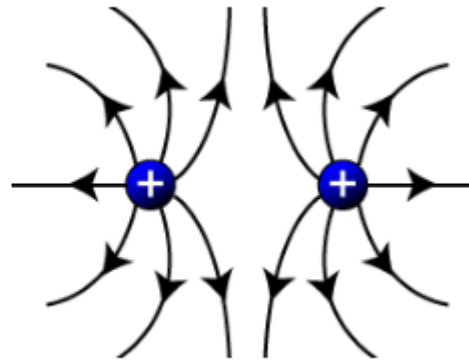
Single negative charge



A positive and negative charge



Two positive charges

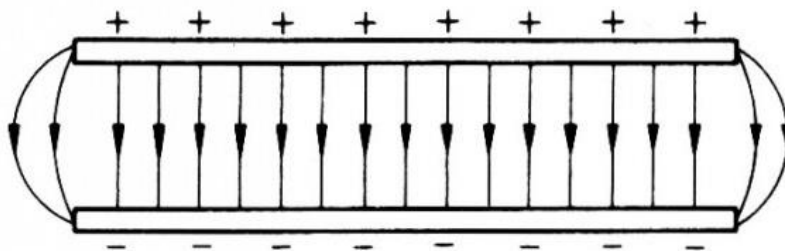


A single electric charge is also called an electric monopole.

The positive and negative charges above which are equal and opposite charges form an "**electric dipole**."

An electric field is called uniform if its strength does not change with distance. The electric field of a point charge is not uniform, because it strongly weakens when distance from the charge increases. It weakens proportional to $\frac{1}{r^2}$ and its field lines **diverge** or **open up** very quickly in space.

Parallel oppositely charged plates



If two parallel metallic plates are separated by a distance and connected to a battery, one plate accumulates some negative charges while the other plate accumulates equal amount of positive charges. The electric field in between the plates and especially away from the edges will essentially be uniform and the electric field lines become parallel.

Strength of an electric field

As it appears in Unit 3

The strength of an electric field can be defined in terms of the force experienced by a particle with a charge of q .

Electric Field Strength (E) = Force (F) per unit charge (q)

$$E = \frac{F}{q} \quad \text{with units newtons per coulomb (NC}^{-1}\text{)}$$

Consider two parallel plates where one plate has a potential (V) and the other plate is earthed (and therefore has a potential of zero). There is an electric potential difference (ΔV) between the two plates. The plates are separated by a distance d

To calculate the electric field strength in this scenario:

$$E = \frac{V}{d} \quad \text{with units Volts per metre (Vm}^{-1}\text{)}$$

By combining the two equations above

$$\frac{V}{d} = \frac{F}{q}$$

As $\text{Work} = \text{Force} \times \text{Distance}$

The work done on a point test charge to move it a distance, d across a potential difference, V :

$$W = qV \text{ or } W = qEd \text{ with units Joules (J)}$$

When calculating work done, which changes the electrical potential energy, it is important to remember that work can be done either by the electric field on a charged object or on the electric field by forcing the object

to move. When work is done by the field the charge particle is moving in the direction of the field, i.e. a positively charged particle moving with an electric field, when a charged object is doing work on a field the object is being forced against its natural direction such as the positively charged particle being forced towards the positive plate. In order to do work the charge needs to move a distance parallel to the field.

Review Questions

Question 1

A charge of $20\ \mu\text{C}$ is moved $50\ \text{cm}$ in a uniform electric field of strength $20\ \text{Vm}^{-1}$. Find

- a. The voltage between the plates when the distance between them is $120\ \text{cm}$
- b. The work done when the charge is moved in the direction of the field lines
- c. The work done when the charge is moved against the direction of the field lines
- d. The work done when the charge is moved perpendicularly to the field lines
- e. which of the answers (from a to d) will change when the charge is negative and what will it change to?

Question 2

How much energy is acquired by an electron whose charge is $1.6 \times 10^{-19}\text{C}$ moving through a potential difference of $2.5 \times 10^4\text{V}$.

Application: Accelerating electrons

As it appears in Unit 3

Electrons being attracted across a set of parallel plates are once example of work done between plates and the subsequent kinetic energy obtained. The electrons are attracted to the positive plate and repelled from the negative plate. When an electron is accelerated by an electric field, the increase of kinetic energy of the electron is equivalent to the work done by the electric field.

$$\Delta E_k = W = qV$$

Where ΔE_k = the increase of kinetic energy (J)

V = voltage or potential difference between two parallel plates (V)

W = work done by the electric field (J)

Review Questions

Question 3

An electron is accelerated from rest by an electric field between two parallel plates. The potential difference across the two plates is 2500 V. The distance between the two plates is 20 cm.

- a. Calculate the strength of the electric field.

- b. Calculate the magnitude of the velocity of the electron after the acceleration.

Question 4

In an experiment, a group of physics students found that electrons emerged from an electron gun with energy of 6 keV.

- a. What is the magnitude of the velocity of these electrons?

- b. If the electrons are accelerated from rest, what is the accelerating voltage in the electron gun?

Coulomb's Law

As it appears in Unit 3

As already defined there is a force that acts between 2 charged objects.

This force is effected by the amount of charge each object has and their proximity to each other. This force can be calculated via Coulombs Law.

Coulombs Law states the electrical force between 2 charged objects is directly proportional to the product of the quantity of charge and inversely proportional to the square of the distance of separation.

$$\text{Force} = \frac{kq_1q_2}{r^2}$$

k is defined as Coulomb's constant and is equal to $\frac{1}{4\pi\epsilon_0}$

where ϵ_0 is the permittivity of free space which equals $8.8542 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

As the permittivity of free space is the same in air or a vacuum
 $k = 9.0 \times 10^9 \text{ Nm}^{-2}$

Review Questions

Question 5

The force of electrostatic repulsion between two small positively charged objects, A and B, is $3.6 \times 10^{-5} \text{ N}$ when $AB = 0.12 \text{ m}$.

What is the force of repulsion if AB is increased to the following?

a. 0.24 m

b. 0.3 m

Question 6

Calculate the force between charges of $5.0 \times 10^{-8} \text{ C}$ and $1.0 \times 10^{-7} \text{ C}$ if they are 5.0 cm apart.

Question 7

What is the magnitude of the force a $1.5 \times 10^{-6} \text{ C}$ exerts on a $3.2 \times 10^{-4} \text{ C}$ located 1.5 m away.

Question 8

Two equal charges of magnitude $1.1 \times 10^{-7} \text{ C}$ experience an electrostatic force of $4.2 \times 10^{-4} \text{ N}$. How far apart are the centres of the two charges?

Electric field at a distance from a charge

As it appears in Unit 3

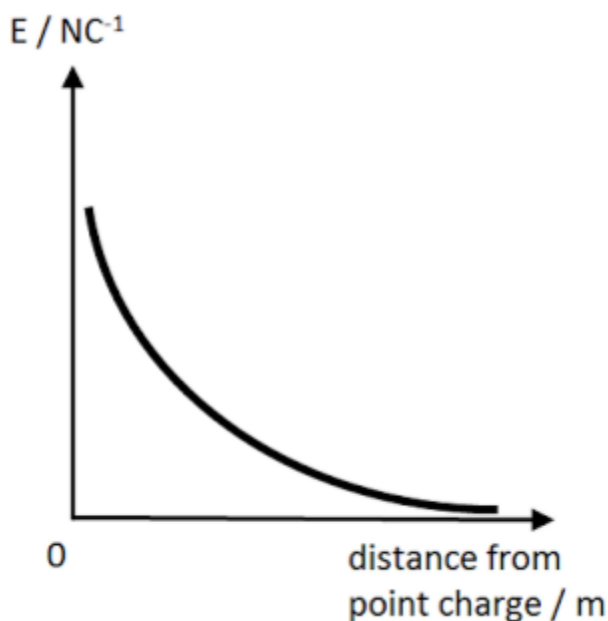
As previously defined, the electric field is the field produced by any electric charge such that when another charge is brought into the area its affects will be observed as a force.

When combining $E = \frac{F}{q}$ and $F = \frac{k q_1 q_2}{r^2}$ we get

$$E = \frac{kq}{r^2}$$

The strength of the electric field created by a source charge is therefore proportional to the size of the source charge and inversely proportional to the square of the distance.

A graph of field vs distance



The area under a field vs distance graph can be defined as the work required to move a unit charge from the point charge to a distance X. As it is energy per unit charge it is also defined as potential or voltage with units JC^{-1} or V.

When comparing electric field strengths use the relationship:

$$\frac{E_1}{E_2} = \left(\frac{r_2}{r_1} \right)^2$$

Review Question

Question 9

Calculate the electric field strength around a $25\mu\text{C}$ point charge at the following different distances:

a. 10.0 cm

b. 30.0 cm

Differences between Electric Fields Gravitational Fields As it appears in Unit 3

A field is a region of space where forces are exerted on objects with certain properties.

- **gravitational fields** affect anything that has mass
- **electric fields** affect anything that has charge

	Electric	Gravitational
Force	$F = \frac{kq_1q_2}{r^2}$	$F = \frac{GM_1m_2}{r^2}$
Field Strength	$E = \frac{F}{q}$	$g = \frac{F}{m}$
Field Strength	$E = \frac{kq}{r^2}$	$g = \frac{GM}{r^2}$
Constants	$k = 9.0 \times 10^9 \text{ Nm}^{-2}$	$G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$
Force in a radial field	Attractive or repulsive	Always attractive
Potential in radial field	JC^{-1}	Jkg^{-1}

Solutions to Review Questions

1.
 - a. $E = \frac{V}{d}$
 $20 = \frac{V}{1.2}$
 $V = 24 \text{ V}$
 - b. $W = qED = 20 \times 10^{-6} \times 20 \times 0.5 = 0.2 \text{ mJ}$
 - c. Same amount of work done only difference is the direction hence $W = -0.2 \text{ mJ}$
 - d. Perpendicular to the field therefore $W = 0 \text{ J}$
 - e. Change to a negative charge will only influence the direction of work done hence:
 answer **b.** becomes -0.2 mJ and answer **c.** becomes 0.2 mJ

2. $W = qV = 1.6 \times 10^{-19} \times 2.5 \times 10^4 = 4.0 \times 10^{-15} \text{ J}$

3. a. $E = \frac{V}{d} = \frac{2500}{0.2} = 12500 \text{ N C}^{-1}$

- b. $\Delta E_k = \frac{1}{2}mv^2 = qV$
 $\Rightarrow v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 2500}{9.11 \times 10^{-31}}} = 2.96 \times 10^7 \text{ m s}^{-1}$

4. a. $E_k = \frac{1}{2}mv^2$
 $\Rightarrow v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 6000 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}} = 4.59 \times 10^7 \text{ m s}^{-1}$

- b. 6000 V. The change in kinetic energy is the same as the accelerating voltage when the energy is in eV.

5. a. As $F = \frac{k q_1 q_2}{r^2}$ by doubling the distance AB the amount of force will decrease by a factor of 4
hence $F = 9.0 \times 10^{-6} \text{ N}$

b. $F = \frac{k q_1 q_2}{r^2}$

$$3.6 \times 10^{-5} = \frac{9.0 \times 10^9 \times q_1 q_2}{0.12^2}$$

$$q_1 q_2 = 5.76 \times 10^{-17}$$

$$F = \frac{9.0 \times 10^9 \times 5.6 \times 10^{-17}}{0.3^2} = 5.76 \times 10^{-6} \text{ N}$$

6. $F = \frac{k q_1 q_2}{r^2}$

$$F = \frac{9.0 \times 10^9 \times 5.0 \times 10^{-8} \times 1.0 \times 10^{-7}}{0.05^2} = 0.018 \text{ N}$$

7. $F = \frac{k q_1 q_2}{r^2}$

$$F = \frac{9.0 \times 10^9 \times 1.5 \times 10^{-6} \times 3.2 \times 10^{-4}}{1.5^2} = 1.92 \text{ N}$$

8. $F = \frac{k q_1 q_2}{r^2}$

$$4.2 \times 10^{-4} = \frac{9.0 \times 10^9 \times (1.1 \times 10^{-7} \times 1.1 \times 10^{-7})}{r^2}$$

$$r = 0.51 \text{ m}$$

9.

a.

$$E = \frac{kq}{r^2} = \frac{9.0 \times 10^9 \times 25 \times 10^{-6}}{0.1^2} = 2.25 \times 10^7 \text{ N C}^{-1}$$

b.

$$E = \frac{kq}{r^2} = \frac{9.0 \times 10^9 \times 25 \times 10^{-6}}{0.3^2} = 2.5 \times 10^6 \text{ N C}^{-1}$$

Physics
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Topic 7: Magnetism, fields and forces (Unit 3)

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What you should know

As it appears in Unit 3

According to the study design you need to know the following:

- Describe magnetism using the field model
- Investigate and apply theoretically and practically a vector field model to magnetic phenomena, including shapes and directions of fields produced by bar magnets, and by current-carrying wires, loops and solenoids
- Analyse the use of a magnetic field to change the path of a charged particle, including:
 - the magnitude and direction of the force applied to an electron beam by a magnetic field: $F = qvB$, in cases where the directions of v and B are perpendicular or parallel
 - the radius of the path followed by a low-velocity electron in a magnetic field: $qvB = \frac{mv^2}{r}$
- Describe the interaction of two fields; allowing that electric charges, magnetic poles and current carrying conductors can either attract or repel, whereas masses only attract each other
- Investigate and analyse theoretically and practically the force on a current carrying conductor due to an external magnetic field, $F = nIB$, where the directions of I and B are either perpendicular or parallel to each other

Magnetic materials

As it appears in Unit 3

All materials exhibit magnetic properties, however, most effects are quite weak. Iron, nickel and cobalt interact much more strongly and are classed as **ferromagnetic** materials. This means that they have the ability to become magnetised.

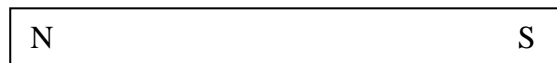
Substances that are easily magnetised and lose their magnetic properties easily are referred to as soft magnets.

Properties of magnets

As it appears in Unit 3

All magnets have the following properties:

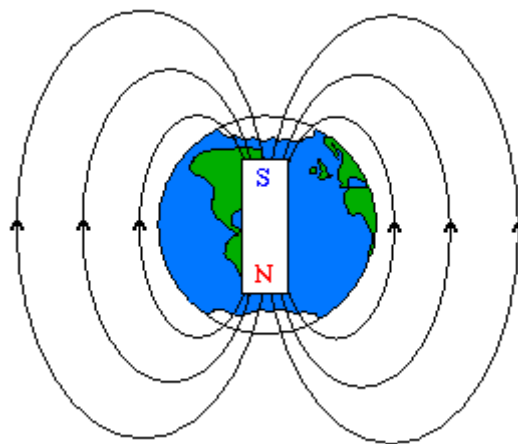
- They attract or repel each other.
- They attract certain metals.
- They align with the poles of the Earth. The North-seeking pole, commonly called the North pole, on a magnet will point towards the Earth's North Pole.
- They point in the direction of a magnetic field.
- They are all dipolar which means that they all have a north and a south pole as shown in the diagram below.



- Like poles repel each other and opposite poles attract.

Why do north poles in compasses point towards the North Pole of the Earth?

- Confusion has arisen because the word 'seeking' is usually removed when referring to the poles of a magnet.
- The Earth itself is acting as a huge magnet. Its Geographic North Pole actually is very close to its Magnetic South Pole and vice versa.
- The north-seeking pole or North pole of a compass points to the "top" of the earth so this is the direction a small North magnet would point towards.

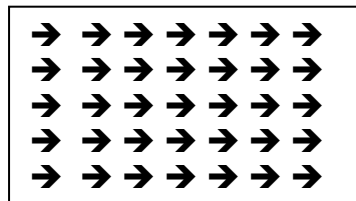


Why are magnets magnetic?

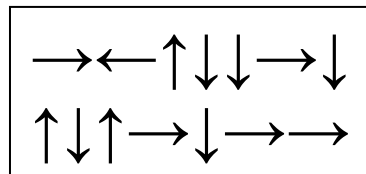
As it appears in Unit 3

Electrons have a magnetic field caused by their spin. Groups of electrons are called domains. When these domains are aligned within the material, a magnetic effect is achieved because their fields are aligned.

In the diagrams below the arrows represent the domains.



In materials that aren't magnetic a random arrangement of domains means that the material does not exhibit magnetic properties because the random direction of the domains causes their fields to cancel out.



Magnetic fields

As it appears in Unit 3

A magnetic field is the property of the space around a magnet that causes an object in that space to experience a force due to the presence of the magnet.

Magnetic fields are produced as the result of current.

The strength and direction of a field depends on where an object is placed: field strength decreases as distance increases.

Magnetic field lines

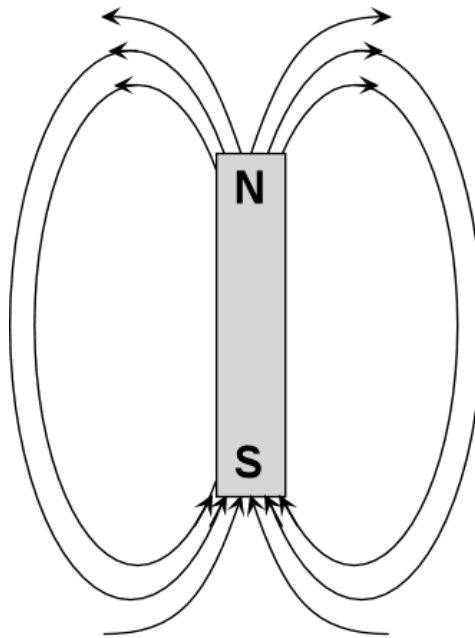
As it appears in Unit 3

Field lines show the direction of the force at any point on an imaginary single north pole that is placed near a magnet.

The strength of the magnetic field is always greatest at the ends of the magnet. This is shown by a greater number of field lines in a small area.

The diagram below shows the magnetic field of a single bar magnet. The arrows show the direction of the magnetic field on an imaginary north pole.

Note: Strictly speaking, magnetic field lines are continuous and pass through the magnet. They have not been drawn through the magnet in the diagram for simplicity



If an object acting as an imaginary north pole is placed around the bar magnet shown above it will experience:

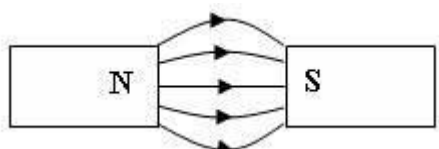
- A downward force when placed directly left or right of the magnet
- An upward force when placed directly above or below the magnet

Magnetic field lines for bar magnets

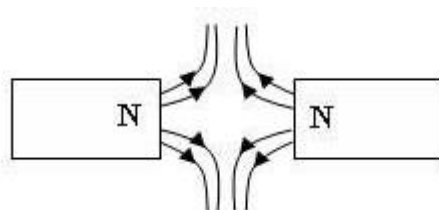
As it appears in Unit 3

There are some rules that are used when drawing field lines:

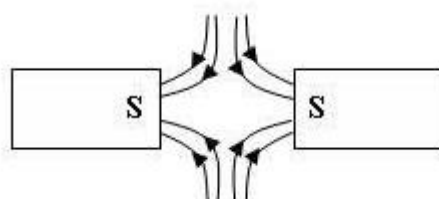
- Field lines are always drawn from North to South; they are a loop which leaves the north end and enters the south end passing through the magnet, returning to the north end.
- The lines never intersect or cross each other.
- Lines are drawn close together if the field is strong, further apart if weak.
- The direction of the field at a point is along the tangent to the field line.



This diagram represents the interactions between 2 bar magnets of opposite polarity.



This diagram represents the interactions between 2 bar magnets where the north poles are facing each other. The field lines are leaving both ends and you can see the repulsion between the ends.



This diagram represents the interactions between 2 bar magnets where the south poles are facing each other. The field lines are entering both ends and you can see the repulsion between the ends.

One mistake made by some students is that they only draw in one field line. It is always best to put in at least 3 lines, although 5 would be preferable. This is because one line doesn't indicate a good level of knowledge regarding the shape of the magnetic field. If you were asked to draw the shape of the magnetic field between the 2 magnets then diagram 1 is clearly a superior answer compared to diagram 2.

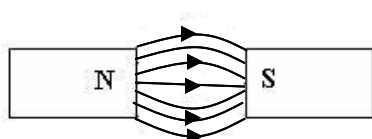


Diagram 1

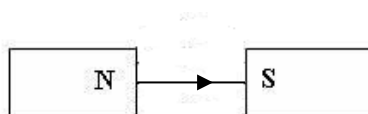
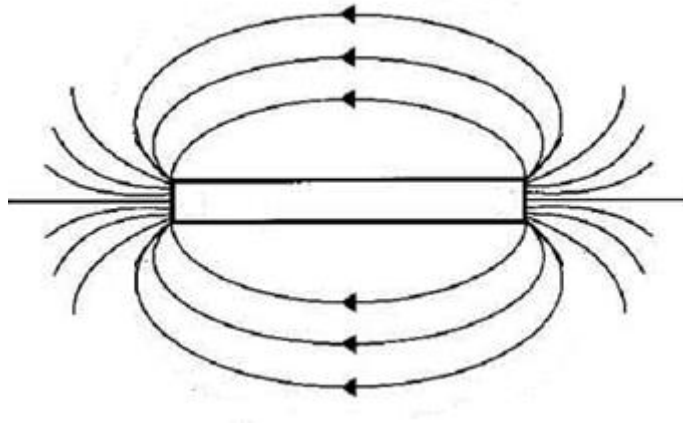


Diagram 2

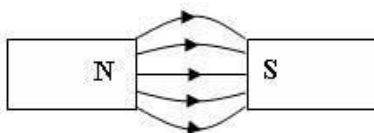
Another common error is to show what appears to be a start and finish line when in fact they are a continuous loop.

Review Questions

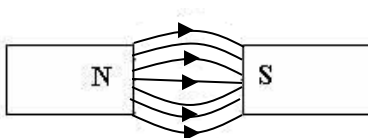
1. The diagram below shows a bar magnet and surrounding magnetic field. Identify the north and south ends of the magnet. Explain how you can tell which end is north and which is south.



2. Compare the field strengths of the 2 sets of magnets shown below. Which diagram shows the stronger magnetic field? How can you tell?



Magnet set 1

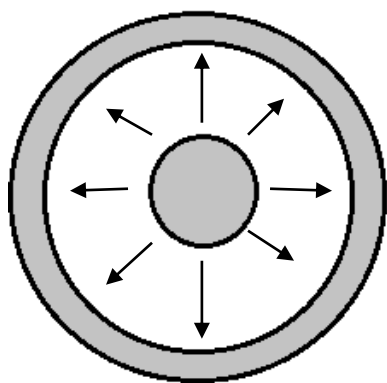


Magnet set 2

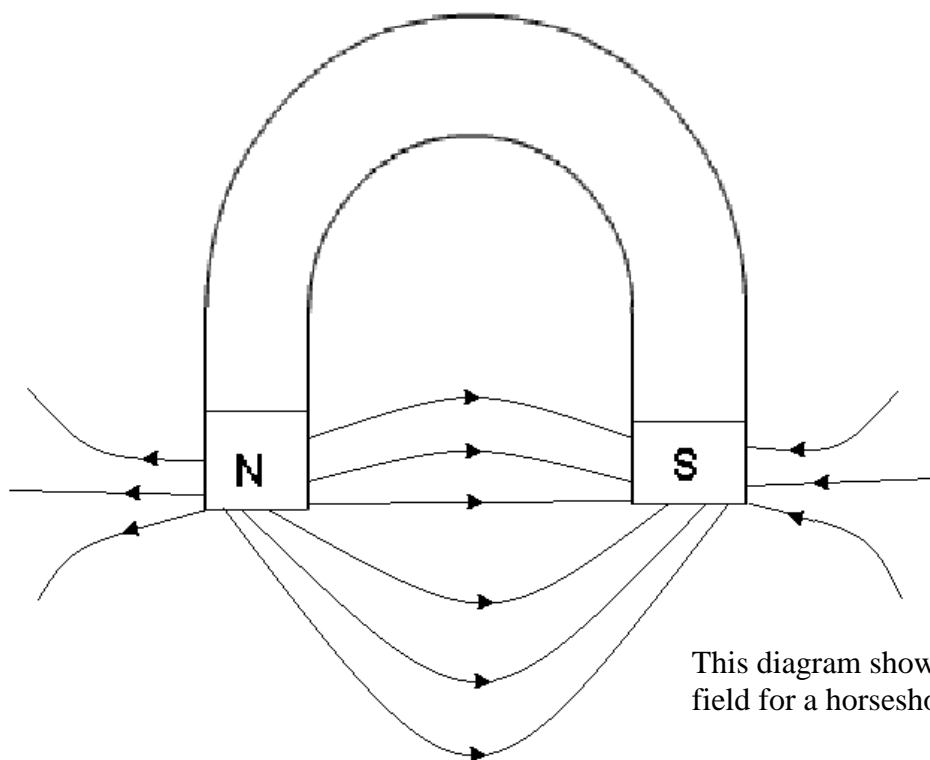
Magnetic field lines for other shaped magnets

As it appears in Unit 3

The same rule applies to magnets that are not bar shaped but there are some slight differences. You need to be familiar with the field shapes for horseshoe magnets and circular magnets.



This is a circular magnet. The central section is north and the outer ring is south.

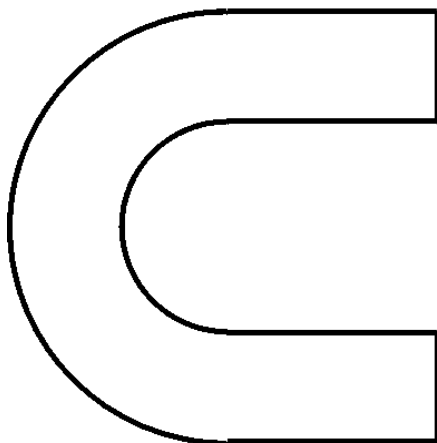


This diagram shows the magnetic field for a horseshoe magnet.

The field lines are still drawn leaving the north pole and entering the south pole.

Review Question

3. Mark in the north and south poles for this magnet and then draw in appropriate field lines.



Magnetic field strength

As it appears in Unit 3

The unit of magnetic field strength (also called magnetic flux density) is the tesla, T, or sometimes, Wb m^{-2} .

The following are some typical magnetic field strengths:

Surface of a neutron star	10^6 T
Strongest magnet produced in a lab	45 T
Strong permanent magnets	10 T
Laboratory bar magnets	0.01 to 1 T
Fridge magnets	$5 \times 10^{-3} \text{ T}$
Earth's Magnetic Field	$5 \times 10^{-5} \text{ T}$

The right hand grip rule

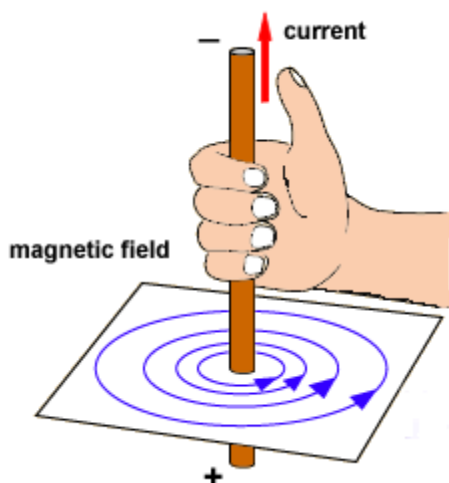
As it appears in Unit 3

A conductor such as a wire which carries an electric current is always surrounded by a magnetic field.

Electromagnetism is a temporary effect caused by the flow of electric current and it disappears when the current flow is stopped.

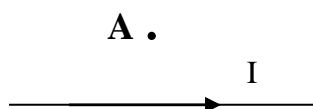
The magnetic field lines due to the current in a straight wire are concentric circles with the wire at the centre. The direction of the magnetic field can be found using the right-hand grip rule.

Wire is gripped with the right hand so that the thumb lines up with the direction of current flow (it is important to remember that if the direction of current flow is not shown, conventional current flows from the positive terminal to the negative terminal). The direction of the magnetic field is given by the curl of the fingers.



Sample question

Identify the direction of the magnetic field at point A.



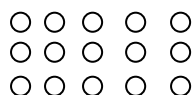
Since the current is flowing from left to right the thumb would point in this direction so the fingers would have to curl up behind and over the wire. Therefore at point A the magnetic field would be coming out of the page.

Drawing 3D direction

As it appears in Unit 3

- Remember: the direction of the current is the direction of positively charged particles. When the electrons are moving in one direction the conventional current is in the opposite direction.
- To represent three dimensional situations on a two dimensional page, use the following convention.

Imagine that an arrow is being used to represent the direction.



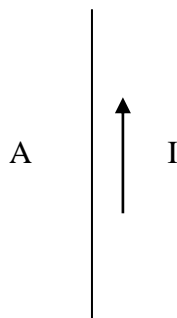
Circles represent the tip of the arrow coming towards you (representing the pointy side of an arrow).



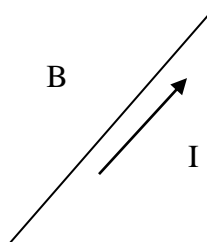
Crosses represent the tail of the arrow going away from you (representing the back side of an arrow).

Review Questions

4. Use the right hand right rule to identify the direction of the magnetic field at point A.



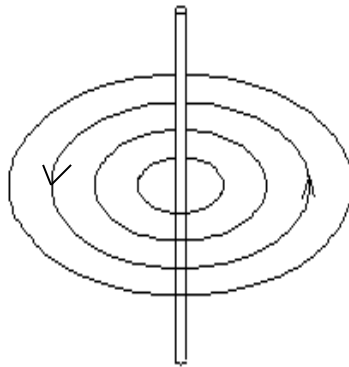
5. Use the right hand right rule to identify the direction of the magnetic field at point B.



6. Use the right hand rule to identify the direction of the magnetic field at point C.

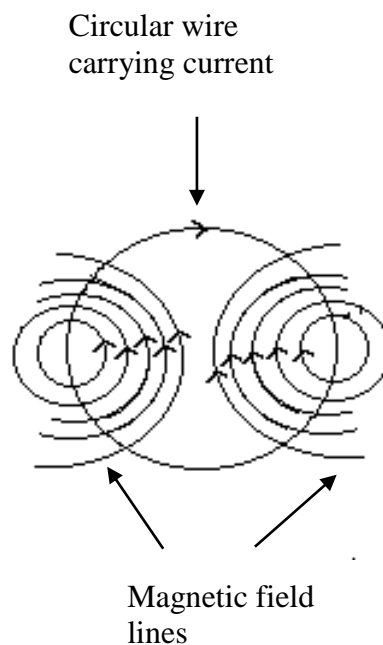
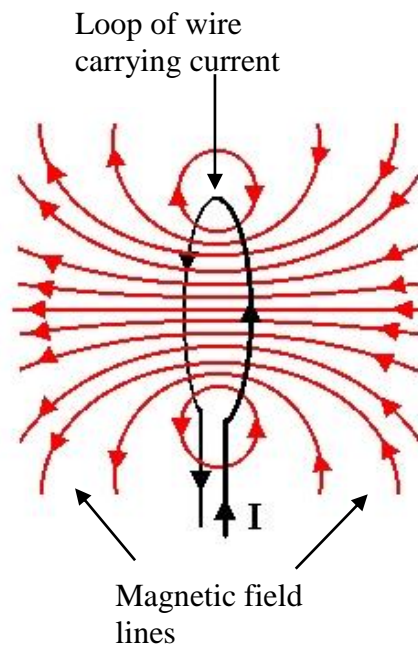


-
7. The diagram below shows a current carrying wire and the direction of the magnetic field that surrounds it.



Identify the direction of the current.

The same rules for fields around straight wires apply to more complex shapes such as the circular shape, the loop and the solenoid shown below. It is important to remember that the direction of the magnetic field changes as the direction of the current changes.

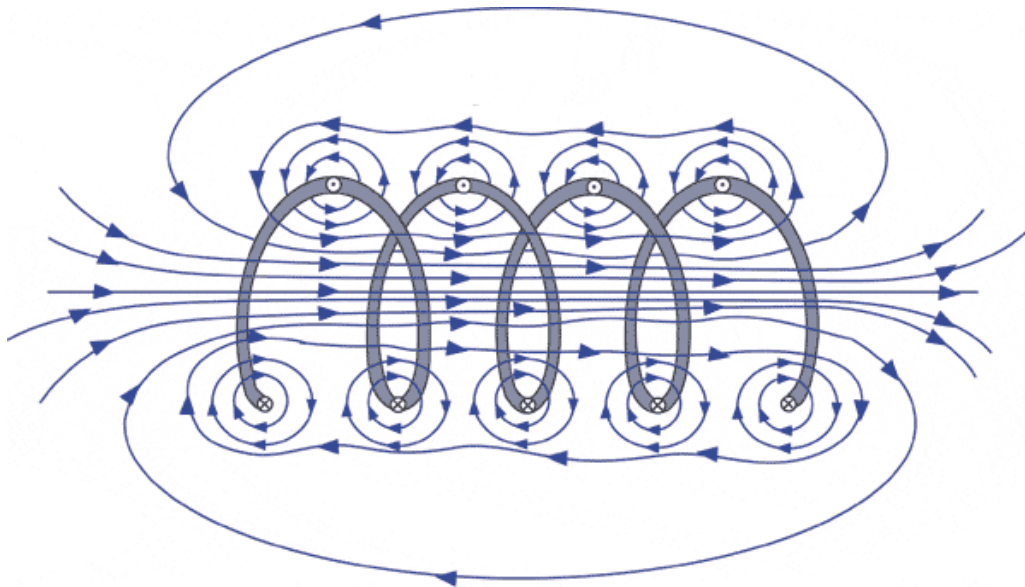


The most complex shape you need to be aware of is the solenoid. A solenoid consists of a long piece of wire usually wrapped around a cylindrical soft iron core.

Solenoids have a stronger field because the magnetic fields from the different loops add together.

The diagram below shows the direction of the magnetic field at different points on the solenoid.

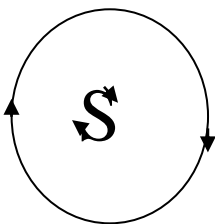
It is important to understand that the cylindrical section of the solenoid has a nearly uniform magnetic field similar in shape to that of a bar magnet.



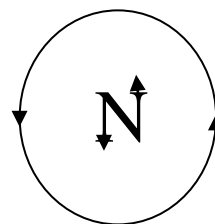
There are a couple of ways to help you to establish the shape of the magnetic field of the solenoid.

Firstly you can modify the right hand rule so that the fingers point in the direction of the current and the thumb shows the direction of the field.

Secondly you can think of the solenoid as acting like a bar magnet.



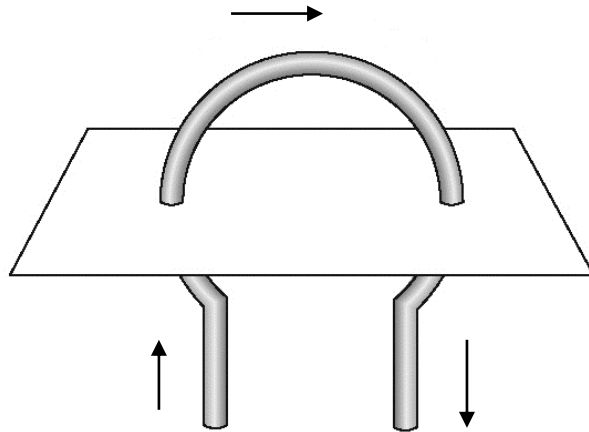
In the diagram above you can see the field lines entering the left hand side. This means that it is acting as the south pole. If you draw an S in a circle then the arrows at the 2 ends of the letter indicate the current direction.



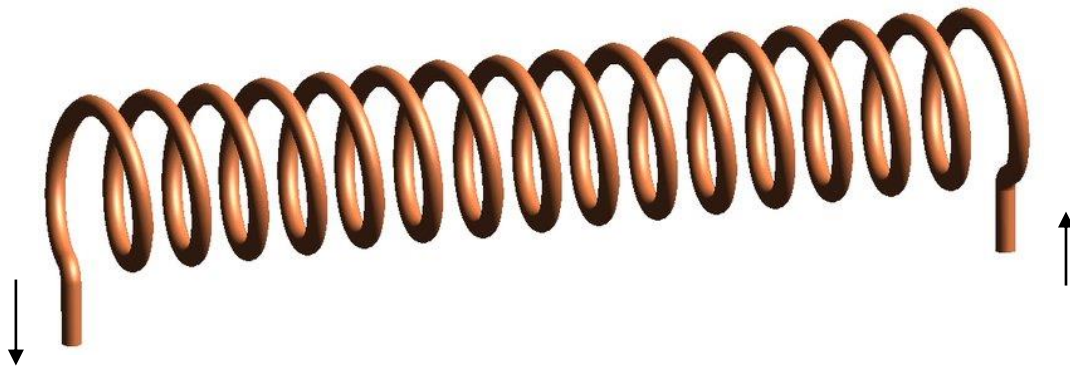
In the diagram above you can see the field lines leave from the right hand side. This means that it is acting as the north. If you draw an N in a circle then the arrows at the 2 ends of the letter indicate the current direction.

Review Questions

8. Draw in the magnetic field lines for the coil shown below. The arrows indicate the direction of current flow.



9. Draw in the magnetic field lines for the coil shown below. The arrows indicate the direction of current flow.

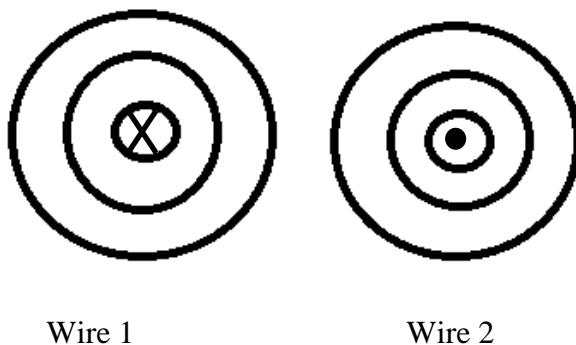


Forces between parallel wires

As it appears in Unit 3

When answering questions about the forces between parallel wires it is essential to consider the direction of the current in both wires and the direction of the forces in order to work out whether the wires are going to be attracted to or repelled from each other.

This diagram shows 2 wires from the end on.



What is this diagram telling us about the direction of the current?

For wire 1 the direction of the current is into the page and for wire 2 the direction of the current is out of the page.

What is the field direction surrounding each wire?

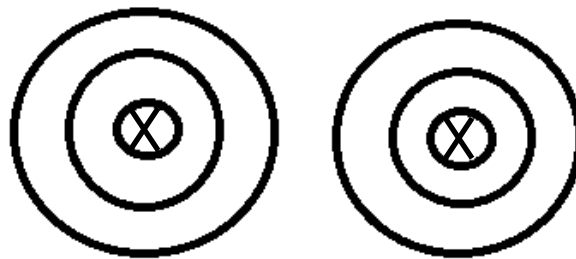
For wire 1 the field direction is clockwise and for wire 2 the field direction is anticlockwise.

Will the wires be attracted to each other or repelled from each other?

The field direction for both wires is the same in the region between the wires, and since like repels like the two wires will be repelled from each other.

Review Questions

10. Explain whether wires 1 and 2 shown below will be attracted to each other or repelled from each other.



Wire 1

Wire 2

11. Draw the magnetic field for the wires in question 10.

Factors which affect magnetic force on a current carrying wire

As it appears in Unit 3

Factors which affect the magnetic force on a current carrying wire include:

- The current (I): increasing the current increases the force.
- The length of the current carrying wire (L): increasing the length increases the force.
- The strength of the magnetic field (B): increasing the strength of the magnetic field increases the force.
- The number of wires involved (n): increasing the number of wires increases the force.

Calculating magnetic force

As it appears in Unit 3

If the field is perpendicular to the flow of current, the force on a current-carrying wire in a magnetic field is proportional to the current, the length of wire in the field, and the strength of the field.

Force is calculated by using the equation:

$$\mathbf{F = nBIL}$$

F = force (N)

n = no of wires (or loops)

B = field strength (T)

I = current (A)

L = length of wire (or length of each loop in case of a solenoid) - (m)

It is important to remember this applies to objects of various shapes, not just straight pieces of wire.

Sample question

A wire of length 20.0 cm carries a current of 1.5 A. If the wire is in a magnetic field with a strength of 0.4 T, calculate the magnitude of the force acting on the wire.

$$n = 1$$

$$B = 0.4 \text{ T}$$

$$I = 1.5 \text{ A}$$

$$L = 0.2 \text{ m}$$

$$F = nBIL$$

$$F = 1 \times 0.4 \times 1.5 \times 0.2$$

$$F = 0.12 \text{ N}$$

Review Questions

- 12.** A 80 cm wire is in a magnetic field of 0.6 T, find the current flowing through the wire if the magnetic force is 2 N?
- 13.** The force on a 35 cm wire is 0.28 N. If the magnitude of current flowing through the wire is 1.2 A, determine the magnitude of magnetic field affect in the wire.
- 14.** A 20 cm lead containing 20 lengths of wire each carrying a current of 0.2 A was placed in a magnetic field of 1.5 T. Calculate the force on the lead.
- 15.** Determine what would happen to the magnitude of the force if the lead was replaced with another consisting of 50 lengths of wire with the same dimensions.

If the field is not at right angles to the wire, then the field is not at right angles to the current, (as the current is restricted to move inside the wire). The force is then given by:

$$F = nBIL\sin\theta$$

Where θ is the angle between the force and the direction of the current.

Note:

When $\theta = 90^\circ$,

$\sin\theta = 1$

So $F = nBIL$

When $\theta = 0^\circ$,

then $\sin\theta = 0$,

So $F = 0$

This means the force is at a maximum when the current is at a right angle to the field and at a minimum when parallel to the field.

Review Questions

16. A wire with a length of 100 cm and carrying a current of 0.1 A is placed into a magnetic field of strength 0.05 T. The wire was originally at an angle of 60° to the magnetic field and was then moved so that it was at an angle of 0° . Calculate the difference in the amount of force on the wire.

17. Briefly explain if the results were as expected.

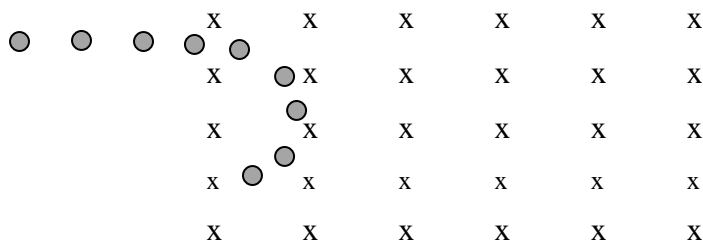
Electrons in a magnetic field

As it appears in Unit 3

It is useful to be aware of the connection between the force on a current carrying wire and the force on an individual electron. This is because the force on a wire is due to the collective forces acting on the individual electrons.

Since electric current consists of moving electrons, a magnetic field will act on the electrons and push them sideways resulting in deflection.

The magnitude of the deflecting force is constant and is always at right angles to the velocity which results in the electron travelling in a circular motion as shown in the diagram below.



The force on the moving electron is calculated using the equation:

$$F = Bqv\sin\theta$$

Where:

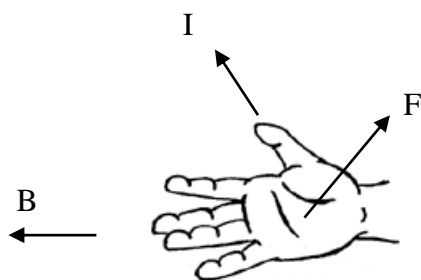
B = strength of the field (T)

q = amount of charge (C)

v = velocity of the charge (m s^{-1})

θ = the angle between the field and charge motion

The force will equal zero when v and B are parallel and the force is at a maximum when v and B are perpendicular.



The direction of the electromagnetic force can be found using the **RIGHT HAND SLAP RULE**
Thumb points in the direction of the current.
Fingers point in the direction of the magnetic field.
Palm faces in the direction of the force acting on a positive charge.
P.S. If the charge is negative, then the force will be going towards your palm (inwards)

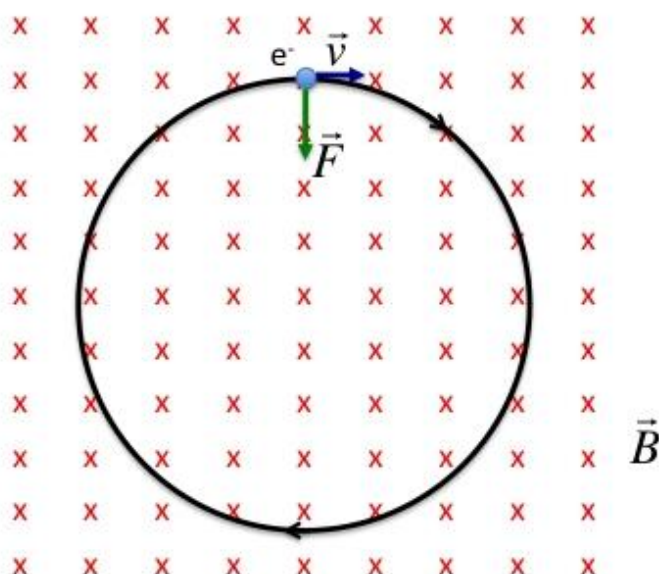
Simplifying this; when the electron velocity is perpendicular to the field it has found that the magnitude of the force (F) on a charge (q) moving with velocity (v) into a magnetic field of strength (B) is given by:

$$F = qvB$$

In the topic of circular motion, the formula for centripetal force has also been introduced.

$$F = \frac{mv^2}{r}$$

Where F = the centripetal force (N)
 m = the mass of the particle (kg)
 v = the velocity of the particle (m s^{-1})
 r = the radius of the circular pathway (m)



Combining these two expressions, an expression for the radius of the path of an electron travelling at right angle to a constant magnetic field is given by:

$$r = \frac{mv}{eB} \quad \text{or} \quad r = \frac{p}{eB}$$

Where p = the momentum of the electron (kg m s^{-1})
 m = the mass of the electron (kg)
 v = the velocity of the electron (m s^{-1})
 r = the radius of the circular pathway (m)
 B = the magnetic field strength (T)
 e = the charge on one electron

Review Questions

18. Explain how firing an electron into a magnetic field can be used to determine the direction of the field.

19. A proton travelling downwards towards the ground enters a magnetic field that is pointing North , explain what will happen to the proton's motion.

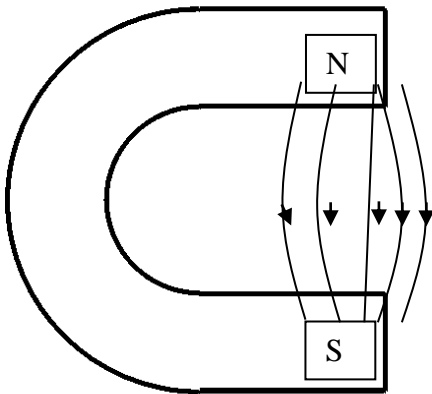
20. In an experiment, a group of physics students found that electrons emerged from an electron gun with energy of 8.7 keV. These electrons then travel through a uniform magnetic field perpendicular to their motion. Given that the strength of the magnetic field is 0.5 T.

- a. Work out the force exerted on the electrons by the magnetic field.

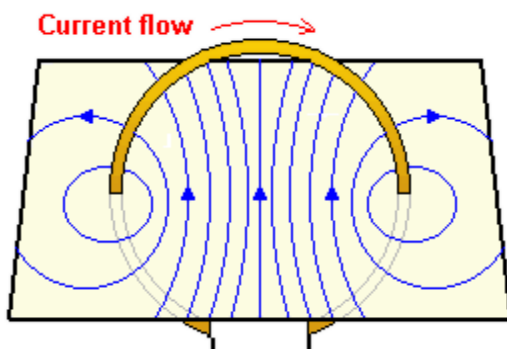
- b. Calculate the radius of the path of the electrons.

