# Topic 1 — Newton's laws of motion

## 1.2 BACKGROUND KNOWLEDGE Motion review

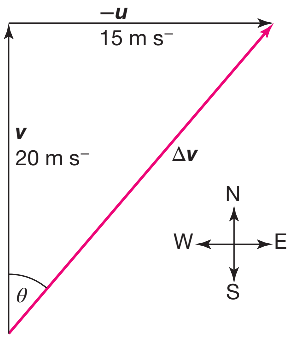
### Sample problem 1

**a** The velocity needs to be expressed in m s$^{-1}$:

60 km h$^{-1}$ = 16.7 m s$^{-1}$

$$\Rightarrow \boldsymbol{a}_{\text{av}} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{16.7}{5.0} = 3.3\ \text{m s}^{-2}\ \text{north}$$

**b** The magnitude of the change in velocity can be found by using Pythagoras's theorem or by trigonometry. Remember, subtracting $\boldsymbol{u}$ is the same as adding the negative vector for $\boldsymbol{u}$ (which is shown in the diagram as going east rather than west):




$$\Delta \boldsymbol{v} = \sqrt{20^2 + 15^2} = 25\ \text{m s}^{-1}$$

The direction can be found by calculating the value of the angle $\theta$:

$$\tan\theta = \frac{15}{20} = 0.75$$

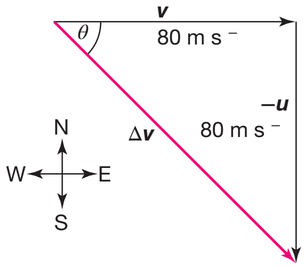
$$\Rightarrow \theta = 37°$$

The direction of the change in velocity is therefore N37°E.

Use the formula $\boldsymbol{a}_{\text{av}} = \frac{\Delta \boldsymbol{v}}{\Delta t}$ to calculate the average acceleration, where $\Delta \boldsymbol{v}$ = 25 m s$^{-1}$ N37°E and $\Delta t$ = 2.5 s:

$$\boldsymbol{a}_{\text{av}} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{25}{2.5} = 10\ \text{m s}^{-2}\ \text{N}\,37°\text{E}$$

**c** The magnitude of the change in velocity can be found by using Pythagoras's theorem or by trigonometry.




$$\Delta \boldsymbol{v} = \sqrt{8.0^2 + 8.0^2} = 11.3\ \text{m s}^{-1}$$

The direction can be found by calculating the value of the angle $\theta$. The triangle formed by the vector diagram shown is a right-angled isosceles triangle. The angle $\theta$ is therefore 45° and the direction of the change in velocity is south-east.

Use the formula $\boldsymbol{a}_{\text{av}} = \frac{\Delta \boldsymbol{v}}{\Delta t}$ to calculate the average acceleration, where $\Delta \boldsymbol{v}$ = 11.3 m s$^{-1}$ S45° E and $\Delta t$ = 4.0 s:

$$\boldsymbol{a}_{\text{av}} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{11.3}{4.0} = 2.8\ \text{m s}^{-2}\ \text{south-east (or S 45° E)}$$

### Practice problem 1

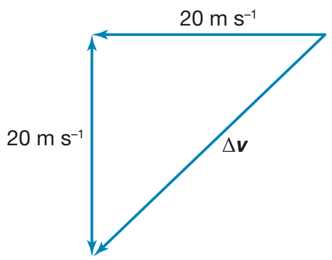
**a** $\boldsymbol{u} = 0;\ \boldsymbol{v} = 15;\ t = 5;\ \boldsymbol{a} = ?$

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$\boldsymbol{a} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t} = \frac{15 - 0}{5.0} = 3.0\ \text{m s}^{-2}\ \text{north}$$

**b** $\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$

$$\Delta \boldsymbol{v} = \boldsymbol{v}_{\text{final}} - \boldsymbol{v}_{\text{initial}} = 20\ \text{m s}^{-1}\ \text{W} - 20\ \text{m s}^{-1}\ \text{N}$$



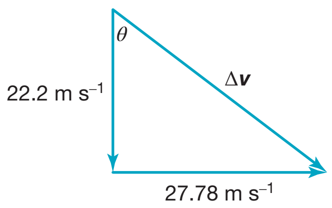

$$\Delta \boldsymbol{v} = \sqrt{20^2 + 20^2} = \sqrt{800}$$

$$\boldsymbol{a} = \frac{\sqrt{800}}{4} = 7.1\ \text{m s}^{-2}$$

The direction is south-west as shown in the diagram.

**c** $\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$

$$\Delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{u} = \frac{80}{3.6} - \frac{100}{3.6} = 22.22 - 27.78$$



$$\theta = \tan^{-1}\left(\frac{27.8}{22.2}\right) = 51°$$

$$\Delta \boldsymbol{v} = \sqrt{22.2^2 + 27.8^2} = \sqrt{1265}$$

$$\boldsymbol{a} = \frac{\sqrt{1265}}{5} = 7.1\ \text{m s}^{-2}$$

The average acceleration is 7.1 m s$^{-2}$ in the direction east 39° south.

## Sample problem 2

**a** The displacement of the car while it was slowing down is given by the area under the graph describing the time interval between 4.0 s and 6.0 s:

$$\text{area} = \frac{1}{2} \times 2.0\ \text{s} \times 10\ \text{m s}^{-1}\ \text{south} = 10\ \text{m south}$$

**b** The acceleration is given by the gradient of the graph describing the first 4.0 s after the lights turned green, that is, the time interval between 12 and 16 s During this time, the velocity increases from 0 m s$^{-1}$ south to 12 m s$^{-1}$

$$\boldsymbol{a} = \frac{\text{rise}}{\text{run}} = \frac{12}{4.0} = 3.0\ \text{m s}^{-2}\ \text{south}$$

**c** The displacement during the whole time interval described by the graph is given by the total area under the graph:

$$\text{area} = 4.0 \times 10 + \left(\frac{1}{2} \times 20 \times 10\right) + \left(\frac{1}{2} \times 4.0 \times 12\right) + (4.0 \times 12\ \text{south})$$
$$= 40 + 10 + 24 + 48$$
$$= 122\ \text{m south}$$

The average velocity is determined by the formula $\boldsymbol{v}_{\text{av}} = \frac{\Delta \boldsymbol{s}}{\Delta t}$:

$$\boldsymbol{v}_{\text{av}} = \frac{\Delta \boldsymbol{s}}{\Delta t} = \frac{122}{20} = 6.1\ \text{m s}^{-1}\ \text{south}$$

## Practice problem 2

**a** $\boldsymbol{a} = \frac{\Delta v}{\Delta t} = \frac{12}{4.0} = 3.0\ \text{m s}^{-2}\ \text{south}$

**b** $\boldsymbol{a} = \frac{\Delta v}{\Delta t} = \frac{-10}{2.0} = -5.0\ \text{m s}^{-2}\ \text{south or } 5.0\ \text{m s}^{-2}\ \text{north}$

**c**

$$\boldsymbol{v}_{\text{av}} = \frac{\text{displacement}}{\text{time}} = \frac{\text{area}}{\text{time}} = \frac{10 \times 4.0 + 0.5 \times 10 \times 2.0}{6.0} = 8.3\ \text{m s}^{-1}\ \text{south}$$

## Sample problem 3

**a** $\boldsymbol{u} = 0$, $\boldsymbol{v} = 12\ \text{m s}^{-1}$, $\boldsymbol{s} = 19\ \text{m}$

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$$
$$12^2 = 0 + 2\boldsymbol{a} \times 19$$
$$144 = 38 \times \boldsymbol{a}$$
$$\Rightarrow \boldsymbol{a} = 3.8\ \text{m s}^{-2}\ \text{down the slope}$$

**b**

$$s = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})\,t$$
$$19 = \frac{1}{2}(0 + 12)\,t$$
$$19 = 6.0 \times t$$
$$\Rightarrow t = \frac{19}{6.0} = 3.2\ \text{s}$$

**c** The magnitude of the average velocity during a period of constant acceleration is given by $\boldsymbol{v}_{\text{av}} = \frac{\boldsymbol{u} + \boldsymbol{v}}{2}$:

$$\boldsymbol{v}_{\text{av}} = \frac{\boldsymbol{u} + \boldsymbol{v}}{2} = \frac{0 + 12\ \text{m s}^{-1}}{2} = 6.0\ \text{m s}^{-1}$$

The distance travelled when Amy reaches an instantaneous velocity of this magnitude can now be calculated:

$\boldsymbol{u} = 0$, $\boldsymbol{v} = 6.0\ \text{m s}^{-1}$, $\boldsymbol{a} = 3.8\ \text{m s}^{-2}$

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$$
$$(6.0)^2 = 0 + 2 \times 3.8 \times \boldsymbol{s}$$
$$36 = 7.6 \times \boldsymbol{s}$$
$$\Rightarrow \boldsymbol{s} = \frac{36}{7.6} = 4.7\ \text{m}$$

Note that this is well short of the halfway mark in terms of distance.

**d** $\boldsymbol{u} = 0$, $\boldsymbol{v} = 6.0\ \text{m s}^{-1}$, $\boldsymbol{a} = 3.8\ \text{m s}^{-2}$

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$6.0 = 0 + 3.8 \times t$$
$$6.0 = 3.8 \times t$$
$$\Rightarrow t = \frac{6.0}{3.8} = 1.58\ \text{s} \approx 1.6\ \text{s}$$

This is the midpoint of the entire time interval. In fact, during any motion in which the acceleration is constant, the instantaneous velocity halfway (in time) through the interval is equal to the average velocity during the interval.

## Practice problem 3

**a**

$$\boldsymbol{x} = \frac{\boldsymbol{u} + \boldsymbol{v}}{2}t$$
$$\Rightarrow \boldsymbol{v} = \frac{2\boldsymbol{x}}{t} - \boldsymbol{u} = \frac{2 \times 400}{16} - 20 = 30\ \text{m s}^{-1}$$

**b** $\boldsymbol{x} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

$\Rightarrow \boldsymbol{a} = \frac{2(\boldsymbol{x} - \boldsymbol{u}t)}{t^2}$

$= 2\frac{(400 - 20 \times 16)}{16^2}$

$= 0.63 \text{ m s}^{-2}$

**c** $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$= 20 + 0.625 \times 16$

$= 30 \text{ m s}^{-1}$

$v_{av} = \frac{30 + 20}{2}$

$= 25 \text{ m s}^{-1}$

**d** **i** $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$= 20 + 0.625 \times 2$

$= 21 \text{ m s}^{-1}$

**ii** The car accelerates at a constant rate, and 8 seconds is halfway through the 16-second period of acceleration Therefore, the instantaneous speed is the average of the initial and final speed: $25 \text{ m s}^{-1}$

## 1.2 Exercise

**1** $\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

$10 = \frac{1}{2} \times \boldsymbol{a} \times 16$

$10 = 8\boldsymbol{a}$

$\Rightarrow \boldsymbol{a} = 1.25 \text{ m s}^{-2}$

Over the first 5.0 m:

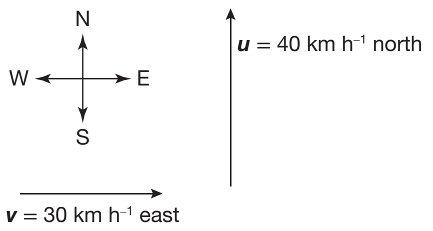
$\boldsymbol{u} = 0 \text{ m s}^{-1}$, $\boldsymbol{s} = 5 \text{ m}$, $\boldsymbol{a} = 1.25$, $\boldsymbol{v} = ?$

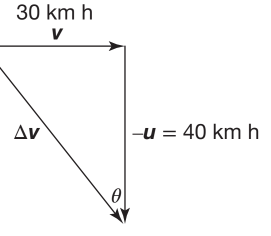
$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$

$= 2 \times 1.25 \times 5.0$

$\Rightarrow \boldsymbol{v} = 3.5 \text{ m s}^{-1}$

**2**




**a** $\Delta\boldsymbol{v}^2 = \boldsymbol{v}^2 + \boldsymbol{u}^2$

$\Delta\boldsymbol{v} = \sqrt{\boldsymbol{v}^2 + \boldsymbol{u}^2}$

$= \sqrt{30^2 + 40^2}$

$= 50 \text{ km h}^{-1}$

$\Rightarrow \tan\theta = \frac{30}{40}$

$\theta = \tan^{-1}\left(\frac{30}{40}\right)$

$= 37°$

$\Rightarrow \Delta\boldsymbol{v} = 50 \text{ km h}^{-1} \text{ s}^{-1} \text{ S } 37° \text{ E}$

$\boldsymbol{a} = \frac{\Delta\boldsymbol{v}}{\Delta t}$

$= \frac{50}{2.0}$

$= 25 \text{ km h}^{-1} \text{ s}^{-1} \text{ S } 37° \text{ E}$

**b** $\boldsymbol{a} = \frac{25}{3.6} \text{ m s}^{-2} \text{ S } 37° \text{ E}$

$\approx 6.9 \text{ m s}^{-2} \text{ S } 37° \text{ E}$

**3** **a** $t_1 = 8\text{s}$, $t_2 = ?$

$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t_2$

$0 = 70 - 4t_2$

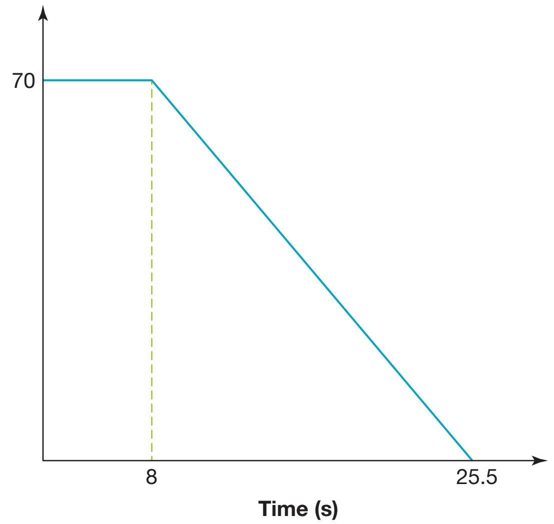
$4t_2 = 70$

$t_2 = 17.5 \text{ s}$

$t_{total} = 8 + 17.5$

$= 25.5 \text{ s}$

**b**




The velocity-time graph should have both axes labelled with their units. For the first 8 s, the velocity is constant at $70 \text{ m s}^{-1}$. From 8 s to 25.5 s (or 26 s, using 2 s.f.), the velocity should decrease at a constant rate, from $70 \text{ m s}^{-1}$ to zero.

**c** Distance is equal to the area under the graph:

$\text{area} = 70 \times 8 + \frac{1}{2} \times 17.5 \times 70$

$= 560 + 612.5$

$= 1172.5$

The length of runway used in the landing process was 1172.5 metres.

**4** $\boldsymbol{p} = m\boldsymbol{v}$

$p = 65 \times \frac{35}{3.6}$

$= 632 \text{ kg m s}^{-1}$

## 1.3 Newton's laws of motion and their application

### Sample problem 4

**a** Determine the net force acting on the car. The forward thrust is 5400 N and the sum of the forces acting in the negative direction is 600 N:

$\boldsymbol{F}_{\text{net}} = 5400 - 600$

$= 4.80 \times 10^3$ N

Apply Newton's second law to determine the acceleration of the car:

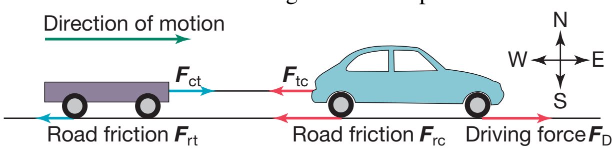
$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$

$\Rightarrow \boldsymbol{a} = \dfrac{\boldsymbol{F}_{\text{net}}}{m}$

$= \dfrac{4.80 \times 10^3}{1600}$

$= 3.00 \text{ m s}^{-2}$ east

**b** **i** A diagram must be drawn to show the forces acting on the car and trailer Assign due east aspositive:




Determine the net force acting on the entire system:

$\boldsymbol{F}_{\text{net}} =$ Driving forces $\boldsymbol{F}_{\text{D}}$ − resisting forces on car $\boldsymbol{F}_{\text{rc}}$

$\boldsymbol{F}_{\text{net}} = 5400 - 600 - 200$

$= 4600$ N

Apply Newton's second law to determine the acceleration of the system:

$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$

$\Rightarrow \boldsymbol{a} = \dfrac{\boldsymbol{F}_{\text{net}}}{m}$

$= \dfrac{4600}{2000}$

$= 2.30 \text{ m s}^{-2}$ east

**ii** Newton's second law can be applied to either the car or the trailer to answer this question.

*Method 1: Applying Newton's second law to the car*

$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$

$= 1600 \times 2.30$

$= 3.68 \times 10^3$ N

Write an expression for the net force acting on the car, and use it to determine the magnitude of the force exerted by the trailer on the car, where $\boldsymbol{F}_{\text{tc}}$ is the magnitude of the force exerted by the trailer on the car:

$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{D}} - \boldsymbol{F}_{\text{rc}} - \boldsymbol{F}_{\text{rt}}$

$\boldsymbol{F}_{\text{net}} = 5400 - 600 - \boldsymbol{F}_{\text{tc}}$

$3680 = 5400 - 600 - \boldsymbol{F}_{\text{tc}}$

$\Rightarrow \boldsymbol{F}_{\text{tc}} = 5400 - 600 - 3680$

$= 1.12 \times 10^3$ N

*Method 2: Applying Newton's second law to the trailer*

$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$

$= 400 \times 2.30$

$= 920$ N

Write an expression for the net force acting on the trailer, and use it to determine the magnitude of the force exerted by the car on the trailer, where $P_{\text{ct}}$ is the magnitude of the force exerted by the car on the trailer.

$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{ct}} - 200$

$920 = \boldsymbol{F}_{\text{ct}} - 200$

$\boldsymbol{F}_{\text{ct}} = 1.12 \times 10^3$ N

### Practice problem 4

**a** **i** $\sum \boldsymbol{F} = 0$ when at constant speed.

$\boldsymbol{F}_{\text{driving force}} - \boldsymbol{F}_{\text{resistive}} = 0$

$\boldsymbol{F}_{\text{driving force}} = 500$ N

**ii** Consider the trailer to have $\sum \boldsymbol{F} = 0$ when at constant speed:

$\boldsymbol{F}_{\text{T}} - \boldsymbol{F}_{\text{friction}} = 0$

$\boldsymbol{F}_{\text{T}} = 100$ N

**b** **i** $\sum \boldsymbol{F} = m\boldsymbol{a} = 600 \times 2.0 = 1.2 \times 10^3$ N

**ii** $\sum \boldsymbol{F} = m\boldsymbol{a}$

$\boldsymbol{F}_{\text{T}} - \boldsymbol{F}_{\text{friction}} = 1.2 \times 10^3$ N

$\boldsymbol{F}_{\text{T}} = 1.2 \times 10^3 + 100 = 1.3 \times 10^3$ N

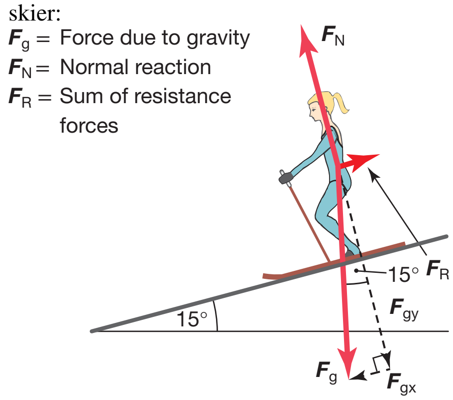
**iii** $\sum \boldsymbol{F} = m\boldsymbol{a}$

$\boldsymbol{F}_{\text{driving force}} - \boldsymbol{F}_{\text{friction}} - \boldsymbol{F}_{\text{T}} = 1.4 \times 10^3 \times 2.0$

$\boldsymbol{F}_{\text{driving force}} = 2.8 \times 10^3 + 1.3 \times 10^3 + 400 = 4.5 \times 10^3$ N

### Sample problem 5

**a** A diagram must be drawn to show the forces acting on the skier:




The net force on the skier has no component perpendicular to the surface of the snow, thus $\boldsymbol{F}_{\text{N}} = \boldsymbol{F}_{\text{gy}}$:

$\boldsymbol{F}_{\text{N}} = \boldsymbol{F}_{\text{gy}}$

$= \boldsymbol{F}_{\text{g}} \cos 15° \left(\text{since } \cos 15° = \dfrac{F_{\text{gy}}}{\boldsymbol{F}_{\text{g}}} \Rightarrow F_{\text{gy}} = \boldsymbol{F}_{\text{g}} \cos 15°\right)$

$= m\boldsymbol{g} \cos 15°$

$= 70 \times 9.8 \times \cos 15°$

$= 663$ N, rounded to $6.6 \times 10^2$ N

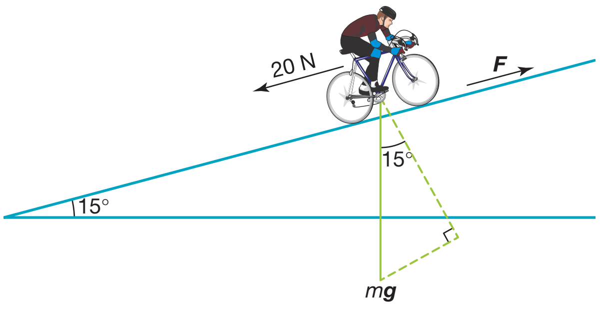
The normal force is therefore $6.6 \times 10^2$ N in the direction perpendicular to the surface as shown.