Solutions to Review Questions

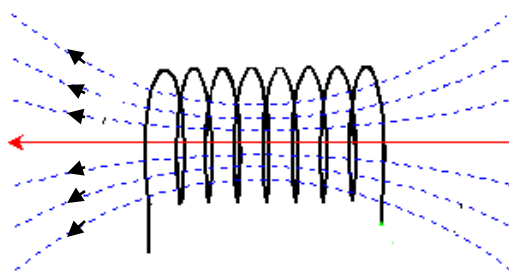
1. The South end is on the left side of the page and the North end is on the right side of the page. Field lines are always drawn from North to South
2. Magnet set 2 shows the stronger field because there are more field lines per unit of area.
3. The diagram should look like this:



4. Out of the page.
5. Out of the page.
6. To the right.
7. Up the wire.
8. The diagram should look like this:

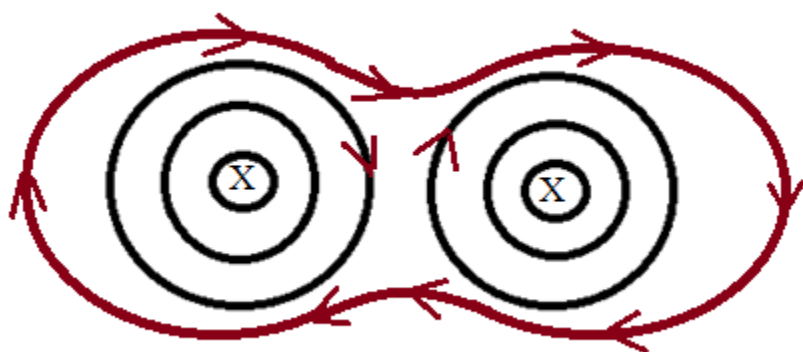


9. The diagram should look like this:



10. Since the current is in the same direction, in the region between the wires the forces will be going in opposite directions from each other and therefore there will be an attractive force between the two wires

11.



12. $L = 0.8 \text{ m}$
 $B = 0.6 \text{ T}$
 $F = 2 \text{ N}$
 $n = 1$

$$I = \frac{F}{nBL} = \frac{2}{0.8 \times 0.6} = 4.17 \text{ A}$$

13. $L = 0.35 \text{ m}$
 $I = 1.2 \text{ A}$
 $F = 0.28 \text{ N}$

$$B = \frac{F}{nIL} = \frac{0.28}{1.2 \times 0.35} = 0.67 \text{ T}$$

14. $L = 20 \text{ cm}$
 $I = 0.2 \text{ A}$
 $B = 1.5 \text{ T}$
 $n = 20$

$$F = nBIL$$

$$F = 20 \times 1.5 \times 0.2 \times 0.20$$

$$F = 1.2 \text{ N}$$

15. $L = 20 \text{ m}$

$$I = 0.2 \text{ A}$$

$$B = 1.5 \text{ T}$$

$$n = 50$$

$$F = nBIL$$

$$F = 50 \times 1.5 \times 0.2 \times 0.20$$

$$F = 3.0 \text{ N}$$

16. For 60°

$$F = nBIL \sin \theta$$

$$n = 1$$

$$B = 0.05 \text{ T}$$

$$I = 0.1 \text{ A}$$

$$L = 1 \text{ m}$$

$$F = nBIL \sin \theta$$

$$= 1 \times 0.05 \times 0.1 \times 1 \sin 60^\circ = 4.3 \times 10^{-3} \text{ N}$$

For 0°

$$F = nBIL \sin \theta$$

$$F = 1 \times 0.05 \times 0.1 \times 1 \times 0$$

$$F = 0 \text{ N}$$

The difference is $4.3 \times 10^{-3} \text{ N}$

17. Yes the results were as expected $\sin 0^\circ = 0$ and therefore the force is zero when the charge is parallel to the field. As the angle increases so does the magnitude of the force.

18. The direction of the deflection can be used to determine the direction of the magnetic field. The direction of conventional current is opposite to the initial direction of the electron and the right hand rule can then be used to determine field direction. The only circumstances where this would not work are if the magnetic field is either parallel or antiparallel to the direction that the electron is travelling.

19. The proton would travel a circular path, initially in an Easterly direction.

20. a. $E_k = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 8700 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}} = 5.53 \times 10^7 \text{ m s}^{-1}$$

$$F = qvB = 1.6 \times 10^{-19} \times 5.53 \times 10^7 \times 0.5 = 4.42 \times 10^{-12} \text{ N}$$

b. $r = \frac{mv}{eB} = \frac{9.11 \times 10^{-31} \times 5.53 \times 10^7}{1.6 \times 10^{-19} \times 0.5} = 6.3 \times 10^{-4} \text{ m}$

Physics

Teach Yourself Series

Topic 8: DC Motors (Unit 3)

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What you should know

As it appears in Unit 3

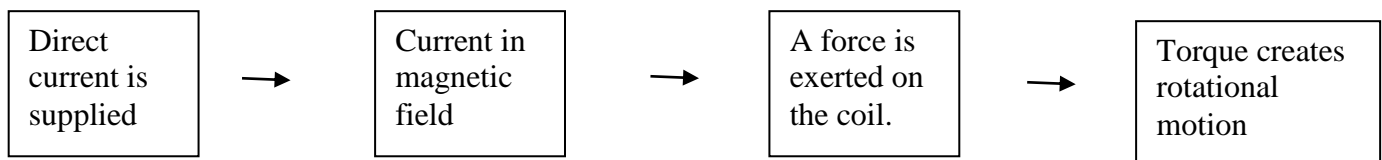
- Investigate and analyse theoretically and practically the operation of simple DC motors consisting of one coil, containing a number of loops of wire, which is free to rotate about an axis in a uniform magnetic field and including the use of a split ring commutator

DC motors

As it appears in Unit 3

A DC motor is a simple motor that uses a magnetic field and electricity in order to create torque which turns a coil.

A DC motor relies on the fact that there is a force produced on a current carrying conductor in a magnetic field, and that this force gives rise to a rotational motion which then turns the axle of the motor.



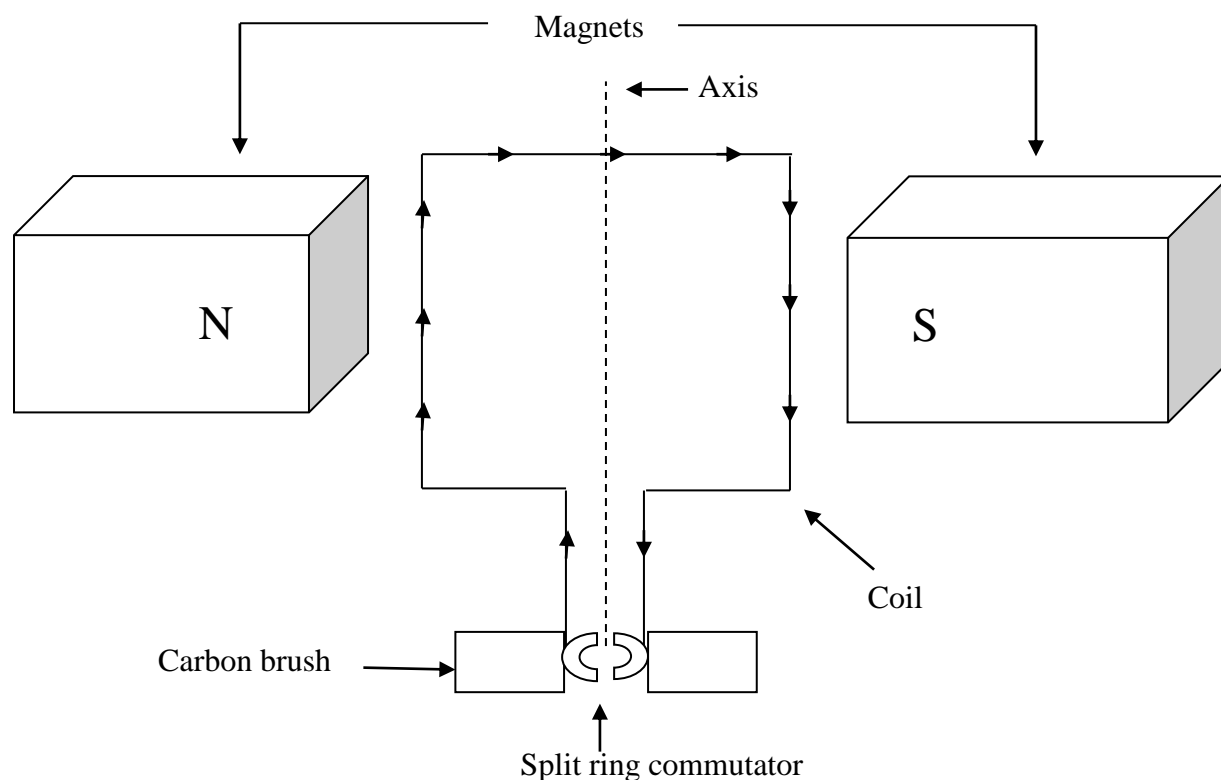
Components of a DC motor

As it appears in Unit 3

The components of a simple DC motor include the following:

COMPONENT	PURPOSE
Battery or power supply	Source of current
Commutator	Reverses the direction of current every 180° (every half turn)
Coil	Carries current through magnetic field
Brushes	Allow current to flow smoothly to the coil
2 magnets of opposite polarity	Provides a magnetic field that interacts with the current producing a force
Axle	Allows the coil to spin

The diagram below shows the components of a DC motor. The arrows on the coil indicate the direction of current flow.



In order to make the coil spin around the axis, it has to be connected to a battery or power supply.

The ends of the coil are connected to the commutator, which is in contact with the brushes, which in turn are connected to the positive and negative leads of the power source.

The brushes allow the smooth transfer of electricity to the coil. This creates a magnetic field around the coil which interacts with the magnetic field of the permanent magnets causing the coil to spin.

Review Question

1. Some students set up a DC motor. They slowly remove the magnets and observe that the coil eventually comes to rest. As a result smoke is produced at the connection between the coil and the commutator. Explain any physics principles and reasons behind these observations.

Force on current carrying wires

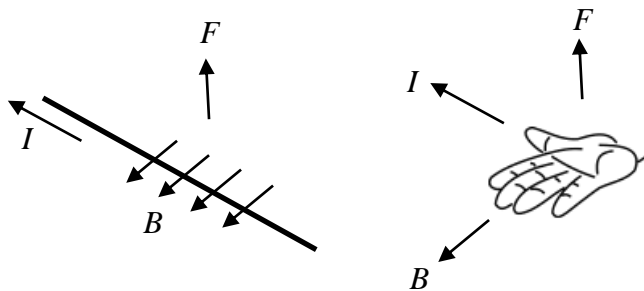
As it appears in Unit 3

There is a magnetic field associated with a current carrying conductor. The direction of the field is given by the right hand grip rule.

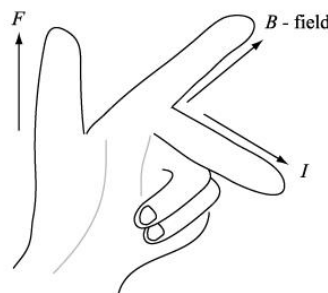
As a result there will be a force on the current carrying conductor when it is in a magnetic field. The direction of the force is determined by the right hand palm rule or Fleming's left-hand rule.

In the right hand palm rule, the hand is opened flat and the fingers are aligned with the magnetic field (B).

The thumb is pointed in the direction of current flow (I) and the palm is now facing the direction of the force (F).



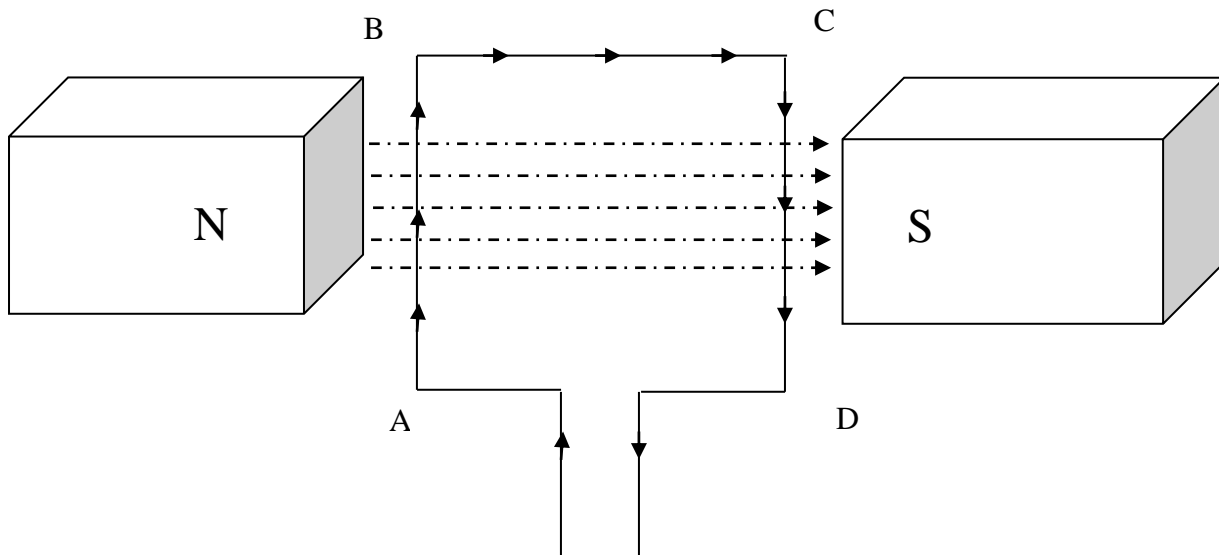
In Fleming's left hand rule, the Fore finger represents the direction of the magnetic Field. The Centre finger represents the direction of the Current. The Thumb represents the direction of the force or Thrust on the conductor.



The current on one side of a coil always flows in the opposite direction to the other side of the coil.

As a result, the forces on the two sides of the coil (AB and CD) will act in opposite directions (either up or down). These opposing forces enable the coil to rotate.

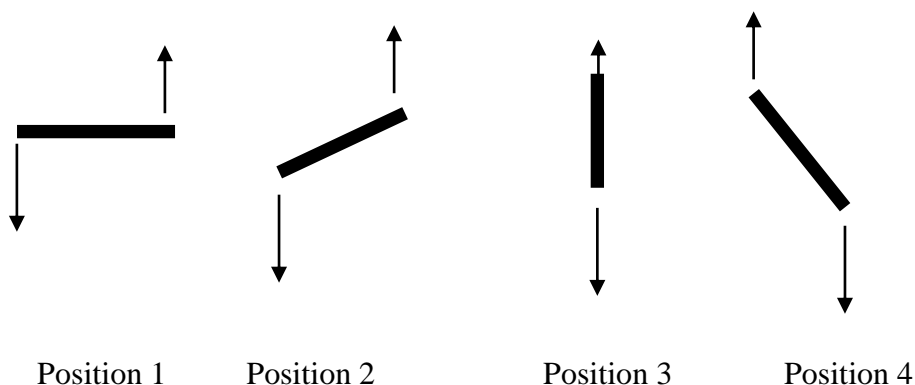
It is important to remember that the turning force is constant at all points in the cycle because the magnetic field and the current are the same size and direction.



In the position shown above, there will be no force acting on the far side of the coil (BC) because the current flowing through the coil is parallel to the direction of the magnetic field.

If the current flows in the direction shown in the diagram above then the coil will rotate in a clockwise direction when viewed from the top of the page. If the current flows in the opposite direction then the coil will rotate in anti-clockwise direction.

In the diagrams below, the bar represents the end on view of a coil and the arrows represent the direction of the force on the wire. The components are arranged in such a way that the coil spins in an anticlockwise direction.



The coil starts off in position 1 and as can be seen the forces make the coil rotate in an anticlockwise direction.

As the coil rotates the amount of torque decreases so the turning effect for position 2 is less than that of position 1.

When the coil reaches position 3, the forces pass through the coil so there will be no turning effect, however, the coil has momentum which enables it to head towards position 4.

The problem is that the current is still travelling in the same direction, and so, the forces would act to bring the coil back to position 3 meaning it would only travel through 90°.

This means that modifications need to be made to enable the motor to travel continuously.

Review Question

2. Explain why the coil would not move if it was in position 3 when the battery was connected to the motor.

The split ring commutator

As it appears in Unit 3

To get a motor to spin continuously, the direction of the forces needs to be changed every half cycle. This could be done by either:

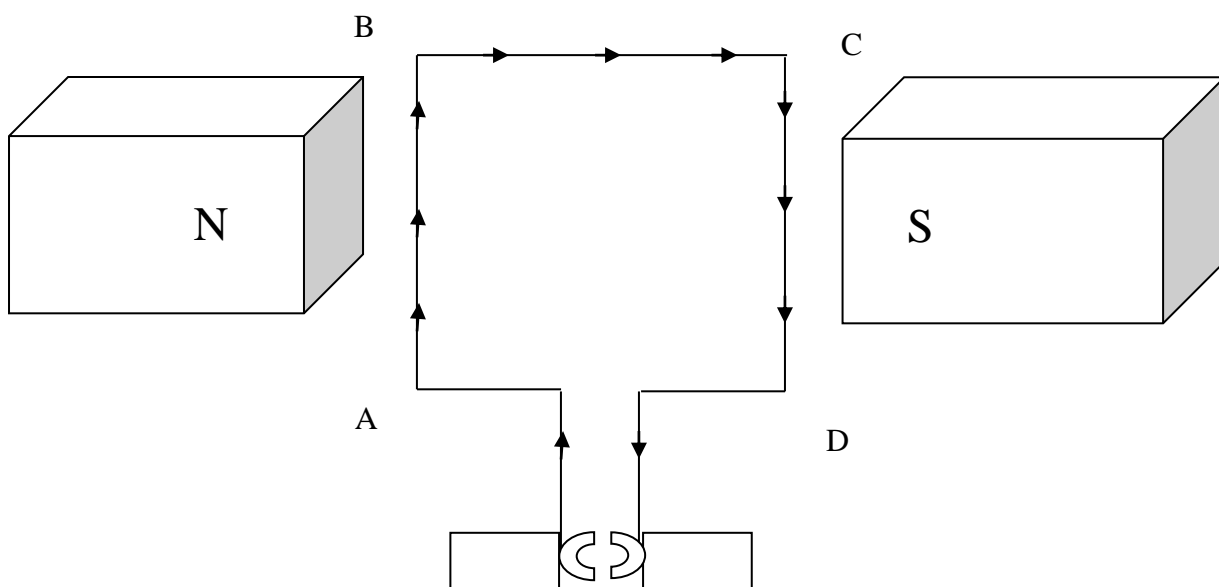
- reversing the direction of the current, or
- reversing the direction of the magnetic field

It is easier to change the direction of the current and this is done using a split-ring commutator; a device consisting of 2 semicircular pieces of metal attached to the axle with an insulating space between them.

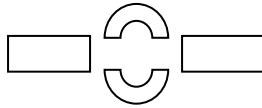
These spaces cut the current to the coil twice every complete cycle.

To see how the commutator works, consider the diagram below.

When the coil is in this position, the D edge of the coil is connected to the right brush and the A edge of the coil is connected to the left brush enabling current to flow from D to A which causes the forces shown previously. This results in the coil turning in the clockwise direction (as seen from the top of the page).



After a quarter of a turn, the non-conductive section of the split ring commutator is aligned with the brushes as shown in the diagram below (looking along the axis of the coil).



When this alignment occurs, there is no current flowing through the coil and, therefore, there is no force acting on the coil. Due to its momentum, the coil continues to move past this position.

After passing through the position shown above, D and A make contact with the left and the right brushes respectively, enabling current to flow from A to D. Since the direction of the current is reversed, the forces on AB and CD are also reversed. This enables the coil to continue to rotate in the clockwise direction.

Review Question

3. Explain the importance of the split ring commutator.

Calculating force

As it appears in Unit 3

If the field is perpendicular to the flow of current, the force on a current-carrying wire in a magnetic field is proportional to the current, the length of wire in the field, and the strength of the field.

Force is calculated by using the equation

$$F = nBIL$$

Where:

F = force (N)

n = no of loops

B = field strength (T)

I = current (A)

L = length of wire (m)

The current is restricted to move inside the wire, hence, if the field is not at right angles to the wire, the field is not at right angles to the current. If the current is at an angle of θ to the field, then the force is given by

$$F = nBIL\sin\theta$$

Note:

When $\theta = 90^\circ$

$\sin\theta = 1$

$$\therefore F = nBIL$$

When $\theta = 0^\circ$

Then $\sin\theta = 0$

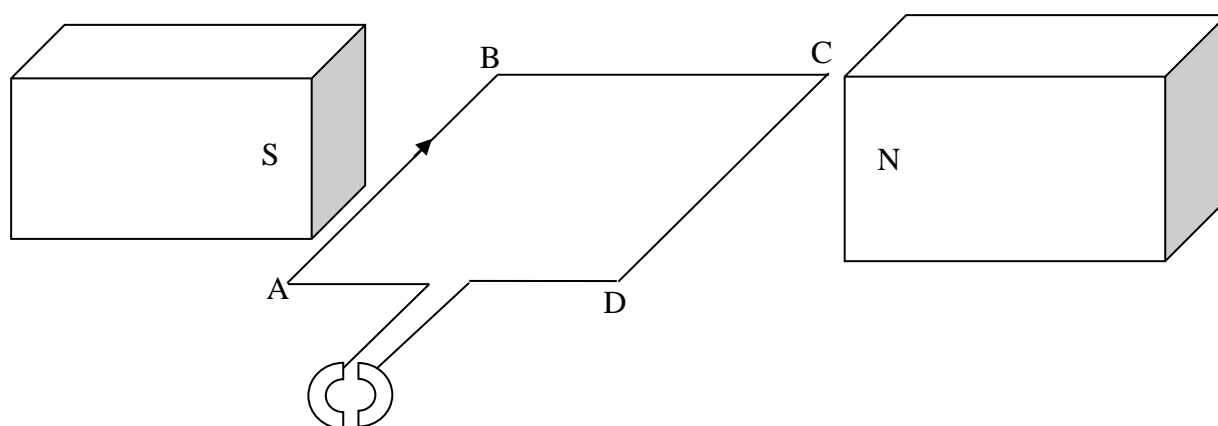
$$\therefore F = 0$$

So force is at a maximum when the current is at a right angle to the field and at a minimum (that is, zero) when parallel to the field.

Review Questions

4. The side of a coil is 10 cm long and a magnetic force of 0.6 N is acting upon it. If the strength of the magnetic field is 1.5 T, determine the magnitude of the current in the coil.

Use the diagram below to answer the following questions



5. Identify the direction of force on the side AB of the coil and explain how the direction of force can be determined.

6. The coil ABCD has 8 turns and sides that are 0.09 m long. It is surrounded by two magnets which supply a field of 0.50 T. If a current of 5 A is supplied, calculate the magnitude of the force acting on side AB.

7. Determine the magnitude of the force acting on side BC.

Torque

As it appears in Unit 3

Torque is the turning effect of a force. It is important to remember there is a difference between the force and the turning effect caused by the force.

The torque depends on the distance between the axis of rotation and the force and therefore unlike the force, torque is not constant.

The formula for calculating torque is:

$$\tau = Fd$$

Where:

τ = torque (N m)

F = force (N)

d = distance of the line of action of the force from the axis (m)

Torque and motors

As it appears in Unit 3

The maximum torque on a motor is calculated using the formula:

$$\tau = nBIA$$

Where:

n = the number of turns in the coil

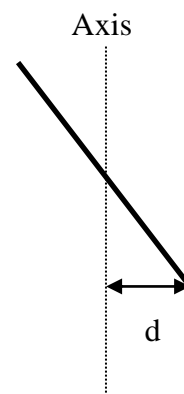
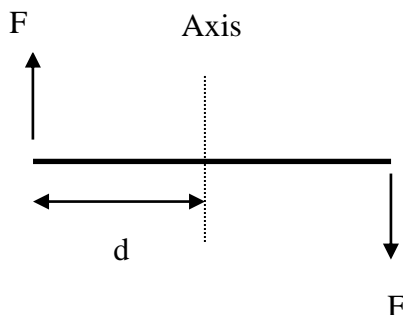
B = the magnetic field (T)

I = the current (A)

A = area of the coil (m²)

The torque on the coil when it is not perpendicular to the field is determined using the equation:

$$\tau = Fd$$



As the coil rotates (as shown in the diagrams above), the distance between the line of action of the force and the axis decreases, which results in the torque decreasing.

Review Question

8. Torque is directly proportional to the speed of a DC motor. The following is a list of methods which enables the speed of an electric motor to be increased.

Identify a potential problem or inconvenience associated with each of these methods.

- i.** Increasing the length of the conductor in the magnetic field

- ii.** Increasing the size of the current that flows through the coil.

- iii.** Increasing the magnetic field of the permanent magnets.

- iv.** Increasing the lever arm of the motor.

Solutions to Review Questions

1. The force on the coil is dependent upon both the current and the magnetic field. When the magnets are removed torque will be reduced to zero, however current is still being supplied and can't be utilized to turn the axle of the motor. As a result the motor overheats causing the smoking.
2. If the coil is in position 3 it is at a right angle to the magnetic field so the lines of action of the forces pass through the axle and therefore have no turning effect. The forces only act to spread the coil.
3. The purpose of the split ring commutator is to reverse the direction of the current twice every cycle. This enables the forces to continuously turn the coil.

4. $F = 0.6 \text{ N}$
 $L = 0.1 \text{ m}$
 $B = 1.5 \text{ T}$
 $n = 1$

$$F = nBIL$$

$$I = \frac{F}{nBL} = \frac{0.6}{1 \times 1.5 \times 0.1}$$

$$I = 4 \text{ A}$$

5. The direction of force is identified using the right hand rule. The magnetic field is from North to South and the current is moving from A to B therefore the force on side AB is upwards.

6. $B = 0.5 \text{ T}$
 $I = 5 \text{ A}$
 $L = 0.09 \text{ m}$
 $N = 8$

$$F = nBIL$$

$$F = 8 \times 0.5 \times 5 \times 0.09$$

$$F = 1.8 \text{ N}$$

7. The magnitude of the force acting on side BC is zero because BC is parallel to the magnetic field.
8. The answers are as follows:
 - i. The most effective means of doing this would be to increase the number of turns in the coil. This would make the motor heavier.
 - ii. Since $V = IR$ the amount of current is limited by the resistance of the wire. Increasing the current could cause overheating.
 - iii. Magnetic field strength is limited by the quality of the magnets used.
 - iv. If the lever arm is increased in size then the whole motor must also be made larger.

Physics
Teach Yourself Series
Topic 9: Generation of Electricity (Unit 3)

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What you should know

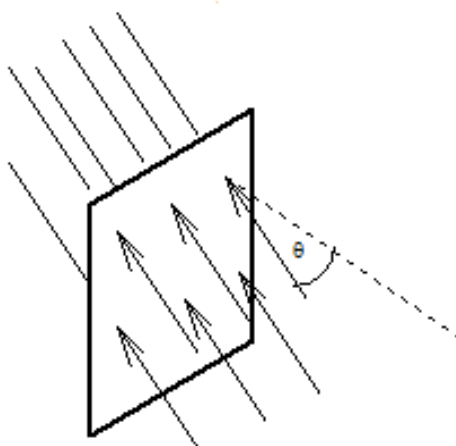
As it appears in Unit 3

- Calculate magnetic flux when the magnetic field is perpendicular to the area, and describe the qualitative effect of differing angles between the area and the field: $\Phi_B = BA$
- Investigate and analyse theoretically and practically the generation of electromotive force (emf) including AC voltage and calculations using induced emf: $\varepsilon = -N \frac{\Delta\Phi_B}{t}$ with reference to:
 - rate of change of magnetic flux
 - number of loops through which the flux passes
 - direction of induced emf in a coil
- Compare alternating voltage expressed as the root-mean-square (rms) to a constant DC voltage developing the same power in a resistive component
- Convert between rms, peak and peak-to-peak values of voltage and current
- Explain the production of DC voltage in DC generators and AC voltage in alternators, including the use of split ring commutators and slip rings respectively.

Magnetic flux

As it appears in Unit 3

Magnetic flux (Φ_B) is a measure of the amount of magnetic field passing through an area as shown in the diagram below.



It is measured in Webers (1 Weber is produced when a field of 1 tesla passes through an area of 1 square metre).

$$\Phi_B = B \times A$$

Φ_B is the magnetic flux (Wb: Weber)
 B_{\perp} is the magnetic field strength (T) perpendicular to the given area A
A is the area (m^2)

So, field strength can be defined as the flux density, or the number of field lines per square metre.

Important things to remember:

- 1) The magnitude of flux is not affected by the number of coils as increasing the number of coils does not have any effect on the area that the field penetrates.
- 2) If you are not given the area in the information, you will need to establish it before calculating the flux.

Sample question

Calculate the magnetic flux passing through a coil with an area of 0.08 m^2 in a magnetic field with a strength of 4.0 T

$$A = 0.08 \text{ m}^2$$

$$B = 4.0 \text{ T}$$

$$\Phi = 4.0 \times 0.08$$

$$\Phi = 0.32 \text{ Wb.}$$

Review Questions

1. Calculate the flux in a circular coil with an area of 0.5 m^2 in a magnetic field with a strength of 0.10 T
2. Calculate the flux in a circular coil with a radius of 4 cm in a magnetic field with a strength of 0.5 T .

3. A square coil with 40 turns has a cross sectional area of 1.2 m^2 . Calculate the magnetic flux if the coil is placed in a magnetic field with a strength of 0.25 T .
4. A rectangular coil has an area of $5.0 \times 10^{-3} \text{ m}^2$. When it passes through a uniform magnetic field the maximum flux that passes through the coil is $6 \times 10^{-5} \text{ Wb}$. Calculate the magnitude of the magnetic field.

Factors affecting magnetic flux

As it appears in Unit 3

Several factors affect the amount of magnetic flux. These include:

Field strength: Increasing the magnetic field increases the amount of flux.

Area: Increasing the area the magnetic field passes through increases the flux.

Movement: moving the magnet or the loop relative to each other can change the flux (for example a loop moving into or out of a magnetic field).

Angle: Magnetic flux is at a maximum when the magnetic field is perpendicular (at 90°) to the area.

If the angle is anything other than 90° then use the formula:

$$\Phi = B A \sin\theta$$

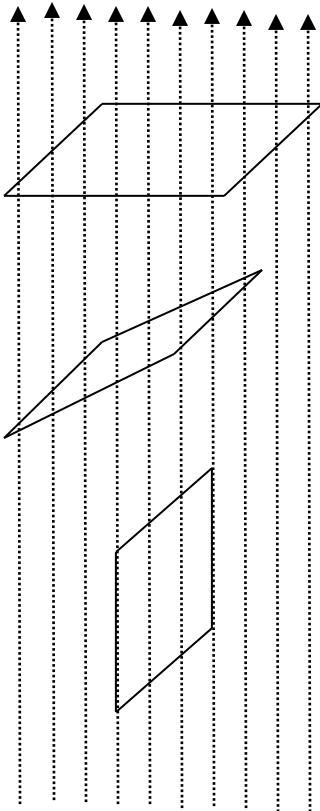
Review Questions

5. Calculate the magnetic flux when a magnetic field of 0.050 T threads 5 rectangular loops of wire of dimensions 50 cm by 75 cm, and such that the plane of the loops and the direction of the field are at an angle of 60° .
6. For the flux and loops given in **Q.5** establish the magnitude of the magnetic flux when the angle between the loops and the field is 30° .
7. Compare the result for **Q.5** and **Q.6**. Were the results as expected? Use your knowledge of magnetic flux to provide a reason to justify your response.

Graphing change in flux

As it appears in Unit 3

The magnitude of flux changes as the area exposed to the magnetic field changes as shown in the diagram below which shows a square coil being rotated in a magnetic field.

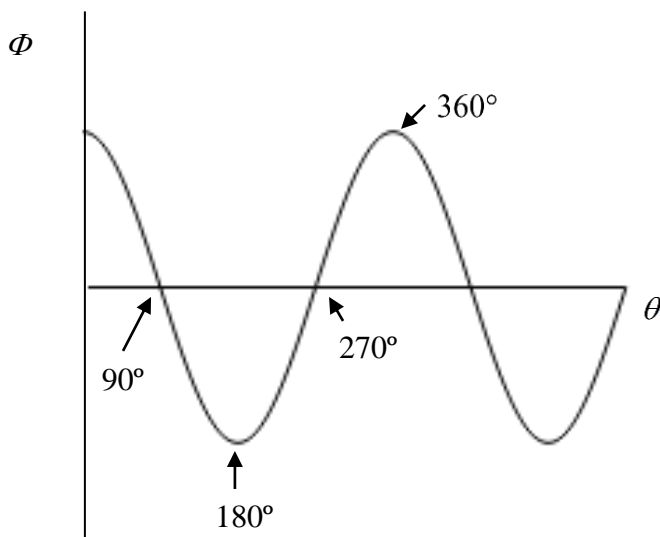


Position 1 shows the surface area of the square exposed to the magnetic field. As it can be seen the entire coil is perpendicular to the magnetic field which cuts through the entire area resulting in maximum flux.

Position 2 shows the coil being rotated. As the surface area exposed to the magnetic field decreases so does flux.

Position 3 shows the coil having been rotated through 90° . The plane of the coil is vertical, so the field lines do not cut through the coil and there is no flux.

The shape of the graph of flux against angle of rotation (measured from the initial position) would be as shown below.

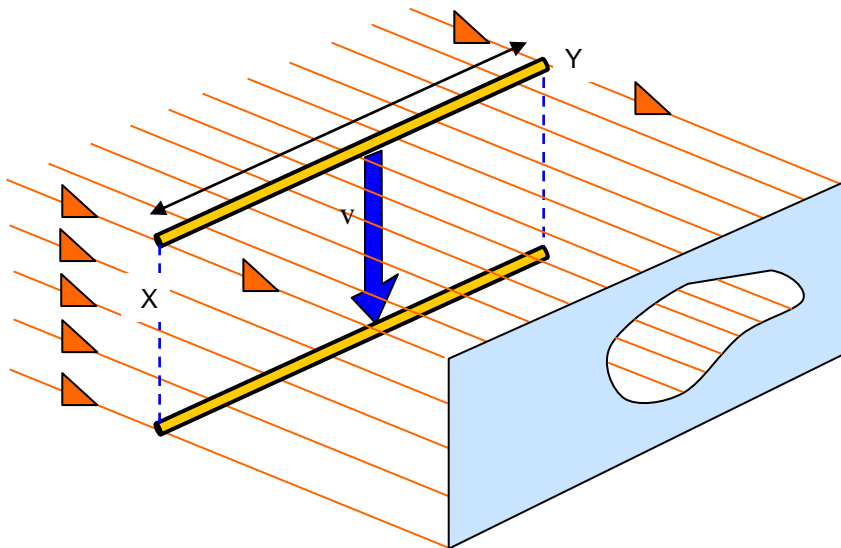


Generation of voltage with a magnetic field

As it appears in Unit 3

Charges moving across a magnetic field experience a force ($F = BIL$).

The diagram below shows a conducting rod XY falling at a steady speed (v) and perpendicular to a magnetic field represented by the lines.



Metals consist of a lattice of positively charged ions and moving electrons. When the metallic rod is placed into the magnetic field, the electrons experience a force that pushes them towards the end Y. The atomic structure of the metal resists the movement of the positively charged ions.

As a result, the electrons build up at end Y which develops a negative charge and end X develops a positive charge. This creates a potential difference ΔV between the two ends of the wire.

So the induced emf, $\mathcal{E} = BLv$

Where:

B = magnetic field (T)

l = length of the rod (m)

v = speed at which the rod is moving in the magnetic field. (m s^{-1})

The result of the rod moving in a magnetic field is an induced emf. Increasing the rate of the movement results in a greater induced emf.

Historically, EMF stands for Electro-Motive Force, however, it is not strictly a force as it is measured in volts not Newtons. Instead, EMF actually represents potential difference.

Induced voltage will only occur as long as charge separation is maintained. When the wire stops moving, the magnetic force is zero, the distribution of electrons throughout the rod equalizes and induced voltage no longer occurs.

Faraday's Law

As it appears in Unit 3

The magnitude of the induced EMF in a loop is directly proportional to the rate of change of magnetic flux cutting through it.

A constant magnetic field will not produce an electric current, but when a conductor moves through the lines of force of a magnetic field, work will be done on the moving charges producing the EMF.

The EMF produced is equal to the negative rate of change of flux passing through the loop.

The magnitude of the induced voltage in a coil is calculated using the formula:

$$\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$$

Where:

ε = induced emf (V)

Φ = flux (Wb)

t = time (s)

N = the number of turns in the coil

This equation can also be written as:

$$\varepsilon = -N \frac{B(\Delta A)}{\Delta t}$$

Review Question

8. If it takes 0.05 s to rotate the coil in **Q.4** on p.5, through an angle of 90°, calculate the magnitude of the average voltage generated.

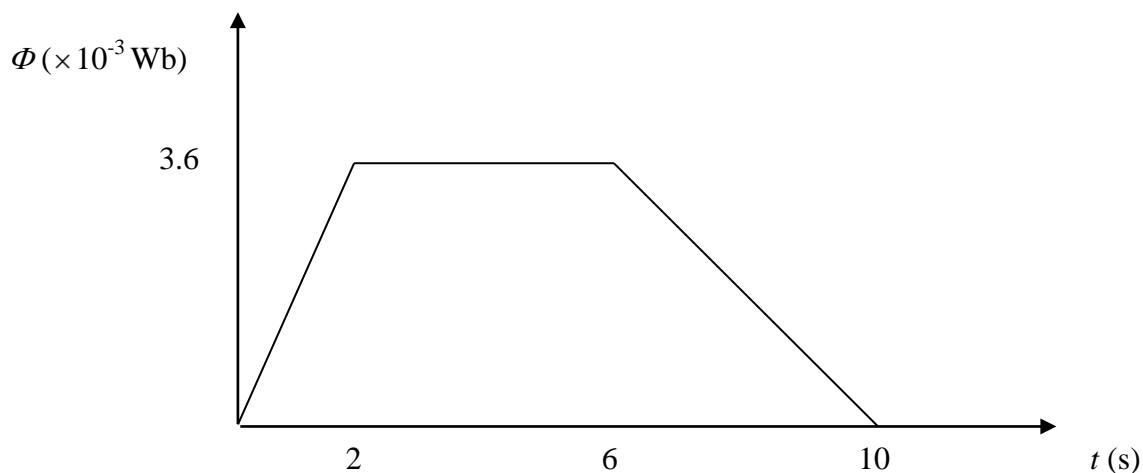
9. A rectangular coil 20 cm long and 15 cm wide with 12 turns enters a perpendicular magnetic field with a strength of 0.5 T. If it takes 1.5 seconds for the coil to enter the magnetic field, calculate the magnitude of the induced emf.

Using a flux graph to graph EMF

As it appears in Unit 3

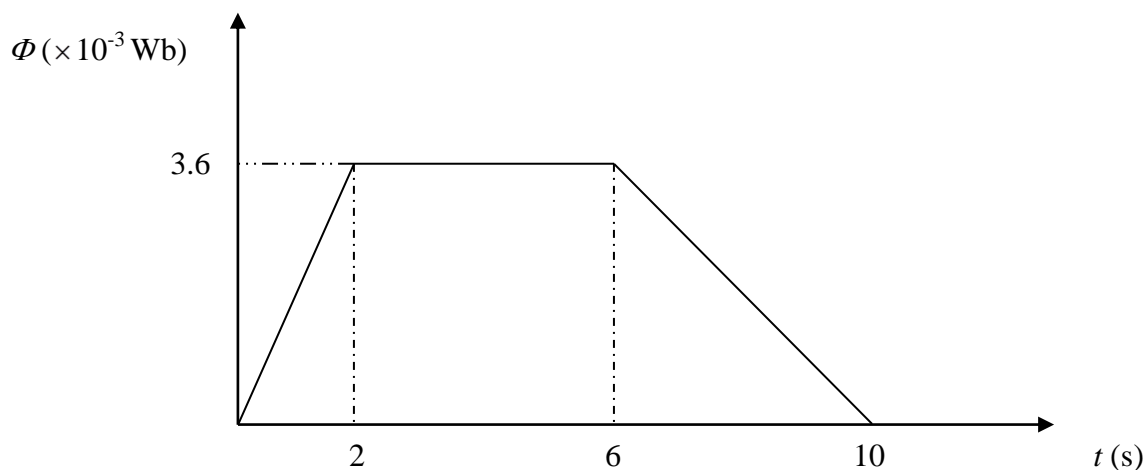
Since EMF is dependent upon flux change, a graph showing a change in flux can be used to produce a graph showing EMF. The EMF is given by the negative gradient of the flux change graph.

Sample question: A bar magnet is moved into and out of a solenoid which has 500 turns. The magnitude of the flux is shown in the graph below.



Use this graph to produce a graph showing the emf.

Step 1: Identify the different areas of the graph.



Step 2: Horizontal lines on the graph indicate constant flux which will mean a zero EMF. There is one region where this occurs: between 2 and 6 seconds.

Straight (but not horizontal lines) such as between 0 and 2 seconds and between 6 and 10 seconds will result in a constant *change* in flux which will lead to a constant but non-zero EMF for those sections

For a graph like this it is important to remember that the first section will result in a negative value (because the gradient is positive) while the last section of the graph will result in a positive value (because the gradient is negative). This is an area where mistakes are commonly made and the result is a graph where the positive and negative values are interchanged.

Step 3: Calculate the flux for each section. It is important to remember to take notice of the units as the magnitude of flux is 3.6×10^{-3} Wb, not 3.6 Wb. Use the formula:

$$\mathcal{E} = -N \frac{\Delta\Phi}{\Delta t}$$

For $t = 0$ to $t = 2$

$$N = 500$$

$$\Delta\Phi = 3.6 \times 10^{-3} \text{ Wb}$$

$$\Delta t = 2 \text{ s}$$

$$\mathcal{E} = \frac{-500 \times 3.6 \times 10^{-3}}{2}$$

$$\mathcal{E} = -0.9 \text{ V}$$

for $t = 2$ to $t = 6$

Since there is no change in flux, $\text{EMF} = 0$.

For $t = 6$ to $t = 10$

$$N = 500$$

$$\Delta\Phi = -3.6 \times 10^{-3} \text{ Wb}$$

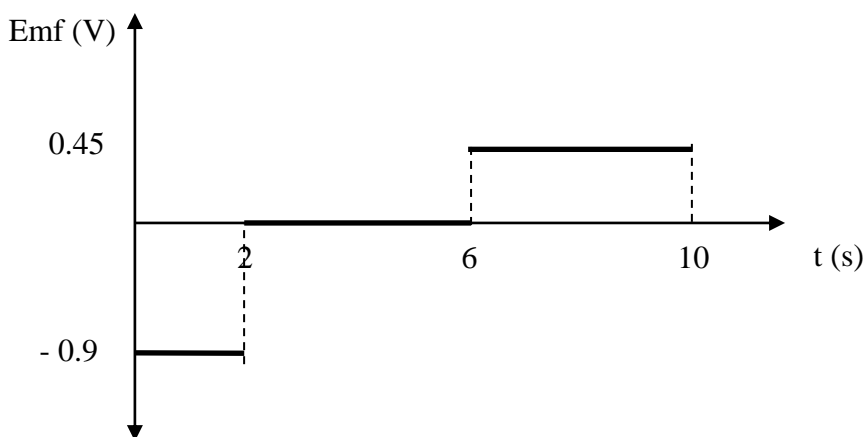
$$\Delta t = 4 \text{ s}$$

$$\mathcal{E} = \frac{-500 \times -3.6 \times 10^{-3}}{4}$$

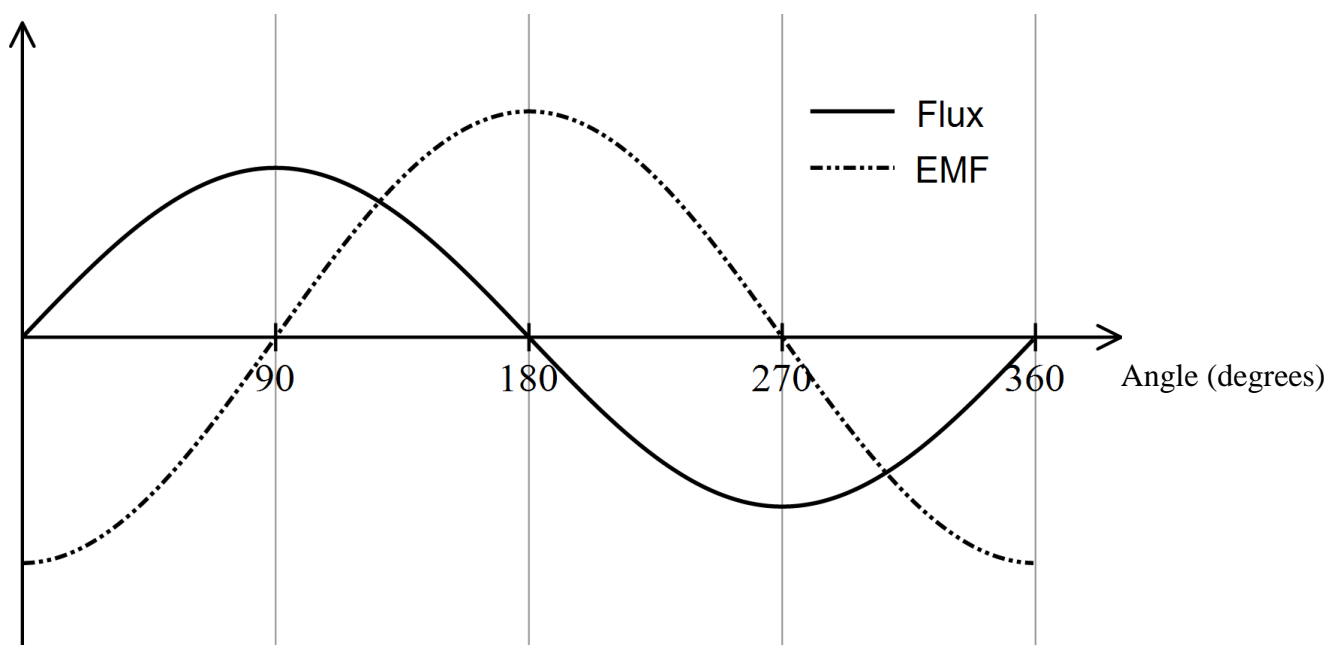
$$\mathcal{E} = 0.45 \text{ V}$$

So between 0 and 2 seconds, $\text{emf} = -0.9 \text{ V}$, between 2 and 6 seconds, $\text{emf} = 0 \text{ V}$ and between 6 and 10 seconds, $\text{emf} = 0.45 \text{ V}$.

Step 4: Now graph emf against time as shown below.



The graph below is an example of the changes in flux and induced emf with respect to each other as a coil is rotated through 360° . Note that the vertical scale will depend on the values of the various parameters such as the magnetic field, speed of rotation etc.

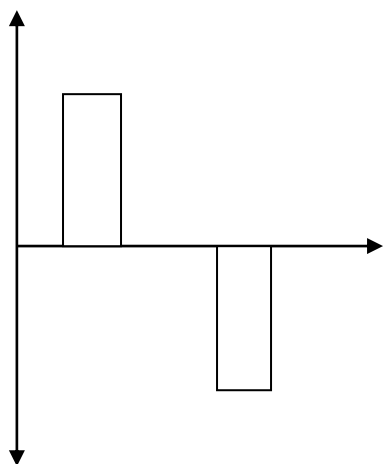


This type of graph is useful for questions involving predictions but it also provides the following information:

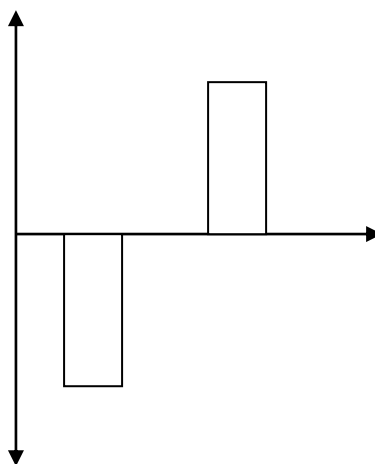
- Induced EMF is at a maximum or minimum when flux = 0
- Induced EMF = 0 when flux is at a maximum or minimum.
- EMF is the negative gradient of the flux graph.

Review Questions

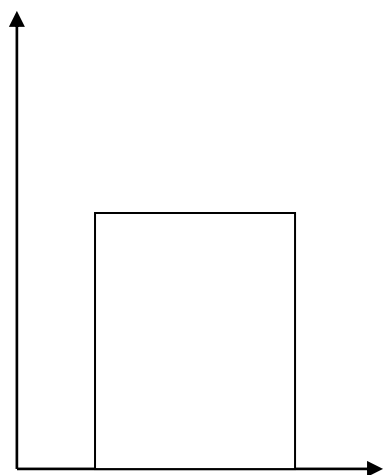
10. A square coil is moved into and then out of a magnetic field. Identify which of the following graphs is the best representation of the output emf as a function of time. Use your knowledge of Faraday's Law to justify your answer.