**b** The net force on the skier in the direction parallel to the surface is zero. This is known because the skier has a constant velocity. The magnitude of the sum of resistance forces therefore must be equal to the component of the force due to gravity that is parallel to the surface, $\boldsymbol{F}_R = \boldsymbol{F}_{gx}$:

$$\begin{aligned}\boldsymbol{F}_R &= \boldsymbol{F}_{gx}\\ &= \boldsymbol{F}_g \sin 15° \left(\text{since } \sin 15° = \frac{F_{gx}}{\boldsymbol{F}_g} \Rightarrow F_{gx} = \boldsymbol{F}_g \sin 15°\right)\\ &= mg \sin 15°\\ &= 70 \times 9.8 \times \sin 15°\\ &= 178\,\text{N, rounded to } 1.8 \times 10^2\,\text{N}\end{aligned}$$

The sum of the resistance forces (air resistance and friction) acting on the skier is $1.8 \times 10^2$ N opposite to the direction of motion.

## Practice problem 5




**a** **i** Resolve parallel to plane:

$$\sum \boldsymbol{F} = 0$$
$$\Rightarrow \boldsymbol{F} - mg \sin\theta - 20 = 0$$
$$\boldsymbol{F} - 90 \times 9.8 \sin 15 - 20 = 0$$
$$\boldsymbol{F} = 2.5 \times 10^2\,\text{N}$$

**ii** Resolve perpendicular to plane:

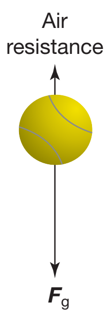
$$\begin{aligned}F_N &= mg \cos\theta\\ &= 90 \times 9.8 \times \cos 15°\\ &\approx 8.5 \times 10^2\,\text{N}\end{aligned}$$

**b** Resolve parallel to plane:

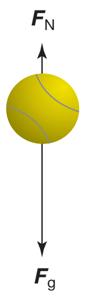
$$\sum F = ma$$
$$\Rightarrow mg \sin\theta - 50 = ma$$
$$90 \times 9.8 \times \sin 15° = 90\boldsymbol{a}$$
$$\boldsymbol{a} \approx 2.0\,\text{m s}^{-2}$$

## 1.3 Exercise

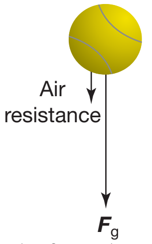
**1 a**




**b**




**c**

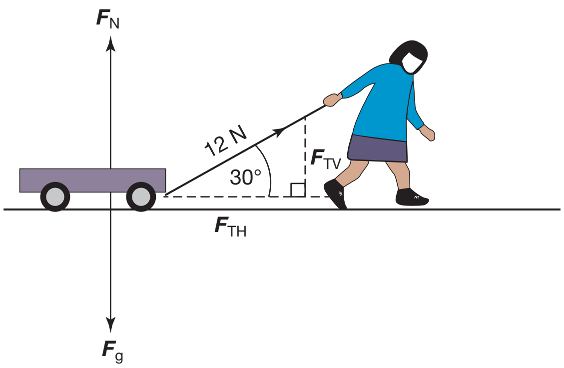



**2 a** The force due to gravity always points towards the centre of the Earth. This direction is represented by the arrow F.

**b** The normal force always acts perpendicularly to the surface. This direction is represented by the arrow C.

**c** X. As the coin is moving with constant velocity, the net force is zero.

**3 a**




$$\begin{aligned}\boldsymbol{F}_g &= mg\\ F_g &= 4.0 \times 9.8\\ &= 39.2\,\text{N}\end{aligned}$$

**b** $\boldsymbol{F}_{TH} = 12 \cos 30°$
$= 10\,\text{N}$

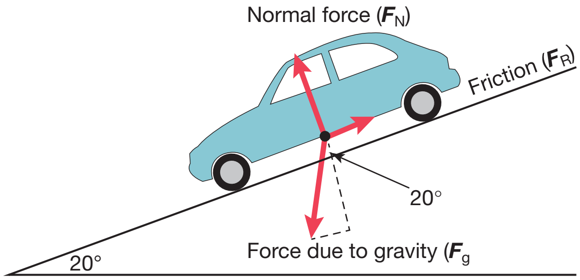
**c** Net vertical force = 0

$$F_N + F_T \sin 30° = F_g$$
$$F_N + 12 \sin 30° = 39.2$$
$$\begin{aligned}F_N &= 39.2 - 12 \sin 30°\\ &= 33.2\,\text{N}\end{aligned}$$

**4 a** 
$$\begin{aligned}\boldsymbol{a} &= \frac{\Delta \boldsymbol{v}}{t}\\ &= \frac{-2.0 - 5.0}{0.20}\\ &= -35\,\text{m s}^{-2}\end{aligned}$$

**b**
$$\begin{aligned}\boldsymbol{F}_{net} &= ma\\ &= 200 \times -35\\ &= -7000\,\text{N}\end{aligned}$$

**5 a**




**b** $F_N = F_g \cos(20°)$

$= mg \cos(20°)$

$= 1500 \times 9.8 \times \cos(20°)$

$\approx 1.4 \times 10^4$ N

**c** The car is stationary, so the net force on it is zero.

**d** $\boldsymbol{F}_{net} = 0$

$mg \sin(20°) - F_R = 0$

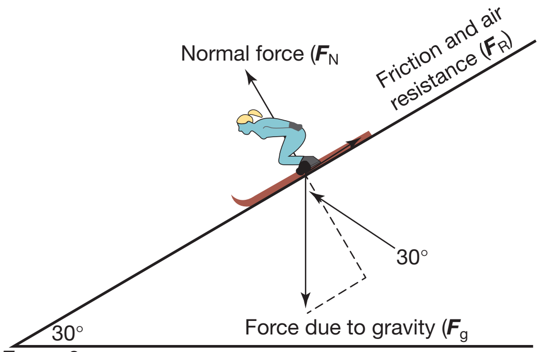
$F_R = mg \sin(20°)$

$= 1500 \times 9.8 \times \sin(20°)$

$\approx 5.0 \times 10^3$ N

**6 a** As the skier is moving at a constant speed there is no net force.

**b**




$\boldsymbol{F}_{net} = 0$

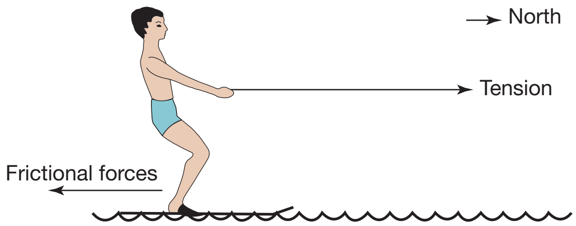
$mg \sin 30° - (\text{air resistance} + \text{friction}) = 0$

$\Rightarrow$ air resistance + friction $= mg \sin 30°$

$= 60 \times 9.8 \times \sin 30°$

$\approx 2.9 \times 10^2$ N

**7 a**




Horizontal forces acting on the skier

$\boldsymbol{F}_{net} = m\boldsymbol{a}$

$= 70 \times 2.0$

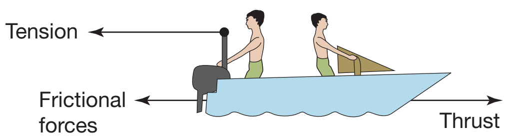
$= 140$ N north

tension − frictional forces = 140

tension − 240 = 140

tension = $3.8 \times 10^2$ N north

**b**




$\boldsymbol{F}_{net} = m\boldsymbol{a}$

$= 350 \times 2.0$

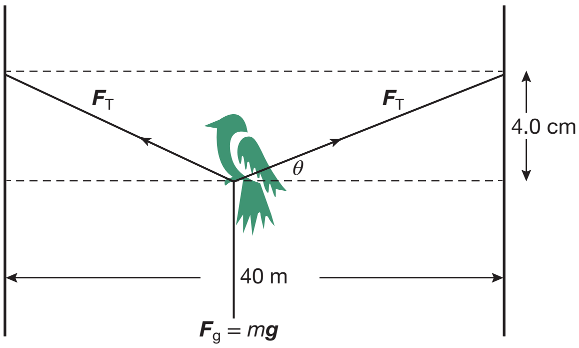
$= 700$ N north

thrust − tension − frictional forces = 700

thrust − $3.8 \times 10^2$ − 600 = 700

thrust = $1.7 \times 10^3$ N north

**8**




The net force on the magpie is zero. Resolving vertically:

$2F_T \sin\theta = mg$

$\Rightarrow F_T = \dfrac{mg}{2\ \sin\theta}$

$\tan\theta = \dfrac{0.040}{2.0}$

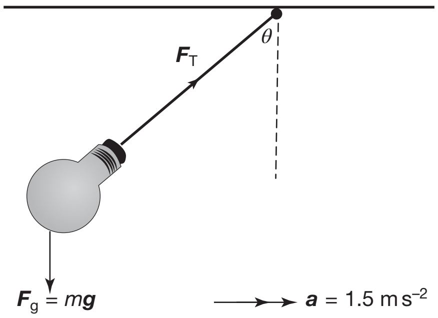
$\Rightarrow \theta = 1.146°$

$\Rightarrow F_T = \dfrac{0.3 \times 9.8}{2 \times \sin\ 1.146°}$

$\approx 74$ N

This answer is based on the assumption that the wire has zero mass and is perfectly flexible.

**9**




Let the mass of the globe be $m$ and assume that the mass of the wire is negligible. Resolving vertically:

$\sum \boldsymbol{F} = 0$

$\Rightarrow F_T \cos\theta = mg$ (equation 1)

$F_T \cos\theta = 9.8\ m$

Resolving horizontally:

$\sum \boldsymbol{F} = m\boldsymbol{a}$

$\Rightarrow F_T \sin\theta = ma$ (equation 2)

$F_T \sin\theta = 1.5\ m$

Divide equation 2 by equation 1:

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{1.5}{9.8}$

$\tan\theta = 0.153$

$\theta = 8.7°$

## 1.3 Exam questions

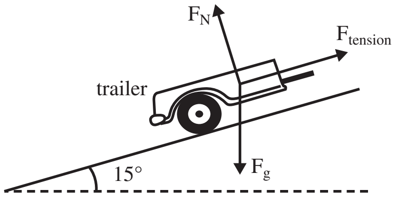
**1** Newton's third law: the two forces are equal in magnitude and opposite in direction.

**2** Newton's third law: the force of the rope on the student is equal and opposite to the force of the student on the rope.

**3** **a** The forces acting on the trailer are:

- The force due to gravity, vertical downward
- The normal force, perpendicular to the slope, upward
- The tension of the coupling, parallel to the slope, in the direction of motion.

Frictions forces are ignored.




Allocate 1 mark per force correctly represented and labelled.

**b** $F_{tow} = mg\sin\theta$

$F_{tow} = 200 \times 9.8 \times \sin 15°$ [1 mark]

$F_{tow} = 5 \times 10^2$ N [1 mark]

**4** Final speed:

$v^2 = u^2 + 2ax$

$= 0 + 2 \times 0.1 \times 20$

$= 4$ [1 mark]

$\Rightarrow v = 2\ \text{m s}^{-1}$ [1 mark]

***VCAA Assessment Report* note:**

The most common mathematical error was to forget to take the square root.

**5** Apply Newton's second law to the system of $M_1$ and $M_2$:

$F_{net} = m_{total}a$

$\Rightarrow m_1 g = (m_1 + m_2)a$

$\Rightarrow 1 \times 10 = (1 + 4)a$ [1 mark]

$\Rightarrow a = \dfrac{10}{5} = 2.0\ \text{m s}^{-2}$ [1 mark]

***VCAA Assessment Report* note:**

Common errors involved mixing up the masses or simple arithmetic errors. Students are advised to become familiar with an Atwood machine.

## 1.4 Projectile motion

### Sample problem 6

**a** $u = 0\ \text{m s}^{-1}$, $s = 100$ m, $a = 9.8\ \text{m s}^{-2}$

$s = ut + \frac{1}{2}at^2$

$100 = 0 \times t + \frac{1}{2} \times 9.8\, t^2$

$t = \sqrt{\dfrac{100}{4.9}}$

$\dfrac{100}{4.9} = t^2$

$\Rightarrow t = 4.5$ s

**b** Determine the distance the package has travelled between $t = 0.00$ s and $t = 0.50$ s:

$s = ut + \frac{1}{2}at^2$

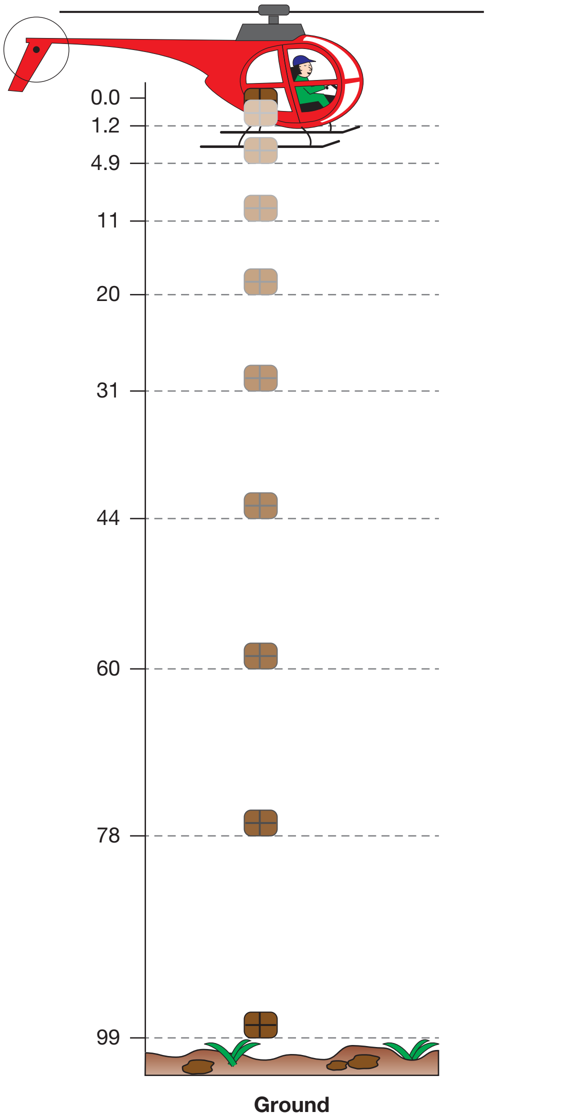
$= 0 \times 0.5 + \frac{1}{2} \times 9.8(0.5)^2$

$= 1.2$ m

Repeat this for $t = 1.0$ s, 1.5 s, 2.0 s and so on until the package hits the ground, and list the results in a table:

| Time (s) | Vertical distance (m) |
|---|---|
| 0.5 | 1.2 |
| 1.0 | 4.9 |
| 1.5 | 11 |
| 2.0 | 20 |
| 2.5 | 31 |
| 3.0 | 44 |
| 3.5 | 60 |
| 4.0 | 78 |
| 4.5 | 99 |

Draw a scale diagram of the ackage's position at half-second intervals:



## Practice problem 6

**a** $\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

$= 0 + \frac{1}{2} \times 9.8 \times 3.0^2$

$\approx 44$ m

**b** $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$= 0 + 9.8 \times 3.0$

$= 29.4\ \text{m s}^{-1}$

$\simeq 2.9 \times 10^1\ \text{m s}^{-1}$

## Sample problem 7

**a** $\boldsymbol{u} = 0\ \text{m s}^{-1}$, $\boldsymbol{s} = 100$ m, $\boldsymbol{a} = 9.8\ \text{m s}^{-2}$

$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

$100 = 0 \times t + \frac{1}{2}(9.8)\,t^2$

$\frac{100}{4.9} = t^2$

$t = \sqrt{\frac{100}{4.9}}$

$\Rightarrow t = 4.5\text{s}$

**b** The range of the package is the horizontal distance over which it travels. It is the horizontal component of velocity that must be used here.

$\boldsymbol{u} = 20\ \text{m s}^{-1}$ (The initial velocity of the package is the same as the velocity of the helicopter in which it has been travelling.)

$\boldsymbol{a} = 0\ \text{m s}^{-1}$ (No forces act horizontally so there is no horizontal acceleration.)

$t = 4.5$ s (from part (a))

$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

$= 20 \times 4.5 + 0$

$= 90$ m

**c** Calculations shown for $t = 0.50$ s.

Vertical component:

$\boldsymbol{u} = 0\ \text{m s}^{-1}$, $t = 0.50$ s, $\boldsymbol{a} = 10\ \text{m s}^{-2}$

$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

$= 0 \times 0.50 + \frac{1}{2} \times 10 \times 0.5^2$

$= 1.2$ m

Horizontal component:

$\boldsymbol{u} = 20\ \text{m s}^{-1}$, $t = 0.50$ s, $\boldsymbol{a} = 0\ \text{m s}^{-2}$

$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

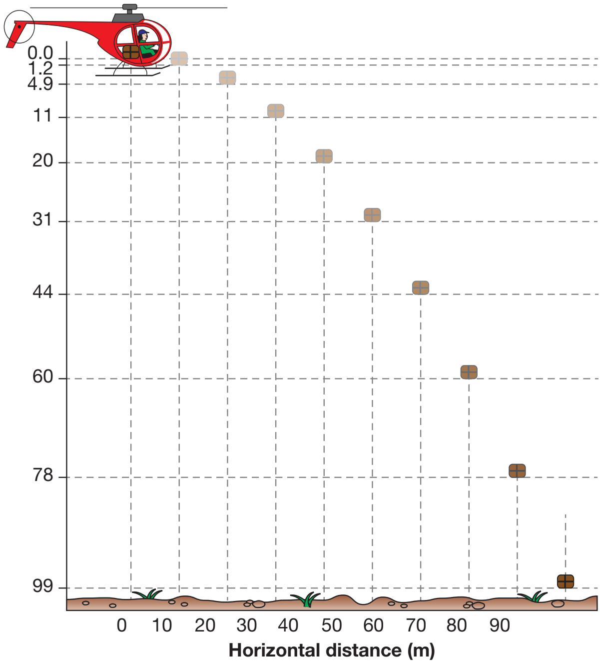
$= 20 \times 0.50 + 0$

$= 10$ m

Results of calculations for $t = 1.0$ s, 1.5 s, 2.0 s, etc. until the package reaches the ground are shown in the table:

| Time (s) | Vertical distance (m) | Horizontal distance (m) |
|---|---|---|
| 0.50 | 1.2 | 10 |
| 1.0 | 4.9 | 20 |
| 1.5 | 11 | 30 |
| 2.0 | 20 | 40 |
| 2.5 | 31 | 50 |
| 3.0 | 44 | 60 |
| 3.5 | 60 | 70 |
| 4.0 | 78 | 80 |
| 4 5 | 99 | 90 |

Scale diagram of the pacage's position at half-second intervals:




## Practice problem 7

**a** Vertically:

$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

$= 0 + \frac{1}{2} \times (-9.8) \times 4.0^2$

$= -78.4$ m

The cliff base is 78.4 – 2 = 76 m below. The cliff is therefore 76 m high.

**b** Horizontally, the velocity is constant:

$s = ut + \frac{1}{2}at^2$
$= 40 \times 4.0$
$= 1.6 \times 10^2$ m

**c** $v = u + at$
$= 0 - 9.8 \times 4$
$\simeq -39$ m s$^{-1}$

**d** $\theta = \tan^{-1}\left(\frac{39}{40}\right)$
$\simeq 44°$

## Sample problem 8

**a** $u = 4.0$ m s$^{-1}$, $a = 9.8$ m s$^{-2}$, $v = 0$ m s$^{-1}$

$v = u + at$
$0 = 4.0 + (-9.8) \times t$
$\Rightarrow t = \frac{4.0}{9.8}$
$= 0.41$ s

The dancer takes 0.41 s to reach her highest point.

**b** $u = 4.0$ m s$^{-1}$, $a = -9.8$ m s$^{-2}$, $v = 0$ m s$^{-1}$

$v^2 = u^2 + 2as$
$(0)^2 = (4.0)^2 + 2(-9.8)s$
$16 = 19.6 \times s$
$\Rightarrow s = 0.82$ m

The maximum displacement of the dancer's centre of mass is 0.82 m.