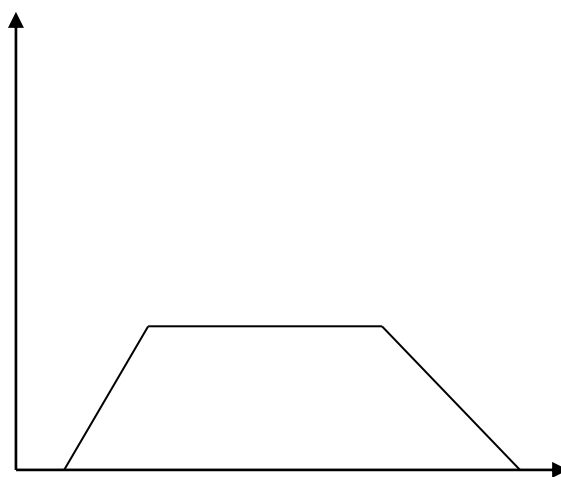
Graph 1



Graph 2

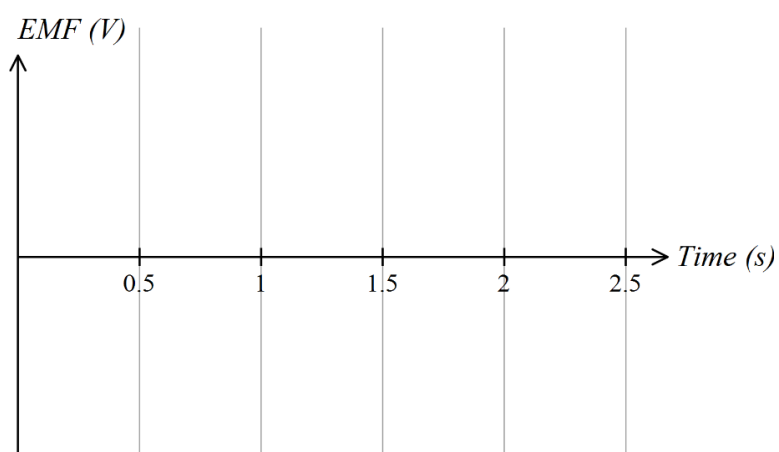
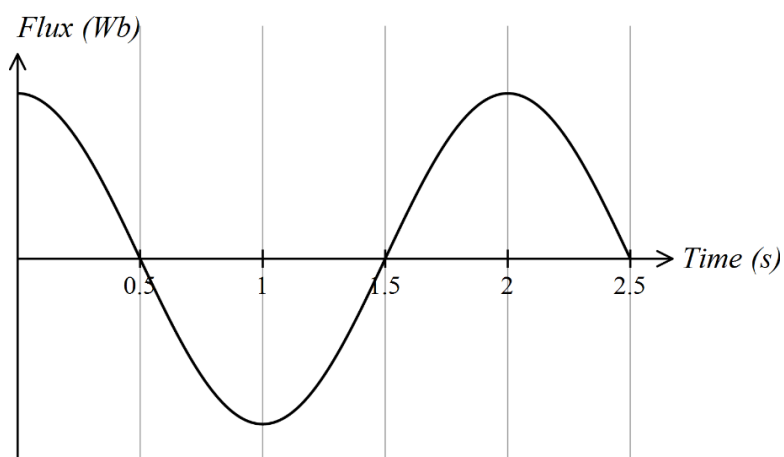


Graph 3

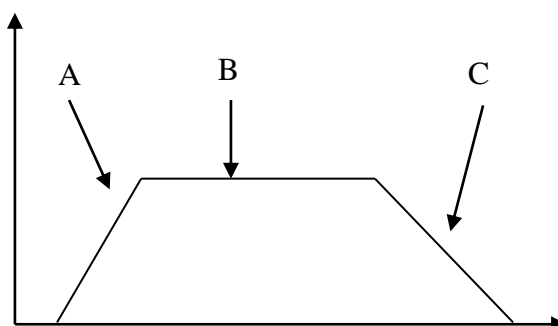


Graph 4

11. The graph below shows flux changing with time. Use this graph to produce a second graph showing the corresponding changes in emf.



12. The graph below represents a change in flux over a period of time. Identify the section of the graph that represents the most rapid change in flux. Provide a reason to justify your response.



Lenz' Law

As it appears in Unit 3

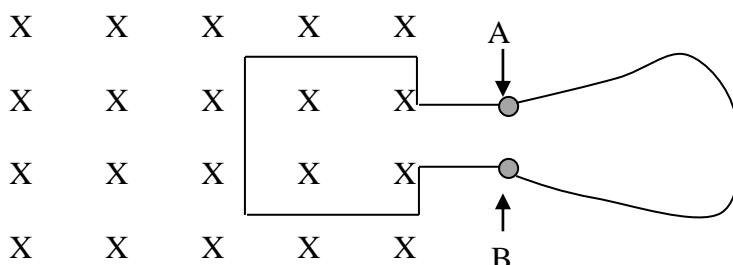
A closed circuit experiencing a changing magnetic flux, and thus an induced EMF, has an induced current whose magnetic flux opposes the external flux change that produced it.

It is likely that you will do an experiment which involves bringing a bar magnet close to a solenoid which is connected to a device like a galvanometer. The easiest way to remember Lenz' law is that a solenoid resists change.

When the magnetic field intensity within the coil changes (due to the change in flux, due to the change in area exposed to the magnetic field), a voltage is created in order to produce a current strong enough to create a magnetic field to compensate for the magnetic field lost due to the movement of the coil. This voltage, or EMF, is "induced" due to necessity, thus the name.

Review Questions

A square coil with sides 3 cm in length is placed into a uniform magnetic field with a strength of $6 \times 10^{-3} \text{ T}$. It is connected to an ammeter with 2 leads at points A and B.



13. If the coil is removed from the field at a velocity of 5 cm s^{-1} , calculate the average induced voltage.

Alternating current and voltage

As it appears in Unit 3

The principle of the AC generator is that it operates when a wire loop forming a closed circuit and immersed in a constant magnetic field, is made to rotate by an external mechanical source such as by steam hitting a turbine

This rotation of the loop causes a sinusoidal change in the area of the loop through which the field threads itself. The resultant sinusoidally varying flux causes a sinusoidal EMF to be induced that fluctuates from positive to negative values and thus induces an alternating current.

The alternating current produced by power stations which is supplied to cities varies sinusoidally at a frequency of 50 Hz. The peak value of the voltage of domestic power is $V_p = 340 \text{ V}$.

For an AC generator, the EMF is given by $\varepsilon = NBAf \sin(2\pi ft)$
(NOTE: this is not required for VCE Physics)

Where

t = time

f = frequency of rotation

B = magnetic field strength

A = area of the loop

N = no of turns in the coil

Although this equation is not required for examination purposes **and should not be used**, it can be useful in developing an understanding of the following 2 points which are needed:

- **Increasing the rotation rate, f , increases the amplitude and frequency (and hence decreases the period of) the electricity generated.**
- **Decreasing the rotation rate, f , decreases the amplitude and frequency (and hence increases the period of) the electricity generated.**

When graphing it is necessary to remember:

Doubling the rotation frequency doubles the amplitude and doubles the number of cycles in that time e.g. if Δt is halved then ε is doubled and the period is halved because the rate of rotation has doubled.

Doubling only one of the magnetic field strength, loop area or number of turns doubles the amplitude only but retains the same frequency.

Information regarding waves

As it appears in Unit 3

Period (T): time taken for a complete cycle (a complete wave passes a given point)

Frequency (f): the number of times a wave repeats itself per second. The unit for frequency is the Hertz (Hz).

The relationship between T and f is $T = \frac{1}{f}$. Standard frequency power is 50 Hz.

Amplitude: the top of the curve; the maximum distance away from 0 on the y axis. Depending on the information in the graph, the amplitude can represent peak voltage (V_p) or peak current (I_p)

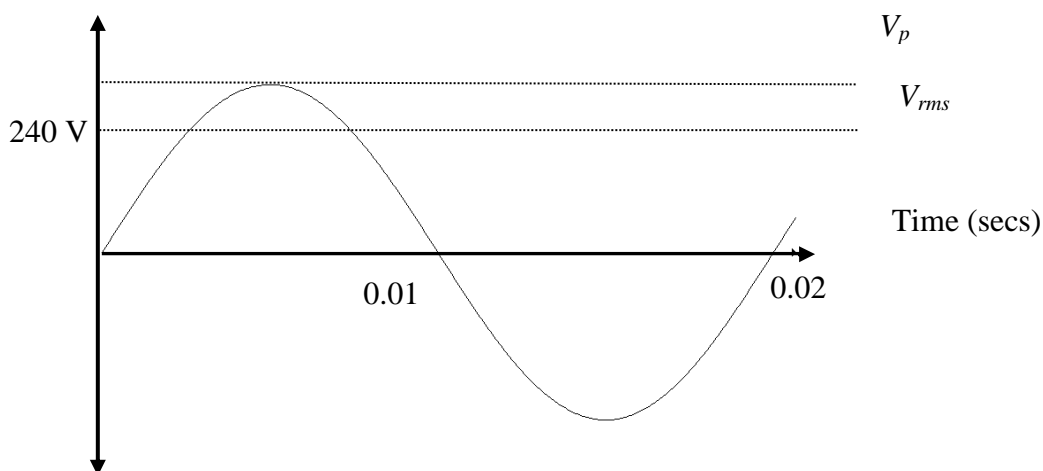
Peak to Peak: the difference between the maximum and minimum.

RMS

As it appears in Unit 3

When comparing DC and AC power supplies, it is necessary to carry out conversions to make a true comparison. The root mean square voltage (V_{RMS}) is the value of the equivalent steady voltage of the direct current which would produce the same power as AC voltage across the same resistance as shown in the graph below.

EMF



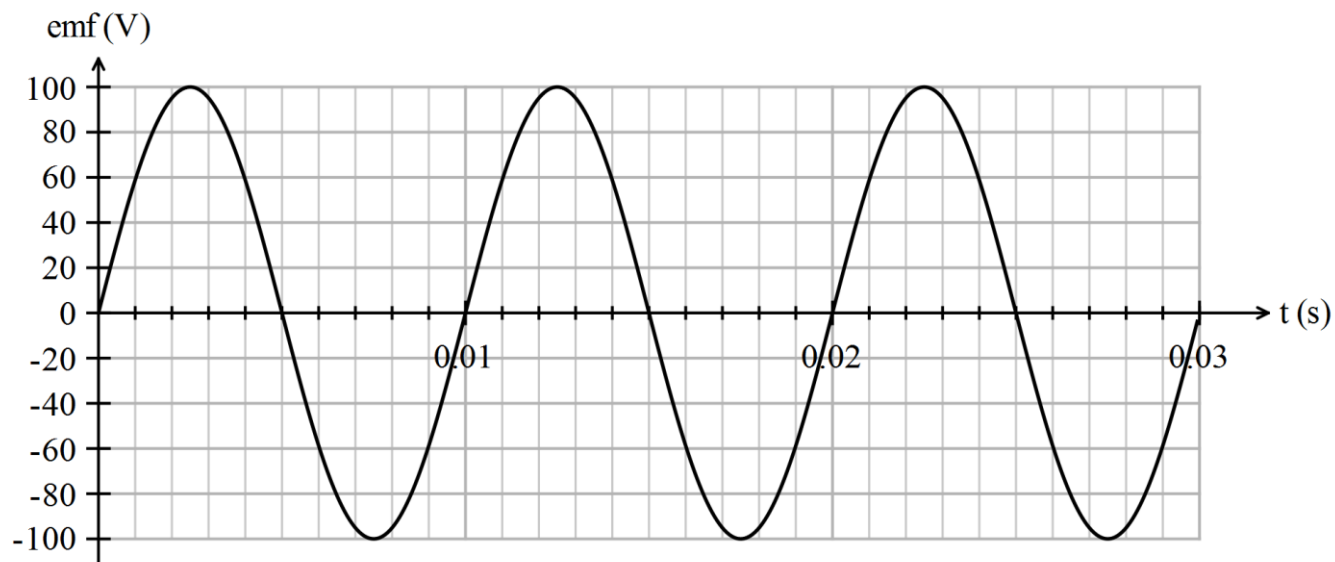
The **Root Mean Square (RMS)** voltage is the square root of the mean of the square of the potential difference.

The DC potential difference that would transfer the same energy is given by $V_{RMS} = \sqrt{\frac{V_p^2}{2}}$

In Australia $V_{rms} = 240$ V for domestic power and in the US $V_{RMS} = 110$ V.

Review Questions

The diagram below shows the graph for an alternator which rotates at 100 revolutions per second and has a peak voltage of 100 V.



15. The rate of rotation is now reduced to 50 revolutions per second. On the graph above draw a second graph showing how the output voltage has changed.

16. Explain why you drew the graph in this way.

Increasing EMF

As it appears in Unit 3

The Faraday equation for average emf is given by:

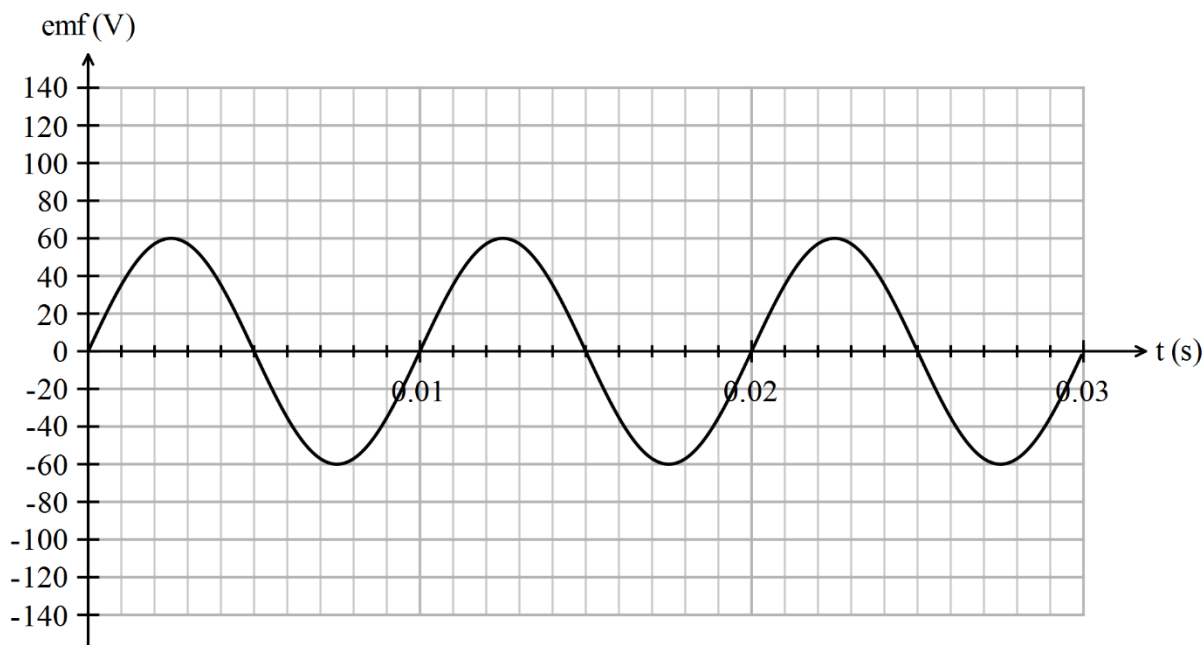
$$\mathcal{E} = -N \frac{\Delta(BA)}{\Delta t}$$

Therefore the emf can be changed by altering any of the components of this equation. This includes:

- Changing the number of coils (N)
- Changing the strength of the magnetic flux (B)
- Changing the area of each coil (A)
- Changing the frequency, which will also change the time (t) taken for one rotation.
- Wrapping the coils around a soft iron core, which will increase the magnetic field through the coils.

Review Questions

The diagram below shows the graph for an alternator which rotates at 100 revolutions per second and has a peak voltage of 60 V.



17. The rate of rotation is now increase to 200 revolutions per second. On the graph above draw a second graph showing how the output voltage has changed.

18. Explain why you drew the graph in this way.

19. A generator is producing a peak output of 1200 V at 50 Hz. The magnetic field through the coils is halved, the area of each coil increased by a factor of 3 and the number of coils increased from 50 to 250. What would be the peak voltage and frequency of the output now?

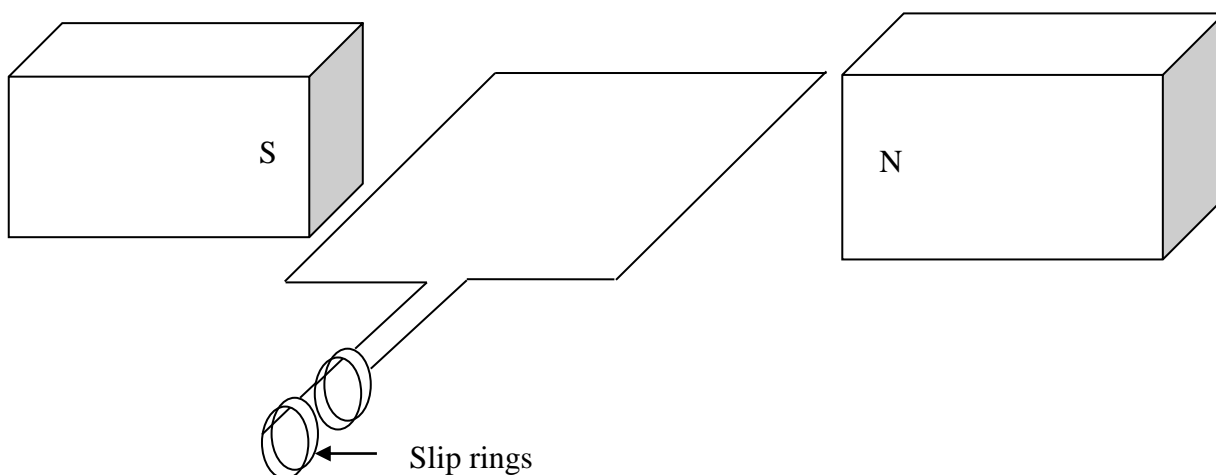
AC generators

As it appears in Unit 3

COMPONENT	PURPOSE
Battery or power supply	Source of current
Slip ring	Allows for Alternating Current (AC) to be created.
Split ring commutator	Allows for Direct Current (DC) to be created
Coil	Carries current through magnetic field
Brushes	Allow current to flow smoothly to the coil
2 magnets of opposite polarity	Provides a magnetic field that interacts with the current producing a force
Axle	Allows the coil to spin

AC generators (also called alternators) have a rotating coil inside a magnetic field resulting in an AC output.

Slip rings are used instead of commutators. Unlike a DC generator, in an AC generator one end of the coil remains attached to the same ring which results in the output varying sinusoidally between positive and negative values which is therefore AC (alternating current) rather than DC (direct current).



If you are given a diagram that could be either a DC generator or an AC generator, the easiest way to tell them apart is to see whether a commutator or a slip ring is present.

The presence of a slip ring would mean the device is an AC generator. The presence of a split ring commutator would mean the device is a DC generator.

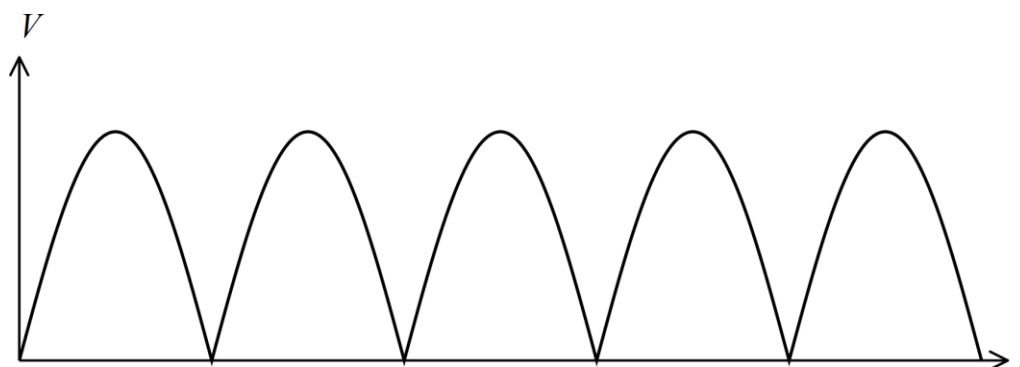
Comparing output graphs

As it appears in Unit 3

The output voltage can be determined by connecting a device such as an oscilloscope to either the slip rings or the commutator.

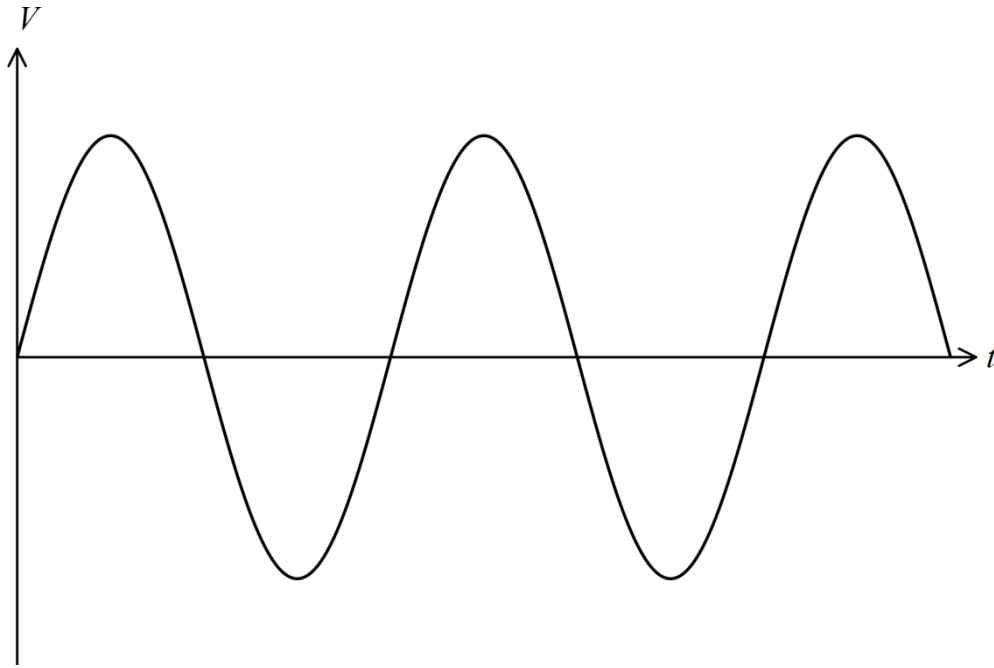
The shape of the output graph can be used to determine the difference between a DC generator and an AC alternator.

The output from a DC generator will look like the diagram below..



Even though the output varies, it is never negative, so current flow is always in the same direction.

The output from an AC alternator is shown below.



The output fluctuates between positive and negative values.

Solutions to Review Questions

1. $\Phi = BA$
 $B = 0.1 \text{ T}$
 $A = 0.5 \text{ m}^2$

$$\Phi = 0.1 \times 0.5$$

$$\Phi = 0.05 \text{ Wb}$$

2. $A = \pi r^2$
 $A = \pi \times (0.04)^2$
 $A = 0.005 \text{ m}^2$
 $B = 0.5 \text{ T}$

$$\Phi = 0.5 \times 0.005$$

$$\Phi = 0.0025 \text{ Wb}$$

3. $B = 0.25 \text{ T}$
 $A = 1.2 \text{ m}^2$

$$\Phi = 0.25 \times 1.2$$

$$\Phi = 0.3 \text{ Wb}$$

4. $A = 5 \times 10^{-3} \text{ m}^2$
 $\Phi = 6 \times 10^{-5} \text{ Wb}$

$$B = \frac{\Phi}{A}$$

$$B = \frac{6 \times 10^{-5}}{5 \times 10^{-3}}$$

$$B = 0.012 \text{ T}$$

5. $B = 0.05 \text{ T}$
 $A = 0.5 \text{ m} \times 0.75 \text{ m}$
 $A = 0.375 \text{ m}^2$
 $\theta = 60^\circ$

$$\Phi = BA \sin \theta$$

$$\Phi = 0.05 \times 0.375 \times \sin 60^\circ$$

$$\Phi = 0.0162 \text{ Wb}$$

6. $B = 0.05 \text{ T}$
 $A = 0.375 \text{ m}^2$
 $\theta = 30^\circ$

$$\Phi = 0.05 \times 0.375 \times \sin 30^\circ$$

$$\Phi = 0.0094 \text{ Wb}$$

7. The results were as expected. As the angle between the loops of wire and the magnetic field decreases the magnitude of the cross sectional area cut by the magnetic field will also decrease, therefore the flux at a 30° angle should be less than that for a 60° as shown in the results.

8. $\Delta\Phi = 6 \times 10^{-5} \text{ Wb}$
 $\Delta t = 0.05 \text{ s}$

$$\varepsilon = \frac{\Delta\Phi}{\Delta t}$$

$$\varepsilon = \frac{6 \times 10^{-5}}{0.05}$$

$$\varepsilon = 0.0012 \text{ V}$$

9. $N = 12$
 $B = 0.5 \text{ T}$
 $A = 0.2 \times 0.15 = 0.03 \text{ m}^2$
 $\Delta t = 1.5 \text{ s}$

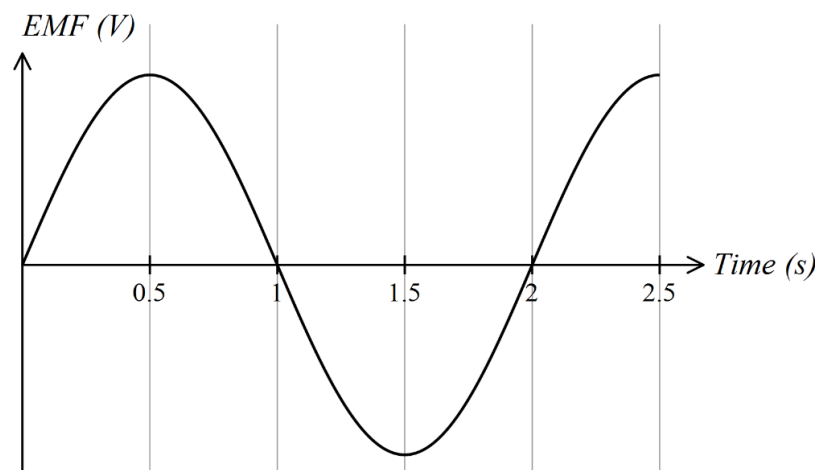
$$\varepsilon = -N \frac{\Delta BA}{\Delta t}$$

$$\varepsilon = \frac{-12 \times 0.5 \times 0.03}{1.5}$$

$$\varepsilon = -0.12 \text{ V}$$

10. Induced emf occurs as flux changes, which occurs when there's a change in the amount of magnetic field passing through the surface and/or the amount of area the magnetic field passes through. When the coil enters or leaves the field, the magnetic flux will change. The sign of the emf will be opposite to the flux changes. As flux increases when the coil enters and decreases when the coil leaves the field the emf should first be negative and then later be positive. This means that only Graph 2 is possible.

11. The graph should look like this



12. Area A. This part of the curve has the steepest slope and so it is showing the greatest change in flux per unit of time.

13. $\varepsilon = Blv$

$$B = 6 \times 10^{-3} \text{ T}$$

$$l = 0.03 \text{ m}$$

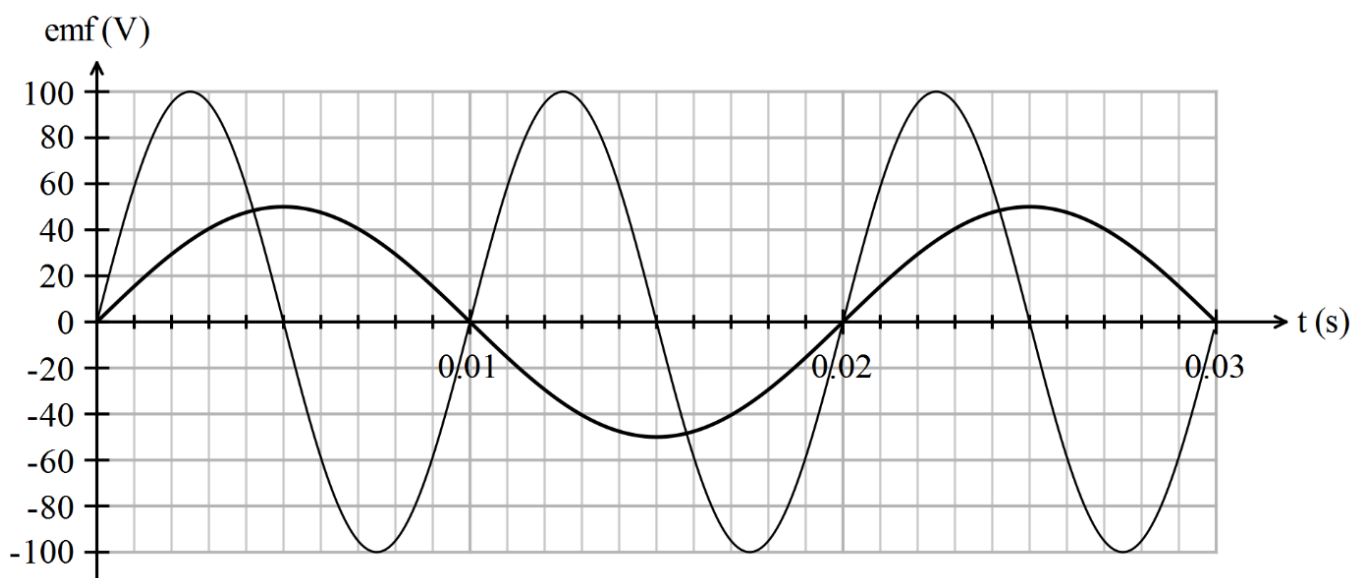
$$v = 0.05 \text{ m s}^{-1}$$

$$\varepsilon = 6 \times 10^{-3} \times 0.03 \times 0.05$$

$$\varepsilon = 9 \times 10^{-6} \text{ V}$$

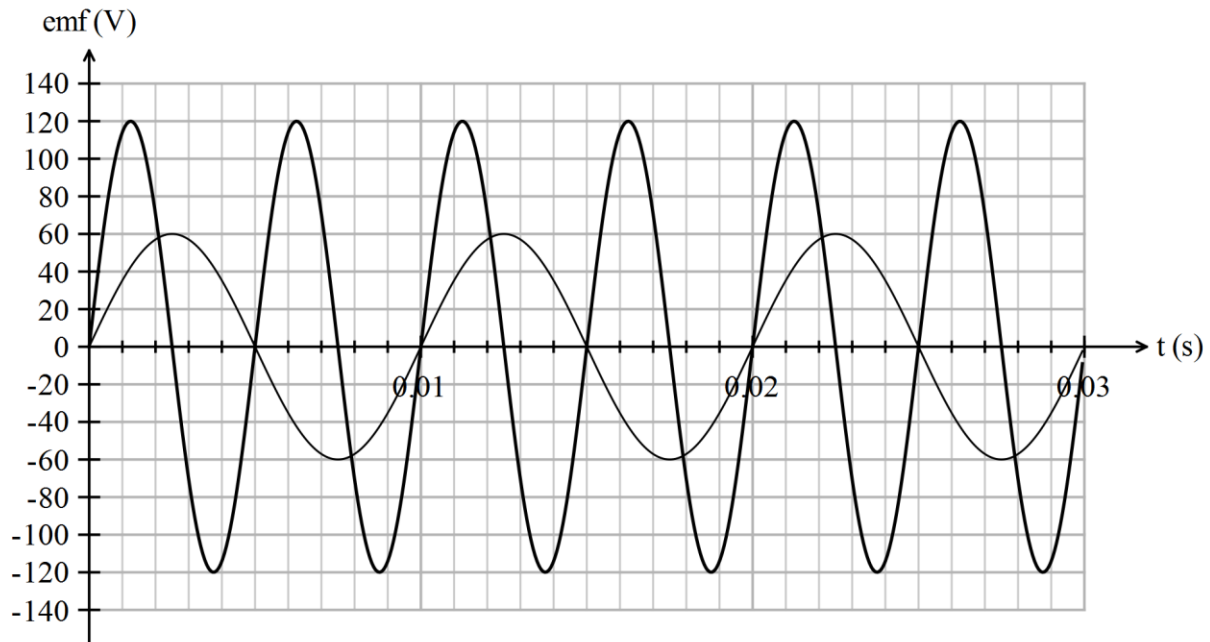
14. As the coil is removed the magnetic flux into the page will be decreasing therefore the direction of the induced magnetic field must also be into the page. The current in the loop would need to be running clockwise to produce the field therefore using the right hand rule we can see that the current must be travelling through the loop from B to A.

15. The graph should look like this



16. Since the rotation has decreased the period has been doubled from 0.01 s to 0.02 s. As the rate of rotation has been halved so is the rate of change of flux. Therefore the voltage has also been halved from 100 V to 50 V.

17. See the graph below



18. Since the rotation has increased, the period has been halved from 0.01 s to 0.005 s. As the rate of rotation has been doubled so is the rate of change of flux. Therefore the voltage has also been doubled from 60V to 120V.

19. N has been **increased** by a factor of $\frac{250}{50} = 5$

A has been **increased** by a factor of 3

B has been **decreased** by a factor of 2

The emf is directly proportional to B , A and N therefore the new peak voltage is

$$\frac{1200 \times 5 \times 3}{2} = 9000 \text{ V}$$

However none of these changes affect the rate of rotation so the frequency will still be 50 Hz.

Physics

Teach Yourself Series

Topic 10: Transmission of electricity (Unit 3)

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What you should know

As it appears in Unit 3

- compare sinusoidal AC voltages produced as a result of the uniform rotation of a loop in a constant magnetic field with reference to frequency, period, amplitude, peak-to-peak voltage (V_{p-p}) and peak-to-peak current (I_{p-p})
- analyse transformer action with reference to electromagnetic induction for an ideal transformer: $\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$
- analyse the supply of power by considering transmission losses across transmission lines
- identify the advantage of the use of AC power as a domestic power supply.

Revision of equations

As it appears in Unit 3

Before starting this topic, you should be aware of the following equations and how to use them. The table below shows the formulae relating to power, voltage, current and resistance and the different ways that they can be calculated.

P = Power	V = Voltage	I = current	R = resistance
$\frac{V^2}{R}$	$R \times I$	$\sqrt{\frac{P}{R}}$	$\frac{V}{I}$
$R \times I^2$	$\frac{P}{I}$	$\frac{P}{V}$	$\frac{V^2}{P}$
$V \times I$	$\sqrt{P \times R}$	$\frac{V}{R}$	$\frac{P}{I^2}$

Note: The resistance of a wire is proportional to its length, l , and inversely proportional to its cross sectional area, A . This can be summarized by the relationship:

$$R = \frac{\rho l}{A}$$

Where ρ depends on the material that the wire is made from.

Sample questions #1

A 60 W light globe has 240 V applied across it.

Determine the current through the globe

$$P = 60 \text{ W}$$

$$V = 240 \text{ V}$$

$$P = IV$$

$$I = \frac{P}{V}$$

$$I = \frac{60}{240}$$

$$I = 0.25 \text{ A}$$

Find the resistance of the globe.

$$V = 240 \text{ V}$$

$$I = 0.25 \text{ A}$$

$$R = \frac{V}{I}$$

$$R = \frac{240}{0.25}$$

$$R = 960 \Omega$$

Sample questions #2

A man has a shed in the backyard which contains the 60 W globe. The globe is attached to the power supply by a 5 m long insulated wire. If the wire has a resistance of $0.030 \Omega \text{ m}^{-1}$, what would be the total resistance of the circuit?

$$R_{\text{total}} = R_{\text{wiring}} + R_{\text{globe}}$$

$$R_{\text{total}} = (5.0 \times 0.030) + 960$$

$$R_{\text{total}} = 960.15 \Omega$$

Transmission of electric power

As it appears in Unit 3

Electric power is produced at an appropriate level for domestic and industrial use.

The main problem is that the power is usually produced at a significant distance from where it is used, and as a result, must be transported by wires.

During the transmission across the country side, the resistance in the wires converts some of the useful power to heat which is dissipated into the environment and thus lost. Even though wires are made of materials of very low resistance, some energy is always lost as heat.

This power loss is negligible over short distances, but becomes significant over long distances to the extent that the voltage eventually delivered would be so low that it would fail to power any devices.

Power delivery and power loss

As it appears in Unit 3

Power delivered for use is the product of current and voltage and is calculated by the formula:

$$P = IV$$

Where P = power (W)

I = current (A)

V = voltage (V)

Power loss (P_{loss}) when the current flows through a transmission line is given by:

$$P_{loss} = I^2 R = \frac{(\Delta V)^2}{R}$$

It is important to remember that R_{total} refers to both lines involved since the supply of electricity is a closed circuit.

Review Questions

1. Calculate the amount of power produced in a situation where there is 12.5 A at 120 V
2. Calculate the amount of power that is dissipated if there is 1.0 mA of current through a 6.0 k Ω resistor
3. Calculate the amount of power that is dissipated if there 12.0 V across a 1.0 k Ω resistor

Minimising power loss

As it appears in Unit 3

There are two main ways of minimizing power loss. These are:

- Reducing the resistance of the wire used in the power lines.
- Reducing the current in the wires.

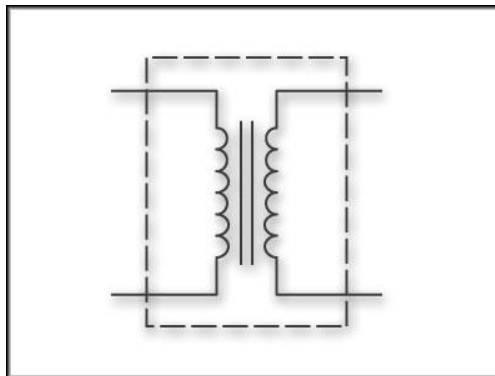
Aluminum is generally used for wires because it is a good conductor (i.e. has low resistance) and because it is light. The length of wire cannot be altered for a particular situation, but thicker wires will lower the resistance. However, thicker wires are more expensive and also require larger more expensive pylons to support them.

Current, on the other hand, can be varied with transformers.

Transformers

As it appears in Unit 3

The circuit symbol for a transformer is:



Transformers are a major component of the electrical distribution system. They:

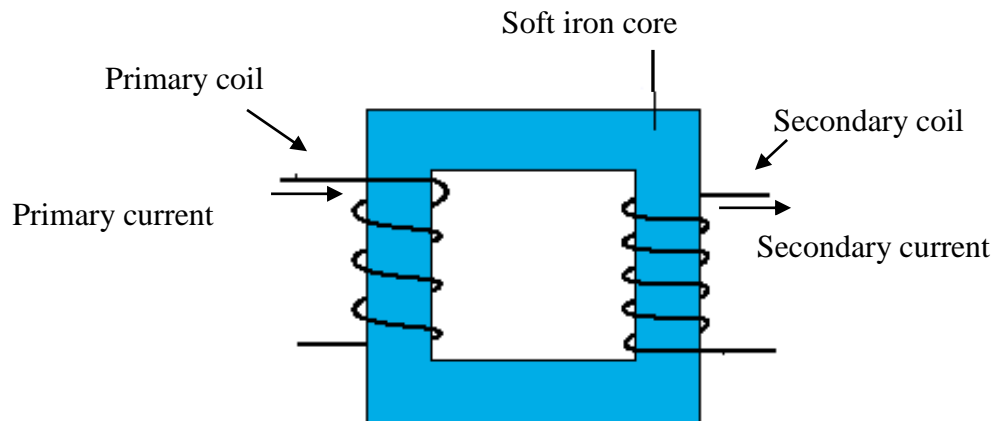
- Enable energy losses in transmission lines to be reduced.
- Are used to change household power supply (240 Volts) to usable voltages in appliances.

Transformer structure

As it appears in Unit 3

A transformer consists of three major components:

- A primary coil wound around one side of the core
- A secondary coil wound around the other side of the core
- A core of soft iron, generally in a square or U shape.



Review Question

4. Explain why the core of a transformer must be a metal that is easy to magnetise.

Transformer operation

As it appears in Unit 3

The purpose of a transformer is to produce an output voltage that is different to the input voltage.

Transformers operate on the principle of EM transduction. The coils are wound around the core in such a way that the magnetic flux generated by one coil passes through the other. When current is passed through the primary coil a magnetic field builds and when the current to the primary coil is turned off the magnetic field collapses. This creates an increasing and then decreasing magnetic field passing through the secondary coil.

The alternating current in the primary coil (input) generates an alternating magnetic field in the iron core. This alternating field passes through the secondary coil and induces an EMF in the secondary coil (output).

Transformers only operate for AC currents because the magnetic flux induced by the current in the primary coil must be variable in order to create a current in the secondary coil.

Review Question

5. Explain why transformers do not operate if the input voltage is DC.

Transformers limit power loss

As it appears in Unit 3

It is important to remember the link between $P_{loss} = I^2 R$ and $P = VI$.

Transformers are used to minimise power loss. As $P_{loss} = I^2 R$, the higher the current in the transmission wires, the greater the loss of power due to the fact that the current is squared.

Since $P = VI$, when the voltage is stepped up in the transmission line the current in the transmission line is stepped down in the same proportion, greatly decreasing power loss due to the fact the amount of power is a fixed value as the transformer is ideal.

Generally the upper limit for conducting voltage is 500 kV.

Review Questions

6. A power station produces 300 MW of power at 20 kV. Determine the current at the power station.

7. Calculate the power loss if the 300 MW was transmitted through lines with a total resistance of 0.8Ω .

8. Determine the current if the voltage is stepped up by a transformer to 250 kV.

9. Determine the power loss after the voltage was stepped up.

10. Is the difference between the answers to questions 7 and 9 as expected? Provide a reason for your answer.

Voltage comparison

As it appears in Unit 3

Transformers are generally considered to be ideal (although this is not completely true), which means that there is no power loss (i.e., $P_p = P_s$)

The relationship between the Power P_p , the voltage V_p and the current I_p in the primary coil and the power P_s , the voltage V_s and the current I_s in the secondary coil is:

$$P_p = P_s \Rightarrow I_p V_p = I_s V_s$$

This can be rearranged as follows:

$$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

N_s and N_p represent the number of turns in the primary and secondary coils.

Sample Question:

A power station, Loy Yang B, that supplies Melbourne, operates at 340 MW @ 20 kV and 17 kA. Transmission occurs at 500 kV and the total resistance in the lines is 5 Ω

- i. What is the ratio of turns in the secondary coil to the primary coil?

In most cases where there is a formula with 4 components you need to know 3 pieces of information to solve the question. In this case the ratio of the turns is the same as the ratio of the voltages. Since we know both the primary and secondary voltages we are able to solve the problem.

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

$V_s = 500$ kV as this is the voltage of transmission

$V_p = 20$ kV as this is the voltage of production

$$\frac{N_s}{N_p} = \frac{500 \times 10^3}{20 \times 10^3} = \frac{25}{1}$$

- ii. Calculate the current in the transmission lines.

$$\frac{I_p}{I_s} = \frac{N_s}{N_p} = 25 \text{ from part i.}$$

$$I_p = 17 \text{ kA}$$

$$\frac{I_p}{I_s} = 25$$

$$I_s = \frac{I_p}{25} = \frac{17000}{25}$$

$$I_s = 680 \text{ A}$$

- iii. Calculate the power loss in the lines

$$P_{loss} = I^2 R$$

$$P_{loss} = 680^2 \times 5$$

$$P_{loss} = 2312000 \text{ W} = 2.3 \text{ MW}$$

- iv. Calculate the percentage of power loss in the lines.

$$\text{Percentage power loss} = \frac{2.3 \times 10^6 \times 100}{340 \times 10^6} = 0.68\%$$

So as a result of using a transformer less than 1% of the power is lost during transmission.

Review Questions

11. Assume that transmission occurred at 100 kV instead of 500 kV. Predict what would happen to the current in the line, the power loss in the lines and the percentage of power lost.

12. If $I_p = 17 \text{ kA}$ and the cable resistance is 5 ohms, calculate the turns ratio, the current in the line, the power loss in the line and the percentage of power lost.

13. Explain the advantage of voltage transmission occurring at 500 kV rather than 100 kV.

Step up transformers

As it appears in Unit 3

A step up transformer produces a voltage output that is greater than the voltage input.

For a step up transformer $V_s > V_p$ or $N_s > N_p$ and $I_s < I_p$

In a step up transformer the secondary coil has more turns than the primary. Since power is constant if voltage increases then current has to decrease by the same ratio.

Step down transformers

As it appears in Unit 3

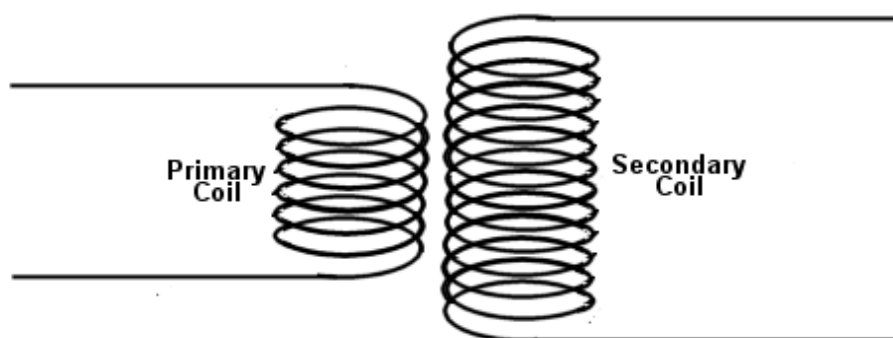
A step down transformer produces a voltage output that is less than the voltage input.

For a step down transformer $V_s < V_p$ or $N_s < N_p$ and $I_s > I_p$

In a step down transformer the primary coil has more turns than the secondary coil.

Review Questions

14. Does the diagram below show a step up transformer or a step down transformer? Provide a reason for your answer.



15. A transformer has 100 turns in the primary coil and 1500 turns in the secondary coil. If the primary coil is connected to a 240 V outlet, determine the voltage across the secondary coil.

16. A transformer has 200 turns in the primary coil and 8 turns in the secondary coil. The secondary voltage was found to be 12 V; what was the primary voltage?

17. A transformer is used to convert 48000 V to 240 V. What type of transformer is it? Provide a reason to justify your response.

18. With regard to **Q.17** above, if there were 10000 turns in the primary coil, how many turns would there be in the secondary coil?
19. A person has a string of Christmas lights but they need to use a transformer to convert the 240 V household supply to 12 V.
- There are 80 turns in the secondary coil. How many turns will there be in the primary coil?
 - The string of lights consists of forty 12 V bulbs connected in parallel, each using 5 W of power. What is the current in the secondary coil?

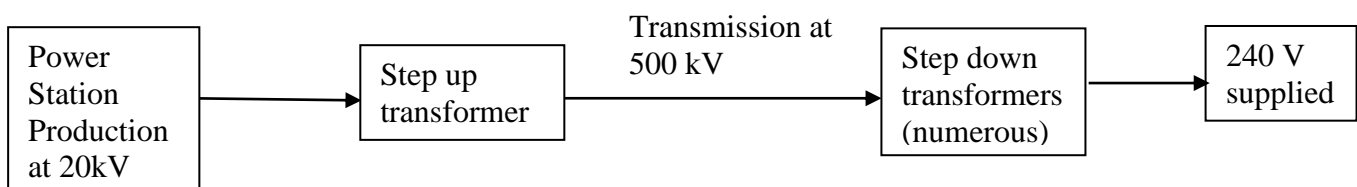
Household electricity supply

As it appears in Unit 3

In Australia, AC power is supplied to houses at 240 V_{RMS} but it often needs to be transmitted long distances from the site of power generation which can lead to significant power losses as power loss is proportional to resistance and resistance of the power lines is proportional to their length; so the longer the wires, the greater the power losses become. As previously discussed, aluminium wires with a relatively large diameter are used to minimise resistance as much as possible.

However, power loss is also affected by current according to: $P_{Loss} = I_{Transmission}^2 \times R$

The advantage of using AC power is that, unlike DC voltages, AC voltages can be easily stepped up near the power station and transmitted for long distances at high voltages and relatively low current thus reducing the amount of power lost. The voltage can then be stepped down again near the areas where the power is required for industrial and domestic use as shown below.