**c** 9.8 m s$^{-2}$ downwards

**d** Only the downward motion needs to be investigated:

$v^2 = u^2 + 2as$
$= (0)^2 + 2(-9.8) \times (-0.82)$
$\Rightarrow v = -4.0$ m s$^{-1}$

The velocity of the dancer's centre of mass when it returns to its original height is 4.0 m s$^{-1}$ downwards.

## Practice problem 8

**a** $v^2 = u^2 + 2as$
$0^2 = u^2 + 2(-9.8)^2 \times 0.50$
$\Rightarrow u = \sqrt{2(-9.8)^2 \times 0.50}$
$= 3.13$ m s$^{-1}$

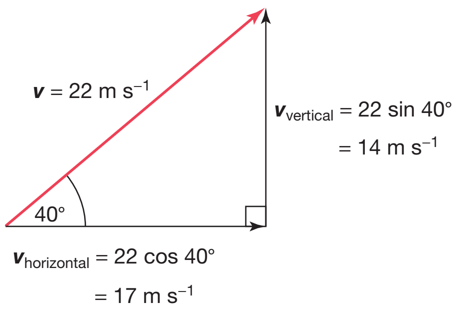
So, $v = 3.1$ m s$^{-1}$ down (due to symmetry).

**b** $s = \frac{u + v}{2}t$
$\Rightarrow t = \frac{2s}{u + v}$
$= \frac{2 \times 0.5}{3.13}$
$= 0.32$ s

So, due to symmetry, the total time taken was $t = 0.64$ s in total.

## Sample problem 9

The vertical and horizontal components of the initial velocity need to be calculated.




The initial vertical velocity is 14 m s$^{-1}$ and the initial horizontal velocity is 17 m s$^{-1}$.

Vertical component:

$u = 14$ m s$^{-1}$, $a = -9.8$ m s$^{-2}$, $v = 0$ m s$^{-1}$ (as the car comes to a vertical halt at its highest point):

$v = u + at$
$0 = 14 + (-9.8) \times t$
$\Rightarrow t = \frac{14}{-9.8 \text{ m s}^{-2}}$
$= 1.4$ s

As this is only half the motion, the total time in the air is 2.8 s. (It is possible to double the time in this situation because air resistance has been ignored. The two parts of the motion are symmetrical.)

Horizontal component:

$u = 17$ m s$^{-1}$, $t = 2.8$ s, $a = 0$ m s$^{-2}$:

$s = ut$
$= 17$ m s$^{-1} \times 2.8$ s
$= 48$ m

Therefore, the unlucky stunt driver will fall short of the second ramp and will land in the river. Perhaps the study of physics should be a prerequisite for all stunt drivers!

## Practice problem 9

**a** $u_{vertical} = u \sin\theta$
$= 32 \sin(25°)$
$\approx 13.5$ km h$^{-1}$

$u_{horizontal} = u \cos\theta$
$= 32 \cos(25°)$
$\approx 29$ km h$^{-1}$

**b** $v = 0$, $a = -9.8$, $u = \frac{13.5}{3.6} \approx 3.76$ m s$^{-1}$

$t = \frac{v - u}{a}$
$= \frac{0 - 3.76}{-9.8}$
$\approx 0.3837$ s

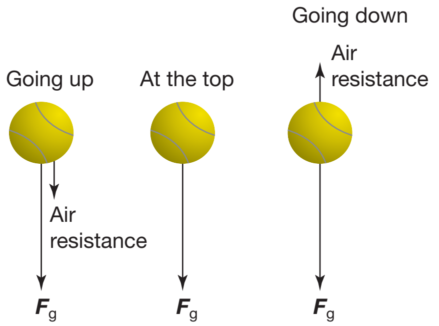
Total: $t = 0.77$ s

**c** The range of the hockey ball is the horizontal distance the ball travels in 0.77 seconds:

$s = ut$
$= \frac{29}{3.6} \times 0.77$
$= 6.2$ m

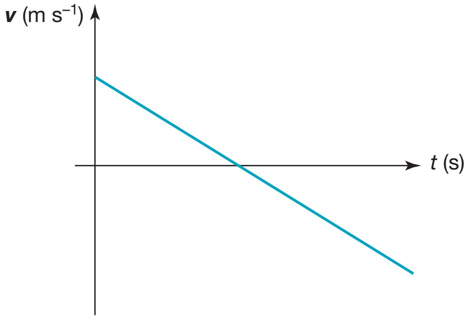
## 1.4 Exercise

**1**

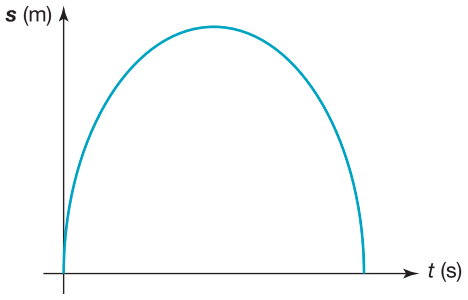



**2** The acceleration of a projectile when in motion is due to gravity, which is a constant near the earth's surface (9.8 m s$^{-2}$ downwards). This is displayed by the following acceleration–time graph:


Acceleration is the change in velocity (or the gradient), so the velocity of a projectile is a straight line of gradient –9.8 as shown in the following diagram:




Velocity is the change in position (or the gradient), so the position of a projectile is a negative parabola This is displayed on the following position–time graph:




**3** **a** Vertical component:

$v_v = 20 \sin 50° \approx 15 \text{ m s}^{-1}$

Horizontal component:

$v_h = 20 \cos 50° \approx 13 \text{ m s}^{-1}$

**b** Vertical component:

$v_v = 11 \cos 23° \approx 10 \text{ m s}^{-1}$

Horizontal component:

$v_h = 11 \sin 23° \approx 4.3 \text{ m s}^{-1}$

**c** Vertical component:

$v_v = 5 \text{ m s}^{-1}$

Horizontal component:

$v_h = 5 \sin 0° = 0 \text{ m s}^{-1}$

**d** Vertical component:

$v_v = 10 \sin 0° = 0 \text{ km h}^{-1}$

Horizontal component:

$v_h = 10 \text{km h}^{-1}$

**e** Vertical component:

$v_v = 33 \cos 60° \text{ or } 33 \sin 30° = 16.5 \text{ m s}^{-1}$

Horizontal component:

$v_h = 33 \sin 60° \text{ or } 33 \cos 30° \approx 29 \text{ m s}^{-1}$

**4** **a** $u = 18 \text{ m s}^{-1}$ (consider up as positive), $a = -9.8 \text{ m s}^{-2}$ (due to gravity), $v = -18 \text{ m s}^{-1}$ (due to symmetry)

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

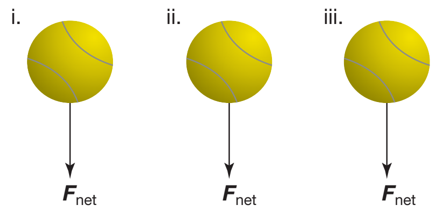
$$\Rightarrow t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}} = \frac{-18 - 18}{-9.8} = \frac{-36}{-9.8} \approx 3.67 \text{ s}$$

**b** Consider the first half of the motion, until $v = 0$ (at top of flight):

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$$

$$\boldsymbol{s} = \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{a}} = \frac{0 - (18)^2}{2 \times (-9.8)} \approx 16.5 \text{ m}$$

**c**




**5** Vertical component:

$\boldsymbol{u} = 7.0 \times \sin 45°$

$\approx 4.95 \text{ m s}^{-1}$

$\boldsymbol{v} = -4.95 \text{ m s}^{-1}$ (due to symmetry)

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$\Rightarrow t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}} = \frac{-4.95 - 4.95}{-9.8} \approx 1.0 \text{ s}$$

Horizontal component:

$\boldsymbol{u} = 7.0 \times \cos 45°$

$\approx 4.95 \text{ m s}^{-1}$

$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$

$= 4.95 \times 1.0$

$= 4.95 \text{ m}$

Distance required to jump over 11 people = 11 × 0.5 = 5.5 m.
You should not lie down as the 11th person, as your friend will not make the jump!

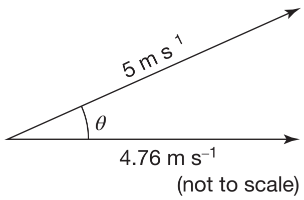
**6** **a** The horizontal velocity is constant, so it can be calculated by the formula for average velocity.

$\boldsymbol{v} = \frac{\boldsymbol{s}}{t}$

$= \frac{2}{0.42}$

$\approx 4.76\ \text{m s}^{-1}$

**b**




$\cos\theta = \frac{4.76}{5}$

$\theta = \cos^{-1}\left(\frac{4.76}{5}\right)$

$\approx 18°$

**c** Vertical component:

$\boldsymbol{u} = 5\sin(18°)$

$\approx 1.545\ \text{m s}^{-1}$

$\boldsymbol{a} = -9.8\ \text{m s}^{-2}$; $\boldsymbol{v} = 0\ \text{m s}^{-1}$

$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$

$\Rightarrow \boldsymbol{s} = \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{a}}$

$= \frac{0 - 1.545^2}{2 \times -9.8}$

$\approx 0.12\ \text{m}$

**7** **a** Vertical component:

$\boldsymbol{u} = 9.8\sin 45°$

$\approx 6.93\ \text{m s}^{-1}$

$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$\Rightarrow t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}}$

$= \frac{0 - 6.93}{-9.8}$

$\approx 0.71\ \text{s}$

**b** Vertical component:

$\boldsymbol{u} \approx 6.93\ \text{m s}^{-1}$

$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$

$\Rightarrow \boldsymbol{s} = \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{a}}$

$= \frac{0 - (6.93)^2}{2 \times (-9.8)}$

$\approx 2.45\ \text{m}$

Horizontal component (horizontal velocity is constant throughout the motion):

$\boldsymbol{v} = 9.8\cos 45°$

$\approx 6.93\ \text{m s}^{-1}$

$\boldsymbol{s} = \boldsymbol{v}t$

$= 6.93 \times 0.71$

$\approx 4.92\ \text{m}$

**c** Horizontal distance remaining:

$7.0 - 4.92 = 2.08\ \text{m}$

Time remaining:

$d = \boldsymbol{v}t$

$\Rightarrow t = \frac{d}{\boldsymbol{v}}$

$= \frac{2.08}{6.93}$

$\approx 0.30\ \text{s}$

**d** Vertical component of downward trajectory:

$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

$= \frac{1}{2} \times 9.8 \times (0.30)^2$

$\approx 0.44\ \text{m}$

Final height of ball $= 2.45 - 0.44 = 2.01\ \text{m}$

The ball will just be out of the reach of the goalkeeper and so will go into the net.