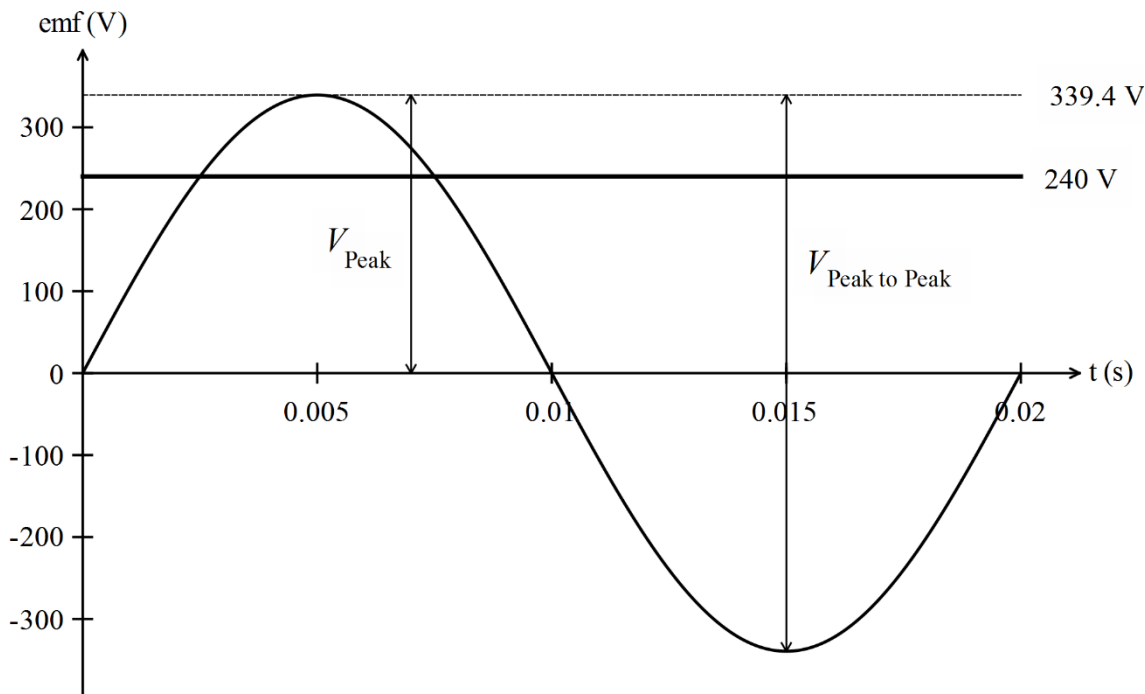


Alternating voltage and current

As it appears in Unit 3

Alternating current, unlike direct current, changes polarity. In Australia, the mains voltage operates at 240 V_{RMS} at 50 Hz. It is different in other countries such as in the US where it is 120 V_{RMS} at 60 Hz.

When comparing DC and AC power supplies, it is necessary to carry out conversions to make a true comparison. The root mean square voltage (V_{RMS}) is the value of the equivalent steady voltage of the direct current which would produce the same power as AC voltage across the same resistance. Below is a regular sinusoidal AC voltage such as could be produced by a generator.



You need to know the following information regarding waves:

Period (T): time taken for a complete cycle (a complete wave passes a given point)

Frequency (f): the number of times a wave repeats itself per second. The unit for frequency is the Hertz (Hz).

The relationship between T and f is $T = \frac{1}{f}$. Standard frequency power is 50 Hz.

Amplitude: the top of the curve; the maximum distance away from 0 on the y axis. Depending on the information in the graph, the amplitude can represent peak voltage (V_p) or peak current (I_p)

Peak to Peak: the difference between the maximum and minimum so peak voltage is given by:

$$V_{peak\ to\ peak} = 2V_{peak}$$

The DC potential difference that would transfer the same energy as V_{peak} AC is given by:

$$V_{RMS} = \frac{V_{peak}}{\sqrt{2}}$$

I_{RMS} is the value of a DC current that generates the same power as an AC current through the same resistance. It is calculated as

$$I_{RMS} = \frac{I_{peak}}{\sqrt{2}}$$

From the graph on the previous page, the following information can be obtained:

$$V_{peak} = 339.4 \text{ V}$$

$$V_{peak\ to\ peak} = 339.4 \times 2 = 678.8 \text{ V}$$

$$\text{Period, } T = 0.02 \text{ s}$$

$$f = \frac{1}{T}$$

$$f = \frac{1}{0.02}$$

$$f = 50 \text{ Hz}$$

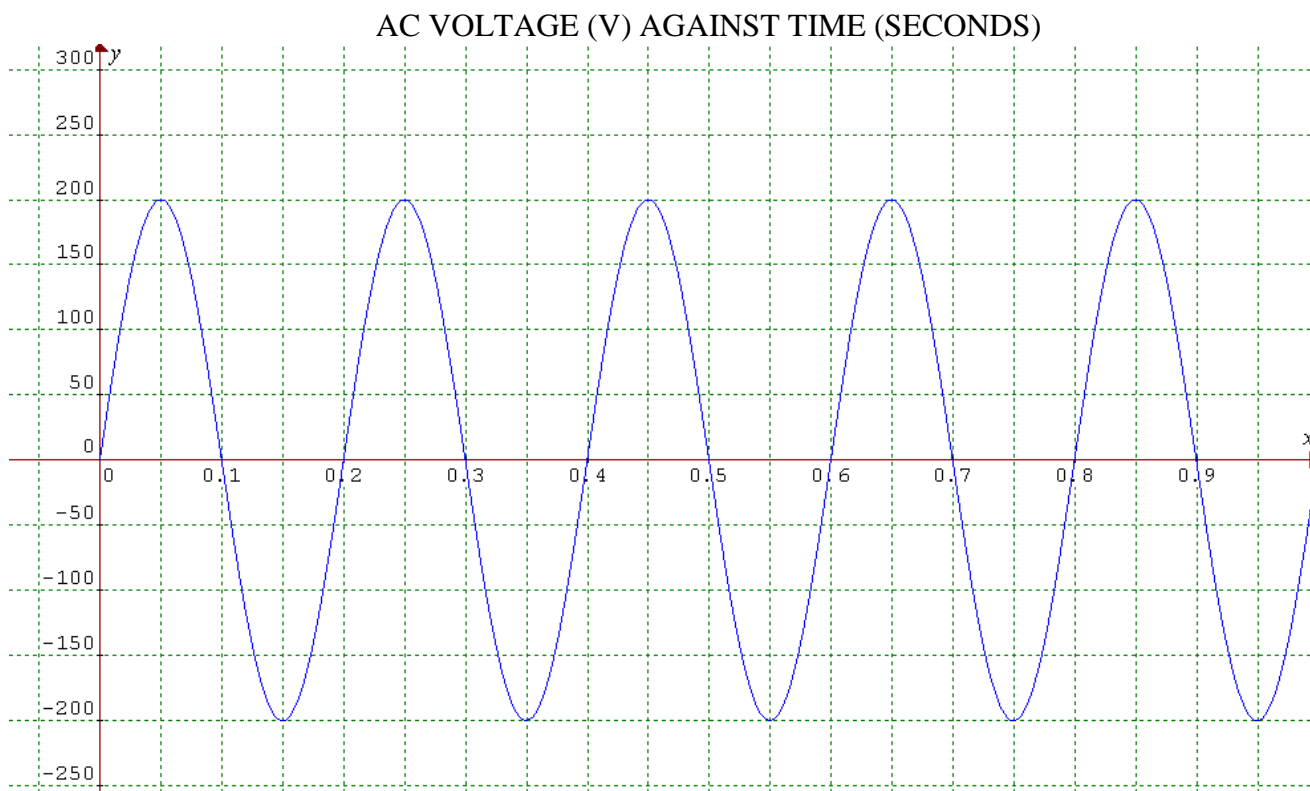
Review Questions

A portable generator is used to power the lights at a showground. It produces voltage at a constant value of 550 V DC. Two lines connect the generator and the lights, each with a resistance of 5 Ω . The distance between the generator and the lights is approximately 300 m. When the lights are on, the generator has an output of 30 A.

20. Calculate the power output of the generators.

21. Calculate the power loss in the transmission lines

22. A data logger was attached to a circuit. The AC current was established over a period of 1 second. The graph produced is shown below.



Use the data from the graph to assist you to calculate the following:

- i. RMS voltage
- ii. Period
- iii. Frequency
- iv. Peak to Peak voltage
- v. Was this circuit connected to a standard Australian power supply? Provide 2 reasons to support your answer.

The following equations are useful for exam purposes.

$$I_{line} = \frac{P_{gen}}{V_{trans}}$$

Where P_{gen} = power generated by the station and V_{trans} = voltage transmitted by the step-up transformer.

$$P_{loss} = I_{line}^2 \times R_{line}$$

Where I_{line} = the current in the transmission lines and R_{line} = the resistance in the transmission lines.

$$V_{loss} = I_{line} \times R_{line}$$

$$V_{city} = V_{trans} - V_{loss}$$

$$P_{city} = P_{gen} - P_{loss}$$

$$P_{loss} = I_{line} V_{loss}$$

Where V_{city} and P_{city} are the voltage and power at the primary side of the first step-down transformer at the city being supplied with the power.

Review Question

23. Complete the following table.

Assume that $P_{gen} = 500 \text{ MW}$ and $R_{line} = 10 \Omega$

$V_{trans}(\text{kV})$	$I_{line}(\text{A})$	$P_{loss}(\text{MW})$	$P_{city}(\text{MW})$	$P_{loss}(\%)$	$V_{loss}(\text{kV})$	$V_{city}(\text{kV})$
500						
300						
150						

Solutions to Review Questions

1. $P = IV$

$$I = 12.5 \text{ A}$$

$$V = 120 \text{ V}$$

$$P = 12.5 \times 120$$

$$P = 1500 \text{ W}$$

2. $P = I^2 R$

$$I = 1 \times 10^{-3} \text{ A}$$

$$R = 600 \Omega$$

$$P = (1 \times 10^{-3})^2 \times 6000$$

$$P = 6 \times 10^{-3} \text{ W}$$

3. $P = \frac{V^2}{R}$

$$V = 12 \text{ V}$$

$$R = 1000 \Omega$$

$$P = \frac{12^2}{1000}$$

$$P = 0.144 \text{ W}$$

4. The core must be easy to magnetise to enable the direction of the magnetic field to change as quickly as the current changes direction.

5. If the current is DC there will be no change in magnetic flux so the primary coil will have no influence on the secondary coil. There will however be a momentary current generated when the DC source is turned on or off

6. $P = VI$

$$P = 300 \text{ MW}$$

$$V = 20 \text{ kV}$$

$$I = \frac{P}{V}$$

$$I = \frac{300 \times 10^6}{20 \times 10^3}$$

$$I = 1.5 \times 10^4 \text{ A}$$

$$7. \quad P_{loss} = I^2 R$$

$$P_{loss} = (1.5 \times 10^4)^2 \times 0.8$$

$$P_{loss} = 1.8 \times 10^8 \text{ W}$$

$$8. \quad I = \frac{P}{V}$$

$$I = \frac{300 \times 10^6}{250 \times 10^3}$$

$$I = 1200 \text{ A}$$

$$9. \quad P_{loss} = I^2 R$$

$$P_{loss} = (1200)^2 \times 0.8$$

$$P_{loss} = 1.15 \times 10^6 \text{ W}$$

10. The difference between the 2 answers is as expected. The power loss is much greater without the transformer.

11. The current, amount of power loss and percentage power loss should all increase.

12. The answers are as follows:

The turns ratio is calculated using:

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

$V_s = 100 \text{ kV}$ as this is the voltage of transmission

$V_p = 20 \text{ kV}$ as this is the voltage of operation

$$\frac{N_s}{N_p} = \frac{100 \times 10^3}{20 \times 10^3} = \frac{5}{1}$$

The current in the transmission lines is calculated using:

$$\frac{I_p}{I_s} = \frac{N_s}{N_p} = \frac{5}{1}$$

and $I_p = 17 \text{ kA}$

$$\frac{I_p}{I_s} = 5$$

$$I_s = \frac{I_p}{5} = \frac{17000}{5}$$

$$I_s = 3400 \text{ A}$$

Calculate the power loss in the lines using:

$$P_{loss} = I^2 R$$

$$P_{loss} = 3400^2 \times 5$$

$$P_{loss} = 57800000 = 57.8 \text{ MW}$$

$$\text{Percentage power loss} = \frac{57.8 \times 10^6 \times 100}{340 \times 10^6} = 17\%$$

13. The advantage of transmission occurring at 500 kV rather than 100 kV is that stepping up the voltage in the transmission lines to the higher level decreases the current by the same proportion. As a result power loss is greatly minimised. In this example 0.7% power was lost when transmission was at 500 kV compared to 17% power being lost when transmission was at 100 kV.

14. This is a step up transformer as the secondary coil has more turns than primary coil.

$$\mathbf{15.} \quad \frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$V_s = \frac{N_s}{N_p} \times V_p$$

$$V_s = \frac{1500}{100} \times 240$$

$$V_s = 3600 \text{ V}$$

$$\mathbf{16.} \quad V_p = \frac{N_p}{N_s} \times V_s$$

$$V_p = \frac{200}{8} \times 12$$

$$V_p = 300 \text{ V}$$

17. A step down transformer. The output voltage is lower than the input.

$$\mathbf{18.} \quad N_s = \frac{V_s}{V_p} \times N_p$$

$$N_s = \frac{240}{48000} \times 10000$$

$$N_s = 50 \text{ turns}$$

19. The answers are as follows:

$$\mathbf{i.} \quad N_p = \frac{V_p}{V_s} \times N_s$$

$$N_p = \frac{240}{12} \times 80$$

$$N_p = 1600 \text{ turns}$$

ii. $I = \frac{P}{V}$
 $P = 40 \times 5 = 200 \text{ W}$
 $V = 12 \text{ V}$
 $I = \frac{200}{12}$
 $I = 16.7 \text{ A}$

20. $P = VI$
 $V = 550 \text{ V}$
 $I = 30 \text{ A}$
 $P = 550 \times 30$
 $P = 16500 \text{ W}$

21. $P_{loss} = I^2 R$
 $I = 30 \text{ A}$
 $R = 2 \times 5 = 10 \Omega$ (each of the lines has a resistance of 5Ω so the total resistance is 10Ω)
 $P_{loss} = 30^2 \times 10$
 $P_{loss} = 9000 \text{ W}$

22. The answers are as follows:

i. $V_{RMS} = \frac{V_{peak}}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.4 \text{ V}$

ii. Period, $T = 0.2 \text{ s}$

iii. $f = \frac{1}{T}$
 $f = \frac{1}{0.2} = 5 \text{ Hz}$

iv. $V_{peak\ to\ peak} = 400 \text{ V}$

v. No this circuit was not connected to an Australian power supply. The Australian power supply is $240V_{RMS}$ @ 50Hz . This circuit is connected to a source of $141.4V_{RMS}$ @ 5Hz

23. The table should have the following information.

$V_{trans}(\text{kV})$	$I_{line}(\text{A})$	$P_{loss}(\text{MW})$	$P_{city}(\text{MW})$	$P_{loss}(\%)$	$V_{loss}(\text{kV})$	$V_{city}(\text{kV})$
500	1000	10	490	2	10	490
300	1667	27.8	472.2	5.6	16.7	283.3
150	3333	111	389	22.2	33.3	116.7



Physics

Teach Yourself Series

Topic 11: Properties of mechanical waves (Unit 4)

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What you should know

As it appears in Unit 4

- explain a wave as the transmission of energy through a medium without the net transfer of matter
- distinguish between transverse and longitudinal waves
- identify the amplitude, wavelength, period and frequency of waves
- calculate the wavelength, frequency, period and speed of travel of waves using:
$$v = f\lambda = \frac{\lambda}{T}$$
- explain qualitatively the Doppler effect
- Describe diffraction as the directional spread of various frequencies in terms of different gap width or obstacle size, including the significance of the magnitude of the $\frac{\lambda}{w}$ ratio;
- explain resonance as the superposition of a travelling wave and its reflection, and with reference to a forced oscillation matching the natural frequency of vibration.
- analyse the formation of standing waves in strings fixed at one or both ends

Mechanical waves

As it appears in Unit 4

Characteristics of waves

Waves are defined as disturbances that travel through a medium without carrying any of the medium along with them.

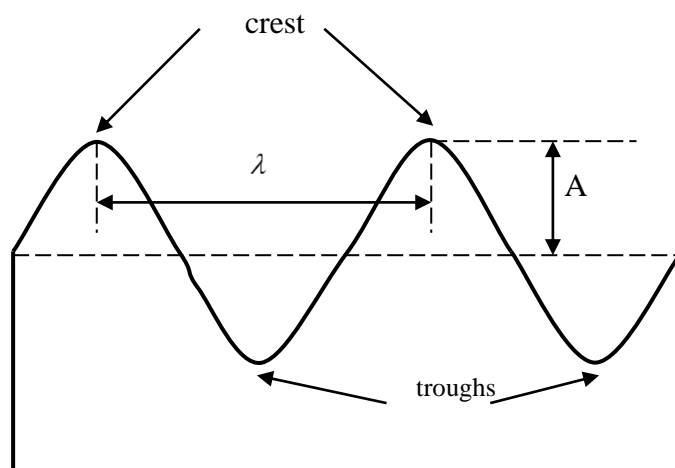
The substance through which the wave travels is called the **medium**. The medium itself is not changed as a result of the wave motion.

Waves can be classified as **mechanical** or **electromagnetic** waves. Mechanical waves, such as water waves or sound waves need a medium to travel through. But electromagnetic waves can travel through vacuum as well.

Waves that repeat themselves at regular intervals are called **periodic or continuous** waves

A source of energy is needed to create such waves.

The **amplitude**, A , of a wave is the **maximum displacement** of the particles of the medium from their rest position.



The **frequency** (f) of a wave is the number of waves generated per second, or the number of waves passing a given point per second. Frequency is measured in hertz (Hz).

The **period**, T , of a wave is the time it takes before a wave repeats itself. It is also the time for a complete oscillation of a particle of the medium to occur. Period is measured in seconds (s).

The period and frequency are related
$$f = \frac{1}{T}$$

The **speed**, v , of a wave is the rate at which a wave covers distance. Speed is measured in metre per second (m s^{-1}).

The **wavelength**, λ , is the distance that one wave covers before it repeats itself. The **wavelength** is also the distance between two consecutive crests or troughs. Furthermore it is also the distance travelled by the wave in one period. Wavelength is measured in metre (m).

Wave equation

It is an important relation between the speed, frequency and wavelength of a wave.

$$v = f\lambda$$

v is the speed of the wave, expressed in (m s^{-1})

f is the frequency of the wave, expressed in (Hz)

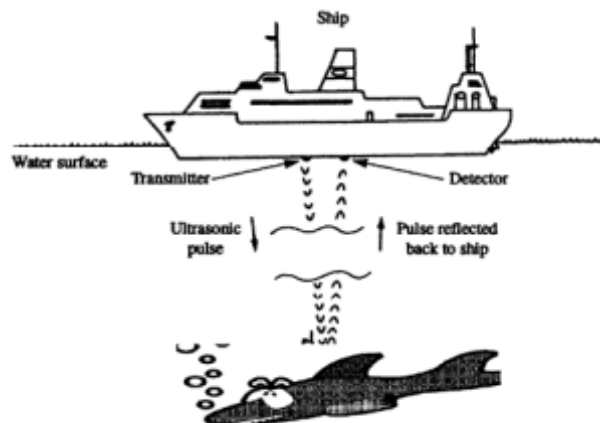
λ is the wavelength of the wave, expressed in (m)

The **wave equation** is true for all types of waves.

Review Questions

Question 1

The diagram shows a ship using an Echo locator (SONAR) to find a school of fish. The pulsed wave is transmitted from the ship, reflected off the school and picked up by a receiver

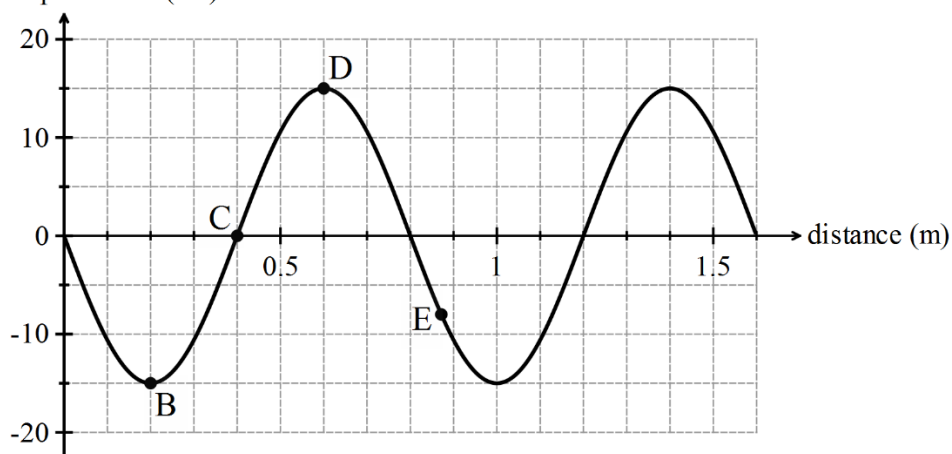


The time taken to receive the echo is 0.2s after transmission. Calculate how deep the ship has to lower its fishing nets to catch the school if the speed of sound in water is 1300 ms^{-1} .

Question 2

The displacement–distance graph shows a snapshot of a transverse wave as it travels along towards the right. The speed of the wave is 12 m s^{-1} .

particle displacement (cm)



- Use the graph to determine the wavelength and the amplitude of this wave.
- At the moment shown, state the direction in which the following particles are moving:
B, C, D and E.
- Find the frequency and the period of the wave.

Question 3

Calculate the period and frequency of a spring that undergoes 16 vibrations in 8 seconds.

Question 4

A wave with a frequency of 500 Hz is traveling at a speed of 200 m s^{-1} . What is the wavelength of the wave?

Transverse and Longitudinal waves

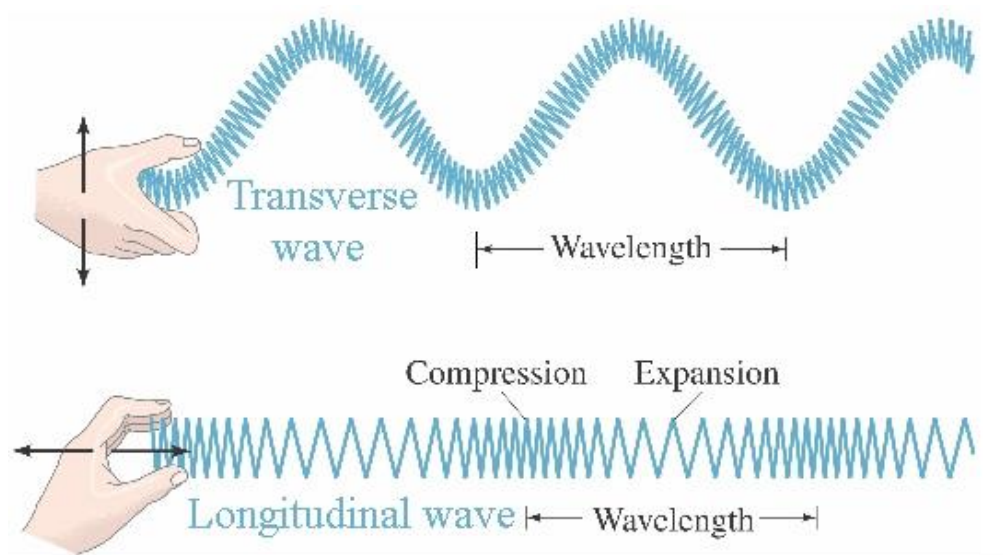
As it appears in Unit 4

Longitudinal waves

Longitudinal waves are waves in which the particles of the medium oscillate about a mean position in the **same** line as the **direction** of travel of the wave. Sound waves are longitudinal waves.

Transverse waves

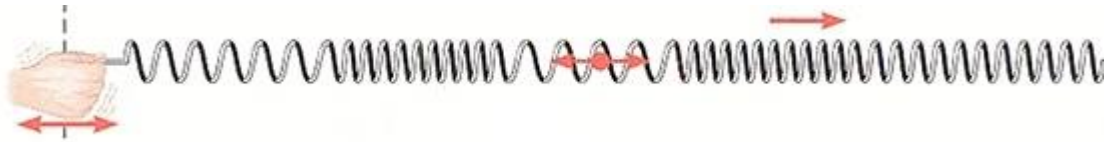
Transverse waves are waves in which the particles of the medium oscillate about a mean position **perpendicular** to the line of travel of the wave. As the individual particles of the medium move up and down about their rest position, a series of **crests** and **troughs** move across the medium. Water waves and electromagnetic waves are transverse waves.



Review Questions

Question 5

Is the wave in the spring longitudinal or transverse? Give a reason for your answer



Question 6

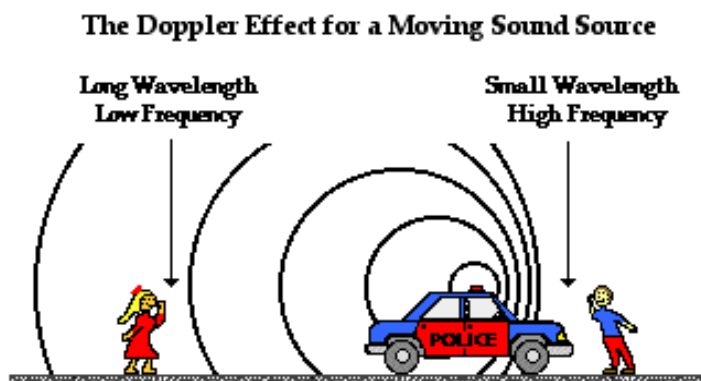
The energy in a wave is travelling from east to west. The particles in the wave medium are oscillating along a north-south axis. Is the wave longitudinal or transverse? Give a reason for your answer.

Doppler Effect

As it appears in Unit 4

The Doppler Effect is a wave phenomenon that can be observed whenever the source of the wave is moving relative to the observer. It can be described as the effect produced by a moving source in which there is an apparent upwards shift in frequency for observers when the sound source is approaching and an apparent downwards shift in frequency when the sound source is receding. It does not result from an actual change in the frequency of the source, just an apparent change in what the observer experiences.

An example you would be familiar with is when you hear an emergency vehicle approaching with its siren on and then you notice an abrupt change in the frequency of the siren when it goes past you. If you are standing still when the vehicle is coming toward you, the frequency is higher than it would be if the vehicle was stationary; when the vehicle moves away from you, the frequency is lower. This is due to the relative velocity of the moving object and the observer. If the observer is stationary, the wavelength of the emitted sound appears to be shortening as the sound approaches, creating a higher frequency, and lengthening as it moves away, hence creating the lower frequency.



Similar changes in observed frequency occur when an observer is walking towards or away from a stationary source of sound. Again, the changes occur because of the relative velocity of the object and the moving observer.

Review Question

Question 7

What will happen to the observed frequency of a sound when an observer is walking towards the sound source?

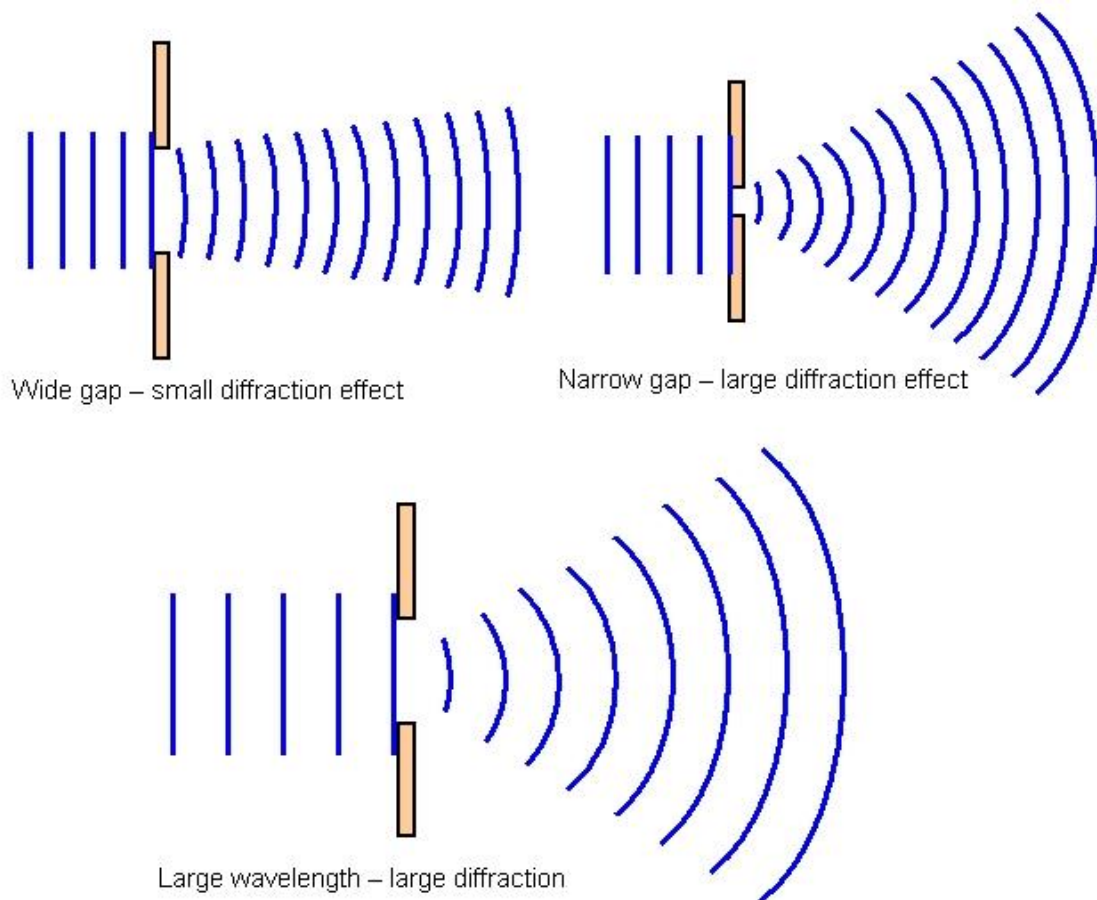
- A It will increase because the apparent wavelength increases.
- B It will increase because the apparent wavelength decreases.
- C It will decrease because the apparent wavelength increases.
- D It will decrease because the apparent wavelength increases.

Diffraction of Waves

As it appears in Unit 4

Diffraction is the bending of waves as they pass the edge (or edges) of an obstacle or pass through an aperture.

Significant Diffraction will occur when the wavelength is of at least the same order of magnitude as the width of the obstacle or aperture ($\frac{\lambda}{w} \geq 1$).



Review Questions

Question 8

A musical instrument is producing a note with a frequency of 1000 Hz, and this sound passes through a half-open slide door with a gap about 0.3 m wide. (Assume that the speed of the sound is 340 m s^{-1})

- a. What is the wavelength of the sound?

Is there significant diffraction when the sound goes through the door?

Question 9

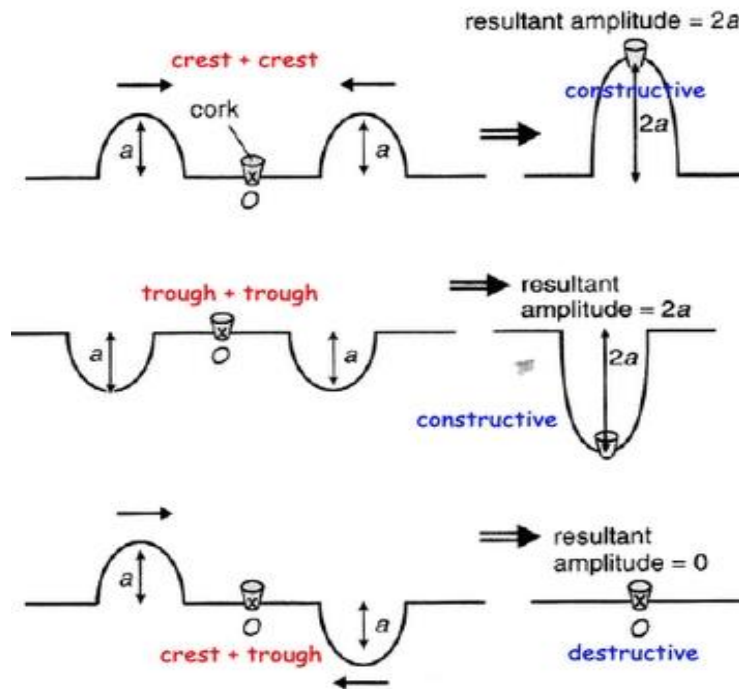
A 150 Hz sound and an 800 Hz sound go through a 2.2 m gap in a wall. Which frequency will be diffracted most and why? Assume the speed of sound is 340 m s^{-1} .

Standing Waves, Harmonics & Resonance

As it appears in Unit 4

Superposition

When two or more longitudinal or transverse waves meet, the resulting displacement at each point will be the vector sum of the displacements of the component waves. This is the principle of **Superposition**. After the interaction, the pulses continue unaltered: they do not permanently affect each other.

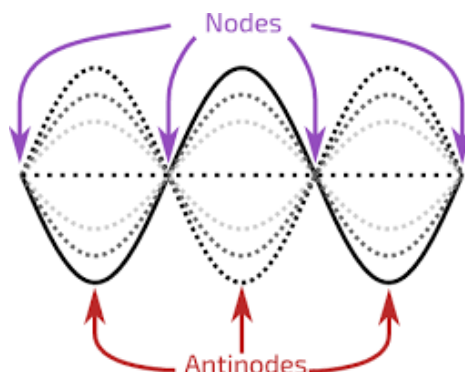


Standing waves

If a set of waves moving along a string is reflected from the end of the string then the two sets of waves (moving in opposite directions) will be superimposed on each other.

Because both sets of waves have the same amplitude and wavelength there are points, called nodes, where they will always superimpose destructively and have no amplitude, and other points, called antinodes, where they will always superimpose constructively and have maximum amplitude.

The resulting waveform is called a standing wave as it appears to be stationary with regions of the string vibrating up and down separated by points of no movement.



Harmonics on strings

If a string is fixed at both ends certain frequencies of vibration can form standing waves on the string as long as the ends of the string are nodes. These frequencies are called harmonics and are the natural frequencies at which the string will vibrate.

The lowest frequency is called the fundamental frequency or first harmonic.

The next highest frequency is called the first overtone or second harmonic.

The next highest frequency is called the second overtone or third harmonic and so on.

The wavelengths and frequencies of the harmonics are related to the length, L , of the string and the speed, v , of the waves as shown in the following diagrams.

first harmonic
(fundamental
frequency)



$$\lambda_1 = 2L \quad f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

second harmonic
(first overtone)



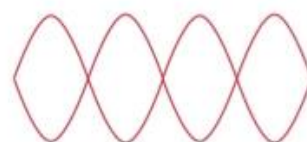
$$\lambda_2 = L \quad f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1$$

third harmonic
(second overtone)



$$\lambda_3 = \frac{2L}{3} \quad f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3f_1$$

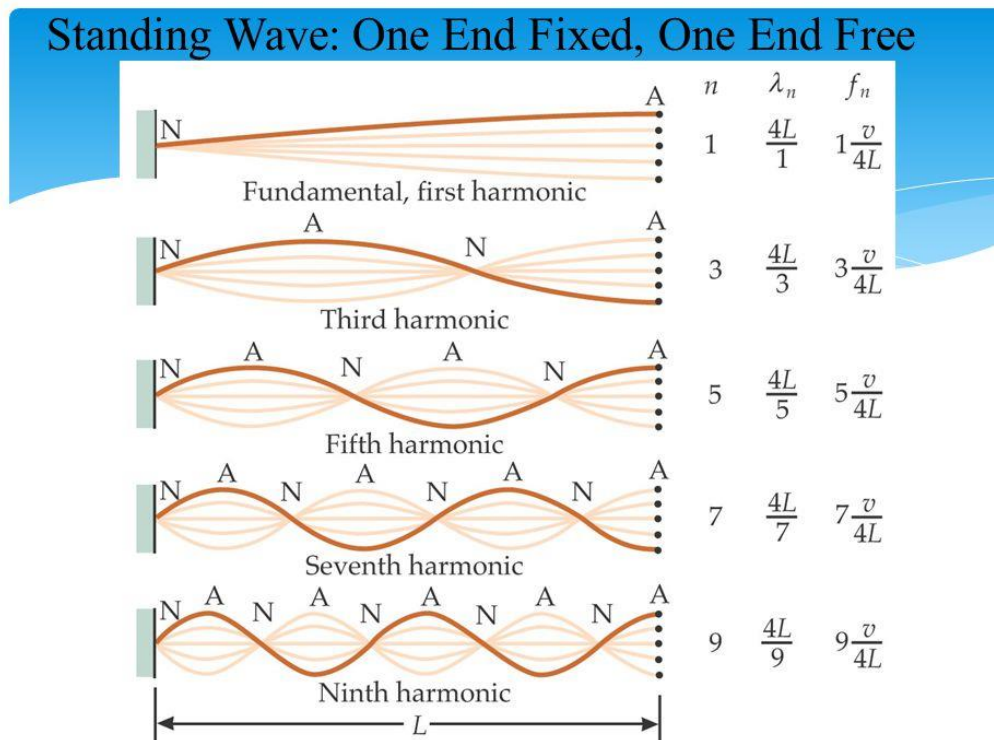
fourth harmonic
(third overtone)



$$\lambda_4 = \frac{L}{2} \quad f_4 = \frac{v}{\lambda_4} = \frac{2v}{L} = 4f_1$$

Note that the frequencies of higher harmonics are integer multiples of the fundamental frequency

If a string is fixed at one end only then the fixed end will be a node but the free end will be an antinode. This leads to a different set of harmonics as shown in the diagram below.



Note that the wavelength of the fundamental frequency is $4L$, rather than $2L$ as in the string fixed at both ends.

Also, only the odd harmonics are present in this case, with the first overtone being the third harmonic, the second overtone being the fifth harmonic and so on.

Resonance

Resonance in an object or string or column of air) occurs when an external vibration causes the object to vibrate. This happens when the frequency of the external vibration (known as the forcing frequency) equals one of the natural frequencies of the object.

Examples of this are when a string of a musical instrument starts vibrating when a note on another instrument is played or when an object in a house vibrates when the washing machine is spinning at a certain rate.

Review Questions

Question 10

A guitar player wants to tune a guitar for his friend. He adjusts a string to an effective length of 34 cm.

a. What is the wavelength of its fundamental frequency?

b. What is the wavelength of its second overtone?

c. What is the wavelength of the fourth harmonic?

After the tuning, the guitar player struck one string and made a note of 550 Hz. He found that the other string with fundamental frequency of 2200 Hz also vibrates.

d. These two strings are not connected and are some distance apart, how can this happen, and why?

Question 11

A steel ruler is pushed so that 48 cm of its length hangs over the edge of a desk. The ruler is clamped to the desk at the edge of the desk. A sound is produced by pulling down the end of the ruler and then releasing it. What is the wavelength of its second overtone?

Solutions to Review Questions

1. $v = \frac{d}{t}$
 $1300 \times 0.1 = \text{distance}$
 $\text{distance} = 130 \text{ m}$

2. a. $\lambda = 0.8 \text{ m}$, amplitude = 15 cm

b. B-not moving, C-down, D-not moving, E-up

c. $v = f\lambda$
 $f = \frac{12}{0.8} = 15 \text{ Hz}$
 $T = \frac{1}{f} = \frac{1}{15} = 0.067 \text{ s}$

3. $T = \frac{8}{16} = 0.5 \text{ s}$
 $f = \frac{1}{T} = \frac{1}{0.5} = 2 \text{ Hz}$

4. $v = f\lambda$
 $200 = 500 \times \lambda$
 $\lambda = 0.4 \text{ m}$

5. Longitudinal wave – particle movement is left and right and wave movement is right.

6. Transverse wave – particle movement is at right angles to the wave movement.

7. B.

8. The 150 Hz sound

$$f = 150 \text{ Hz}, \lambda = \frac{340}{150} = 2.27 \text{ m}$$

$$\frac{\gamma}{w} \approx 7.3 \therefore \text{significant diffraction}$$

$$f = 800 \text{ Hz}, \lambda = \frac{340}{800} = 0.425 \text{ m}$$

$$\frac{\gamma}{w} < 1 \therefore \text{some diffraction but less than for the 150 Hz sound.}$$

9.

a. 0.34 m

$$\lambda = \frac{v}{f} = \frac{340}{1000} = 0.34 \text{ m}$$

b. Yes, because $\frac{\lambda}{w} \geq 1$, there is significant diffraction.

10.

a. 0.68 m

First harmonic: $\lambda_1 = 2L = 2 \times 0.34 = 0.68 \text{ m}$

b. 0.23 m

second overtone = third harmonic

$$\lambda_3 = \frac{2L}{3} = \frac{2 \times 0.34}{3} = 0.23 \text{ m}$$

c. 0.17 m

$$\lambda_4 = \frac{L}{2} = \frac{0.34}{2} = 0.17 \text{ m}$$

d. This situation involves resonance. $550 \text{ Hz} \times 4 = 2200 \text{ Hz}$. It corresponds to the fourth harmonic of the forcing vibration by the 550 Hz.

11. For a vibrating object fixed at one end, the second overtone is the fifth harmonic.

$$\lambda_5 = \frac{4L}{5} = \frac{4 \times 0.48}{5} = 0.384 \text{ m}$$



Physics

Teach Yourself Series

Topic 12: Light as a wave (Unit 4)

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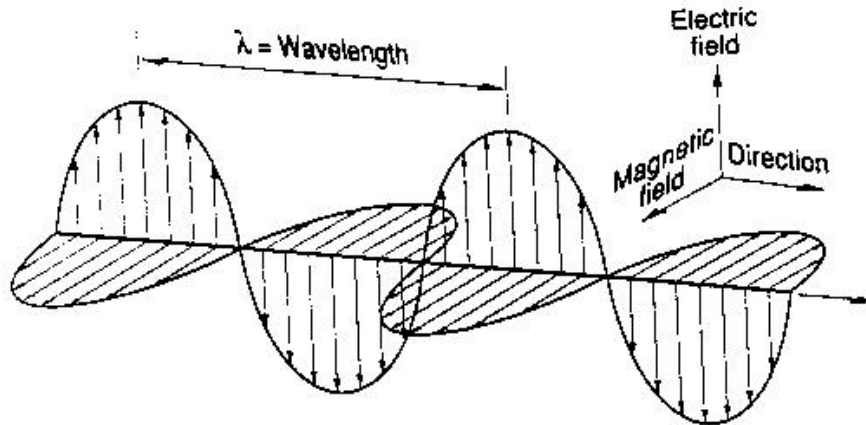
What you should know

As it appears in Unit 4

- describe light as an electromagnetic wave which is produced by the acceleration of charges, which in turn produces changing electric fields and associated changing magnetic fields
- identify that all electromagnetic waves travel at the same speed, c , in a vacuum
- compare the wavelength and frequencies of different regions of the electromagnetic spectrum, including radio, microwave, infrared, visible, ultraviolet, x-ray and gamma, and identify the distinct uses each has in society
- explain polarisation of visible light and its relation to a transverse wave model
- investigate and analyse theoretically and practically the behaviour of waves including:
 - refraction using Snell's Law $n_i \sin \theta_i = n_r \sin \theta_r$ and $n_1 v_1 = n_2 v_2$
 - total internal reflection and critical angle including applications: $\sin \theta_c = \frac{n_r}{n_i}$
- investigate and explain theoretically and practically colour dispersion in prisms and lenses with reference to refraction of the components of white light as they pass from one medium to another

Electromagnetic waves

As it appears in Unit 4



Electromagnetic waves are made up of oscillating electric and magnetic fields that generate each other. An electromagnetic wave travels in three dimensions simultaneously. The electric and magnetic vectors oscillate in mutually perpendicular planes.

Electromagnetic waves are **transverse waves**, as the electric and magnetic fields both vibrate at right angles to the direction of propagation of the waves. Electromagnetic waves can travel through vacuum. Electromagnetic waves are produced by the acceleration of electric charges.

Visible light can be produced in a synchrotron when electrons are accelerated in circular orbits by deflecting magnetic fields. All electromagnetic waves travel through vacuum with the same speed, $c = 3 \times 10^8 \text{ m s}^{-1}$

Frequency and wavelength can be measured for all electromagnetic waves

The wave equation for electromagnetic waves is

$$c = f\lambda$$

The Electromagnetic Spectrum

As it appears in Unit 4

- compare the wavelength and frequencies of different regions of the electromagnetic spectrum, including radio, microwave, infrared, visible, ultraviolet, x-ray and gamma, and identify the distinct uses each has in society

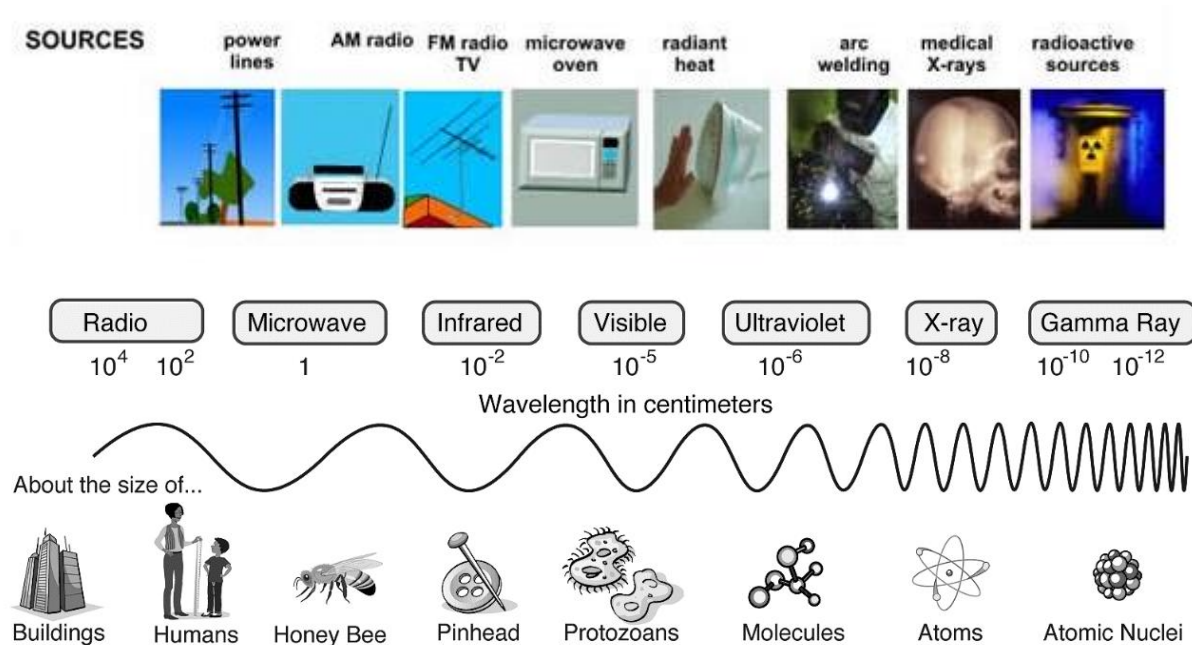
The *electromagnetic spectrum* comprises all of the known types of electromagnetic radiations such as:

radiowaves ultraviolet radiation

microwaves X-rays

infrared gamma rays

visible spectrum



Review Question

Question 1

Which 2 electromagnetic waves lay between Visible light and Gamma waves in the Electromagnetic Spectrum?

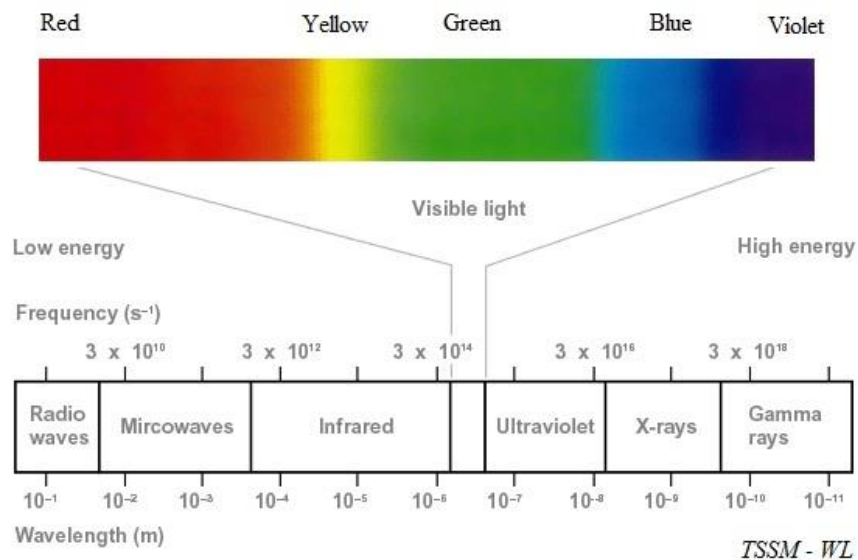
Electromagnetic model of light

In its interactions with matter, light acts like an electromagnetic wave. Visible light occupies a small band within the electromagnetic spectrum. It has a continuous range of wavelengths from 400 nm to 750 nm.