**8** **a** Vertical component:

$\boldsymbol{u} = \frac{50}{3.6} \times \sin(35°)$

$\approx 7.97\ \text{m s}^{-1}$

$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$\Rightarrow t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}}$

$= \frac{0 - 7.97}{-9.8}$

$\approx 0.81\ \text{s}$

**b** Vertical component:

$\boldsymbol{u} \approx 7.97\ \text{m s}^{-1}$

$\boldsymbol{s} = \boldsymbol{v}t - \frac{1}{2}\boldsymbol{a}t^2$

$= -\frac{1}{2} \times (-9.8) \times (0.81)^2$

$\approx 3.2\ \text{m}$

Horizontal component:

$\boldsymbol{u} = \frac{50}{3.6} \times \cos 35°$

$\approx 11.38\ \text{m s}^{-1}$

Horizontal velocity remains constant throughout flight:

$\boldsymbol{s} = \boldsymbol{u}t$

$= 11.38 \times 0.81$

$\approx 9.2\ \text{m}$

**c** Downward motion:

$\boldsymbol{s} = 3.2 + 0.8$

$= 4.0\ \text{m}$

$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

$4.0 = 0 + \frac{1}{2}(9.8) \times t^2$

$\Rightarrow t^2 = \frac{4}{4.9}$

$\Rightarrow t = +\sqrt{\frac{4}{4.9}}$

$= 0.90\ \text{s}$

**d** Horizontal velocity remains constant throughout flight:

$\boldsymbol{s} = \boldsymbol{u}t$

$= 11.38 \times (0.81 + 0.9)$

$\approx 19.5\ \text{m}$

**9 a** Upward flight — consider the vertical component:

$$\boldsymbol{u} = \frac{50}{3.6} \times \sin 30°$$
$$= 6.94\ \text{m s}^{-1}$$
$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$$
$$\Rightarrow \boldsymbol{s} = \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{a}}$$
$$= \frac{0 - (6.94)^2}{2 \times (-9.8)}$$
$$\approx 2.46\ \text{m}$$
$$\Rightarrow \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$\Rightarrow t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}}$$
$$= \frac{0 - 6.94}{-9.8}$$
$$\approx 0.71\ \text{s}$$

So the upward flight lasts for 0.71 s and reaches a height of 2.46 + 1.7 = 4.16 m.

Downward flight — consider the vertical component:

$$\boldsymbol{s} = -4.16\ \text{m}$$
$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$
$$-4.16 = 0 + \frac{1}{2}(-9.8) \times t^2$$
$$\Rightarrow t^2 = \frac{4.16}{4.9}$$
$$\Rightarrow t = +\sqrt{\frac{4.16}{4.9}}$$
$$= 0.92\ \text{s}$$

The total flight time is 0.71 + 0.92 = 1.63 s.

To find the range of the jump consider the horizontal component:

$$\boldsymbol{u} = \frac{50}{3.6} \times \cos 30°$$
$$= 12.03\ \text{m s}^{-1}$$

The horizontal velocity remains constant throughout flight:

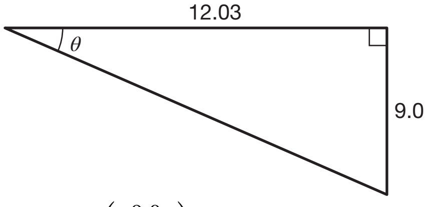
$$\boldsymbol{s} = \boldsymbol{u}t$$
$$= 12.03 \times 1.63$$
$$= 19.6\ \text{m}$$

The range of the jump is 19.6 metres.

**b** Final velocity:

Horizontal velocity: $\boldsymbol{u} = 12.03\ \text{m s}^{-1}$

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$\text{Vertical velocity}: = 0 + (-9.8) \times 0.92$$
$$\approx -9.0\ \text{m s}^{-1}$$
$$|\boldsymbol{v}| = \sqrt{\boldsymbol{v}_{\text{H}}{}^2 + \boldsymbol{v}_{\text{V}}{}^2}$$
$$\text{Total velocity}: = \sqrt{(12.03)^2 + (-9.0)^2}$$
$$\approx 15.0\ \text{m s}^{-1}$$




$$\theta = \tan^{-1}\left(\frac{9.0}{12.03}\right)$$
$$\approx 37°$$

The velocity of the water skier when they hit the water is 15.0 m s$^{-1}$ in the direction of 37 degrees below the horizontal.

**10** Horizontally:

$\boldsymbol{u} = \boldsymbol{u} \times \cos 28°$; $\boldsymbol{s} = 2.5\ \text{m}$

$$t = \frac{\boldsymbol{s}}{\boldsymbol{u}}$$
$$= \frac{2.5}{\boldsymbol{u} \times \cos 28°}$$

Vertically, apply $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$ to the first half of the flight, with:
$\boldsymbol{v} = 0\ \text{m s}^{-1}$, $\boldsymbol{a} = -9.8\ \text{m s}^{-2}$, $\boldsymbol{u} = \boldsymbol{u} \sin 28°\ \text{m s}^{-1}$

$$\Rightarrow \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$0 = \boldsymbol{u} \times \sin(28°) + \frac{-9.8 \times 2.5}{2\boldsymbol{u} \times \cos(28°)}$$
$$9.8 \times 2.5 = 2\boldsymbol{u}^2 \sin(28°)\cos(28°)$$
$$\Rightarrow \boldsymbol{u}^2 = \frac{9.8 \times 2.5}{2\sin(28°)\cos(28°)} \approx 29.55$$
$$\Rightarrow \boldsymbol{u} \approx +\sqrt{29.55}$$
$$= 5.4\ \text{m s}^{-1}$$

The initial velocity of the gymnast was 5.4 m s$^{-1}$

**11** Horizontally:

$\boldsymbol{u} = 7 \times \cos\theta\ \text{m s}^{-1}$, $\boldsymbol{s} = 3.0\ \text{m}$

$$t = \frac{\boldsymbol{s}}{\boldsymbol{u}}$$
$$= \frac{3.0}{7 \times \cos\theta}$$

Vertically, apply $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$ to the first half of the flight, with:
$\boldsymbol{a} = -9.8\ \text{m s}^{-2}$, $\boldsymbol{v} = 0\ \text{m s}^{-1}$, $\boldsymbol{u} = 7 \times \sin\theta\ \text{m s}^{-1}$,

$$t = \frac{3.0}{2 \times 7 \times \cos\theta}\text{s}$$
$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$0 = 7\sin\theta - 9.8 \times \frac{3.0}{14\cos\theta}$$
$$\Rightarrow 7\sin\theta = \frac{2.1}{\cos\theta}$$
$$\sin\theta\cos\theta = 0.3$$
$$\frac{\sin 2\theta}{2} = 0.3$$
$$\sin 2\theta = 0.6$$
$$\Rightarrow 2\theta = \sin^{-1}(0.6)$$
$$2\theta \approx 36.87$$
$$\theta \approx 18.5°$$

## 1.4 Exam questions

**1 a** Horizontal component of velocity

$v_{\text{H}} = 7.0\cos 50° = 4.5\ \text{m s}^{-1}$ [1 mark]

$$t = \frac{d}{v}$$
$$= \frac{3.2}{4.5} \quad \text{[1 mark]}$$
$$t = 0.71\ \text{s}$$

**b** Vertical component of velocity:

$V_{\text{V}} = 7.0\sin 50° = 5.36\ \text{m s}^{-1}$ [1 mark]

$$x = ut + \frac{1}{2}at^2$$
$$x = (5.36 \times 0.71) + \left(0.5 \times -9.8 \times 0.71^2\right) \quad \text{[1 mark]}$$
$$x = 1.34\ \text{m} \quad \text{[1 mark]}$$

Add launch height of 2.2 m.

The top of the basket is 134 + 2.2 = 3.5 m above the ground.

2 a $d = vt$

$d = 3.0 \times 0.4$

$d = 1.2$ m [1 mark]

b $x = \frac{1}{2}at^2$

$x = 0.5 \times 10 \times 0.4^2$ [1 mark]

$x = 0.8$ m [1 mark]

The most common error was to use the initial horizontal velocity as the initial vertical velocity.

c The speed at which the ball hits the floor is the vector sum of the horizontal and vertical velocities. [1 mark]

$v_v = at$

$= 10 \times 0.4$

$= 4\ \text{m s}^{-1}$ [1 mark]

Given that the initial horizontal velocity was $3.0\ \text{m s}^{-1}$, the final speed required the use of Pythagoras formula.

$v = \sqrt{v_h^2 + v_v^2}$

$= \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$v = 5\ \text{m s}^{-1}$ [1 mark]

It was also possible to solve this using a conservation of energy approach.

$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2$ [1 mark]

$0.5 \times 0.2 \times 3^2 + 0.2 \times 10 \times 0.8 = 0.5 \times 0.2 \times v_f^2$

$0.9 + 1.5 = 0.1v_f^2$ [1 mark]

$v_f = \sqrt{\frac{2.5}{0.1}}$

$v_f = 5\ \text{m s}^{-1}$ [1 mark]

The most common error was to simply find the vertical component.

3 $t = \frac{d}{v}$

$= \frac{26}{20 \cos 30°}$

$= 1.50$ s [1 mark]

$x = ut + \frac{1}{2}at^2$

$x = (20 \sin 30° \times 1.5) + (0.5 \times -9.8 \times 1.5^2)$ [1 mark]

$x = 3.98$

$x = 4.0$ m [1 mark]

Students demonstrated a range of inappropriate techniques, suggesting that they did not have a clear strategy for solving these types of projectile problems.

Students should be mindful of how they set out their working so they can clearly demonstrate their understanding of how to solve multi-step problems such as this.