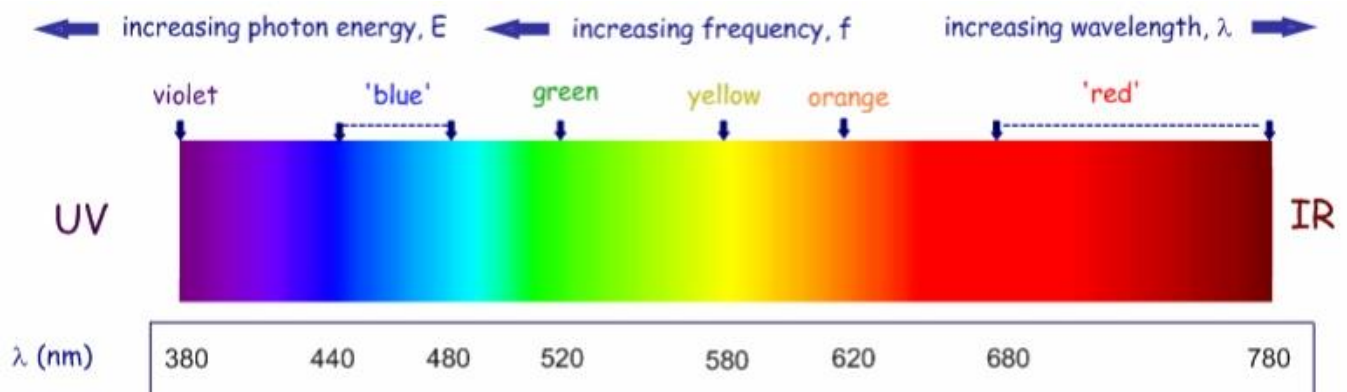
The red end of the spectrum of visible light has the longest wavelength. $\lambda_r = 750 \text{ nm}$

$1 \text{ nm} = 10^{-9} \text{ m}$, *nm* stands for nanometer.

The violet end of the spectrum has the shortest wavelength. $\lambda_v = 400 \text{ nm}$



Further information on the visible spectrum



Review Questions

Question 2

What is common between different types of electromagnetic radiations

Question 3

To which part of the spectrum does a wave with a frequency of $3.0 \times 10^{13} \text{ Hz}$ belong?

Question 4

Which part of the spectrum has the least penetrating power? Explain

Polarisation

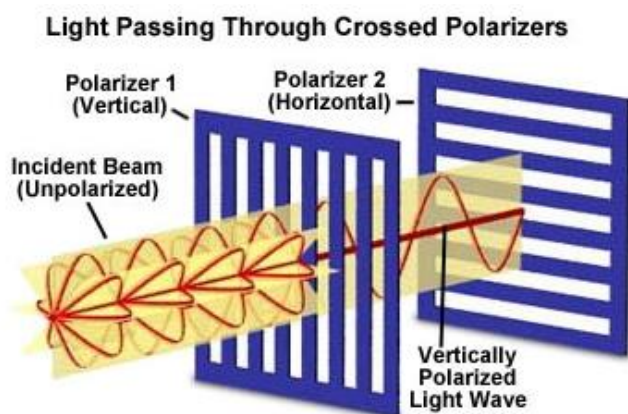
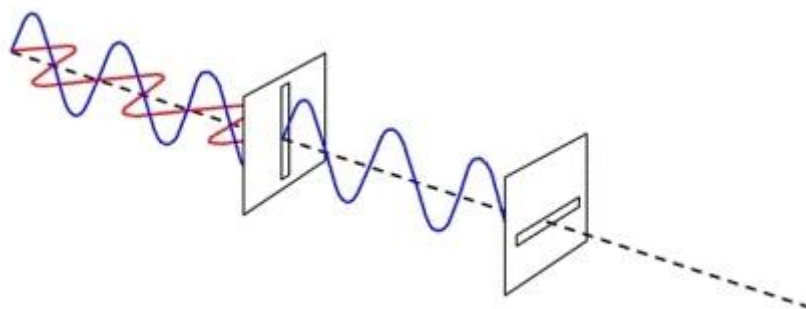
As it appears in Unit 4

The electric field vector of light emitted from a luminous source is constantly changing direction because the atoms emitting the light are in random motion. Hence the natural light is ***unpolarised***.

If the direction of the electric field vector doesn't change in time, then the light is ***polarised***. Only transverse waves can be polarised. Therefore polarisation of light gives evidence of the transverse nature of electromagnetic waves.

Natural unpolarised light becomes polarised when passing through a ***polaroid sheet*** or when reflected. When two polaroid sheets have their polarising planes parallel, light is transmitted. When two polaroid sheets have their polarising planes at right angles, light is not transmitted.

A polaroid is made from many small polarising and transparent crystals on a polyvinyl plastic base. Polaroid lenses in sunglasses reduce the glare of light reflected from surfaces.



Review Questions

Question 5

Explain, with the aid of a diagram, how the transverse wave model can account for how unpolarised light can become polarised.

Question 6

Can a sound wave be polarised? Explain

Reflection

As it appears in Unit 4

Straight-line propagation of light

Light travels in straight lines through any given medium.

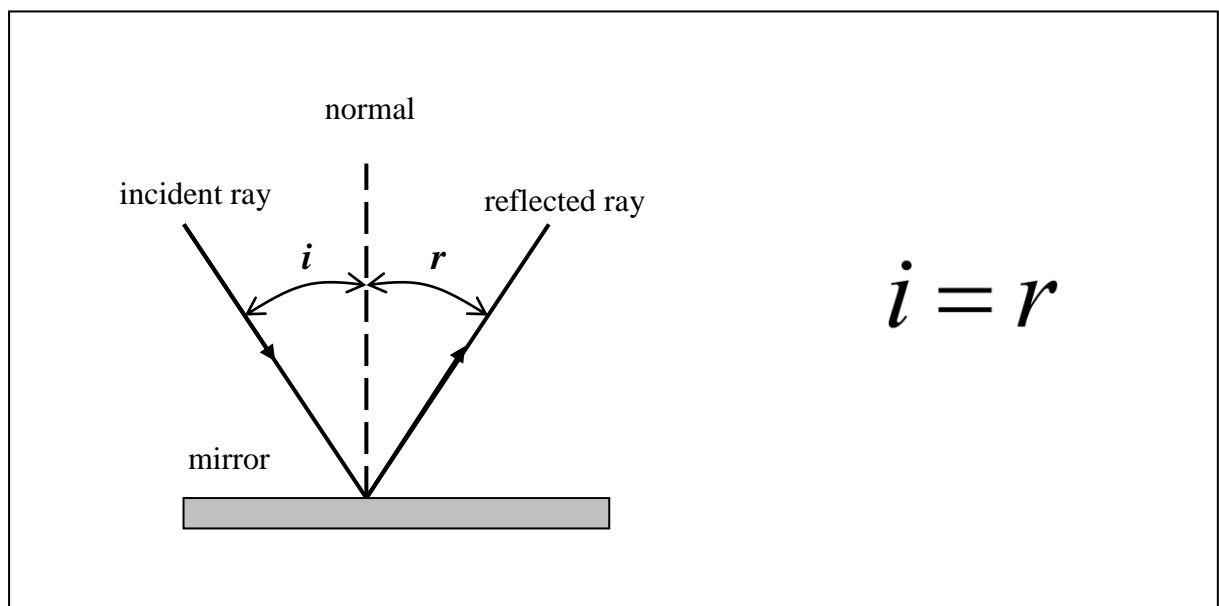
It enables:

1. The formation of shadows to be explained.
2. The position of images in mirrors and lenses to be determined.
3. Optical instruments to be designed.
4. Us to see things clearly.

Laws of reflection

First Law of reflection states that:

The incident ray, the reflected ray and the normal to the surface at the point of incidence are coplanar and are on the same side of the reflecting surface.



Second Law of reflection states that:

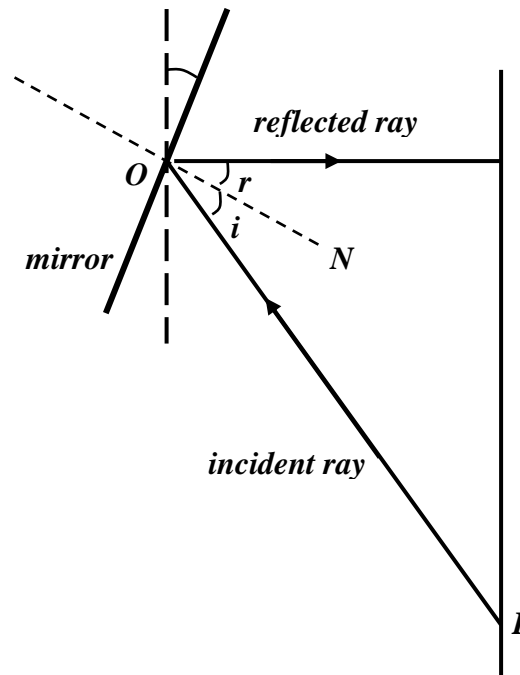
The angle of incidence is equal to the angle of reflection

$$i = r$$

Review Question

Question 7

The diagram shows a small plane mirror that is tilted at an angle of 15° to the vertical. The mirror reflects an incident ray IO upon a vertical screen, as in the diagram. The reflected ray is horizontal and the distance IO is equal to 2 m.



- Determine the value of the angle of incidence i .
- Determine the value of the angle of reflection r .

Refraction

As it appears in Unit 4

Index of refraction

When a ray of light travels from one transparent medium into another, it changes direction. This phenomenon is called **refraction**. The **refraction** is caused by a **change in the speed** of light as it passes from one medium into another.

Each transparent medium has an **absolute index of refraction**. It is equal to the ratio between the speed of light in vacuum and the speed of light in the medium.

$$\text{index of refraction} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} \quad n = \frac{c}{v}$$

For vacuum the index of refraction $n = 1$ (exactly).

For air the index of refraction $n = 1$ (approx.)

As $v < c$, the index of refraction for all transparent media is greater than one, $n > 1$.

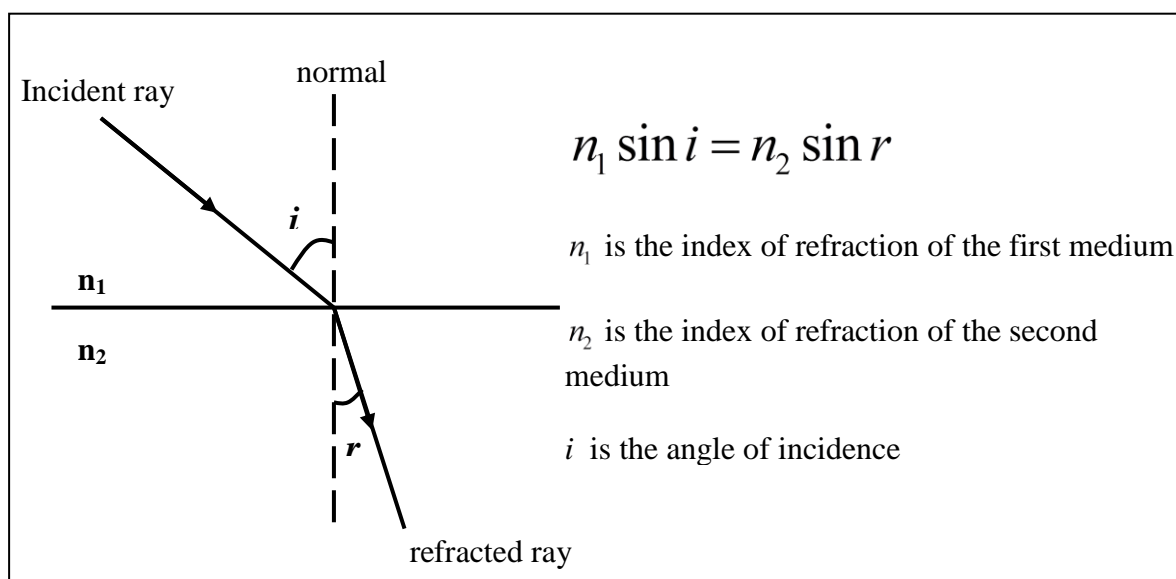
Laws of refraction

First Law of Refraction

The incident ray, the normal and the refracted ray are coplanar.

Second Law of Refraction or Snell's Law

Snell's law is the quantitative expression of the relationship between the angle of incidence and the angle of refraction.



Snell's Law can also be written as:

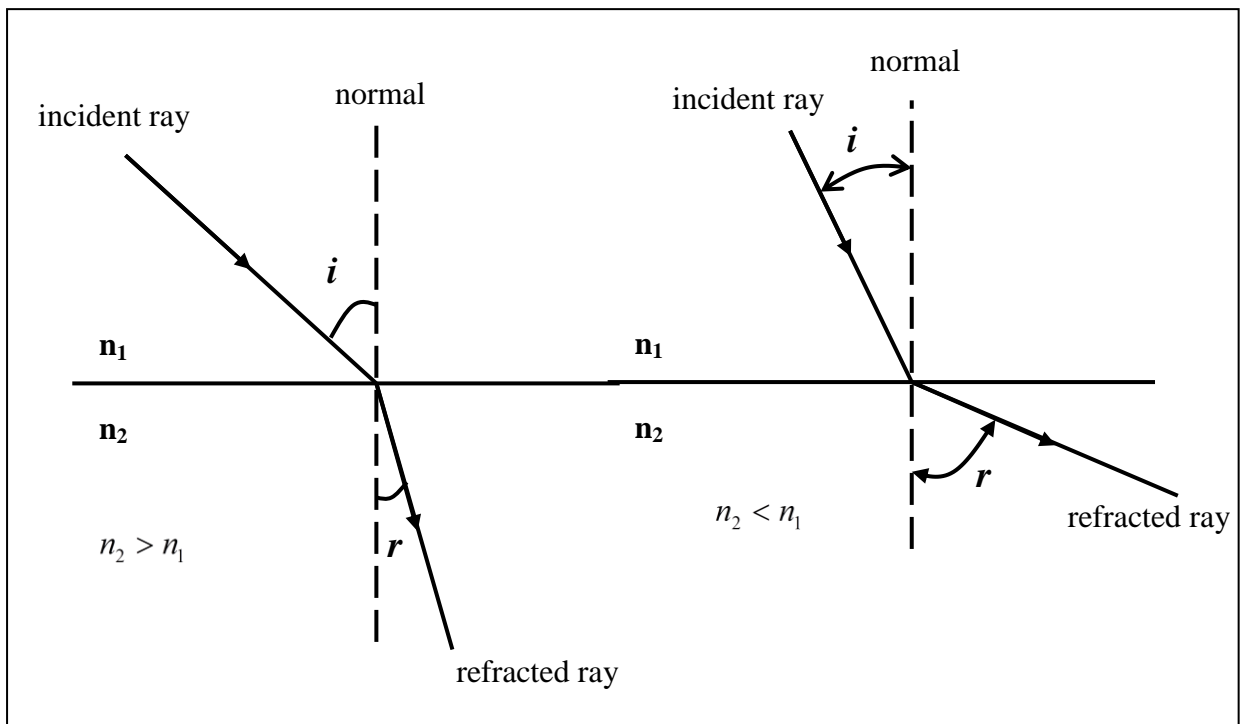
$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

$\frac{n_2}{n_1}$ is called the relative index of refraction of the second medium with respect to the first.

Refraction towards or away from normal

When light enters a medium in which it travels more slowly ($n_2 > n_1$), it is refracted towards the normal.

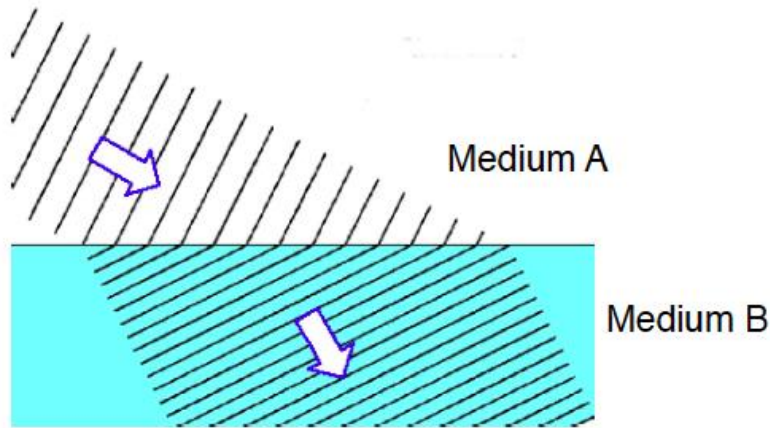
When light enters a medium in which it travels more quickly ($n_2 < n_1$), it is bent away from the normal.



Review Questions

Question 8

The diagram below shows a diagram of waves meeting the boundary between two media of different optical densities



Which medium would you expect to have the higher refractive index? Explain your answer.

Question 9

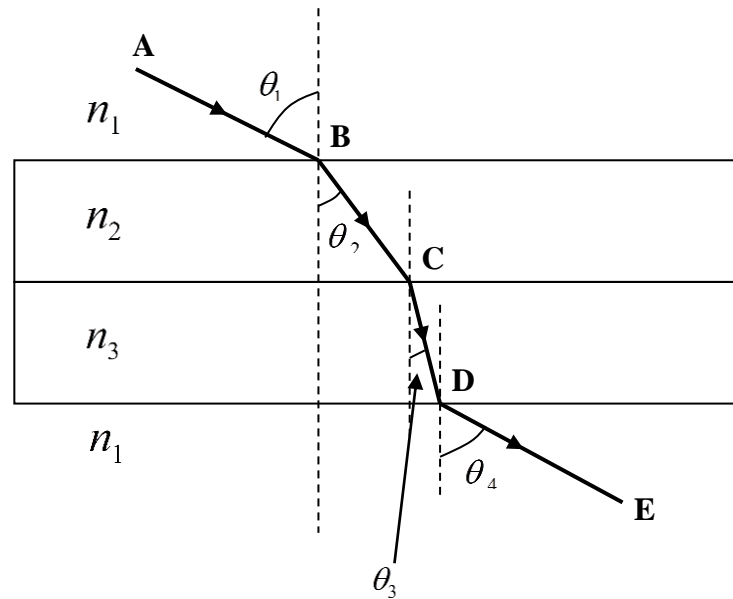
Light travels from crown glass ($n = 1.52$) into air ($n = 1.00$). The angle of refraction in air is 60° . What is the angle of incidence in glass?

Question 10

A ray of light in air strikes a block of quartz at an angle of incidence of 30° . The angle of refraction is 20° . What is the refractive index of the quartz?

Multiple refractions

When a ray of light undergoes multiple successive refractions, each angle plays a double role: the angle of refraction and the angle of incidence for the next refraction.



The Snell's Law for all the refractions involved can be written as follows:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 = n_3 \sin \theta_3 = n_1 \sin \theta_4 = \text{constant}$$

Review Question

Question 11

A ray of light is directed from medium 1 into medium 2 and then medium 3 and then into medium 1 again along the ABCDE path as shown in the diagram above.

- a. Which of the following statement(s) regarding the indices of refraction is/are correct (one or more answers)?

A $n_1 < n_2 < n_3$

B $n_1 > n_2 > n_3$

C $n_1 < n_2 > n_3$

D $n_1 > n_2 < n_3$

- b. Let v_1 , v_2 and v_3 be the speed of light in medium 1, medium 2 and medium 3 respectively. Which of the following statement(s) regarding the speed of light is/are correct (one or more answers)?

A $v_1 < v_2 < v_3$

B $v_1 > v_2 > v_3$

C $v_1 < v_2 > v_3$

D $v_1 > v_2 < v_3$

- c. Medium 2 is replaced with a more optically dense medium, therefore the index of refraction n_2 increases. As a result of this, the angles θ_2 , θ_3 , θ_4 might change. Which of the following statement(s) regarding the way these angles change is/are correct (one or more answers)?

A θ_2 increases but θ_3 and θ_4 stay the same.

B θ_2 decreases but θ_3 and θ_4 stay the same.

C All three angles θ_2 , θ_3 and θ_4 change.

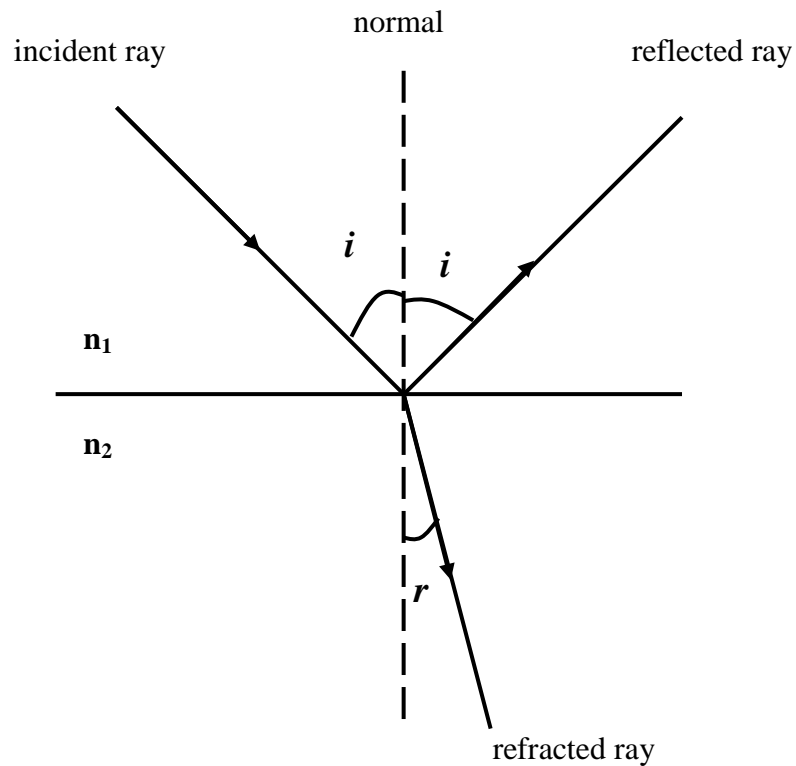
D θ_2 and θ_3 change but θ_4 stays the same.

Total internal reflection

As it appears in Unit 4

Partial reflection and partial refraction

When a ray of light reaches the boundary between two media, in general, it is partially reflected and partially refracted.

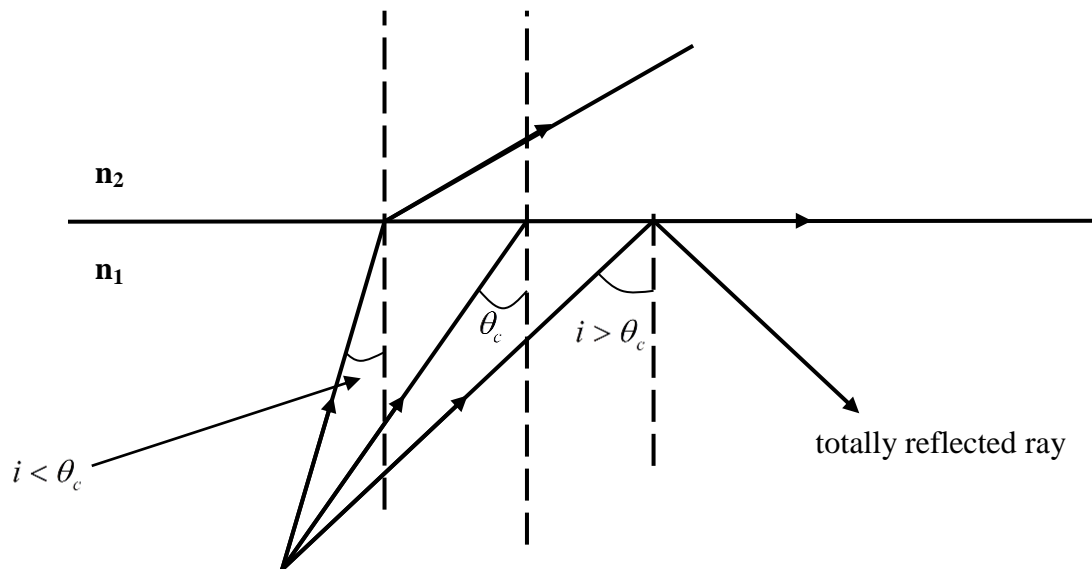


But, under some conditions, the ray can be totally reflected. That means there will be no refraction.

Critical angle

Total internal reflection occurs if:

1. The second medium is less dense than the first, $n_2 < n_1$
2. The angle of incidence exceeds the **critical angle** θ_c , $i > \theta_c$



The **critical angle** is the angle of incidence for which the angle of refraction equals 90° .

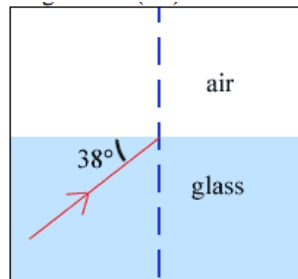
$$n_1 \sin \theta_c = n_2 \sin 90^\circ \quad \therefore \quad \sin \theta_c = \frac{n_2}{n_1}$$

If n_1 and n_2 are given and $n_2 < n_1$, a value for the critical angle can always be calculated.

Review Questions

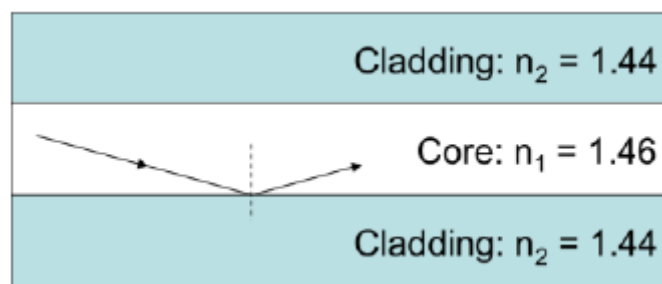
Question 12

Will light traveling from glass to air reach the air if it approaches at a 38° angle to the (flat) surface? Explain. (Take the refractive index of the glass to be 1.52).



Question 13

The diagram below shows a section of optic fibre. A laser source is directed along the fibre so that it meets the core-cladding boundary at an angle of 82° .

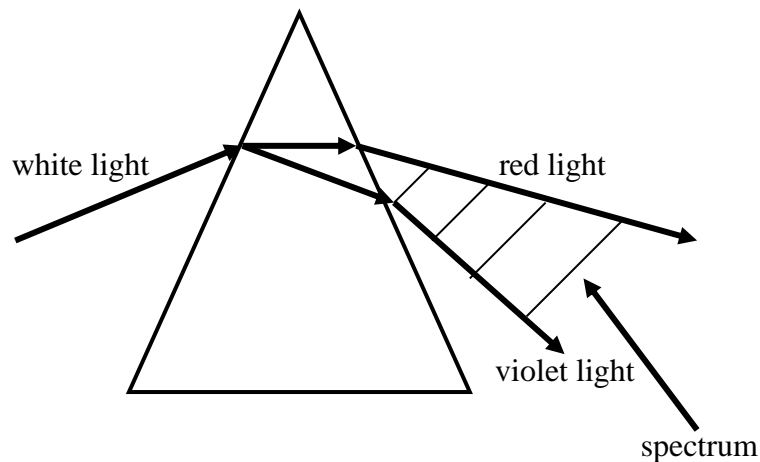


Find the critical angle for this situation and explain why total internal reflection is achieved.

Colour dispersion

As it appears in Unit 4

When white light is allowed to fall on a glass prism, it is broken up into its component colours. The component colours of white light are (in order): red, orange, yellow, green, blue, indigo and violet. This collection of colours is called the *spectrum* of white light. The process by which white light is broken up into its component colours is called *dispersion*.



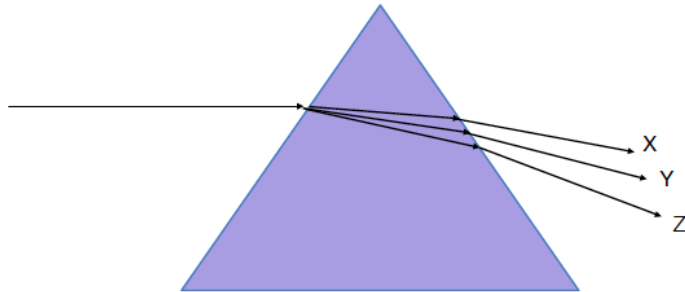
Light of different colours travel at slightly different speeds. As a result the prism has slightly different values of the index of refraction for different colours. $n_{\text{violet}} > n_{\text{red}}$

Hence different colours will be refracted at different angles and thus be dispersed.

Review Question

Question 14

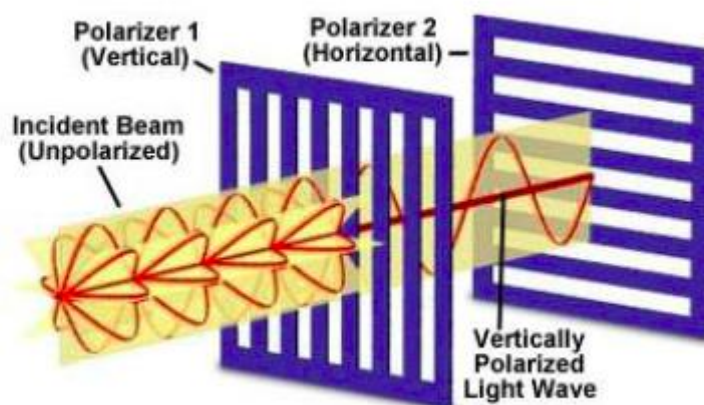
Using your understanding of refraction and dispersion, which of the following options best matches the scenario shown in the diagram below?



Option	Ray X	Ray Y	Ray Z
A	Red	Blue	Green
B	Green	Blue	Red
C	Blue	Red	Green
D	Red	Green	Blue

Solutions to Review Questions

1. Ultraviolet and X-ray.
2. Speed stays constant.
3. $v = f\lambda$
 $3.0 \times 10^8 = 3.0 \times 10^{13} \times \lambda$
 $\lambda = 1.0 \times 10^{-5} \text{ m}$
 Therefore from the diagram on page 6 this would be infrared.
4. Penetrating power is proportional to energy and as $E = hf$ as the frequency increases the amount of energy increases therefore so does the penetrating power. Therefore the low frequency or radio end of the spectrum would be the least penetrating.
5. Polarisers are made up of either vertical or horizontal filters. They absorb all waves that are not travelling in their line of propagation. The first filter below only allows waves travelling on a vertical plane through while the second one allows none through due to the horizontal plane.



6. Sound can not be polarised as sound is a longitudinal wave. Sound will pass through a polarising filter and will not be affected by the alignment of the filters
7.
 - a. 15°
 - b. 15°

8. Medium 2 must have a higher refractive index, meaning light travels more slowly within it. For a given frequency (this remains constant) this would reduce the wavelength as shown in the diagram.

9. $n_i \sin \theta_i = n_r \sin \theta_r$

$$1.52 \sin \theta_i = 1.0 \sin 60$$
$$\theta_i = 34.73^\circ$$

10. $n_i \sin \theta_i = n_r \sin \theta_r$
 $1.0 \sin 30 = n_r \sin 20$
 $n_r = 1.46$

11.

- a. A
- b. B
- c. B

12. $\sin \theta_c = \frac{n_2}{n_1} = \frac{1}{1.52}$
 $\theta_c = 41.14^\circ$

The incident angle of $38^\circ < \theta_c$, therefore refraction will occur and some light will enter the air.

13. $\sin \theta_c = \frac{n_2}{n_1} = \frac{1.44}{1.46}$
 $\theta_c = 80.5^\circ$

The incident angle of $82^\circ > \theta_c$, so at this angle light no longer refracts. It will be totally reflected.

14. D

Physics
Teach Yourself Series
Topic 13: Interference and diffraction (Unit 4)

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What you should know

As it appears in Unit 4

- Explain the result of Young's double slit experiment in terms of:
 - evidence for the wave-like nature of light
 - constructive and destructive interference of coherent waves in terms of path differences, $pd = n\lambda$, $pd = (n - \frac{1}{2})\lambda$ respectively
 - effect of wavelength, distance of screen and slit separation on interference patterns; $\Delta x = \frac{\gamma L}{d}$
- Explain the effects of varying the width of gap or diameter of an obstacle on the diffraction pattern produced by light of appropriate wavelength in terms of the ratio $\frac{\lambda}{w}$;
- Interpret electron diffraction patterns as evidence for the wave-like nature of matter;

Models of light

As it appears in Unit 4

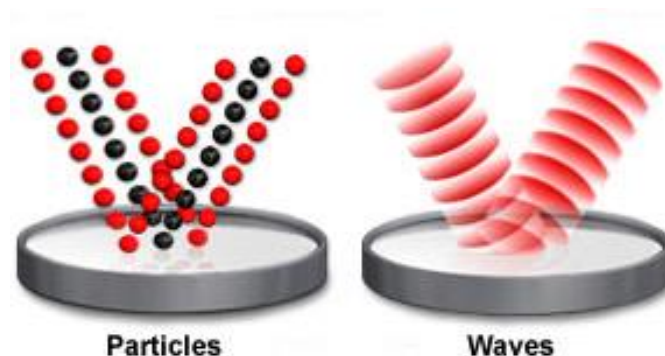
There are 2 models of light:

The Particle Model: this model treats light as particles that behave according to Newton's Laws. Light interaction is explained by forces acting on particles during collisions which cause energy and momentum transfers.

The Wave model: this model treats light as having wavelike behavior. Light wave interactions are explained by wave interference and Huygens principle.

The diagram below shows the difference between the two models; particles are located in one position, while waves spread out.

Waves and particles have some properties in common such as the ability to carry energy; however individual models cannot always be used to explain all properties of light.



Properties of waves

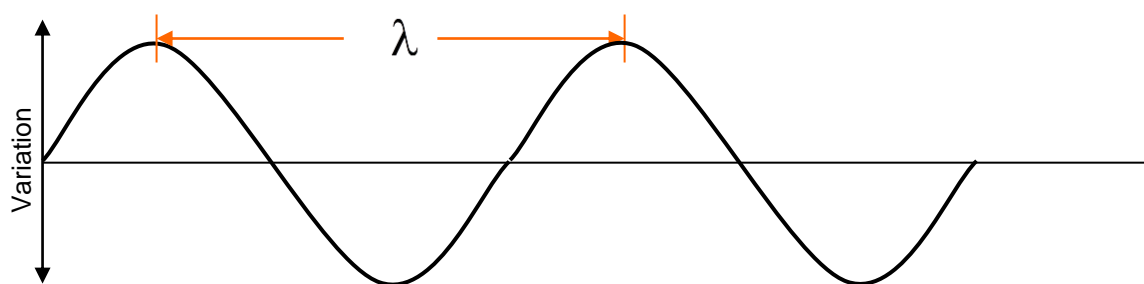
As it appears in Unit 4

Period (T): the time taken for a complete cycle (a complete wave passes a given point).

Frequency (f): the number of times a wave repeats itself per second. The unit for frequency is the Hertz (Hz). The relationship between T and f is $T = \frac{1}{f}$.

Amplitude: the top of the curve; the maximum distance away from the x -axis.

Wavelength (λ): the distance between two points that are in phase. e.g. 2 peaks, 2 troughs or any other two points that are one cycle away.



The wave equation is used to link velocity and frequency to the wave length. The formula for this equation is:

$$v = f\lambda$$

Where:

v = velocity in m s^{-1}

f = frequency in Hz

λ = wavelength in m

If we are working with the speed of light the formula is altered to $c = f\lambda$

where c is the speed of light $= 3 \times 10^8 \text{ m s}^{-1}$.

Review Question

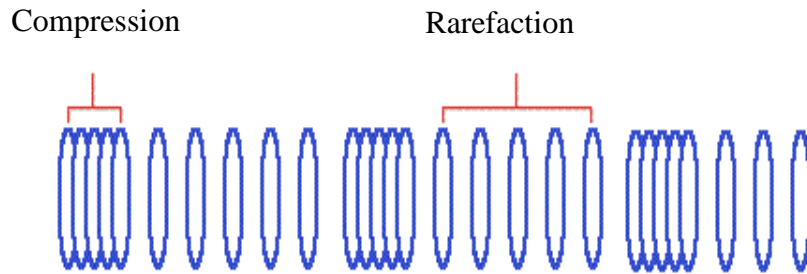
1. If the frequency of light is 6.2×10^{14} Hz, what is its wavelength and period?

Types of waves

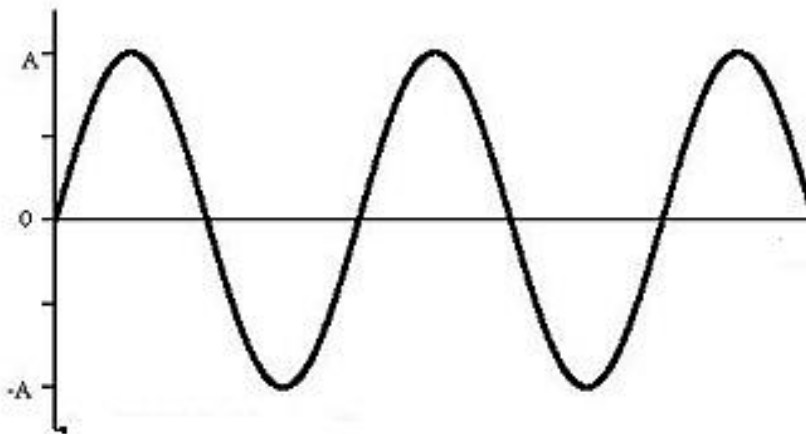
As it appears in Unit 4

There are 2 different kinds of waves.

Longitudinal waves: these are waves that oscillate along the same line as the direction of travel of the wave. They have areas of compression and areas of rarefaction. This type of wave will not be relevant to your studies except where it relates to sound.



Transverse waves: these are waves that vibrate in a direction that is perpendicular to the direction of travel of the wave such as in the diagram shown below.



Your studies in the area of light and matter will focus on transverse waves.

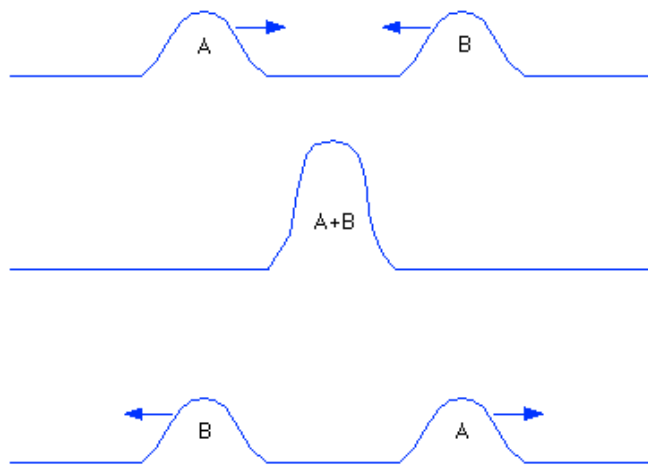
Superposition of waves

As it appears in Unit 4

Waves are able to pass through each other without being disturbed. As they pass through each other, the amplitude is the sum total of the amplitude of each wave added together.

If the waves are in phase (their crests and troughs coincide with each other), the amplitude of the resultant wave is greater. This is called **constructive interference**.

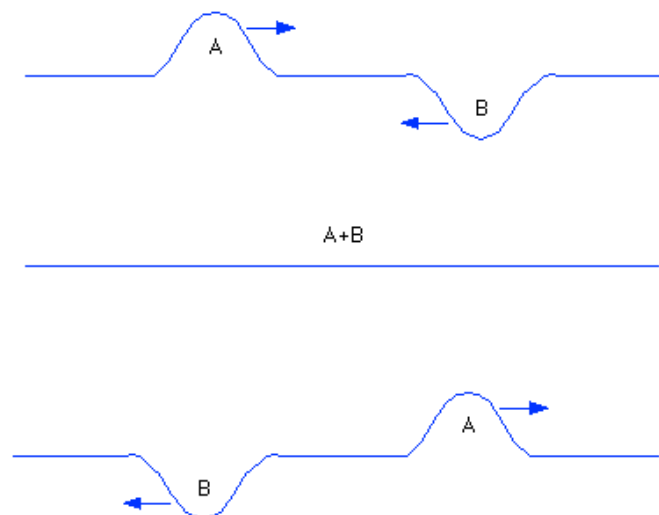
The diagram below shows two waves of equal amplitude travelling in opposite directions.



When the two waves meet, the resultant wave has an amplitude that is the size of the sum of both waves. Then the two waves continue to move through the medium with the same properties that they had to start with.

If the waves are exactly out of phase, the waves will effectively cancel each other out during interference.

The diagram below shows two waves that are exactly out of phase when they meet.

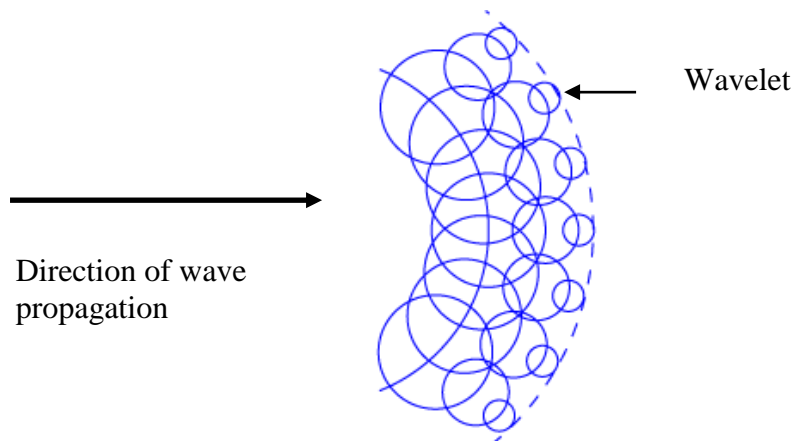


When the two waves meet, the resultant wave has an amplitude that is the size of the difference between the amplitudes; this is called **destructive interference**. The two waves continue to move through the medium with the same properties that they had to start with.

Huygens principle

As it appears in Unit 4

Huygens Principle states that every point along a wave front acts as a source of small spherical wavelets which proceed in the original direction and at the original speed of the wave. The new wave front is the resultant envelope of the small individual wavelets.



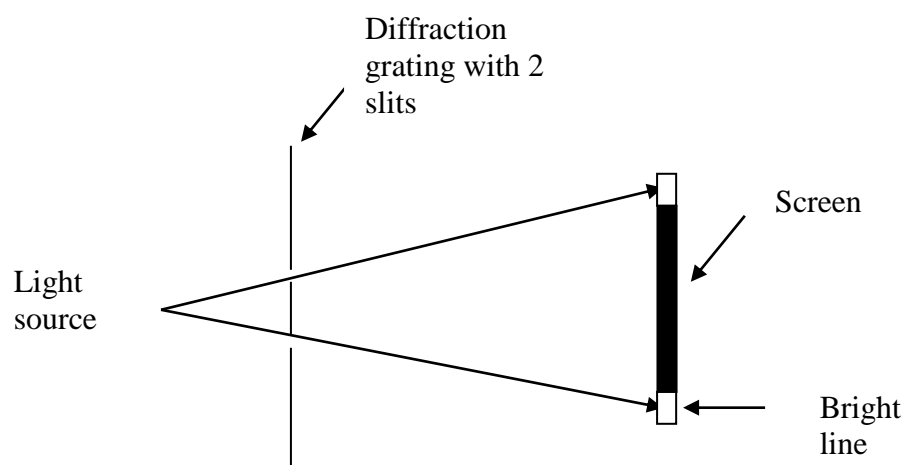
Huygen's principle is used to support the wave model of light.

Young's double slit experiment

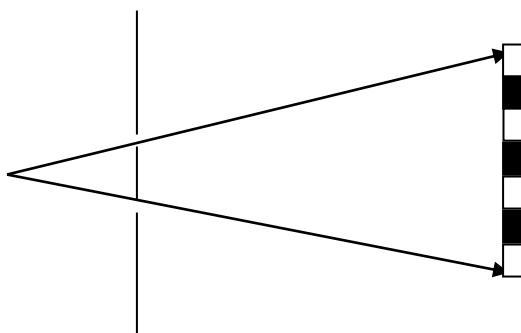
As it appears in Unit 4

In 1801, Thomas Young completed an experiment in which he projected light through a grating with 2 openings (slits). A screen was placed some distance behind the slits.

If the particle theory of light was correct, then he would have expected to see 2 bright lines on the screen; one corresponding to each slit as shown in the diagram below.

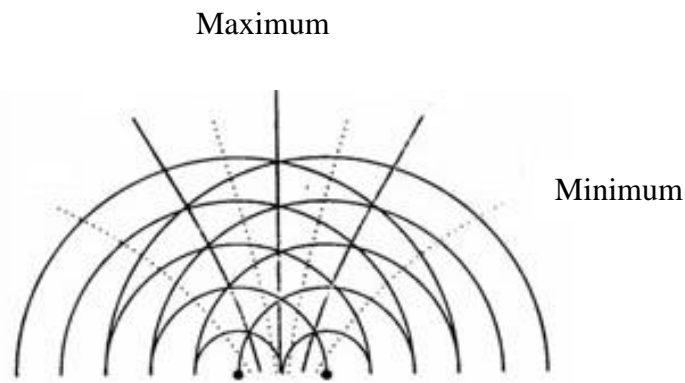


However, what was actually observed was a series of bright and dark lines called fringes as shown below.



These results can only be explained using the wave model. Each slit acts as the source for a series of secondary wavelets which spread out behind the grating as per Huygens principle.

The wavelets interfere with each other constructively and destructively as shown in the diagram below.



The dark lines represent constructive interference or maxima: waves from both sources arrive in phase. The result is a bright line on a screen. These lines are called **antinodal lines**. Path difference refers to the difference between the lengths of the 2 paths from each slit.

The path difference for maxima is given by:

$$PD = n\lambda, \quad n = 0, 1, 2, 3, \dots$$

This means that for a maxima, the path differences are a whole number of wavelengths. For example, the 4th bright band from the central maximum occurs when $n = 4$, that is, for a path difference of 4λ .

Note: the central maxima (also called the central bright band) occurs at a path difference of 0λ .

The dotted lines represent destructive interference or minima; waves from both sources arrive half a cycle out of phase. The result is a dark line on a screen. These lines are called **nodal lines**.

The path difference for minima is given by:

$$PD = \left(n - \frac{1}{2}\right) \lambda, \quad n = 1, 2, 3, \dots$$

This means that the minima occur when the path difference is an odd number of half wavelengths

e.g. $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2} \dots$

For instance the 2nd minima occurs when $n = 2$, that is for a path difference of $\left(2 - \frac{1}{2}\right)\lambda = \frac{3\lambda}{2}$.

Sample question

A light has a wavelength of 400 nm. Work out the path differences for the following:

- i. 3rd bright fringe from the centre.
- ii. 1st dark fringe.

- i. $n = 3$, so the path difference to the 3rd bright fringe is:

$$PD = 3\lambda$$

$$PD = 3 \times 400$$

$$PD = 1200 \text{ nm}$$

- ii. $n = 1$ so the path difference to the 1st dark fringe is:

$$PD = \left(1 - \frac{1}{2}\right)\lambda = \frac{\lambda}{2} = \frac{400}{2} = 200 \text{ nm}$$

Review Questions

2. Explain the difference between an antinodal line and a nodal line.

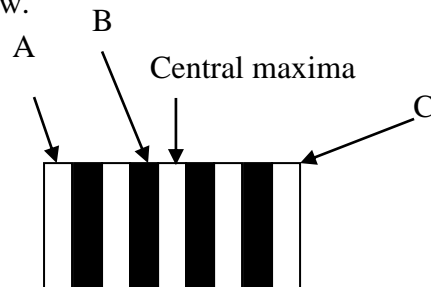
3. If the path difference to the 2nd bright fringe from the centre is 400 nm, determine the wavelength of the light.

4. If the path difference to the 3rd dark fringe is 1500 nm, determine the wavelength of light.

Determining wavelength based on the position of the fringes

As it appears in Unit 4

As previously stated, the result of Young's experiment was an alternating pattern of bright and dark fringes like the pattern shown below.



From before, we know the following information:

- The path difference between any 2 adjacent maxima or any 2 adjacent minima is one whole wavelength.
- The path difference between any maxima and the adjacent minima is half a wavelength.

Although it is helpful if the central maximum is indicated, it is not always essential.

Sample question

Refer to the diagram above. The path difference to point A is 300 nm greater than the path difference to point B. Find the wavelength of the light.

The difference between the path differences in this case = $1\frac{1}{2}$ wavelengths = 300 nm.

$$\text{Therefore } 300 = \frac{3\lambda}{2}$$

$$3\lambda = 600$$

$$\text{So } \lambda = 200 \text{ nm}$$

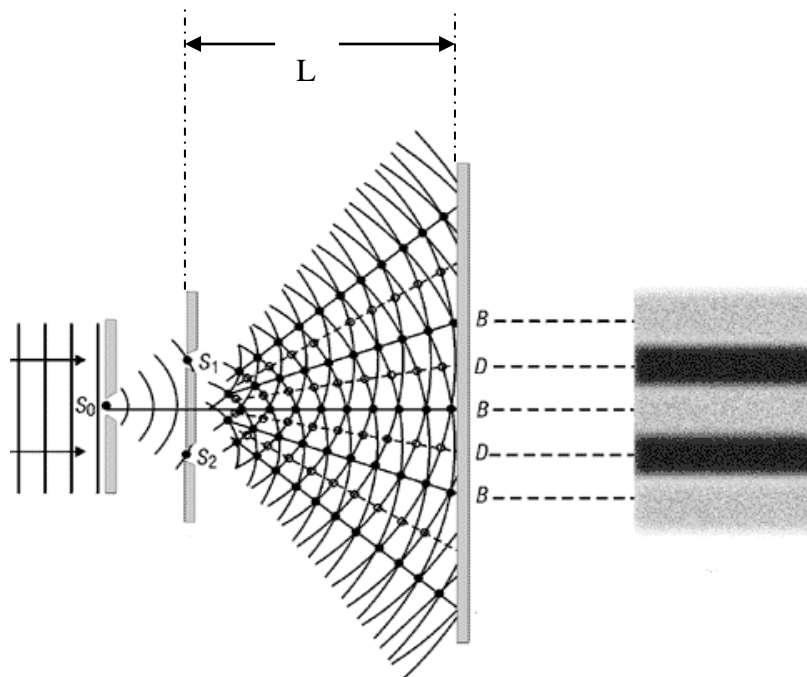
Review Question

5. Use the diagram above to answer this question. If the path differences for point C is 1200 nm greater than the path difference for point B, what is the wavelength?

Determining wavelength using fringe and slit separation

As it appears in Unit 4

The diagram below shows how the double-slit experiment was set up and the results of the experiment.



This information can be used to determine the wavelength of the light source using the equation:

$$\lambda = \frac{Wd}{L}$$

Where:

L = the distance from the grating to the screen

W = the distance between fringes

d = the distance between the slits (S_1 and S_2 in the diagram above)

Review Questions

6. Use the relationship above to explain why interference patterns for light are not usually seen in everyday situations.

7. Explain how the light and dark bands are formed in the experiment shown on p.12 and how the results from this experiment are used to support the wave model for light

8. Students attempt to duplicate Young's experiment and they set up their equipment as shown in the diagram on p.12. They obtain the following data:

The distance between slit S1 and the 2nd minimum is 1.095×10^{-5} m

The distance between slit S2 and the 2nd minimum is 1.000×10^{-5} m

- i. Determine the wavelength of the light in nanometers.

- ii. Determine the path difference for the 3rd maxima from the centre.

9. The student then conducts a second experiment using light with a wavelength of 225 nm. They achieve the following results.

The distance between slit S1 and position X is 1.045×10^{-5} m

The distance between slit S2 and position X is 1.090×10^{-5} m

Determine the type of interference that occurs at position X.

Factors that alter the interference pattern

As it appears in Unit 4

There are several factors that can alter the patterns either by causing the fringes to become closer (bunch up) or spread out.

Since the equation $W = \frac{\lambda L}{d}$ applies, this means that altering the wavelength, the distance between the slits or the distance to the screen will alter the distance between the fringes. The fringes will either bunch up (come closer to each other) or spread out.

Fringes will bunch up under the following conditions:

- Light frequency increases
- Wavelength decreases
- The distance between the slits increases
- The distance between the source and the screen decreases

Fringes will spread out under the following conditions:

- Light frequency decreases
- Wavelength increases
- The distance between the slits decreases
- The distance between the source and the screen increases

Sample question

An interference pattern has a fringe separation of x m. The distance between slits is then decreased. What can be done to restore the original fringe separation.

Decreasing the slit separation will increase the fringe separation, therefore either the wavelength must be decreased or the distance from the slits to the screen must be decreased.

Review Question

- 10.** An experiment is being conducted. Initially red light is used, but this is then changed to blue light. State what will happen to the pattern and briefly explain why this will occur.

- 11.** A student uses green light in a double-slit interference experiment and obtains a fringe separation of 2 mm. She then moves the screen closer to the slits (while keeping the same slit separation) and repeats the experiment with orange light and violet light. Which colour could possibly give her an interference pattern with a 2 mm fringe separation. Justify your answer.

Diffraction

As it appears in Unit 4

- Diffraction occurs when waves bend around obstacles in their pathway or pass through narrow openings.
- As long as the wave remains in the same medium the properties of the wave such as its frequency (f), wavelength (λ) and speed (v) also remain the same.
- The only thing that is changed is the direction of the wave.
- It is important to remember that low frequency waves have long wavelengths and high frequency waves have short wavelengths.

Diffraction: the effect of obstacles

As it appears in Unit 4

If the obstacle is much smaller than the wavelength of the light there will be little diffraction. If the obstacle is comparatively larger than the wavelength, then there will be much greater diffraction.

Diffraction: the effect of openings

As it appears in Unit 4

When discussing waves travelling through openings, you have to apply Huygens's principle.

When a wave travels through an opening, the waves bend spread out and are diffracted into the region beyond the barrier.

The extent of diffraction is proportional to the ratio $\frac{\lambda}{w}$

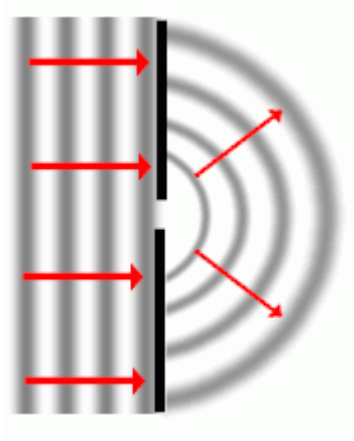
Where

λ = the wavelength of the wave

w = the width of the opening

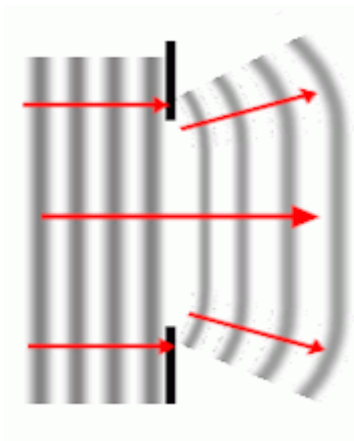
Circular wave fronts like the one shown below are caused when the opening is narrow. The wavelength is larger than the opening.

$$\lambda \geq w$$



Wave fronts remain straight with slight bending at the ends when the opening is wide. The wavelength is much smaller than the opening.

$$\lambda < w$$



So:

If the ratio $\frac{\lambda}{w} \geq 1$, significant diffraction will occur

If the ratio $\frac{\lambda}{w} \ll 1$, there is very little if any diffraction occurring

Or

The greater the ratio, the greater the extent of diffraction, however, generally **if the ratio is much less than 1 then the amount of diffraction is not considered to be significant.**

Note that when the wavelength is much greater than the gap size, that is $\frac{\lambda}{w} \gg 1$, the amount of diffraction is at a maximum but so little of the wave actually passes through the gap that it becomes unobservable.

Review Questions

- 12.** An experiment is set up which involves light being shone through a narrow slit of $1.1\ \mu\text{m}$. Initially red light with a wavelength of $650\ \text{nm}$ is shone through the slit and then violet light with a wavelength of $420\ \text{nm}$ is shone through the slit. Determine which would diffract the most. Show calculations to support your answer.
- 13.** Radiation with a frequency of $5 \times 10^{15}\ \text{Hz}$ is shone through a narrow slit with a width of $2\ \mu\text{m}$. Determine whether an observer would expect to see a significant amount of diffraction.

Electron diffraction patterns

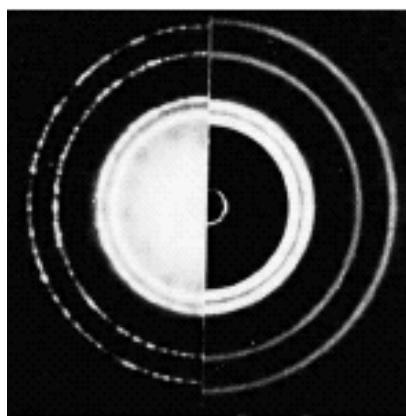
As it appears in Unit 4

Electron diffraction patterns support the wave model for matter. If electrons are fired through a thin polycrystalline metallic foil, a pattern is formed as the wave front travels between the atoms of the foil. The wave fronts interfere with each other forming bright and dark rings.

Half of this pattern is projected onto a screen in the diagram below.

X-rays are then projected through the same foil and they produce a pattern in the same way as the electrons do.

Half of this pattern is also projected onto the screen. The screen now shows half of each pattern as shown in the diagram below.



Electrons

X-rays

If the rings on both sides align, then the x-rays and the electrons must have the same wavelength.

Review Questions

14. A beam of electrons that has a wavelength of 3.1×10^{-10} m is fired into a crystal. The spacing between atoms in the crystal is 3.0×10^{-10} m.

Compare the spacing with the wavelength and determine whether there would be significant diffraction

15. If x-rays with a similar wavelength to the electrons were then fired through the same crystal, explain what you would expect to observe if the two diffraction patterns were compared.

Solutions to Review Questions

$$\begin{aligned} 1. \quad f &= 6.2 \times 10^{14} \text{ Hz} \\ c &= 3 \times 10^8 \text{ m s}^{-1} \\ c &= f\lambda \end{aligned}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{6.2 \times 10^{14}} = 4.84 \times 10^{-7} \text{ m} = 484 \text{ nm}$$

$$t = \frac{1}{f} = \frac{1}{6.2 \times 10^{14} \text{ Hz}} = 1.6 \times 10^{-15} \text{ s}$$

2. Antinodal lines are the result of constructive interference between two wave fronts and nodal lines are the result of destructive interference between two wave fronts.

3. For the 2nd bright fringe $n = 2$
 $PD = 2\lambda = 400$

$$\lambda = \frac{400}{2} = 200 \text{ nm}$$

4. For the 3rd dark fringe $n = 3$

$$PD = \left(3 - \frac{1}{2}\right)\lambda = \frac{5\lambda}{2} = 1500.$$

$$\lambda = \frac{2 \times 1500}{5} = 600 \text{ nm}$$

5. Path differences are normally taken as the absolute value of the difference between the 2 paths so they are always positive. Therefore care must be taken when comparing fringes on opposite sides of the central maximum.

C is the second bright band from the centre so its path difference = 2λ .

B is the first dark band from the centre so its path difference = $\frac{\lambda}{2}$

The **difference** in their path differences is therefore $2\lambda - \frac{\lambda}{2} = \frac{3\lambda}{2}$

$$\frac{3\lambda}{2} = 1200$$

$$\lambda = \frac{2 \times 1200}{3} = 800 \text{ nm}$$

6. The wavelength of light is very small, hence, to achieve a fringe separation which is large enough that our eyes can distinguish the fringes, a very small separation between the light sources is required. This does not occur very often in the scale of most everyday objects.

7. The light bands are produced as the result of constructive interference and the dark bands are produced as the result of destructive interference.

The results seen in Young's experiment can only be produced if light is acting as a wave because particles would not interfere with each other in this manner. These patterns can only be produced as the result of waves interfering with each other.

8. The answers are as follows:

i. $PD = (1.095 \times 10^{-5}) - (1.00 \times 10^{-5}) = 9.5 \times 10^{-7} \text{ m}$
 $PD = 950 \text{ nm}$

The point referred to was the 2nd minimum ($PD = \frac{3\lambda}{2}$)

$$\frac{3\lambda}{2} = 950 \text{ nm}$$

$$\lambda = 633.3 \text{ nm}$$

ii. $3^{\text{rd}} \text{ maxima} = 3\lambda$
 $PD = 633.3 \times 3$
 $PD = 1900 \text{ nm}$

9. $PD = (1.090 \times 10^{-5}) - (1.045 \times 10^{-5}) = 4.5 \times 10^{-7} \text{ m}$
 $PD = 450 \text{ nm}$

The light being used has a wavelength of 225 nm.

$$\frac{PD}{\lambda} = \frac{450}{225} = 2$$

Since the result is an exact multiple of the wavelength of light, this has to be an example of constructive interference.

10. The fringes will bunch up when the light is changed from red to blue. The wavelength of light has decreased from approximately 650 nm to approximately 400nm and as a result the distance between the fringes (W) decreases.

11. Consider $W = \frac{L\lambda}{d}$

Moving the screen closer to the slits will decrease the fringe separation so to increase the separation again will require light of a longer wavelength. Thus orange light is required as orange has a longer wavelength than green light while violet has a shorter wavelength.

12. For 650 nm

$$\frac{\lambda}{w} = \frac{650 \times 10^{-9}}{1.1 \times 10^{-6}} = 0.59$$

For 420nm

$$\frac{\lambda}{w} = \frac{420 \times 10^{-9}}{1.1 \times 10^{-6}} = 0.38$$

Since the ratio for the red light is greater than that for the violet light the red light will diffract more than the violet light will, though neither will diffract to a great extent.

13. $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^{15}} = 6 \times 10^{-8} \text{ m}$

$$\frac{\lambda}{w} = \frac{6 \times 10^{-8}}{2 \times 10^{-6}} = 0.03$$

Since the ratio is considerably less than 1 there would not be a significant amount of diffraction.

14. The difference between the wavelength and the spacing between the atoms is very small

(1×10^{-11}) and the ratio $\frac{\lambda}{w} = 1.03$. Therefore the observer would expect to see significant diffraction.

15. If the wavelength of the electrons and the x rays is similar then the ring shaped diffraction patterns should align with each other.

Physics
Teach Yourself Series
Topic 14: Photoelectric effect (Unit 4)

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What you should know

As it appears in Unit 4

According to the study design you need to know the following:

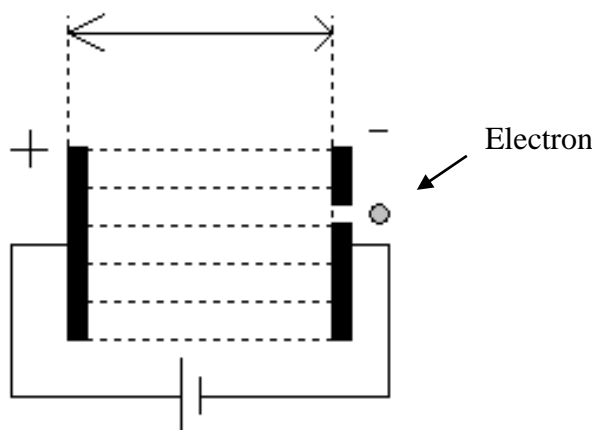
Analyse the photoelectric effect in terms of:

- evidence for the particle-like nature of light;
- experimental data in the form of graphs of photocurrent versus electrode potential, and of kinetic energy of electrons versus frequency;
- kinetic energy of emitted photoelectrons, $E_{K_{\max}} = hf - W$, using energy units of joule and electron-volt;
- effects of intensity of incident irradiation on the emission of photoelectrons;
- describe why the wave model of light cannot account for the experimental photoelectric effect results.

Motion of an electron in an electric field

As it appears in Unit 4

If two metal plates are placed at some distance apart in an evacuated chamber, and if a potential difference is applied across the plates, then the difference in the potential of the plates produces a constant electric field.



When the electron drifts into the electric field between the plates, the constant electric force (\mathbf{F}_E) acts on the electrons. Since electrons have a negative charge, they will accelerate towards the positive plate.

Electrons moving directly between the plates will have a uniform force acting on them. The electric force does work on these electrons and they gain kinetic energy as they move towards the positive plate.

When electrons are accelerated or decelerated travelling along the lines of an electric field set up by a potential difference, V , the work done on an electron is given by the equation:

$$W = q\Delta V = \Delta E_k$$

Where:

W = work done in joules (J)

q = charge of electron in coulombs (C)

ΔV = the potential difference that the electron moves through in volts (V).

Review Question

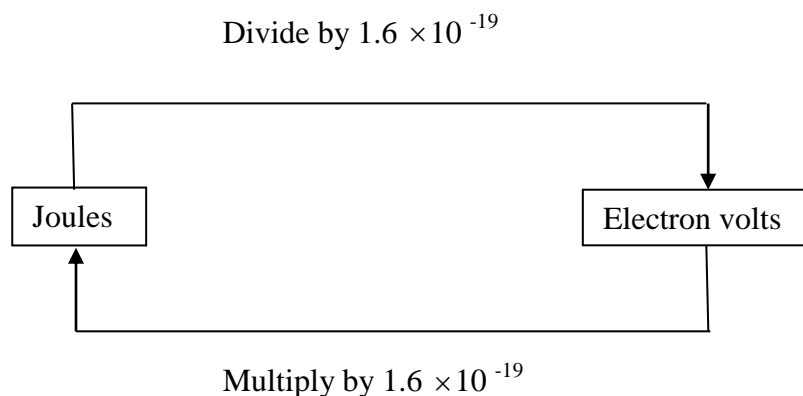
1. The voltage drop between 2 plates is 80 V. If an electron is accelerated from one plate to the other, calculate the work done in Joules.

The electron volt

As it appears in Unit 4

An electron volt (eV) is the quantity of energy acquired by an electron ($q = 1.6 \times 10^{-19}$ C) as it passes through a potential difference of 1 volt. Electron volts are generally used in preference to joules because it is a unit that is more convenient.

The diagram below shows how to convert from Joules to electron volts and vice versa.



Review Question

2. Complete the following conversions.

i. How many electron volts in 8×10^{-19} J?

ii. How many joules in 15 eV?

Photons

As it appears in Unit 4

Planck developed the idea that light energy is not continuous but travels in packets called photons

The energy provided by a photon is calculated using the formula:

$$E_{\text{photon}} = hf$$

Where:

E = energy in J

h = Planck's constant = 6.63×10^{-34} J s or 4.14×10^{-15} eV s

f = frequency of the light in Hz

The momentum of a photon of light is established using the formula

$$p = \frac{E}{c}$$

Where:

p = momentum of the quanta of light in N s

E = kinetic energy in J

c = the speed of light = 3.0×10^8 m s⁻¹

Other derivations of this formula include:

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

Points to remember:

Wavelengths are usually given in nm and need to be converted to m. The factor involved is 10^{-9} . For example, if a wavelength of light is 450 nm then it is 450×10^{-9} m. If you wish to have this in proper scientific notation, move the decimal point 2 places so $450 \text{ nm} = 4.5 \times 10^{-7} \text{ m}$

- Since there are two values for Planck's constant, you need to use the value that works with what you are trying to do. For example, if you are working out the energy of a quanta of light in joules you would need to use the value $6.63 \times 10^{-34} \text{ J s}$ to come up with an answer in Joules. You would still get an accurate answer if you used the other value, but you would come up with an answer in eV that would then need to be converted to Joules.
- If you are trying to calculate the kinetic energy of an electron, you need to use $4.14 \times 10^{-15} \text{ eV s}$, which would give an answer in eV.
- To relate the frequency of a photon to the wavelength of a photon use the formula $c = f\lambda$

Review Questions

3. Light has a frequency of 5.4×10^{14} Hz. Calculate the energy (in Joules) and momentum associated with a quantum of the light.
4. If light has a wavelength of 600 nm, calculate the momentum of the light.

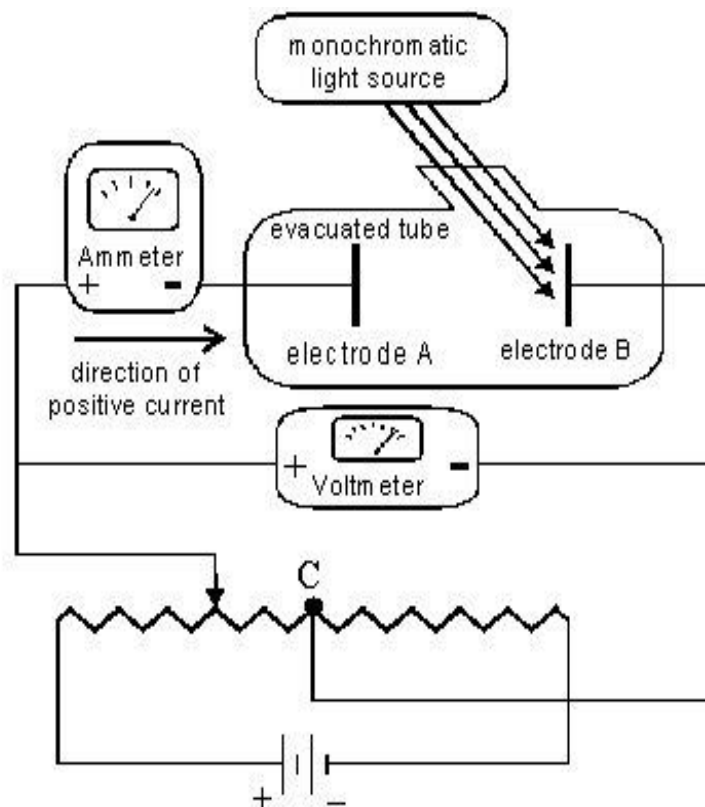
The apparatus used in demonstrating the photoelectric effect

As it appears in Unit 4

The apparatus used to conduct experiments involving the photoelectric effect includes:

- A photoelectric tube (also called a phototube). This is a vacuum tube which has a photoconductive metal plate at one end (electrode B in the diagram) and a collecting plate (electrode A in the diagram) at the other. Different tubes can be used which have different metal plates.
- A monochromatic light source is the source of the photons. Generally the sources used can produce light of different frequencies and/or intensities.
- An ammeter which is used to detect current.
- A voltage supply that can be used to create an electric field to oppose the electrons. The level of voltage can be varied as well.
- A voltmeter to measure the voltage between the electrodes.

The diagram shown below shows how the apparatus for this type of experiment is typically set up.



Review Questions

5. Under what circumstances does current register on the ammeter?

6. Why is it important that the light is monochromatic?

What happens during the photoelectric effect

As it appears in Unit 4

When the light is shone onto the metal plate, energy is provided which causes electrons to be emitted from the metallic plate. These electrons are called **photoelectrons**. They cross the tube and hit the collector at the other end of the tube.

The electrons hitting the collector complete the circuit and a current is measured by the ammeter. This means that the ammeter is the detector of the emission of photoelectrons. The ammeter also allows the experimenter to identify the lowest frequency of light that causes electrons to be ejected with a minimum amount of kinetic energy.

The reading on the ammeter reflects the number of electrons that cross the tube; since $I = \frac{Q}{t}$, increasing the number of electrons crossing results in a greater current.

Stopping potential/voltage

As it appears in Unit 4

The kinetic energy of the photoelectrons can be experimentally measured by using a battery and variable resistor to make the collecting plate negatively charged.

The electrons can still hit the collecting plate if they have sufficient kinetic energy. As the plate is made, more and more negative, fewer and fewer electrons have enough energy to hit the plate.

Ultimately, the collecting plate can be made so negative that even the electrons with the most energy have insufficient energy to cross the tube and hit the collecting plate.

If no electrons hit the collecting plate, then the circuit is incomplete and the current measured by the ammeter goes to zero.

The voltage that causes the current to go to zero is called the **stopping potential/voltage**.

Review Questions

7. What effect does increasing the voltage have on the ability of the electrons to reach the collecting plate?

8. Explain what will happen to the current as the voltage is increased.

Einstein's explanation of the photoelectric effect

As it appears in Unit 4

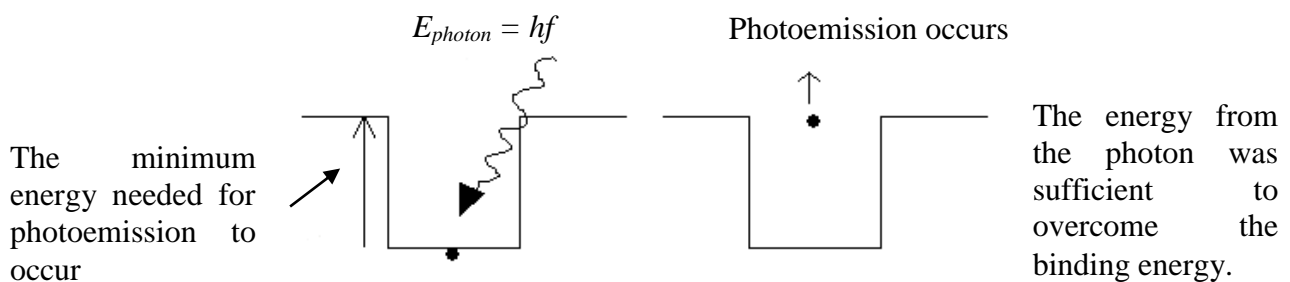
Einstein's interpretation of the photoelectric effect resulted in equations which relate photon energy to the energy of the emitted electrons

Energy of photon = energy needed to remove an electron + kinetic energy of the emitted electron.

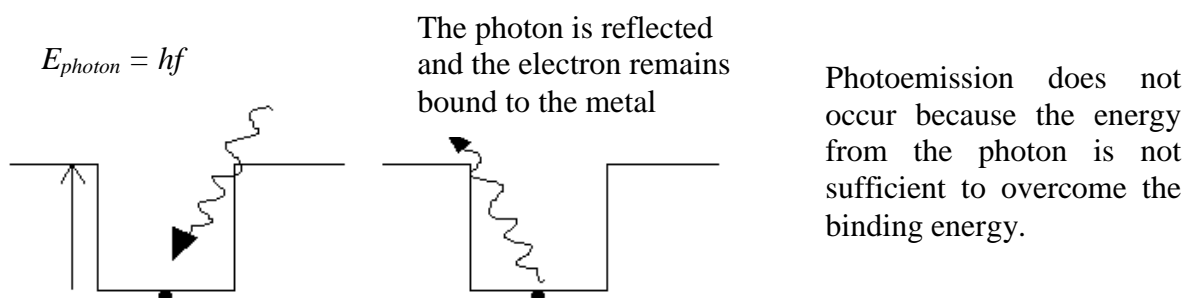
A good analogy for this is like playing football. If a member of the opposite team has the ball, it will take you a certain amount of energy to take the ball from them (just like the energy needed to remove the electron from the metal plate) and then more energy to run away with the ball (just like the kinetic energy of the emitted electron).

Einstein made the following assumptions when accounting for the interaction of light with the electrons in a metal.

- There is an attraction between the electrons and the nucleus of an atom and it will take energy to overcome this binding energy.
- The minimum amount of energy needed to free the electron from the metal is called the **work function**.
- The work function will be different for all metals (the more active a metal the lower its work function will be).
- If a photon interacts with an electron then the photon will give up all of its energy to the electron by being absorbed by it. Photoemission occurs and the metal is ionised.



- If a photon collides with an electron and is not absorbed by it then it will reflect off the electron with no loss of energy (conservation of energy applies).



So, each photon gives energy to the electron it collides with causing it to leave the surface of the metal. Some energy is lost in ripping the electron free (overcoming the binding energy) this is called the work function of the metal (W).

The **threshold frequency** (f_o) is the frequency of photons of the minimum energy needed to release photoelectrons from the surface of the metal.

If the frequency is less than the threshold frequency ($f < f_o$), no electrons will be emitted because the energy supplied by the photons is insufficient to overcome the binding energy holding the electrons to the metal plate.

The equation for calculating the work function is:

$$W = hf_o$$

Where

W = work function

h = Planck's constant

f_o = threshold frequency

Quite often you will be given the wavelength of light rather than the frequency of the light. If that is the case, then you will need to use the formula $c = f\lambda$ to calculate the frequency of the light source.

Review Question

9. Green light with a wavelength of 530 nm is the minimum frequency that will eject electrons from a metallic surface. Calculate the work function (in J) of the metallic surface.

Calculating the kinetic energy of electrons

As it appears in Unit 4

If the frequency is greater than the threshold frequency ($f > f_o$) then photoelectrons will be released with a maximum kinetic energy as per the following equation.

$$hf = W + E_k$$

This can be rearranged to:

$$E_k = hf - W$$

And

$$W = hf - E_k$$

Where:

h is Planck's constant

W is the work function

E_k is the maximum kinetic energy of ejected electrons:

Sample question

Photons with an energy of 4.8 eV are shone onto a metallic surface. If the electrons in the metal have a work function of 3.5 eV, determine whether photoemission will occur, and, if so determine the kinetic energy of the photoelectrons.

Photoemission can occur because the energy of the photon is greater than the work function.

$$E_k = E_{\text{photon}} - W$$

$$E_k = 4.8 - 3.5$$

$$E_k = 1.3 \text{ eV}$$

Review Questions

10. Why is it necessary to subtract W from hf when determining the kinetic energy of photoelectrons?

11. A stream of light whose photons have an energy of 5.3 eV is shone onto a metallic plate. Determine if photoemission will occur under the following circumstances, and if so what will be the kinetic energy of the electrons?

i. The electrons have an energy of – 4.1 eV

ii. The electrons have an energy of – 5.2 eV

iii. The electrons have an energy of – 6.2 eV

12. The work function of a certain metal is 3×10^{-19} J. If light of wavelength 500 nm shines on the metal, find the maximum kinetic energy of the electrons emitted.

13. When a metal surface is illuminated by radiation of frequency 6.4×10^{14} Hz, the maximum kinetic energy of the photoelectrons emitted is 0.24 eV.

i. Calculate the work function of the metal.

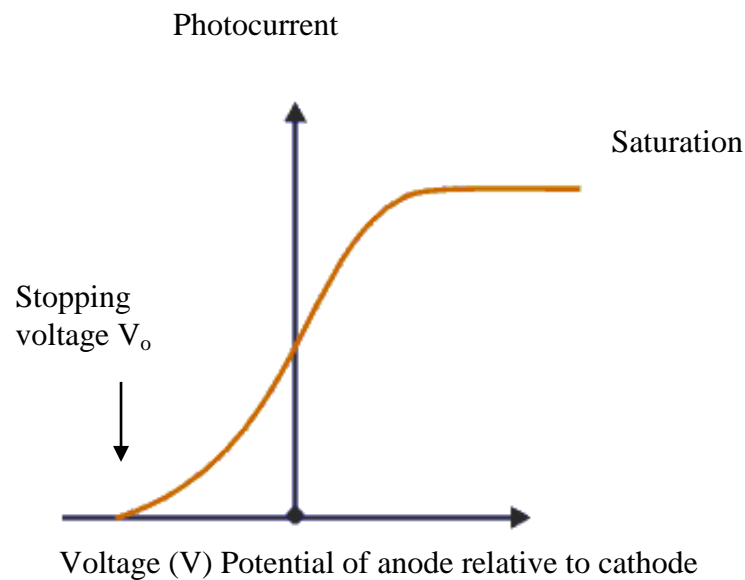
ii. Establish the threshold frequency for the metal.

Interpreting graphs of photocurrent vs stopping voltage

As it appears in Unit 4

The curve shown below shows how the current measured depends on the potential difference between the anode and the cathode.

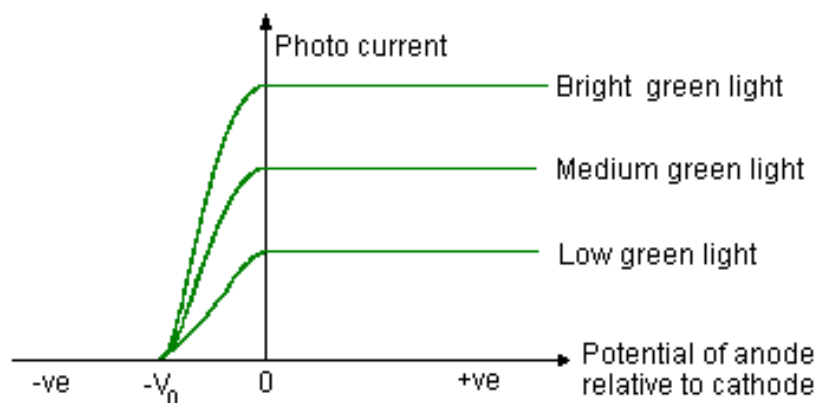
All graphs relating to this concept will have the same kind of shape as this one.



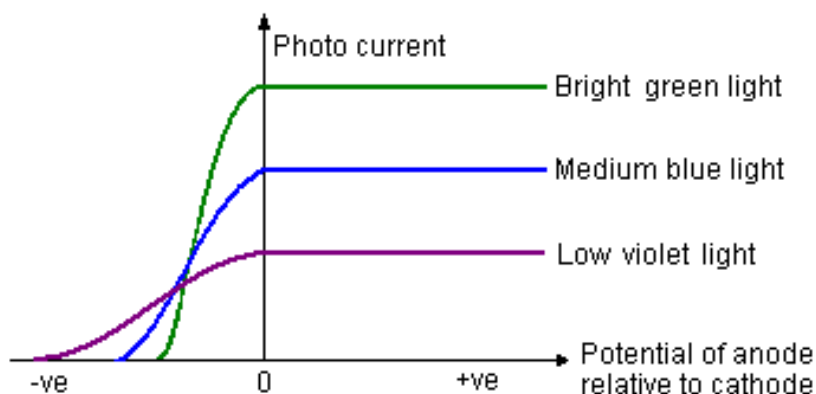
The maximum current occurs at the saturation point. The curve flattens out because the accelerating voltage has caused all of the emitted electrons to be collected at the anode.

When voltage opposes the motion of the electrons, the number of electrons that reach the anode decreases. If the retarding voltage is high enough, none of the emitted electrons reach the anode as they all reverse direction. As a result, the current will be zero because the circuit is not complete.

Increasing the intensity of the light will increase the number of photons reaching the cathode per unit of time. As a result, there will be more electrons emitted and the photocurrent will be higher, however, the energy of the photons has not been changed, so the voltage required to stop the electrons will not change.



Increasing the frequency of the light means the energy of the photons is increased, resulting in higher energy photoelectrons. Therefore it will take more energy to stop their movement and the stopping voltage will be higher.



The following table shows the wavelengths of light associated with the colours of light in the visible spectrum.

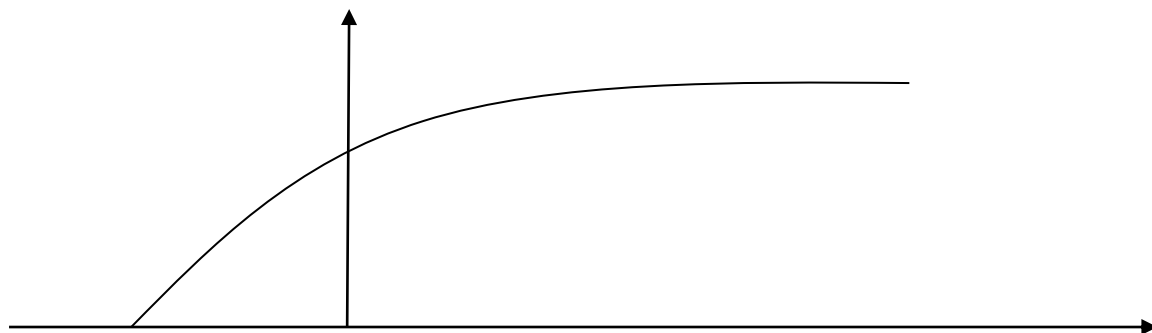
COLOUR	WAVELENGTH (nm)
Violet	400 – 420
Blue	420 – 490
Green	490 – 560
Yellow	560 – 600
Orange	600 – 630
Red	630 – 750

Violet light has the shortest wavelength and the highest frequency. Red light has the longest wavelength and the lowest frequency.

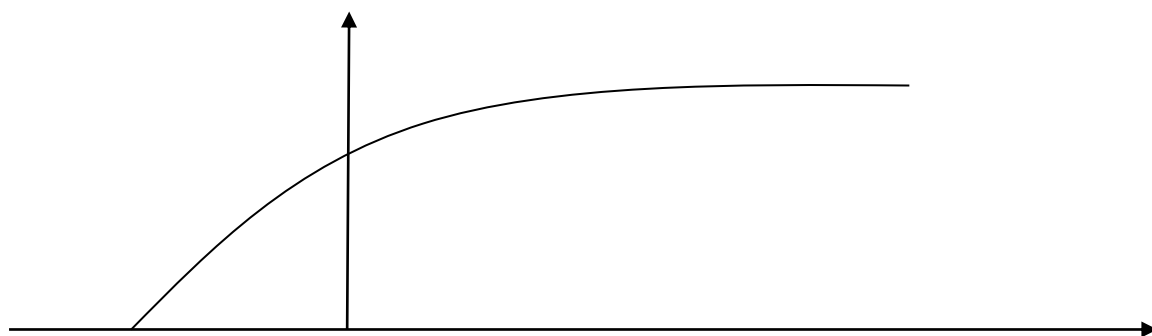
Review Questions

14. A student begins an experiment using red light. Explain what will happen to the photocurrent and the stopping voltage if they then change the colour of the light to blue without affecting the intensity.

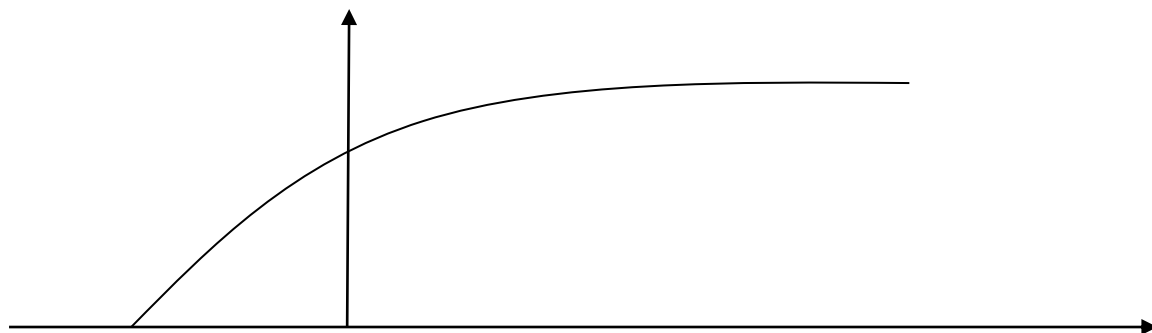
15. On the graph below draw what the curve would look like if the intensity of the light is increased but the frequency remains the same. Explain why you have drawn the curve this way.



16. On the graph below draw what the curve would look like if the frequency of the light is increased but the intensity is decreased. Explain why you have drawn the curve this way.



17. On the graph below draw what the curve would look like if metal is changed without the intensity or frequency of the light being changed. Explain why you have drawn the curve this way.



Interpreting graphs of maximum kinetic energy vs frequency

As it appears in Unit 4

It is common for questions to be asked that require you to interpret information from graphs of maximum kinetic energy versus frequency.

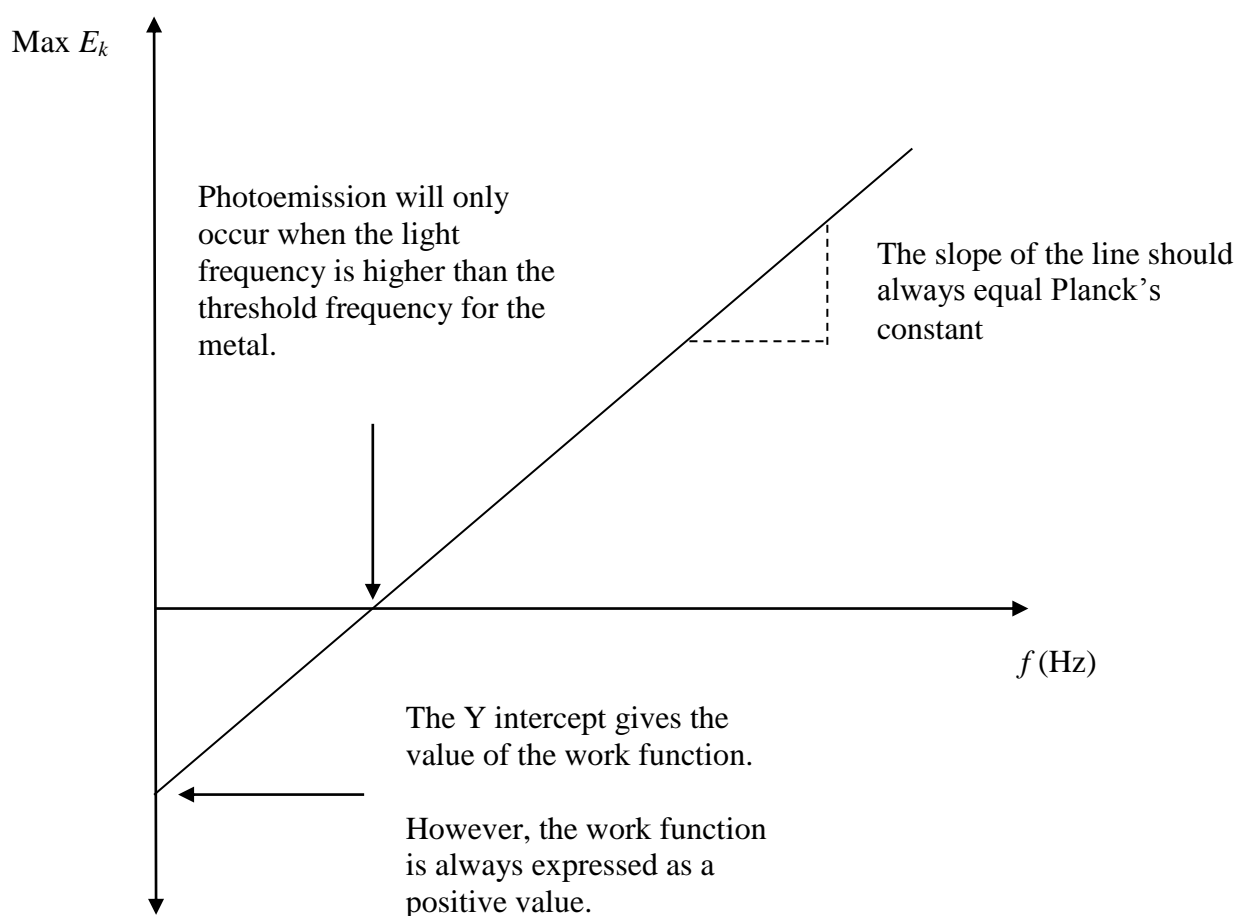
You should be able to determine several pieces of information:

Planck's constant

The work function of the metal

The threshold frequency

Comparison of the work function of different metals (if two or more metals are shown on the graph)

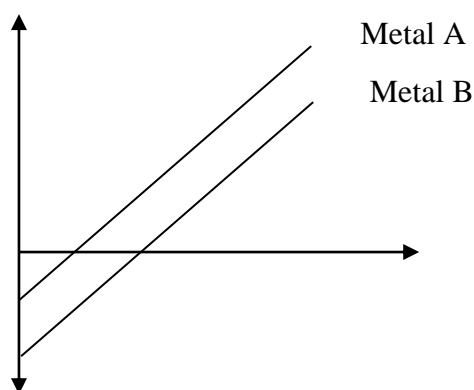


These graphs can take two different forms.

If the graph shows kinetic energy in joule against frequency, the Planck's constant value will be in J s.

However, if the y axis is in eV then the Plancks constant value will be in eV s.

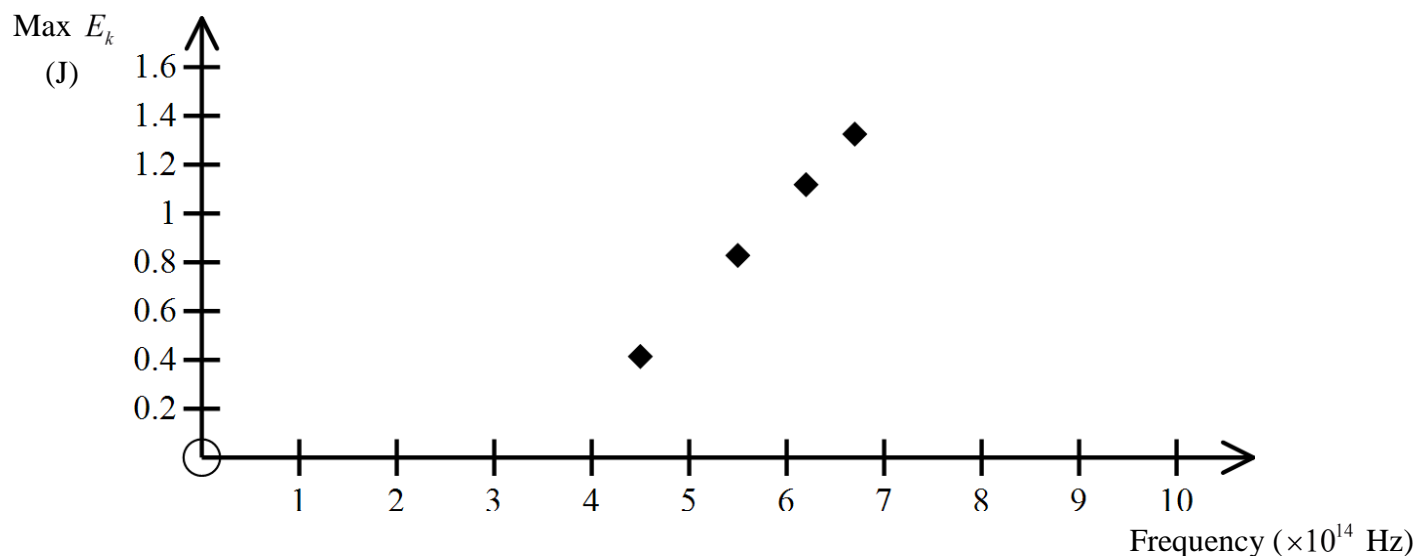
Since the gradient will always be the same, the lines for all metals will be parallel with each other. Higher work functions will be shown further down the Y axis.



This graph shows that metal B has a higher work function than metal A (it will take more energy to remove electrons from metal B than metal A)

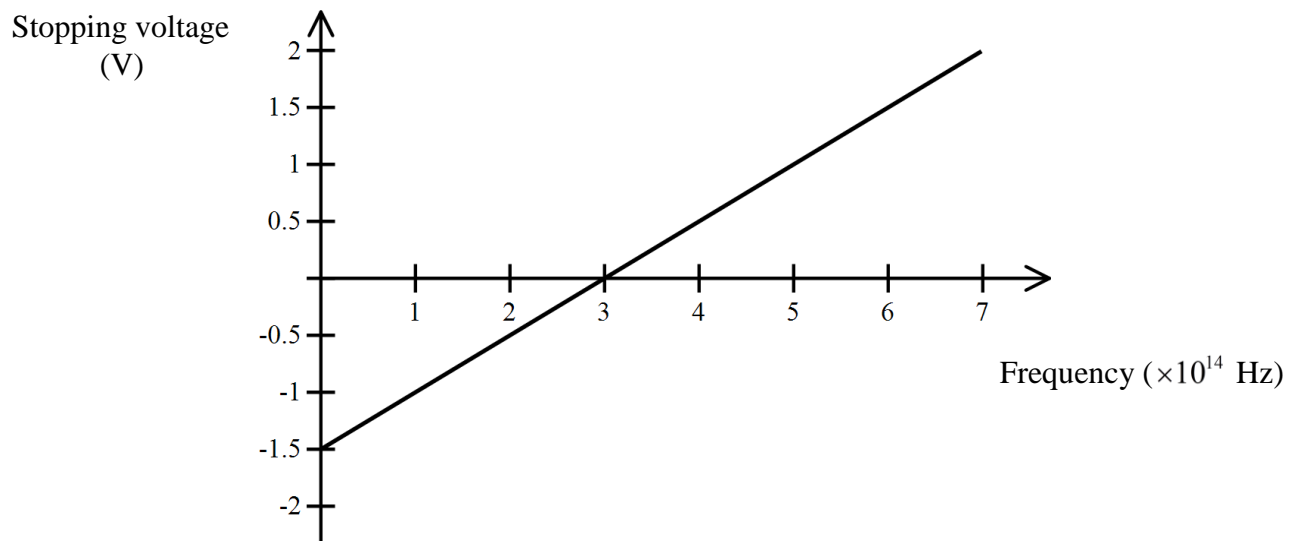
Review Questions

18. Observe the graph below.



Explain how this graph can be used to determine the minimum energy required to remove electrons from the metallic plate.

19. Observe the graph below.



- What is the threshold frequency in this situation?
- Find the work function for this metal.
- Use the graph to calculate the value of Planck's constant. Is it what you expected?
- Draw an additional line onto this graph to indicate a metal with a smaller work function.
- Photons with a wavelength of 1.2×10^{-6} m are shone onto the plate. Determine whether photoelectrons will be emitted or not.