4 a $x = ut + \frac{1}{2}at^2$ [1 mark]

$-20 = 0.5 \times 9.8 \times t^2$ [1 mark]

$t = 2.0$ s [1 mark]

b $x = v \times t$ [1 mark]

$x = 25 \times 2.0$

$x = 50$ m [1 mark]

c $Ek_{\text{final}} = Ek_{\text{initial}} + mgh$ [1 mark]

$Ek_{\text{final}} = 0.5 \times 2 \times 25^2 + 2 \times 9.8 \times 20$

$Ek_{\text{final}} = 1017$ J [1 mark]

5 Method is to calculate the total time of flight = time up and down; then calculate the horizontal distance travelled.

Vertical component of velocity:

$\boldsymbol{u_v} = 40 \times \sin 30°$ [1 mark]

$= 20\ \text{m s}^{-1}$ [1 mark]

Rises until speed is zero:

$\boldsymbol{v = u + at}$

$\Rightarrow 0 = 20 - 10\boldsymbol{t}$

$\Rightarrow t = 2$ s

$\Rightarrow$ total time of flight $= 2 \times 2$

$= 4$ s [1 mark]

Horizontal motion has constant speed

$\boldsymbol{v_x} = 40 \cos 30° = 34.6$

$\Rightarrow d = v_x t_{\text{total}}$

$= 34.6 \times 4$

$= 139$ m [1 mark]

***VCAA Assessment Report* note:**

A number of students used the range equation incorrectly. As has been advised in previous examination reports, students who wish to use derived formulas must ensure that they are used correctly.

Other students used $v = u + at$ to find the time of flight, but many students who used this approach forgot to double the time. It is recommended that students do not use the strategy of finding the time to the top of flight and doubling it, as the students who did so frequently forget to double the result.

## 1.5 Uniform circular motion

### Sample problem 10

a distance $= 2\pi r$

$= 2\pi \times 7.0$ m

$= 44$ m

speed $= \frac{d}{t}$

$= \frac{44}{9}$

$= 5\ \text{m s}^{-1}$

b After three laps, Ralph is in exactly the same place as he started, so his displacement is zero:

$\boldsymbol{v}_{\text{av}} = \frac{\Delta \boldsymbol{x}}{\Delta t}$

$= \frac{0}{3 \times 9}$

$= 0\ \text{m s}^{-1}$

c At point A, Ralph's velocity is $5\ \text{m s}^{-1}$ north.

### Practice problem 10

a $\boldsymbol{v} = \frac{2\pi r}{T}$

$\Rightarrow r = \frac{\boldsymbol{v}T}{2\pi}$

$= \frac{15 \times 1.3}{2\pi}$

$= 3.1$ m

**b** The magnitude of velocity is speed. The car's speed is constant at 1.3 m s$^{-1}$.

**c** $v_{av} = \dfrac{\text{displacement}}{\text{time}}$

$= \dfrac{2r}{7.5}$

$= \dfrac{6.2}{7.5}$

$= 0.83 \text{ m s}^{-1}$

**d** $v_{av} = \dfrac{\text{displacement}}{\text{time}}$

$= \dfrac{0}{7.5}$

$= 0 \text{ m s}^{-1}$

## Sample problem 11

**a** $a = \dfrac{v^2}{r}$

$= \dfrac{(5.6)^2}{3.5}$

$= 9.0 \text{ m s}^{-2}$

The car accelerates at 9.0 m s$^{-2}$ towards the centre of the roundabout.

**b** Method 1: use the answer to (a) and substitute into $\boldsymbol{F}_{net} = m\boldsymbol{a}$:

$\boldsymbol{F}_{net} = m\boldsymbol{a}$

$= 1200 \times 9.0$

$= 1.1 \times 10^4 \text{ N}$

Method 2: use the formula $\boldsymbol{F}_{net} = \dfrac{m\boldsymbol{v}^2}{r}$:

$\boldsymbol{F}_{net} = \dfrac{m\boldsymbol{v}^2}{r}$

$= \dfrac{1200(5.6)^2}{3.5}$

$= 1.1 \times 10^4 \text{ N}$

Both methods give the force on the car as $1.1 \times 10^4$ N towards the centre of the roundabout.

## Practice problem 11

**a** $a = \dfrac{v^2}{r}$

$= \dfrac{4\pi^2 r}{T^2}$

$= \dfrac{4\pi^2 \times 3.5}{2.5^2}$

$\approx 22 \text{ m s}^{-2}$ south

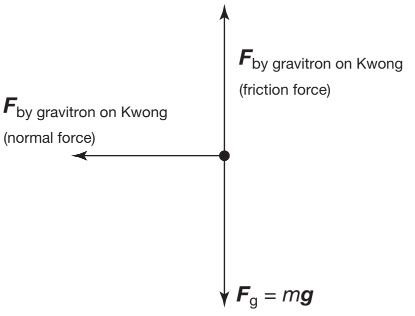
**b** $\boldsymbol{F}_{net} = m\boldsymbol{a}$

$= 60 \times 22$

$= 1.3 \times 10^3 \text{ N}$

The direction of the force is the same direction as the acceleration: towards the centre of the circle. Net force = $1.3 \times 10^3$ N towards the centre of the circle.

**c**




## Sample problem 12

$\boldsymbol{F}_{net} = \dfrac{m\boldsymbol{v}^2}{r}$

$= 16\,400 \text{ N}$

$= 1280 \times \dfrac{\boldsymbol{v}^2}{12.0}$

$\boldsymbol{v}^2 = 16\,400 \times \dfrac{12.0}{1280}$

$= 153.75 \text{ m}^2 \text{ s}^{-2}$

$\Rightarrow \boldsymbol{v} = 12.4 \text{ m s}^{-1}$

The maximum constant speed at which the vehicle can be driven around the bend is 12.4 m s$^{-1}$.

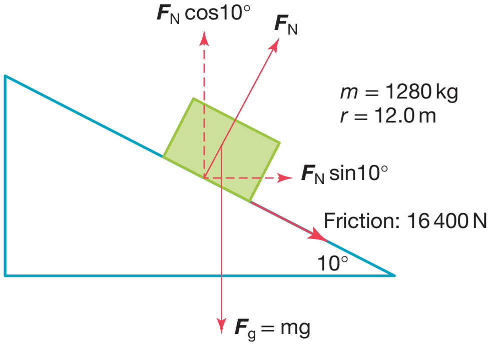
## Practice problem 12

$\boldsymbol{F}_{net} = \dfrac{m\boldsymbol{v}^2}{r}$

$= \dfrac{200 \times 30^2}{100}$

$= 1.8 \times 10^3 \text{ N}$

## Sample problem 13



The vertical forces are balanced:

$$F_N \cos 10° = 16\,400 \sin 10° + 1280 \times 9.8$$
$$F_N = \frac{15\,392}{\cos 10°} = 15\,629\ \text{N}$$

The net force is equal to the sum of the horizontal forces:

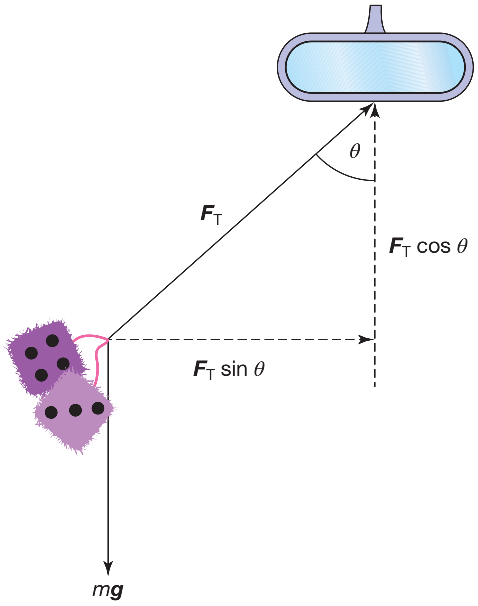
$$F_{net} = \frac{mv^2}{r}$$
$$F_N \sin 10° + 16\,400 \cos 10° = \frac{1280 \times v^2}{12}$$
$$15\,629 \sin 10° + 16\,400 \cos 10° = \frac{1280 \times v^2}{12}$$
$$\Rightarrow v = 13.3\ \text{m s}^{-1}$$

## Practice problem 13

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{8^2}{25 \times 9.8}\right) \approx 15°$$

## Sample problem 14

When John enters the roundabout, the dice, which are hanging straight down, will begin to move outwards. As long as John maintains a constant speed, they will reach a point at which they become stationary at some angle to the vertical. At this point, the net force on the dice is the centripetal force. Because the dice appear stationary to John, they must be moving in the same circle, with the same speed, as John and his car:




Vertical component:

$$mg = T \cos\theta$$
$$\Rightarrow F_T = \frac{mg}{\cos\theta} \ldots (1)$$

Horizontal component:

$$F_{net} = \frac{mv^2}{r} = F_T \sin\theta$$
$$\Rightarrow \frac{mv^2}{r} = F_T \sin\theta \ldots (2)$$

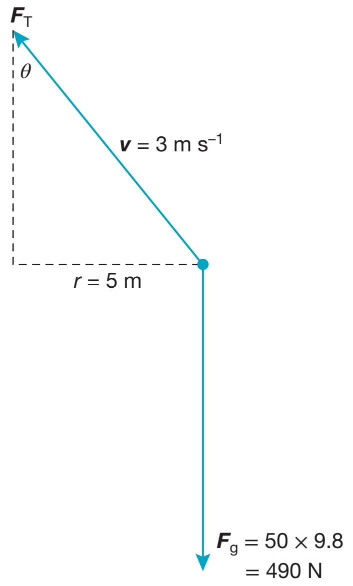
To solve the simultaneous equations, substitute for $T$ (from equation (1)) into equation (2):

$$\frac{mv^2}{r} = \frac{mg}{\cos\theta} \times \sin\theta = mg \tan\theta$$
$$\Rightarrow \tan\theta = \frac{v^2}{rg} = \frac{(8.0\ \text{m s}^{-1})^2}{5.0\ \text{m} \times 9.8\ \text{N kg}^{-1}}$$
$$\Rightarrow \theta = 53°$$

## Practice problem 14

**a** $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{3^2}{5 \times 9.8}\right) \approx 10°$

**b** $F_{net} = \frac{mv^2}{r} = \frac{50 \times 3^2}{5} = 90\ \text{N}$




$$F_g = mg = 50 \times 9.8 = 490\ \text{N}$$
$$\Rightarrow F_T = \sqrt{90^2 + 490^2} \approx 5.0 \times 10^2\ \text{N}$$

## 1.5 Exercise

**1 a** $v = 6.0\ \text{km h}^{-1} \approx 1.7\ \text{m s}^{-1}$

$$a = \frac{v^2}{r} = \frac{(1.7)^2}{120} = 0.024\ \text{m s}^{-2} \text{ towards the centre of the circle}$$

**b** $F_{\text{net}} = ma = 65 \times 0.024 \approx 1.6\ \text{N}$ towards the centre of the circle

**2 a** $v = 15\ \text{km h}^{-1} = \frac{15}{3.6}\ \text{m s}^{-1} \approx 4.2\ \text{m s}^{-1}$

$$\Rightarrow a = \frac{v^2}{r} = \frac{4.2^2}{350} \approx 0.050\ \text{m s}^{-2}$$

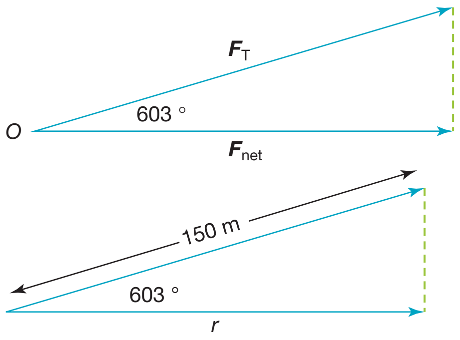
**b** $F_{\text{net}} = ma = 35 \times 0.05 = 1.75\ \text{N}$ towards the centre of the circle

**c** $F_{\text{net}} = ma = 1500 \times 0.05 = 75\ \text{N}$ towards the centre of the circle

**d** To move along the same path, the child and the train require the same acceleration. As the masses of the child and the train are different, different forces are needed to produce identical accelerations.