The photoelectric effect supports the particle model

As it appears in Unit 4

If the wave theory was correct then 3 main conditions should have been met

- 1) The photoelectric effect should occur for any light regardless of its frequency or wavelength.
- 2) The maximum kinetic energy should be proportional to the intensity of the radiation because increasing the brightness should cause the electrons to absorb more energy and so be emitted at a greater speed.
- 3) There should be a delay between the contact of the radiation with the metal and the initial release of photoelectrons.

However the experimental results were opposite to what was expected.

- 1) For any given metal there is a minimum frequency of light that is required for photoemission to occur. Frequencies below this value will not cause emission no matter how bright the light.
- 2) The intensity of the light had no effect on the maximum kinetic energy of the photoelectrons. For frequencies that cause electron emission a dim light is enough to cause emission.
- 3) The delay between the contact of the radiation with the metal and the emission of the photoelectrons was found to be insignificant, being less than 10^{-9} seconds.

Review Question

20. Explain how the photoelectric effect supports the particle model for light.

Solutions to Review Questions

1. $V = 80 \text{ V}$

$$q = 1.6 \times 10^{-19} \text{ C}$$

$$W = qV$$

$$W = 1.6 \times 10^{-19} \times 80$$

$$W = 1.28 \times 10^{-17} \text{ J}$$

2. The answers are as follows:

i. $\frac{8 \times 10^{-19}}{1.6 \times 10^{-19}} = 5 \text{ eV}$

ii. $15 \times 1.6 \times 10^{-19} = 2.4 \times 10^{-18} \text{ J}$

3. $f = 5.4 \times 10^{14} \text{ Hz}$

$$E = hf$$

$$E = 6.63 \times 10^{-34} \times 5.4 \times 10^{14}$$

$$E = 3.6 \times 10^{-19} \text{ J}$$

$$p = \frac{E}{c} = \frac{3.6 \times 10^{-19}}{3 \times 10^8} = 1.2 \times 10^{-27} \text{ N s}$$

4. $\lambda = 6.0 \times 10^{-7} \text{ m}$

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{6.0 \times 10^{-7}} = 1.1 \times 10^{-27} \text{ N s}$$

5. A current will only be detected if the circuit is complete. For this to occur, the photons need to have sufficient energy to overcome the binding energy and then have sufficient kinetic energy to reach the collecting plate.
6. Different colours of light have different frequencies which provide different energy levels. To achieve accurate results it is important to produce photons with only one energy level.
7. Increasing the voltage inhibits the electrons from reaching the collecting plate.

8. The current will decrease until it reaches zero. As the voltage increases fewer electrons are able to cross to the collecting plate.

$$9. f_0 = \frac{3 \times 10^8}{5.3 \times 10^{-7}} = 5.66 \times 10^{14} \text{ Hz}$$

$$W = hf_0$$

$$W = 6.63 \times 10^{-34} \times 5.66 \times 10^{14}$$

$$W = 3.75 \times 10^{-19} \text{ J}$$

10. W represents the work function which is the amount of energy needed to remove the electron from the metal. Therefore the energy does not contribute to the motion of the electron and has to be subtracted from the total energy.

11. The answers are as follows.

In each case the $E_k = E_{\text{photon}} + \text{initial electron energy}$

$$\text{i. } E_k = 5.3 - 4.1 = 1.2 \text{ eV}$$

Photoemission will occur and the electrons will have a kinetic energy of 1.2 eV.

$$\text{ii. } E_k = 5.3 - 5.2 = 0.1 \text{ eV}$$

Photoemission will occur and the electrons will have a kinetic energy of 0.1 eV.

$$\text{iii. } E_k = 5.3 - 6.2 = -0.9 \text{ eV}$$

Photoemission did not occur. The negative answer indicates that the energy supplied by the photons was insufficient to overcome the binding energy.

$$12. f = \frac{3.0 \times 10^8}{5.00 \times 10^{-7}} = 6.0 \times 10^{14} \text{ Hz}$$

$$E_k = hf - W$$

$$E_k = (6.63 \times 10^{-34} \times 6.0 \times 10^{14}) - 3 \times 10^{-19}$$

$$E_k = 9.8 \times 10^{-20} \text{ J}$$

13. The answers are as follows:

$$\text{i. } W = hf - E_k$$

$$W = (4.14 \times 10^{-15} \times 6.4 \times 10^{14}) - 0.24$$

$$W = 2.41 \text{ eV}$$

$$\text{ii. } W = hf_0$$

$$f_0 = \frac{W}{h}$$

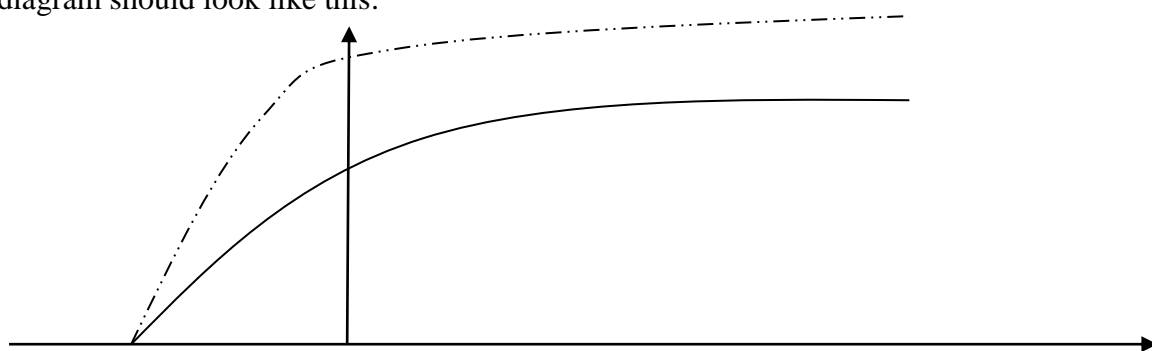
$$f_0 = \frac{2.41}{4.14 \times 10^{-15}}$$

$$f_0 = 5.82 \times 10^{14} \text{ Hz}$$

14. By changing the colour of the light from red to blue, the frequency of the light is being increased from approximately $4.3 \times 10^{14} \text{ Hz}$ to $7.13 \times 10^{14} \text{ Hz}$. If the frequency is increased then the photoelectrons will have higher energy and it will take more energy to stop their movement so the stopping voltage will

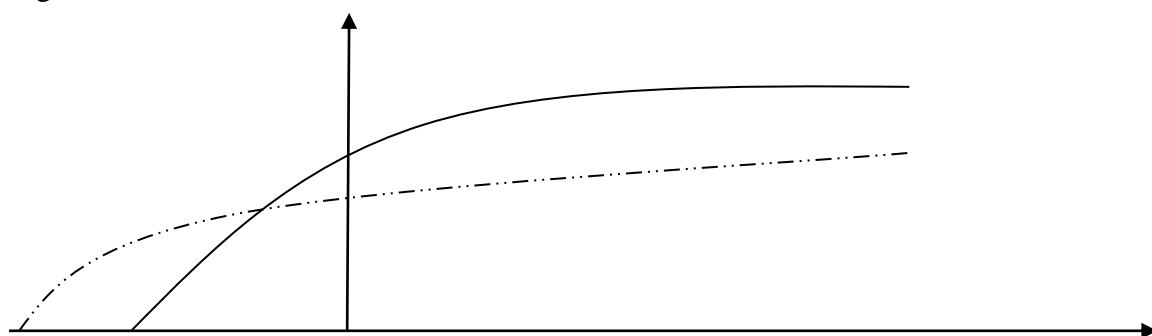
increase. The photocurrent will remain the same since the intensity has not been changed and therefore the same number of electrons will reach the collecting plate per unit time.

15. The diagram should look like this:



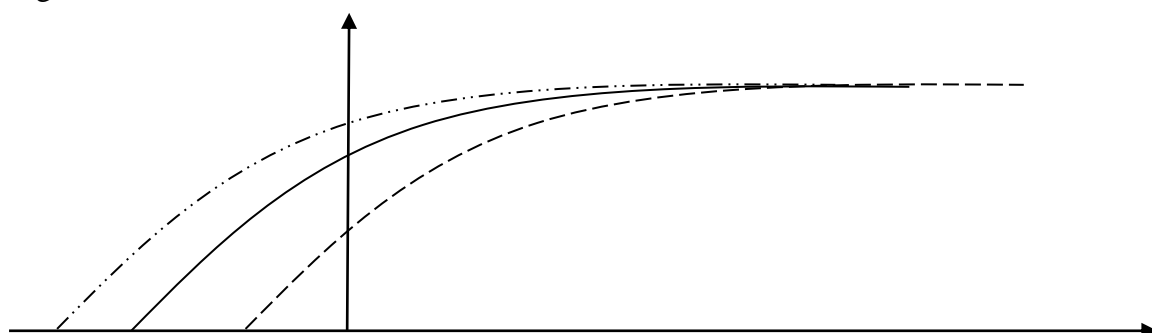
Increasing the intensity will increase the number of photons reaching the cathode per unit of time, as a result there will be more electrons emitted in any given time and the photocurrent will be higher. However, the energy of the photons has not been changed so the voltage required to stop the electrons will not change so the stopping voltage is the same.

16. The diagram should look like this:



If the frequency is increased then the energy of the photons is also increased resulting in higher energy photoelectrons. Therefore it will take more energy to stop their movement and the stopping voltage will increase. Since the intensity has decreased the number of photons per second will decrease so there will be fewer electrons per second reaching the electrode so the photocurrent will be lower.

17. The diagram should look like this:

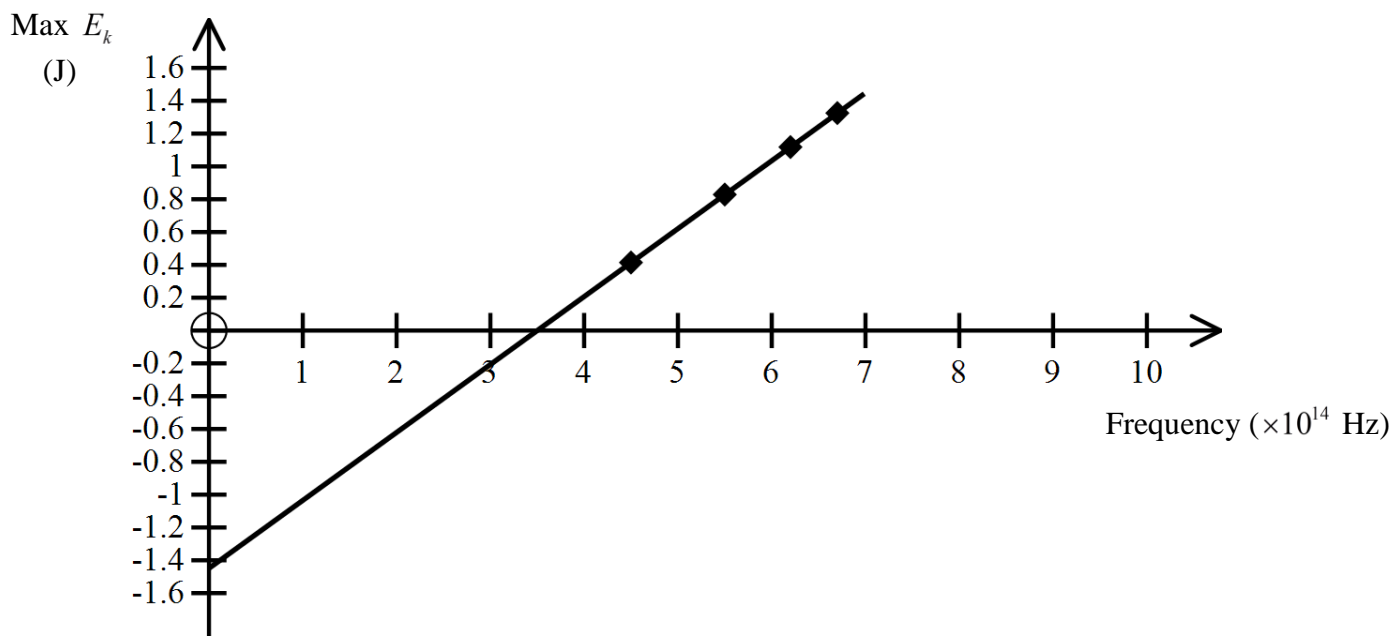


There will be no effect on the photons – the same number will be released and they will have the same energy. This means that the same number of electrons will cross and the photocurrent will not be changed.

The only difference will be due to the work function of the new metal. If the material has a higher work function it has to have a lower maximum E_k and therefore a lower stopping voltage. If the material has a lower work function it has to have maximum E_k and therefore a higher stopping voltage.

- 18.** The y axis of the graph would need to be extended into the negative. Then draw a line through the plotted points and extend it line downwards and find the y intercept. This equals the work function or the minimum energy required to remove photoelectrons from the plate.

The resulting graph is shown below.



19. The answers are as follows:

(Note that the stopping voltage (in V) is equal to the max kinetic energy (in eV))

a. 3.0×10^{14} Hz (From the x -intercept of the graph)

b. 1.5 eV (From the y -intercept of the graph)

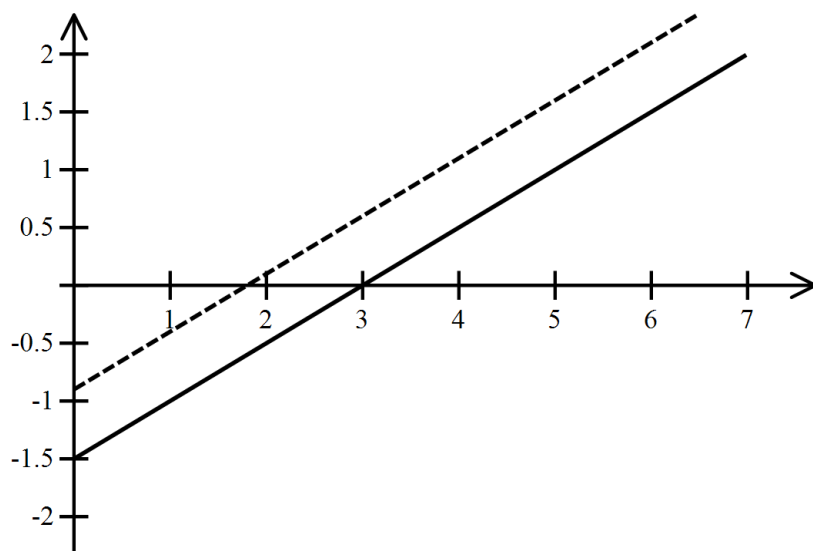
c. The gradient of the line should equal Planck's constant.

Using the two intercepts

$$\text{Gradient} = \frac{0 - (-1.5)}{3 \times 10^{14} - 0} = 5.0 \times 10^{-15} \text{ eV s}$$

The accepted value for Planck's constant is $4.14 \times 10^{-15} \text{ eV s}$. The value obtained from this experiment is reasonably close to the expected value.

d. A metal with a smaller work function would produce a line with the same slope but above the original line and with a less negative y -intercept such as the dashed line below.



e. $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{1.2 \times 10^{-6}} = 2.5 \times 10^{14} \text{ Hz}$

The frequency is lower than the threshold frequency therefore photoemission will not occur.

20. There are several reasons why the photoelectric effect supports the particle model for light. If the wave model were correct then the photoelectric effect should eventually occur for any frequency of light, but it was found that photoemission only occurs above specific frequencies. There should have been a delay between exposure to the light and photoemission occurring, but this did not happen. Also maximum kinetic energy is proportional to the frequency as predicted by a particle model of light, not proportional to the intensity as predicted by the wave model.



Physics

Teach Yourself Series

Topic 15: Wavelike nature of matter (Unit 4)

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What you should know

As it appears in Unit 4

- Calculate the de Broglie wavelength of matter, $\lambda = \frac{h}{p}$
- Compare the momentum of photons and of matter of the same wavelength including calculations using $p = \frac{h}{\lambda}$
- Interpret electron diffraction patterns as evidence for the wave-like nature of matter
- Compare the diffraction patterns produced by photons and electrons
- Explain the production of atomic absorption and emission spectra, including those from metal vapour lamps;
- Interpret spectra and calculate the energy of photons absorbed or emitted, $\Delta E = hf$
- Analyse the absorption of photons by atoms in terms of
 - the particle-like nature of matter
 - the change in energy levels of the atom due to electrons changing state
 - the frequency and wavelength of emitted photons, $E = hf = \frac{hc}{\lambda}$
- Describe the quantised states of the atom in terms of electrons forming standing waves, recognising this as evidence of the dual nature of matter

De Broglie Wavelength

As it appears in Unit 4



Louis de Broglie
(1892-1987)

It has been found that the **momentum** of particles (matter) is given by $p = mv$.

De Broglie suggested that the expression studied in the topic of photoelectric effect, which was used

to obtain the momentum of a photon, also applied to matter, that is $p = mv = \frac{hf}{c} = \frac{h}{\lambda}$

By simply rearranging the equation, we can obtain the formula of the **de Broglie Wavelength or the Wavelength of a Matter Wave**:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Where λ = the de Broglie wavelength of a particle (m)

h = Planck's constant = 6.63×10^{-34} (J s)

p = the momentum of the particle (kg m s^{-1})

m = the mass of the particle (kg)

v = the velocity of the particle (m s^{-1})

Warning: the *Electron-volt* value of Planck's constant, 4.14×10^{-15} eV s, **does not apply** when using this formula.

Please note that matter waves are not noticeable every day because the wavelike properties of matter will be noticeable only if the size of the de Broglie wavelength is comparable to the size of the relevant aperture and obstacle.

Review Questions

1. Find the de Broglie wavelength of the following objects.

a. A car with a mass of 1000 kg, travelling at speed of 20 km h^{-1} .

b. A 3 kg cat walking at 2 m s^{-1} .

c. Will the car or the cat diffract when they pass through a fully open door (about 1 m wide)?
Can you notice the wave-like properties of these objects. Why or why not?

2. Calculate the wavelength of an electron travelling at a speed of $5.1 \times 10^6 \text{ m s}^{-1}$.

Alternative formula for de Broglie Wavelength

As it appears in Unit 4

It has been found that the Kinetic Energy of a particle is given by $E_k = \frac{1}{2}mv^2$. Rearranging the formula, we obtain $v = \sqrt{\frac{2E_k}{m}}$. Substituting the derived v into $p = mv$. $\Rightarrow p = m \times \sqrt{\frac{2E_k}{m}} = \sqrt{2mE_k}$. Substituting the momentum expression into de Broglie's wavelength formula, we have the alternative formula for the de Broglie Wavelength:

$$\lambda = \frac{h}{\sqrt{2mE_k}}$$

Where λ = the de Broglie wavelength of a particle (m)

h = Planck's constant = 6.63×10^{-34} (J s)

m = the mass of the particle (kg)

E_k = the Kinetic Energy of the particle (J)

Review Questions

3. A 400 g tennis ball has kinetic energy of 760 J. What is the de Broglie wavelength of this tennis ball?

4. Calculate the wavelength of an electron that has been accelerated from rest through a potential difference of 100 V. The mass of an electron is 9.11×10^{-31} kg.

Diffraction

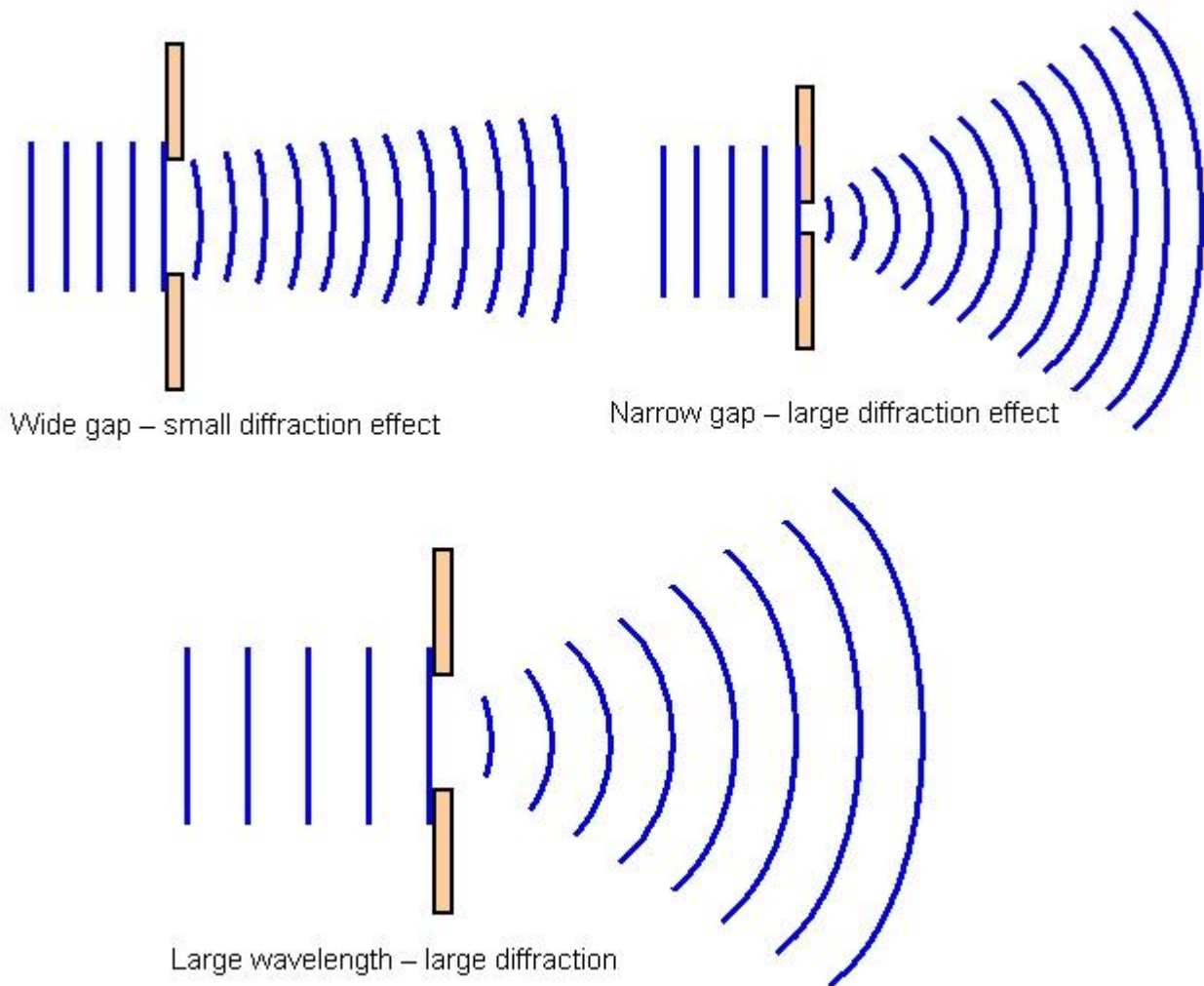
As it appears in Unit 4

Important Concept Review:

Here is what we learned in previous topics about the diffraction of waves.

Diffraction is the bending of waves as they pass the edge (or edges) of an obstacle or pass through an aperture.

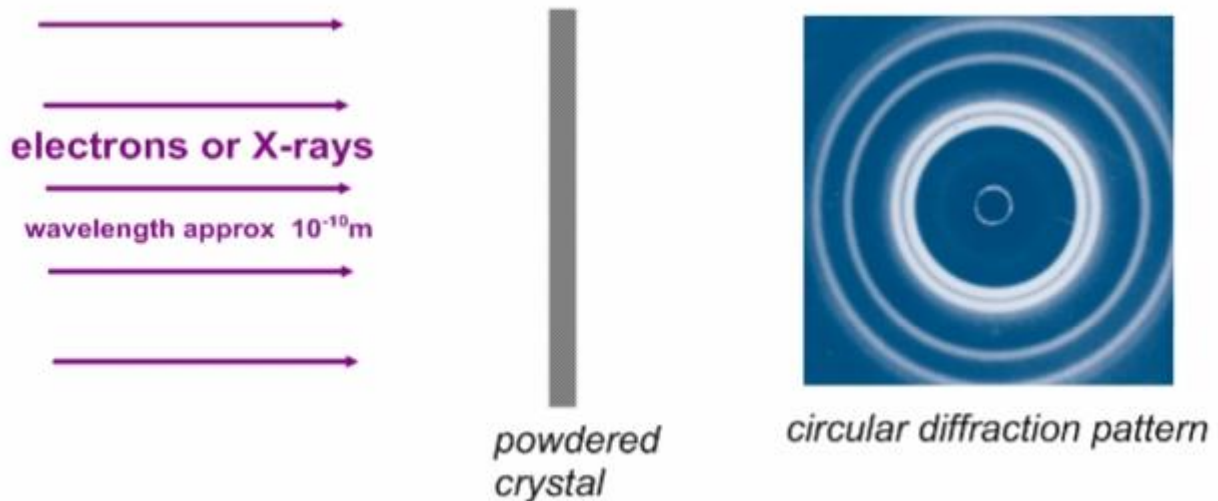
Significant Diffraction will occur when the wavelength is of at least the same order of magnitude as the width of the obstacle or aperture ($\frac{\lambda}{w} \geq 1$).



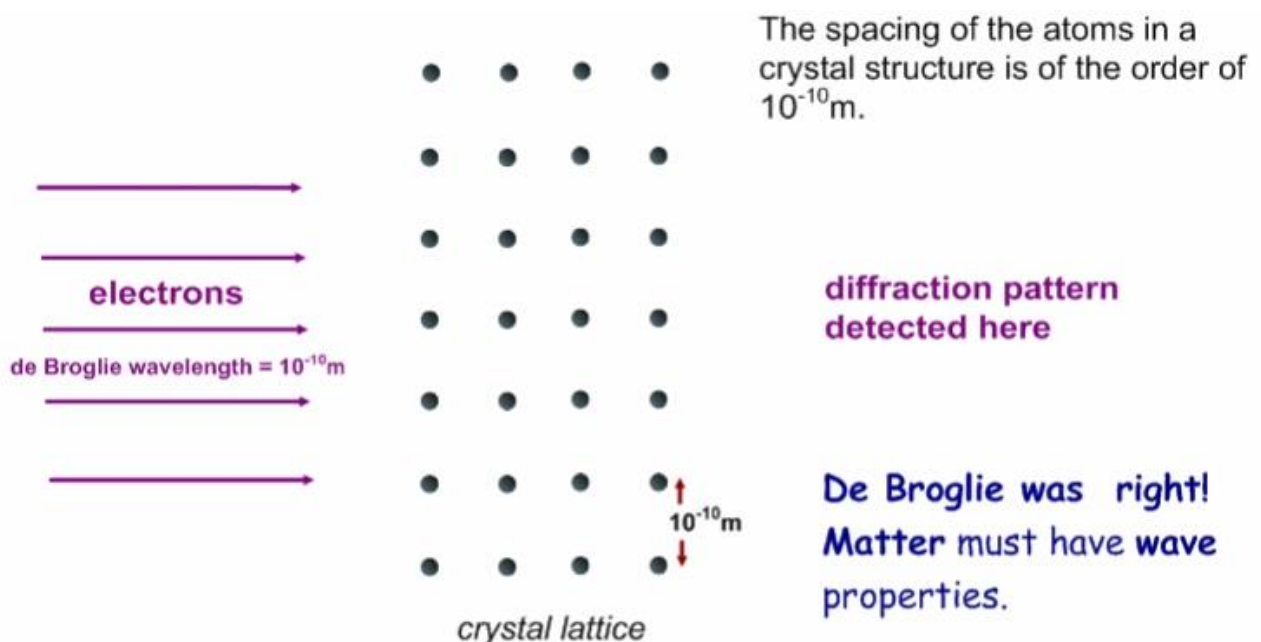
Electron Diffraction Patterns

As it appears in Unit 4

In 1927, G.P Thompson directed a stream of electrons at a small crystal of nickel and produced a scattering of the electrons. The pattern produced was very similar to the pattern that occurred when an X-ray beam was pointed at the crystal. This experiment verified de Broglie's ideas of the wave nature of matter or particles.

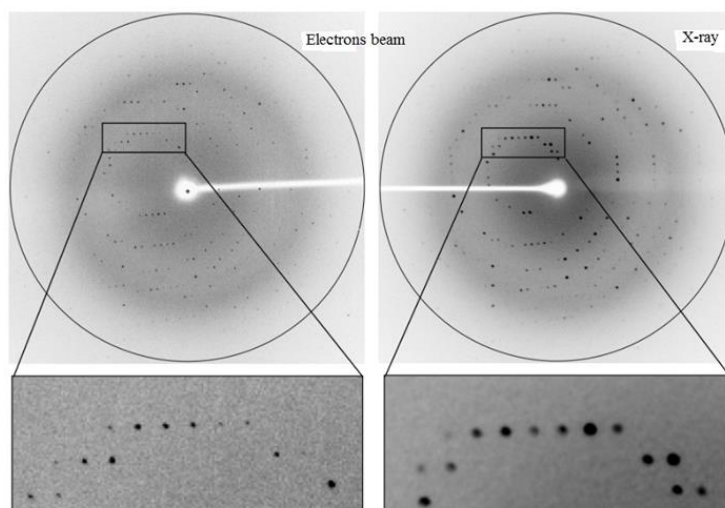


According to the calculations of *review questions 2 and 4* on *pages 4 and 5*, we can see that the de Broglie wavelength of fast moving electrons is approximately of the order of 10^{-10} m , which is comparable to the spacing between the atoms in a crystal. Therefore, it is conceivable that a beam of these electrons might produce a diffraction pattern after passing through the gap between atoms in a crystal.



Review Questions

5. Shown below are two images that have been obtained by passing electrons and X-rays through the same sample of powdered crystal. As displayed, assume that two images show exactly the same diffraction patterns and fringe spacing.



a. The energy of the X-ray photons is 65 keV. Find the wavelength of the X-ray.

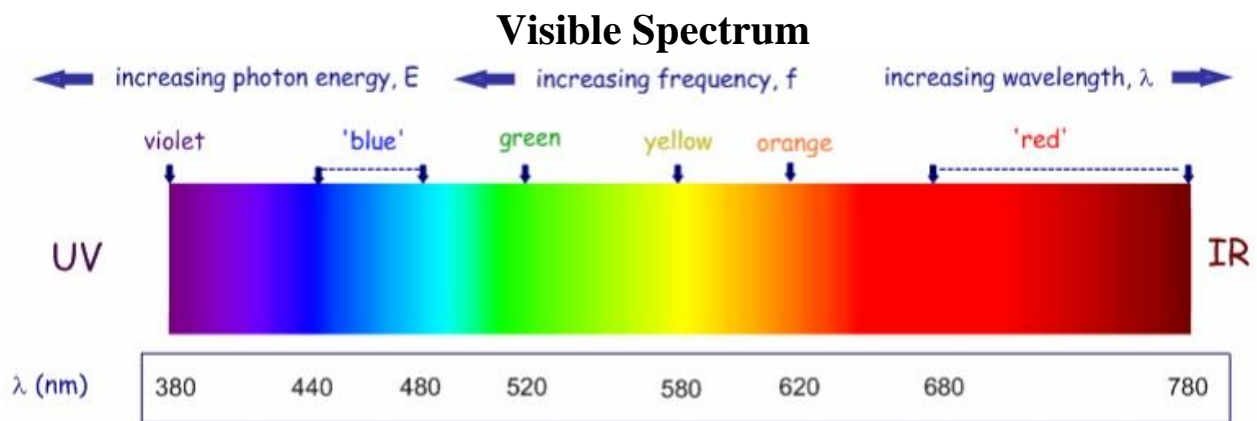
b. What conclusions can be made about the electrons and X-rays that were used?

c. What is the de Broglie wavelength of the electrons?

d. The mass of an electron is 9.11×10^{-31} kg. Calculate the kinetic energy of the electrons in both J and keV.

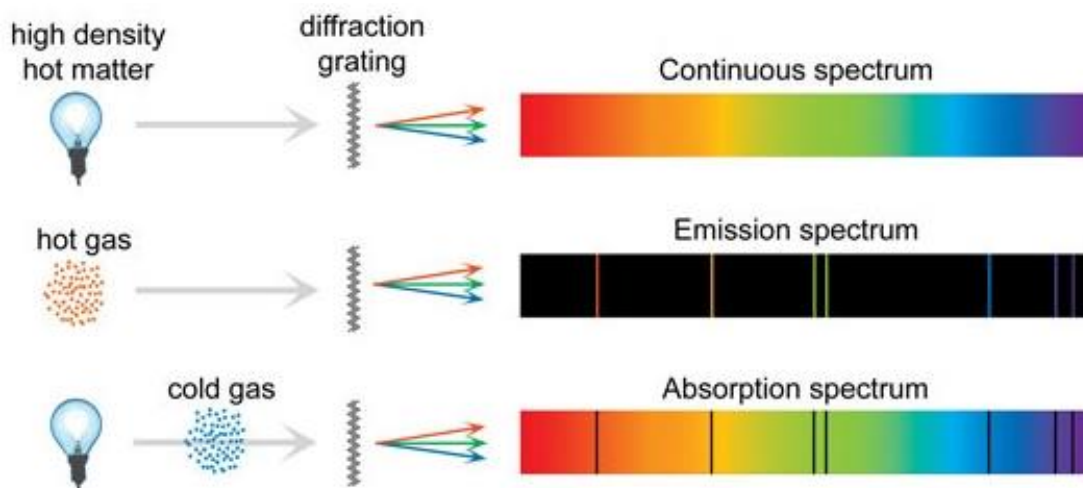
Photons and Atomic Structure

As it appears in Unit 4



In unit 2 physics, it was found that visible light is one type of Electro Magnetic Radiation (EMR). The spectrum of visible light contains a myriad of colours. Each different colour corresponds to a different wavelength or frequency.

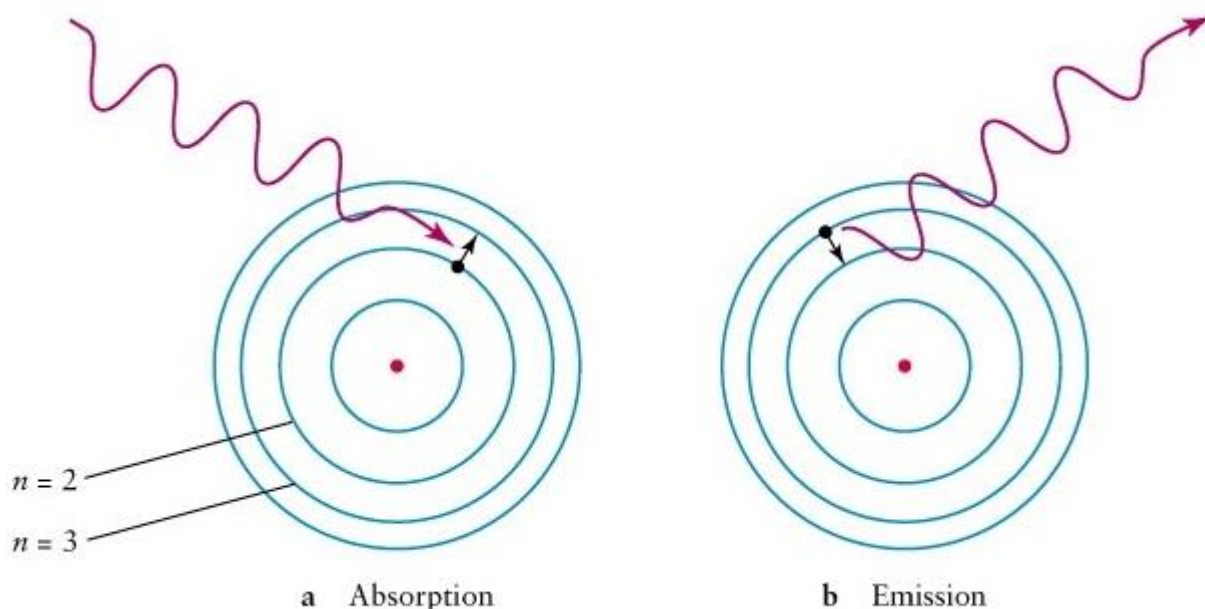
Emission and Absorption Spectra



In this topic, emission and absorption spectra have been discussed. If a spectrum of light passed through a cloud of gas, there would be black bands existing in the resultant spectrum. This is because many of the photons with these colours (corresponding wavelength or frequencies) were absorbed by the atoms of the gas.

On the other hand, if these same gas atoms were excited by being heated, they could emit light. When the emitted light is passed through a diffraction grating or spectroscope, the emission spectrum could be observed. As shown in the figure, **missing lines in the absorption spectrum (for a particular gas) correspond exactly to the lines in the emission spectrum.**

Bohr Model for Emission and Absorption Spectra



The electrons can only move in certain energy levels or circular orbits around the nucleus of the atom. These allowable orbits or energy levels are labeled $n = 1, 2, 3, \dots$; Electrons ordinarily occupy the lowest available energy level or orbit. For an electron to jump up to a higher energy level or orbit, it requires an input of energy equal to the difference in energy levels. Only incident light, carrying just the right amount of energy to raise an electron to an allowed level, can be absorbed. Vice versa, when electrons fall down through orbits either across one level or many levels, the emitted photon will have energy corresponding exactly to the differences between the energy levels or orbits.

If an electron in an atom moves between energy levels m and n , the energy of the photon that is either emitted or absorbed is given by:

$$E_{\text{photon}} = hf = E_m - E_n$$

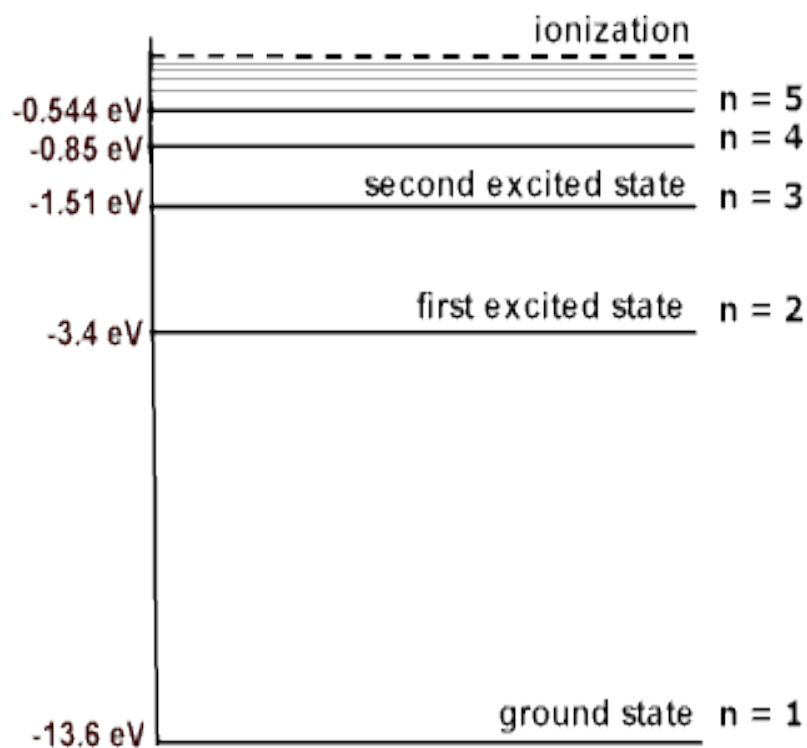
Where $h = \text{Planck's constant} = 6.63 \times 10^{-34} \text{ (J s)}$

$f = \text{the frequency of the photon (Hz)}$

Review Questions

The following information applies to questions 6-11.

Light is passed through hydrogen gas and an absorption spectrum is observed. Energy levels for atomic hydrogen are depicted in the diagram below.



6. What is the energy in eV of a light photon that would cause an electron to jump from the $n = 1$, ground state, to the $n = 4$ excited state?

7. Calculate the frequency and wavelength of this photon.

8. Indicate with an arrow on the energy level diagram, two further possible photon absorptions.

9. What will be the energy in eV of photons that move electrons from energy state

a. $n = 1$ to $n = 2$?

b. $n = 2$ to $n = 3$?

10. What happens to photons whose energy is in between the values in *Question 9*?

11. How could hydrogen atoms be ionized?

Bohr's Energy Levels of Hydrogen & Standing Waves

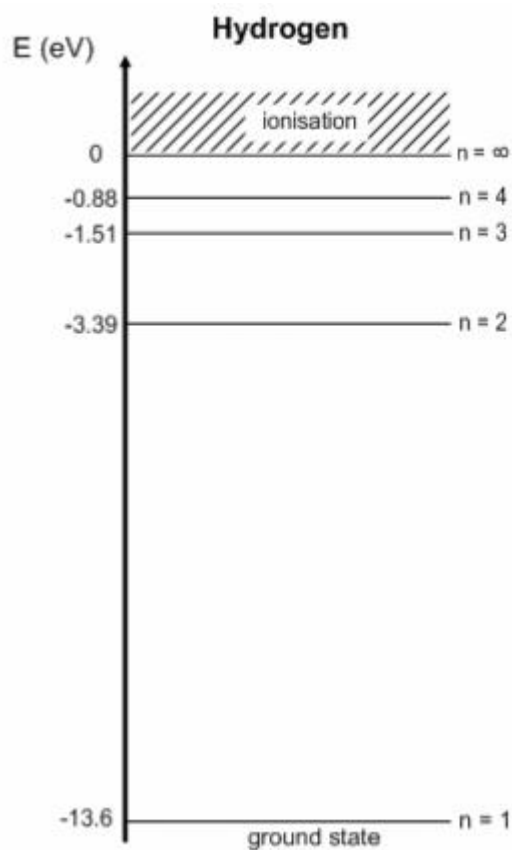
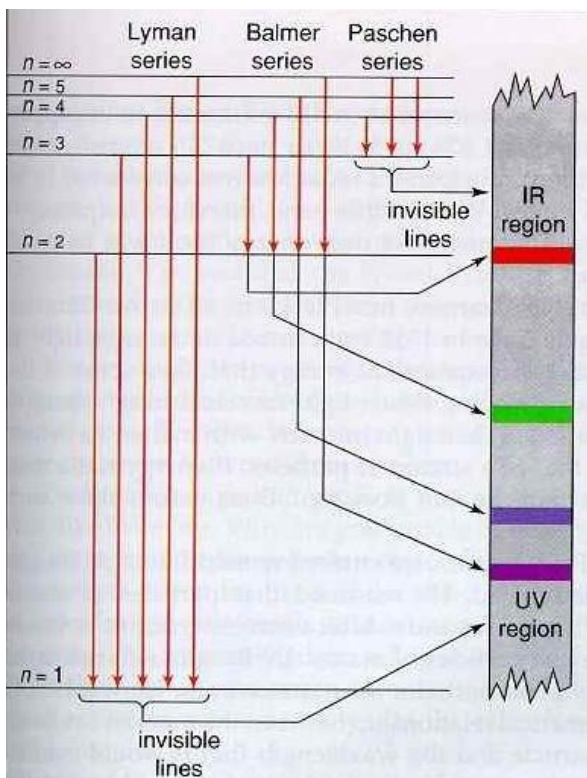
As it appears in Unit 4

The value of the various allowed energy levels for the hydrogen atom can be represented by the equation:

$$E_n = \frac{-13.6}{n^2}$$

Where E_n = the energy of the n th level for hydrogen (eV)

n = the energy level number: 1, 2, 3 ...



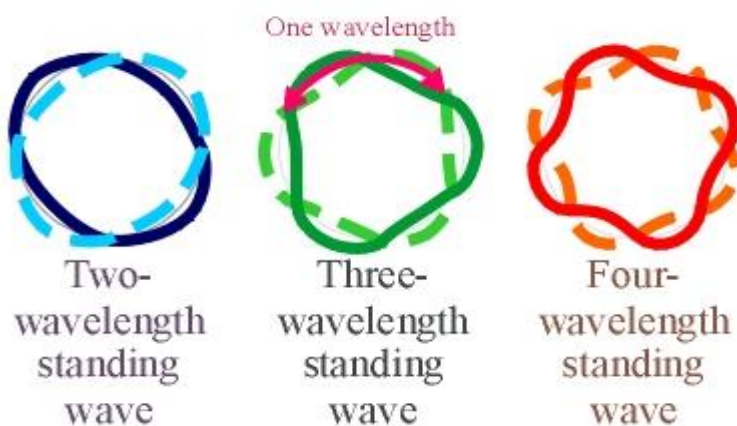
De Broglie proposed that matter had wavelength, which has been called the de Broglie wavelength. He applied this theory to explain Bohr's energy levels in a hydrogen atom. He viewed the electrons that were orbiting the hydrogen nucleus as matter waves and suggested that the electron could only maintain a steady energy level if it established a **standing wave**.

The only wavelength values that the electrons could maintain were the wavelengths that fitted perfectly into the orbit. Therefore the circumference of the orbit must be equal to a whole number multiple of its de Broglie wavelength.

$$2\pi r = n\lambda$$

Where r = the radius of the circular orbit (m)
 λ = the de Broglie wavelength of a particle (m)
 $n = 1, 2, 3, 4, 5 \dots$

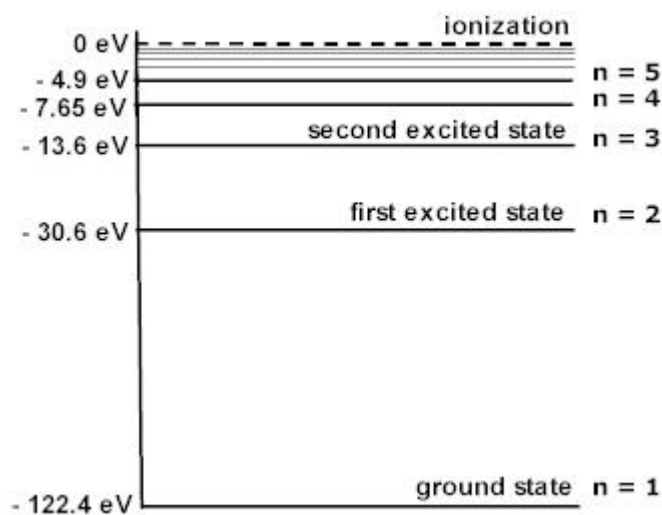
Some Allowed Circular Standing Wave Modes



Review Questions

The following information applies to questions 12-14.

The diagram shows the energy levels of a Lithium atom.



12. An electron of the Lithium atom is in an $n = 5$ excited state.

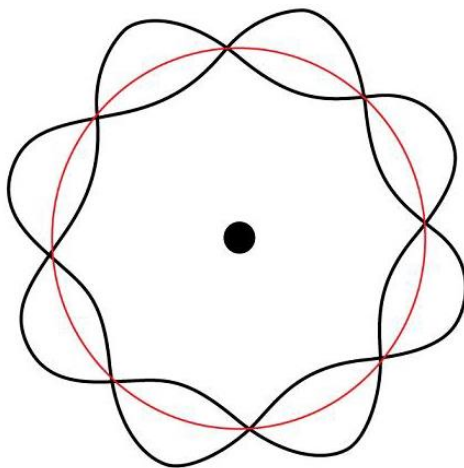
- a. What is the possible transition for the electron to release a photon with lowest energy? Calculate this photon energy.

b. Calculate the corresponding wavelength of the photon emitted in part **a**

13. What is the shortest wavelength of light that this atom could emit, when a Lithium atom is in the second excited state?

14. Many excited Lithium atoms have electrons in the $n = 4$ level. How many different photon energies may be observed as these atoms de-excite?

15. The diagram below represents de Broglie's "standing wave state" of an electron in a hydrogen atom.



a. What value of n would correspond to this pattern?

b. How much energy will be required to ionize this atom?

16. The fact that electrons of an atom are only stable at very specific orbits or energy levels, is best explained by:

- A. Wave properties of photons.
- B. Particle properties of electrons.
- C. The photoelectric effect.
- D. Wave properties of electrons.
- E. Particle properties of photons.

Solutions to Review Questions

1.

$$\text{a. } 20 \text{ km h}^{-1} = \frac{20}{3.6} = 5.56 \text{ m s}^{-1}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1000 \times 5.56} = 1.19 \times 10^{-37} \text{ m}$$

$$\text{b. } \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{3 \times 2} = 1.1 \times 10^{-34} \text{ m}$$

c. No, because their de Broglie wavelengths are much smaller than the gap.

No, because their de Broglie wavelengths are too small.

$$2. \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 5.1 \times 10^6} = 1.43 \times 10^{-10} \text{ m}$$

$$3. \lambda = \frac{h}{\sqrt{2mE_k}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 0.4 \times 760}} = 2.69 \times 10^{-35} \text{ m}$$

$$4. W = \Delta U_e = qV = E_k$$

$$E_k = eV$$

$$E_k = 1.6 \times 10^{-19} \times 100 = 1.6 \times 10^{-17} \text{ J}$$

$$\lambda = \frac{h}{\sqrt{2mE_k}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-17}}} = 1.23 \times 10^{-10} \text{ m}$$

5. a. $E_{\text{photon}} = hf = \frac{hc}{\lambda}$

$$\Rightarrow \lambda = \frac{hc}{E_{\text{photon}}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{65 \times 10^3 \times 1.6 \times 10^{-19}} = 1.91 \times 10^{-11} \text{ m}$$

b. It can be concluded that the electrons have the same wavelength and momentum as the X-rays.

c. $\lambda_{\text{electron}} = \lambda_{\text{Xray}} = 1.91 \times 10^{-11} \text{ m}$

d. $\lambda = \frac{h}{\sqrt{2mE_k}}$

$$\Rightarrow E_k = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31} \times (1.91 \times 10^{-11})^2} = 6.61 \times 10^{-16} \text{ J}$$

$$E_k = \frac{6.61 \times 10^{-16}}{1.6 \times 10^{-19}} = 4.13 \text{ keV}$$

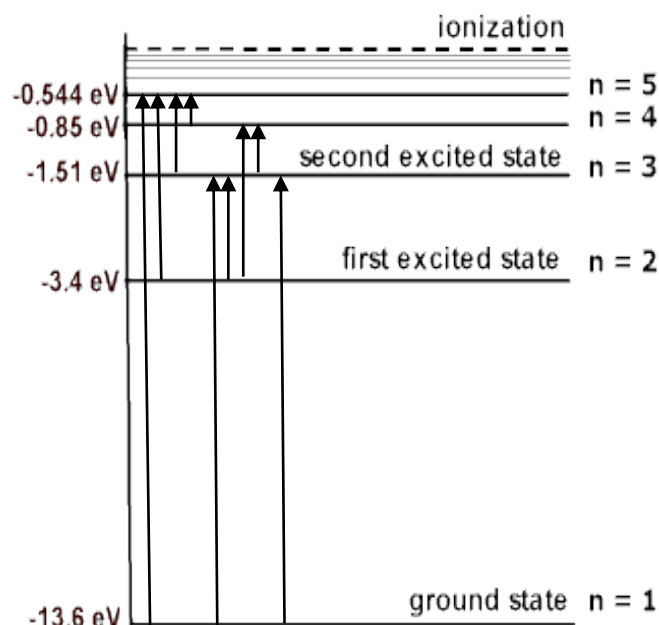
6. $E_{\text{photon}} = hf = E_m - E_n = |-13.6 - -0.85| = 12.75 \text{ eV}$

7. $f = \frac{E_{\text{photon}}}{h} = \frac{12.75}{4.14 \times 10^{-15}} = 3.08 \times 10^{15} \text{ Hz}$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3.08 \times 10^{15}} = 9.74 \times 10^{-8} \text{ m}$$

8.

Any two of the arrows shown in the diagram on the right. (There are other possible absorptions to higher energy levels as well.)



9.

a. $E_{\text{photon}} = hf = E_m - E_n = |-13.6 - -3.4| = 10.2 \text{ eV}$

b. $E_{\text{photon}} = hf = E_m - E_n = |-3.4 - -1.51| = 1.89 \text{ eV}$

10. They pass through the gas unchanged.

11. Photons, with energy greater than 13.6 eV, could supply sufficient energy to ionize the hydrogen atoms.

12. a. $n = 5 \rightarrow n = 4$ (lowest photon energy)

$$E_{\text{photon}} = hf = E_m - E_n = |-4.9 - -7.65| = 2.75 \text{ eV}$$

b. $E_{\text{photon}} = hf = \frac{hc}{\lambda}$

$$\Rightarrow \lambda = \frac{hc}{E_{\text{photon}}} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{2.75} = 4.52 \times 10^{-7} \text{ m}$$

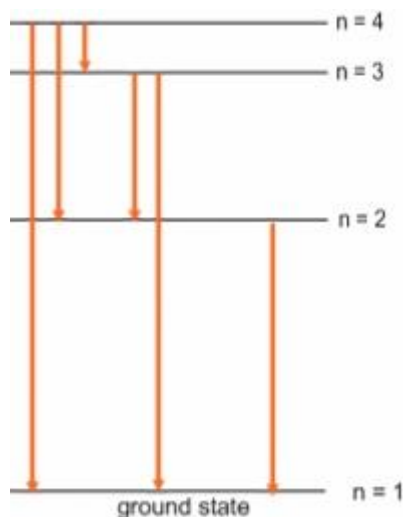
13. Transition of $n = 3 \rightarrow n = 1$ corresponds to shortest wavelength (highest photon energy) of light emitted.

$$E_{\text{photon}} = hf = E_m - E_n = |-13.6 - -122.4| = 108.8 \text{ eV}$$

$$E_{\text{photon}} = hf = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{E_{\text{photon}}} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{108.8} = 1.14 \times 10^{-8} \text{ m}$$

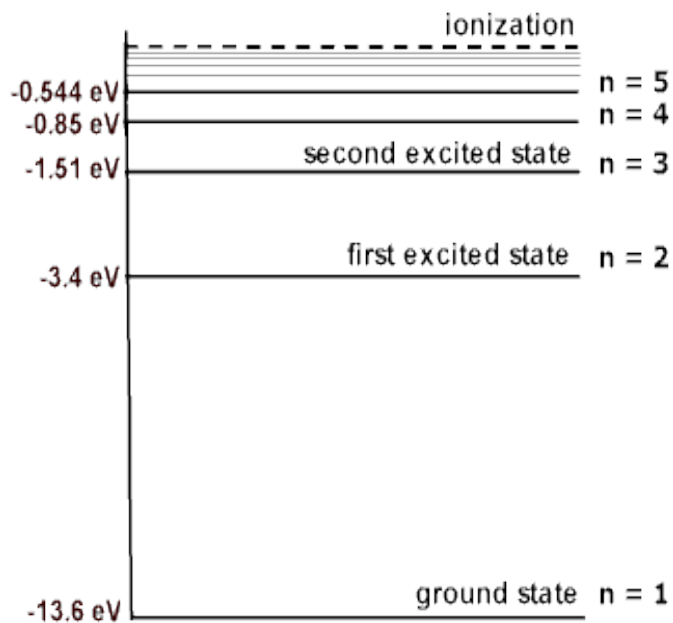
14. 6



15. a. $n = 4$

b. 0.85 eV

$n = 4$ (-0.85 eV) \rightarrow ionization (0 eV)



16. Answer is D.

De Broglie viewed the electron that was orbiting the hydrogen nucleus as matter waves and suggested that the electron could only maintain a steady energy level if it established a standing wave.



Physics

Teach Yourself Series

Topic 16: Production of light from matter (Unit 4)

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What you should know

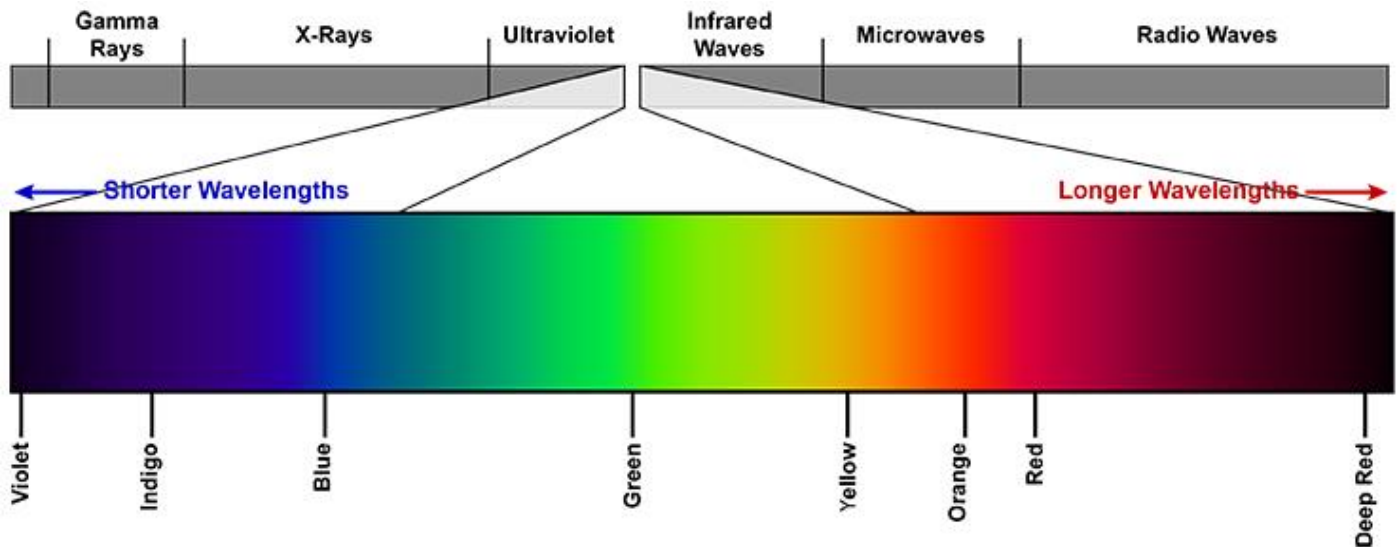
As it appears in Unit 4

- Interpret spectra and calculate the energy of emitted photons: $\Delta E = hf$
- Compare the production of light in lasers, synchrotrons, LEDS and incandescent lights

Light as an electromagnetic wave

As it appears in Unit 4

Light is a form of energy that travels as an electromagnetic wave. When we refer to light, we are generally referring to the section of the electromagnetic spectrum that we classify as “visible light”

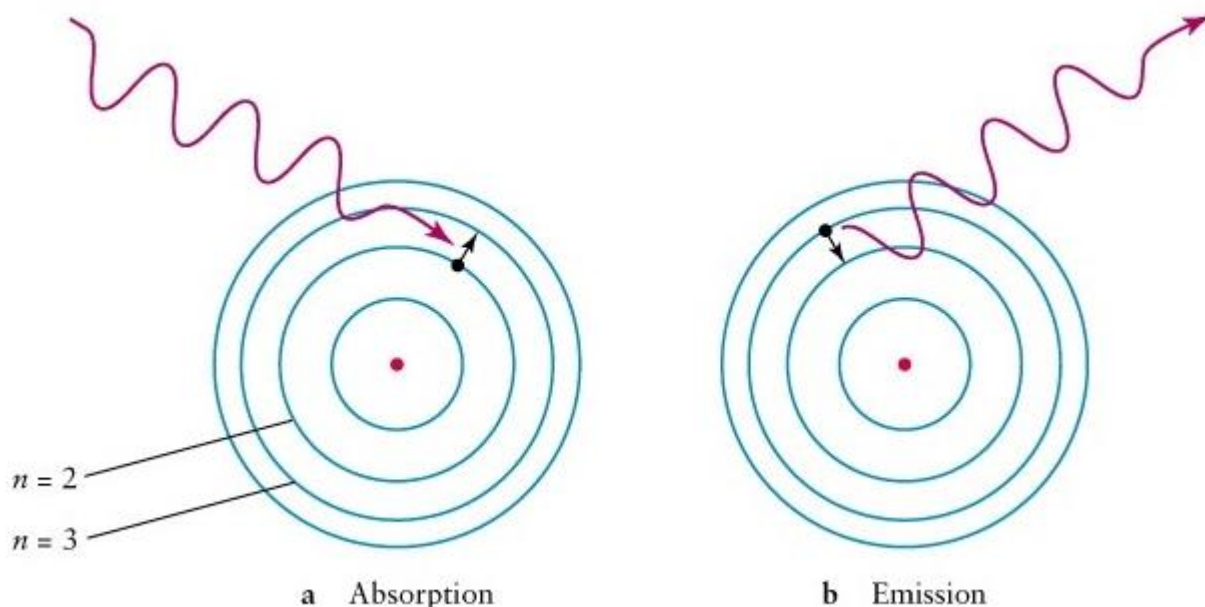


From the spectrum, we can see there are other forms of “light” ranging from Gamma Rays to Radio Waves. Each region of the spectrum varies due to the frequency of the photons contained and therefore the energy they possess as their energy is dependent on the frequency, $E = hf$.

Within this topic we are going to explore the production of light from matter. There are two methods in which light can be produced either via incandescence or luminescence.

Review of Bohr's Model for Emission and Absorption Spectra

As it appears in Unit 4



Electrons can only move in certain energy levels or circular orbits around the nucleus of the atom. These allowable orbits or energy levels are labelled $n = 1, 2, 3, \dots$; Electrons ordinarily occupy the lowest available energy level or orbit. For an electron to jump up to a higher energy level or orbit, it requires an input of energy equal to the difference in energy levels. Only incident light, carrying just the right amount of energy to raise an electron to an allowed level, can be absorbed. Vice versa, when electrons fall down through orbits either across one level or many levels, the emitted photon will have energy corresponding exactly to the differences between the energy levels or orbits.

If an electron in an atom moves between energy levels m and n , the energy of the photon that is either emitted or absorbed is given by:

$$E_{\text{photon}} = hf = E_m - E_n$$

Where h = Planck's constant = 6.63×10^{-34} (J s)

f = the frequency of the photon (Hz)

Incandescence

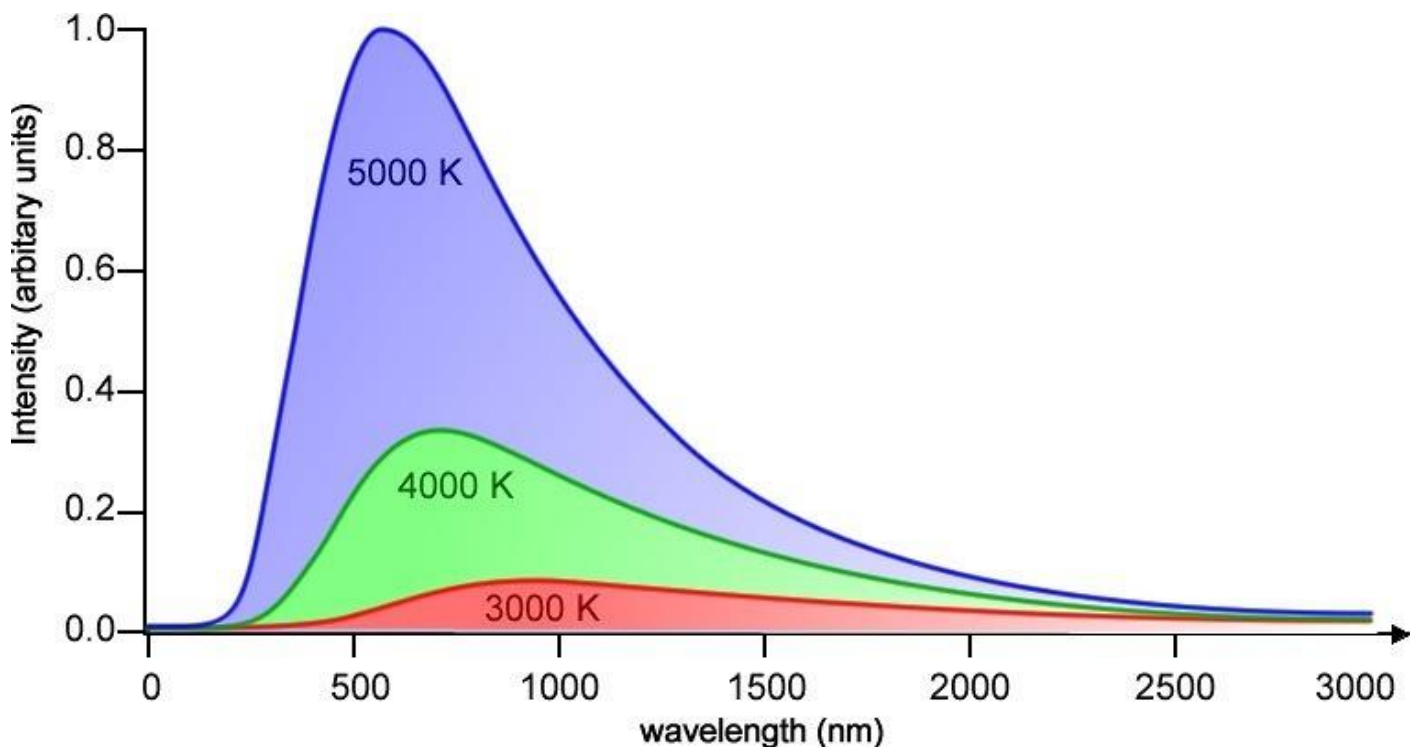
As it appears in Unit 4

Incandescence occurs when a hot object becomes luminous and radiates light. In every material atoms vibrate producing infrared radiation, however when the temperature of these materials is increased as it is in a light bulb by sending a current through it, the increase in temperature increases the vibrational energy of the atoms and the radiation they release shifts to the visible region of the spectrum. Light emitted by a hot object is called “blackbody radiation”.

An interesting property of blackbody radiation is the colour of the light produced does not depend strongly on the material being heated, it's more strongly correlated to the temperature that the body gets to. The hotter the material, the faster the atoms are moving, the higher their kinetic energy the higher the energy emitted is and hence the higher the frequency of the photons produced.

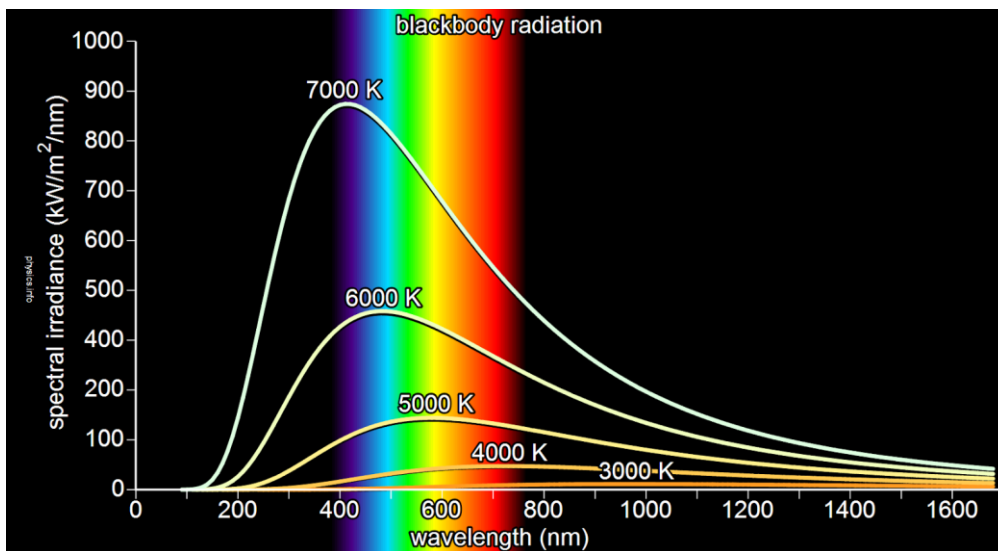
A cool object may emit light that is dim red, a hotter one might be bright orange and a hotter one still will be emitting radiation across the whole visible spectrum resulting in the white appearance.

Therefore, for a given temperature blackbody the spectrum has a peak at a particular wavelength. The hotter the body the shorter or “bluer” the wavelength at the peak.



From this graph, we can observe 5 features:

- The intensity of the emitted radiation is finite regardless of the temperature of the black body.
- There is a peak intensity of the radiation emitted.
- The peak rises as temperature of the black body increases.
- The peak shifts towards a lower wavelength as temperature increases.
- The spectrum is continuous and visible light only forms one small component of it.



Incandescent Light Bulbs

As it appears in Unit 4

Within the filament of the bulb, atoms vibrate and kinetic energy is created as a result, this releases photons of electromagnetic radiation over a range of frequencies. This range of frequencies results from the random nature of the atomic motion over a range of different energy levels.

In order for a human to be able to see the light produced from the filament, the temperature needs to be at least 500°C - this causes the object to glow dull red. For temperatures below this, the electromagnetic radiation is not visible to the human eye.

Light bulbs are generally heated to 2500°C . At this temperature, a fraction of the electromagnetic radiation falls into the visible spectrum. However, the majority is still contained in the infrared region. In order for light bulbs to be more efficient, they would need to be heated to 6300°C , which is hotter than the surface of the sun.

Review Questions

1. What does incandescence mean?

2. Why is an incandescent light bulb not an efficient device for producing light?

Luminescence

As it appears in Unit 4

Luminescent emission occurs after an appropriate material has absorbed energy from a source such as an X-ray, UV or electron beam. The energy they receive lifts the atoms to an excited state which is unstable. Because of the instability within this state, the material undergoes a transition back to the unexcited ground state and the absorbed energy is released as a photon.

Luminescence – Lasers

As it appears in Unit 4

Laser is an acronym for **Light Amplification by Stimulated Emission of Radiation**. A laser is created when electrons in atoms in specialised glass, crystals or gases absorb energy from an electrical current or another laser. This causes the excited electrons to move from the lower energy state (ground) to an excited state. When they return to ground they emit energy in the form of photons. These photons are all of the same wavelength, and hence, are defined as monochromatic. They are also coherent, meaning they are all in phase. The wavelength of the photons is dependent on the energy released as the electron returns to ground.

$$E_{\text{photon}} = hf = \frac{hc}{\lambda}$$

Laser light is also directional.

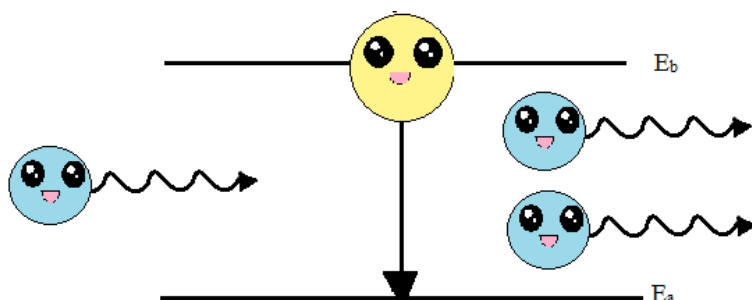
Lasers emit light within the visible light range of the spectrum, they also can emit infrared and ultraviolet radiation.

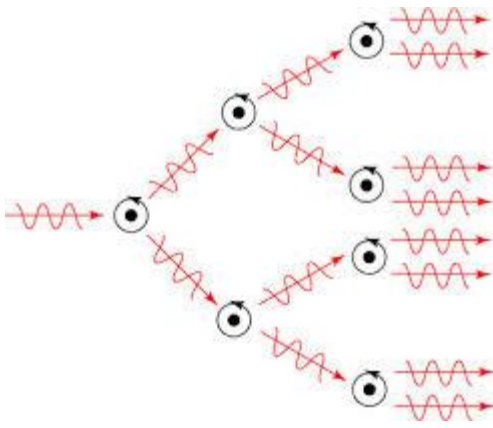
Stimulated emission

Normally an atom would have more electrons in their ground state than in excited states. When energy is added the population of excited electrons becomes bigger than the population in the ground state hence there are plenty of electrons ready to give off their energy – **population inversion**.

If these electrons are made to stay this way for time, stopping them from automatically dropping back to ground they form a **meta-stable state**.

If an additional photon with just the right amount of energy is then added, the excited electrons give off their energy as photons as well as releasing the original photon of light that was added in. The electrons are being stimulated to get energy out of them – **stimulated emission**. Two photons of light are being released for each one that is being added in hence an amplification process. These two photons stimulate another atom to give off more photons soon forming a chain reaction. Hence coherent laser light.





Review Questions

3. Explain how the name LASER indicates how light is produced within the device.

4. Explain the differences between laser light and an incandescent light bulb.

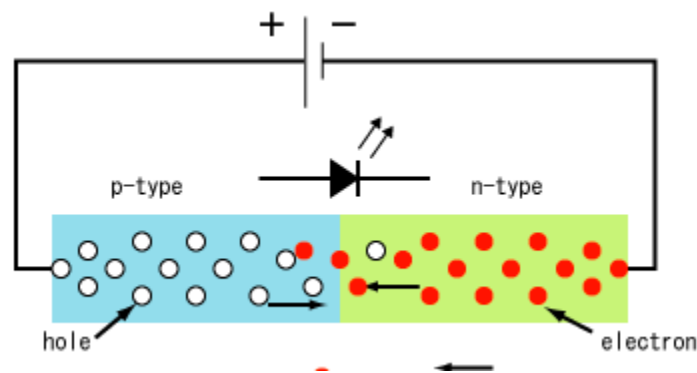
5. What makes laser light monochromatic and coherent?

Luminescence – LED's

As it appears in Unit 4

A light emitting diode (LED) is a device which uses semiconductor material and electroluminescence to create light. LED's are efficient light producing devices. Within an LED, an electric current is used to produce light. Electrons are sent through the material and used to fill electron holes (An electron hole exists where an atom lacks electrons – it is positively charged).

Certain materials such as silicon and germanium can be “doped” to create and control the number of electron holes they contain. Doping is the adding of additional element to the semiconductor, hence, changing its properties. By doing this, two separate semiconductor materials are contained in the one crystal. The p type semiconductor is the positively charged material and the n type has the excess electrons. The boundary between the two materials is called a p-n junction. This junction only allows current to flow in one direction. As an electron passes through one material into the other to fill the electron holes they emit a photon of light.



When a battery is connected across the diode with the positive terminal connected to the p-type layer, only a small potential difference (known as the threshold voltage) is needed to create a drift of electrons through the material. This is referred to as forward biasing. When a forward potential is applied, some of the electrons will diffuse across the junction from the n-type to the p-type material and fill some of the vacant holes. This process is called recombination. When this junction is formed, electrons located in the diode change their state and start to emit photons: the electrons change their orbit by moving from a higher orbit to a lower one to do that they automatically lose energy in the form of light. The more energy is lost, the more powerful the photons of light.

LEDs are specifically designed to make light of a certain wavelength, hence, they are monochromatic via spontaneous emission. Red LEDs produce photons with a wavelength in the range of 630 – 660 nm hence are seen as red, blue LEDs produce photons around 430-500 nm, which are seen as blue. LEDs can also be designed to produce infrared light.

The benefits of LEDs: they are cheap to make, they are easy to control, last a long time and save energy as they do not get hot.

Review Questions

6. With the aid of a diagram, explain how light is produced in an LED.

7. The band gap in an LED is 1.8 eV. Calculate the average wavelength of light emitted by this LED.

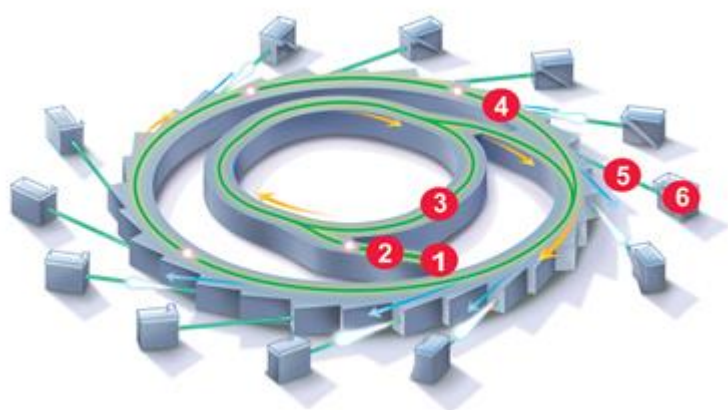
8. The light produced by a red LED and a red HeNe laser is being compared. Identify **three** features of the red HeNe laser light that are superior to the light produced by the red LED.

Synchrotron Light

As it appears in Unit 4

The synchrotron has been developed from particle accelerators. In particle accelerators, electrons, positrons or protons are accelerated to near light speeds in a circular path using magnetic fields. A consequence of accelerating charged particles is that electromagnetic radiation is emitted, which causes a loss of energy from the particle. This process is called synchrotron radiation. The electromagnetic radiation is emitted in a narrow cone in the forward direction, at a tangent to the electron's orbit.

Synchrotron light is unique in its intensity and brilliance and it can be generated across the range of the electromagnetic spectrum: from infrared to x-rays.



1. electron gun
2. linac
3. booster ring
4. storage ring
5. beamline
6. end station

The Australian Synchrotron is a 3 GeV synchrotron radiation facility built in Melbourne, Victoria and opened on 31 July 2007. The circular building was designed by Architectus in conjunction with Thiess, while the lattice design was performed substantially by Professor John Boldeman. The Synchrotron building is located in Clayton near the Monash University Clayton Campus.

The Australian Synchrotron is a light source facility. Electrons are generated in the centre (electron gun) and accelerated to 99.9997% of the speed of light by the linear accelerator (linac). The electrons are then transferred to the booster ring, where they are increased in energy. They are then transferred to the outer storage ring.

The electrons are circulated around the storage ring by a series of magnets separated by straight sections. As the electrons are deflected through the magnetic field created by the magnets, they give off electromagnetic radiation, so that at each bending magnet a beam of synchrotron light is produced. These beams can be captured and focused to a specific wavelength appropriate for a particular technique.

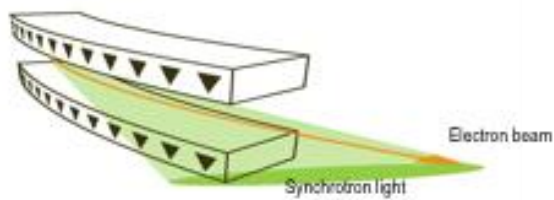


Illustration of a bending magnet.

At each deflection of the electron path a beam of light is produced. The effect is similar to the sweeping of a search light.

Synchrotron radiation spans a wide frequency range, from infrared up to the highest-energy X-rays. Synchrotron light has a number of unique properties. These include:

1. **High brightness:** synchrotron light is extremely intense (hundreds of thousands of times more intense than that from conventional x-ray tubes) and highly collimated.
2. **Wide energy spectrum:** synchrotron light is emitted with energies ranging from infrared light to hard x-rays.
3. **Tunable:** it is possible to obtain an intense beam of any selected wavelength.
4. **Highly polarised:** the synchrotron emits highly polarized radiation, which can be linear, circular or elliptical.
5. **Emitted in very short pulses:** pulses emitted are typically less than a nano-second (a billionth of a second), enabling time-resolved studies.

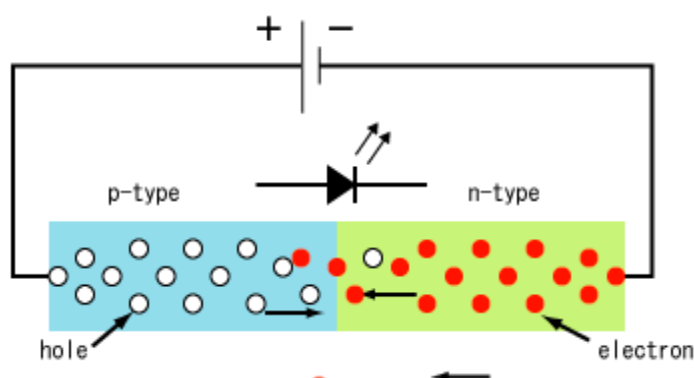
Review Questions

9. Compare the brightness, spectrum and divergence of synchrotron radiation with that of light from a laser beam

10. What are the special characteristics of the light produced by a synchrotron?

Solutions to Review Questions

1. Incandescence is the emission of light as a result of the material being heated. Atoms in the material are vibrating rapidly with large amounts of kinetic energy and as a result they release radiation in the form of infrared and visible light.
2. Incandescent light bulbs are not very efficient. They convert less than 5% of the energy to visible light the rest is converted as heat. The majority of the radiation they emit is in the infrared range which our eyes cannot detect.
3. Electrons within an atom are given energy so they jump to an excited state, where they are made to remain. An additional photon is sent in so a stimulate emission occurs producing two photons of light of the same wavelength that are coherent. These two photons stimulate further emission hence a chain reaction. Light amplification by stimulated emission of radiation.]
4. Laser light is coherent, monochromatic and directional. It is also intense. Incandescent light is a continuous spectrum where then light is made up of a combination of wavelengths. It is not polarised and spreads out in all directions therefore not intense.
5. Stimulated emission in lasers makes electrons produce a cascade of identical photons—identical in energy, frequency, wavelength therefore monochromatic. The photons produced are equivalent to waves of light whose crests and troughs line up hence "in phase" therefore coherent light.
6. Current is made to move through two different semiconductor materials a p-type material that is positively charged and an n-type material that contains electrons. When the electrons move through the p-n junction the electrons jump from an excited state back to ground hence releasing a photon of light.



$$7. E = \frac{hc}{\lambda}$$

$$1.8 = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{\lambda}$$

$$\lambda = 6.9 \times 10^{-7} \text{m}$$

8. Laser light is coherent, LEDs are incoherent

Lases therefore are concentrated and intense where LEDs spread light out.

Lasers are monochromatic whereas an LED produces light with a range of wavelengths.

9. Light produced by a laser has a single wavelength, whereas that produced by a synchrotron is continuous and has a range of wavelengths. In terms of brightness and divergence, beams of light produced by a synchrotron are extremely intense and have a much smaller divergence than those produced by a laser, which produces intense beams of light.

10. The characteristics of synchrotron light include;

Extremely intense and highly collimated.

Wide energy spectrum: synchrotron light is emitted with energies ranging from infrared light to hard x-rays.

Tunable: it is possible to obtain an intense beam of any selected wavelength.

Highly polarized: the synchrotron emits highly polarized radiation, which can be linear, circular or elliptical.

Emitted in very short pulses: pulses emitted are typically less than a nano-second (a billionth of a second), enabling time-resolved studies.



Physics

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What you need to know

As it appears in Unit 4

- independent, dependent and controlled variables
- the physics concepts specific to the investigation and their significance, including definitions of key terms, and physics representations
- the characteristics of scientific research methodologies and techniques of primary qualitative and quantitative data collection relevant to the selected investigation, including experiments (gravity, magnetism, electricity, Newton's laws of motion, waves) and/or the construction and evaluation of a device; precision, accuracy, reliability and validity of data; and the identification of, and distinction between, uncertainty and error
- identification and application of relevant health and safety guidelines
- methods of organising, analysing and evaluating primary data to identify patterns and relationships including sources of uncertainty and error, and limitations of data and methodologies
- models and theories, and their use in organising and understanding observed phenomena and physics concepts including their limitations
- the nature of evidence that supports or refutes a hypothesis, model or theory
- the key findings of the selected investigation and their relationship to concepts associated with waves, fields and/or motion
- the conventions of scientific report writing and scientific poster presentation, including physics terminology and representations, symbols, equations and formulas, units of measurement, significant figures, standard abbreviations and acknowledgment of references

Designing and Planning

Many practical activities you have completed in the past have been prewritten for you with clear goals and outcomes in mind. In this task, the experiment is up to you. The extended practical investigation is designed so you can focus on an area of Unit 3 or 4 you would like to investigate further. It allows you the opportunity to explore through experimenting and investigating something that interests you. It is important that you keep the ideas manageable and achievable.

Formulating a question

Formulating a good question allows you to focus on a particular idea. It provides you with a statement on what you are investigating. It needs to allow you to achieve an answer, be related to an appropriate area of study and be feasible in regards to your school resources.

Hypothesis and Aim

A hypothesis is a prediction based on your knowledge and previous experimental evidence. It generally takes the form of a statement predicting a certain outcome of the experiment. The aim is then the key steps required to test the hypothesis. It should be directly related to one of the variables you are testing and should describe the way you are going to test it.

Example: A student wishes to investigate the rebound height of a bouncing ball. Formulate a suitable hypothesis and aim for this experiment.

Hypothesis: This practical activity will be testing the rebound height of a bouncing tennis ball by dropping it from a variety of different heights and calculating the bounce efficiency on a variety of different surfaces. We would expect balls from dropped from a greater height onto a harder substance such as concrete to have a greater bounce efficiency than a ball dropped from a smaller height onto a softer substance such as carpet. This is due to greater potential energy being converted into greater elastic potential energy and hence in turn increasing the rebound height.

Aim: The aim of this experiment is to measure the rebound height of a tennis ball and calculate the bounce efficiency. The ball will be dropped from a set height of 1m and its rebound height recorded and repeated. The release height will then be changed to investigate if the release height has an impact on bounce efficiency.

Review Question

Question 1

A group of students are performing an experiment to test Hooke's Law by hanging different masses from a spring vertically.

The results they obtained are shown in Table 1 below

Mass (g)	Force (N)	Extension (cm)
0	0	0
100	0.98	1.0
200	1.96	1.97
300	2.94	3.0
400	3.92	3.9
500	4.9	4.9
600	5.88	6.3
700	6.86	6.86
800	7.84	7.85
900	8.82	8.85
1000	9.8	9.8

Construct an aim and a suitable hypothesis for this experiment.

Variables

As it appears in Unit 4

Independent variables

This is a variable that you chose to investigate. In many experiments, there will be numerous independent variables that can be examined, for these experiments you will need to choose two, both of which will be continuous. Independent variables will fall into two different categories:

Quantitative Variables

Continuous – these take any numerical value such as drop height of a ball and can be graphed.

Discrete - these are integer numerical values only such as the energy levels in an atom.

Qualitative Variables

These are observed and not measured. They can only be sorted into categories such as the type of ball.

Dependent variables

These are the variables that are changed due to your independent variable – they come from your experiment.

Example: For the bouncing ball experiment, identify some suitable independent and dependent variables. Record them in the correct classifications.

Independent Variables: release height – continuous, type of ball – qualitative, type of surface the ball is dropped onto – qualitative.

Dependent Variables: Bounce efficiency.

Review Question

Question 2

For the data set from Question 1, which variable would be the independent variable?

Question 3

Which variable would be the dependent variable?

Methodology

As it appears in Unit 4

Validity:

A procedure is valid if it tests what it is supposed to be testing. A procedure is invalid if the method of the experiment is incorrect or partially incorrect.

In a valid experiment, all variables are kept constant apart from those being investigated, all systematic errors have been eliminated and random errors have been reduced by taking multiple measurements.

In determining validity, students should consider the degree to which **evidence** supports the assertion or claim being evaluated. This may be done by making comparisons or conducting further experiments.

Reliability:

Reliability refers to the consistency with which we can confirm a result. Consistency is usually achieved by repetition.

Some good questions to ask:		
	first-hand information and data	secondary information and data
reliability	Have I tested with repetition?	How consistent is the information with information from other reputable sources? Is the data presented based on repeatable/repetition of processes?
validity	Does my procedure experiment actually test the hypothesis that I want it to? Have all variables been identified and controlled?	Do the findings relate to the hypothesis or problem?

Accuracy/Uncertainty

Most measurements contain some uncertainty. Accuracy refers to the exactness of a measurement.

We can measure a small distance with a metre rule or with much greater accuracy using a micrometer.

Every instrument has a limit to how precise it is in its measurements and has a tolerance of its measurements. This tolerance is the difference between a reading and the next reading. For example, using a ruler that measures in mm, if something measures between 1 and 2 mm, it would be hard to be exactly precise with the measurement. Therefore, to account for this, if the reading was closer to 1 mm we would write 1 ± 0.5 mm

Errors: Random vs Systematic

Random errors in experimental measurements are caused by unknown and unpredictable changes in the experiment. These changes may occur in the measuring instruments or in the environmental conditions. These errors can generally be eliminated by repeating the measurements.

Systematic errors in experimental observations usually come from the measuring instruments. They may occur because:

- there is something wrong with the instrument or its data handling system, or
- because the instrument is wrongly used by the experimenter.

Examples of systematic errors caused by the wrong use of instruments are:

- errors in measurements of temperature due to poor thermal contact between the thermometer and the substance whose temperature is to be found,
- errors in measurements of solar radiation because trees or buildings shade the radiometer.

Modifications

Your methodology will more than likely need to be modified during your experiment. In many experiments, problems will be encountered and it is how you deal with and record the problems that is important. Record everything you do, the steps you have taken, the problems you encountered including things like equipment not functioning as expected, results that are not what you have expected and how you overcame these issues. The key here is not to panic, go back through your results, talk to other students and your teacher about what's going wrong and take advice on how to rectify the situation.

Safety

As it appears in Unit 4

The safety of yourselves and others is paramount and it is important that you plan for this. Identify potential risks within your methodology and explain how you would go about eliminating or minimizing them. It is also essential that you get your method checked by your teacher then stick to the plan. Any modifications should also be approved by your teacher.

Keeping a log book

The scientific log book is a key way your teacher will authenticate your work. Your log book is to be handwritten in an exercise book and should be a reflection of your research, findings and thoughts throughout your experiment. It should record:

- initial and any subsequent research
- researchable questions and the hypothesis to be tested
- method and equipment used to conduct your experiment (include materials safety and data sheet (MSDS) and safety information)
- data collected from your experiment
- analysis and explanation of data
- personal thoughts on your progress (successes and difficulties)
- bibliography or reference list of information sources,

The log book is to be submitted with your scientific poster and should be regularly reviewed by your teacher.

Organising and Presenting Data

As it appears in Unit 4

Presenting raw data in tables

It is important to clearly outline what information each table to trying to convey. All tables should contain the following information:

- Title
- Headings of columns including units
- Independent variables listed on left hand side
- Dependent variables on right
- Calculations based on two variables in subsequent columns on right.
- Sample calculations located near the table.
- Consistent use of significant figures
- Clear display of replication of results
- Measurements showing uncertainty

Displaying trends

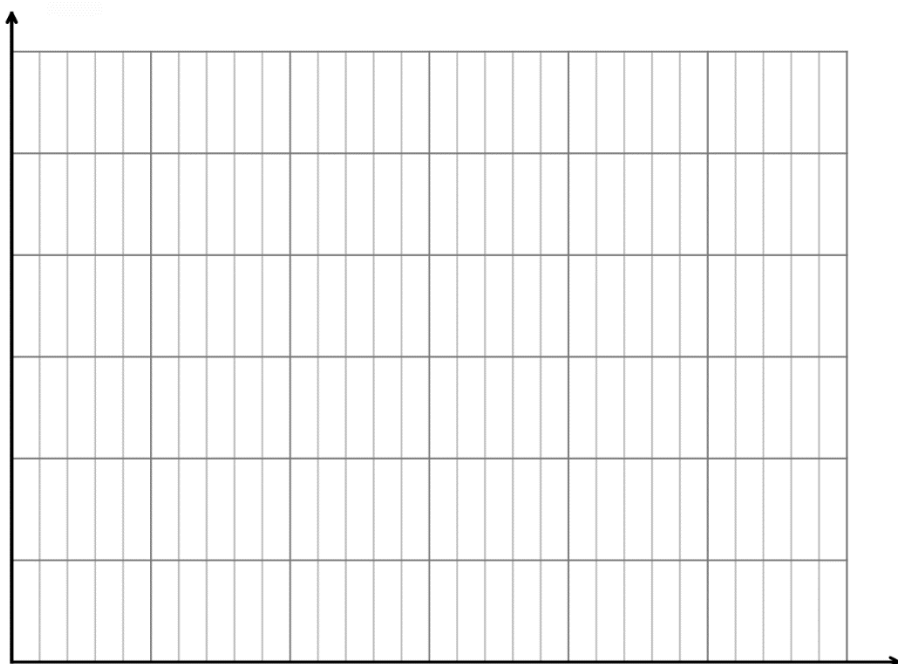
There are many ways trends can be displayed within data sets. Statistical analysis in terms of finding mean and mode might be a useful way. Graphing data is another. In regards to graphs the following information needs to be evident:

- Title
- Independent variable on x – axis, dependent variable on the y – axis.
- Axis should be clearly labelled including units
- Scales of axis need to be appropriate
- Trend lines (lines of best fit) should be included where appropriate
- Identification of outliers (points that are away from the rest of the data)

Review Question

Question 4

On the graph below graph the force vs extension data set from Question 1 including a line of best fit.



Analysing data and drawing relevant conclusions

As it appears in Unit 4

Within this section you are analysing your data. The following points are what you should be looking for:

- Show how your results are related to your original aim/s and the hypothesis described in the introduction.
- What do the results tell you? Are there any relationships, patterns or trends? Support all statements with reference to your data.
- Explain any identified relationships, patterns or trends in your results. Explain the scientific theory behind your results.
- What did you find out about this investigation? Was the outcome different from your prediction? How do your findings relate to scientific literature? Explain.
- Discuss problems you encountered as you conducted your investigation. Did you have to make any changes to the experiment that you described in the method section?
- Could anything be improved or changed to make the results more useful

Review Questions

Question 5

Does the data presented in Question 4 support the hypothesis? Explain.

Question 6

Was there any form of error present within the results? If yes, explain if it was random or systematic.

Question 7

Identify a suitable uncertainty for these results.

Question 8

Determine the spring constant (in N m^{-1}) for these results.

Question 9

Identify 2 safety aspects that needs to be taken into consideration when performing this experiment.

Presenting the Scientific Poster

As it appears in Unit 4

There are many different scientific posters available online. Your teacher may prescribe one or leave it up to you. The key headings that should be included are:

- Title
- Hypothesis/Aim
- Physics concepts and relationships
- Methodology
- Results
- Analysis
- Conclusion
- Bibliography

Solutions to Review Questions

Question 1

Hypothesis: Hookes Law states that the extension of a spring is directly proportional to the force applied.

Aim: Within this practical investigation by applying a series of masses to a spring hanging vertically and measuring the extension of the spring we aim to prove Hookes Law.

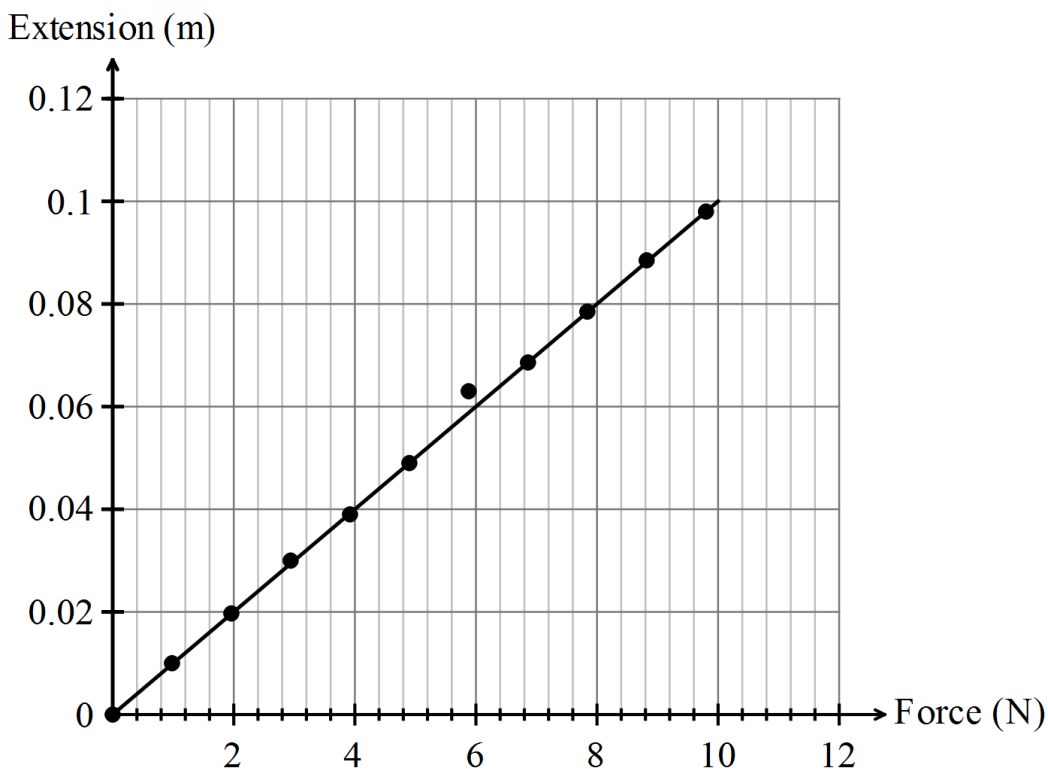
Question 2

Mass or force

Question 3

Extension

Question 4



Note that force is the independent variable so it is plotted on the horizontal axis. Extension is the dependent variable so it is plotted on the vertical axis.

Question 5

Yes.

Straight line which is indicative of the linear relationship we were expecting.

Line passes through the origin, therefore for no force there is no extension.

Question 6

The data points fell on or very close to the line apart from one point (5.88 N, 6.3 cm) – this would indicate a random error.

Question 7

Uncertainty could be due to the weights – estimate 5%

Within the measurements – length ± 0.05 cm

Question 8

In Hooke's Law, $F = kx$

\Rightarrow spring constant, $k = \frac{F}{x}$

For this graph, k will be the inverse gradient of the line of best fit.

$$k = \frac{\text{run}}{\text{rise}} = \frac{10}{0.1} = 100 \text{ N m}^{-1}$$

Question 9

Dropping weights onto feet

Spring flying off retort standing and flicking someone in the face