**3** To go around a bend, a motorcyclist needs a horizontal force acting on the bike towards the centre of the curve of the bend. This is provided by the road acting on the tyres. The force of the road on the tyres needs to act through the centre of mass of the cyclist, otherwise the force will act to tip the bike over. As the horizontal component of this force is acting towards the centre of the curve, the motorcyclist must lean into the curve to avoid falling off.

**4 a**




$r = 1.50 \cos(6.03°) \approx 1.4917\ \text{m}$

$$v = \frac{2\pi r}{T} = \frac{2\pi \times 1.4917}{0.800} \approx 11.7\ \text{m s}^{-1}$$

**b** $$a = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 \times 1.4917}{(0.800)^2} \approx 92.0\ \text{m s}^{-2} \text{ towards the centre of the circle}$$

**c** $F_{\text{net}} = ma = 0.0500 \times 92.0 = 4.60\ \text{N}$ towards the centre of the circle

**d** $$F_{\text{tension}} = \sqrt{F_g{}^2 + F_{\text{net}}{}^2} = \sqrt{(mg)^2 + 4.6^2} = \sqrt{(0.0500 \times 9.8)^2 + 4.6^2} = \sqrt{21.4} \approx 4.63\ \text{N}$$

**5 a** $v = 15\ \text{km h}^{-1} = \frac{15}{3.6}\ \text{m s}^{-1} \approx 4.2\ \text{m s}^{-1}$

$$F_{\text{net}} = \frac{mv^2}{r} = \frac{(90)(4.2)^2}{4.5} \approx 353\ \text{N towards the centre of the circle}$$

**b** The sideways frictional force is the only force in the horizontal direction. The vertical forces cancel out, so the frictional force is equal to the net force: 353 N.

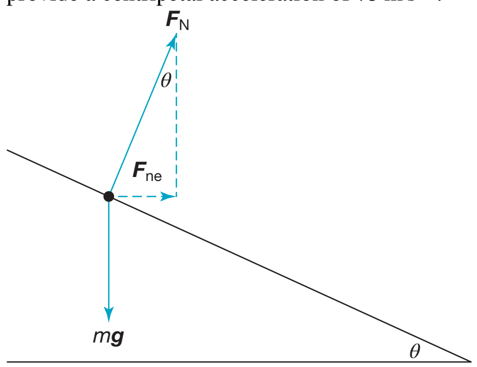
**c** $F_{\text{net}} = 0.9 \times 353 \approx 318\ \text{N}$

$$\Rightarrow r = \frac{mv^2}{F_{net}} = \frac{90 \times 4.2^2}{318} \approx 5.0\ \text{m}$$

The radius of the circle will increase to 5.0 metres.

**6** $$a = \frac{v^2}{r} = \frac{30^2}{12} = 75\ \text{m s}^{-2}$$

The horizontal component of the normal force must therefore provide a centripetal acceleration of $75\ \text{m s}^{-2}$.



$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$

$= 75\,m$

$\Rightarrow \boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{N}} \sin\theta$

$\Rightarrow \boldsymbol{F}_{\text{N}} \sin\theta = 75\,m$

$\boldsymbol{F}_{\text{N}} \cos\theta = m\boldsymbol{g}$

Dividing these equations results in:

$$\tan\theta = \frac{75m}{m\boldsymbol{g}} = \frac{75}{9.8}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{75}{9.8}\right) \approx 82.6°$$

The road should be banked at an angle of 82.6°.

## 1.5 Exam questions

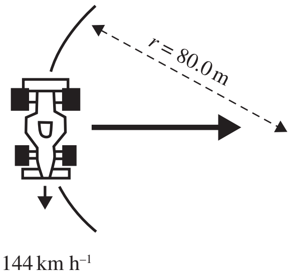
**1 a** $F = \dfrac{mv^2}{r}$

$F = \dfrac{800 \times 40^2}{80}$ [1 mark]

$F = 1.6 \times 10^4$ N [1 mark]

The most common errors were to forget to square the velocity or to attempt to use a different force formula.

**b** The direction of the net force is toward the centre of the circle.




A number of students put the arrow on figure 8a, rather than on figure 8b. Students must ensure they read the question carefully and respond appropriately.

**c** A net horizontal force is required to maintain circular motion [1 mark]. The horizontal force is provided by the friction force on the tyres, by the road. [1 mark]

While most students were able to identify the tyre/road interface as the source of the friction, many were unable to link this to circular motion. Students also seemed confused as to the direction that the friction force acted and had difficulty linking the frictional force to a centripetal force.

**2 a** $r = \dfrac{v^2}{g}$

$r = \dfrac{180^2}{9.8}$ [1 mark]

$r = 3306$ m

$r = 3.3 \times 10^3$ m [1 mark]

The most common errors were mathematical as most students who made an attempt knew the problem involved circular motion.

**b** The force of gravity is not zero at the top of the flight. [1 mark]

The 'zero gravity experience' is due to the lack of a contact or normal reaction force. [1 mark]

Many students stated that the force of gravity was zero at the top and then struggled to explain how that might occur. Others stated that if the gravitational force was still present it would be very small, which was also incorrect.

A number of students made reference to 'apparent weightlessness'. It should be pointed out that 'apparent weightlessness' is no longer part of the study design.

**3 a** The height of the mass is $2.0\cos 60° = 1.0$ m [1 mark].

$mgh = \frac{1}{2}mv^2$ [1 mark]

$2.0 \times 9.8 \times 1.0 = 0.5 \times 2.0 \times v^2$

$v = \sqrt{19.6}$

$v = 4.4\text{ m s}^{-1}$

**b** The mass will reach its maximum velocity at the bottom of the arc. [1 mark] At this point, the maximum amount of gravitational potential energy will have been converted to kinetic energy. [1 mark]

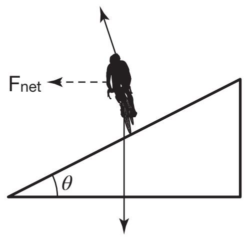
**c** The tension is greatest at the bottom of the arc where

$T = mg + \dfrac{mv^2}{r}$ [1 mark]

$T = 2.0 \times 9.8 + \dfrac{2.0 \times 4.4^2}{2.0}$ [1 mark]

$T = 39$ N

**4 a**




Award a total of 2 marks for drawing all three arrows correctly.

Award a total of 1 mark for drawing only two arrows correctly.

***VCAA examination report* note:**

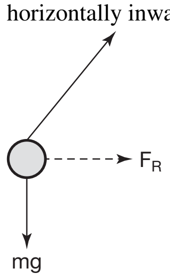
The most common errors were the inclusion of extra forces or drawing $\boldsymbol{F}_{\text{net}}$ as down the slope.

**b** $\tan\theta = \dfrac{v^2}{rg} = \dfrac{10^2}{20 \times 9.8}$ [1 mark]

$= 0.51$

$\Rightarrow \theta = 27°$ [1 mark]

**5 a** There are only two forces on the ball: tension and the gravitational weight force. As a result of the direction of the application of the tension there is an unbalanced force horizontally inwards — the resultant force.



Award 1 mark for the tension force along the string plus weight force.

Award 1 mark for the dotted resultant force.

***VCAA Assessment Report* note:**

The most common errors were to mislabel the forces or misrepresent the resultant force as a force that would exist without the other two. Many students also added extra arrows incorrectly.

**b** From the data given,

$mg = 2 \times 10$

$= 20$ N

If the angle between the string and the vertical is $\theta$, then:

$\sin\theta = \frac{0.5}{1.0}$

$p\theta = 30°$ [1 mark]

Then balancing the vertical forces on the ball:

$T\cos\theta = mg$

$T\cos 30° = 20$ [1 mark]

$pT = \frac{20}{\cos 30°}$

$= 23$ N [1 mark]

There are several other methods to solve this problem, all equally valid.

***VCAA Assessment Report* note:**

The most common errors were to treat the motion as vertical circular motion. Other errors were mathematical.

## 1.6 Non-uniform circular motion

### Sample problem 15

**a** At point A, the skateboarder has potential energy but no kinetic energy. At point B, all the potential energy has been converted to kinetic energy. Once the kinetic energy is known, it is easy to calculate the velocity of the skateboarder. The decrease of potential energy from A to B is equal to the increase of kinetic energy from A to B.

$\Delta\text{GPE} = \Delta\text{KE}$

$-mg(h_B - h_A) = \frac{1}{2}mv^2$

Cancelling $m$ from both sides:

$-g(h_B - h_A) = \frac{1}{2}v^2$

$-9.8\,(0\text{ m} - 4.0\text{ m}) = \frac{1}{2}v^2$

$\Rightarrow v^2 = 78.4\text{ m}^2\text{ s}^{-2}$

$\Rightarrow v = 8.854\text{ m s}^{-1}$

$\approx 8.9\text{ m s}^{-1}$

The skateboarder's speed at B is 8.9 m s$^{-1}$.

**b** $F_{net} = \frac{mv^2}{r}$

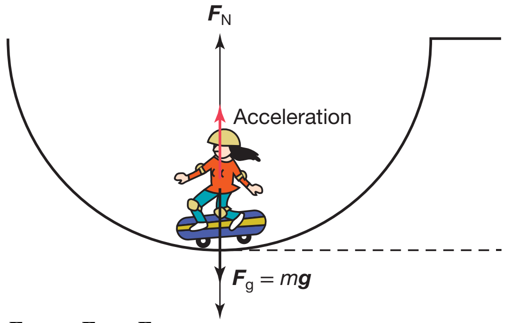
$= \frac{60 \times (8.854)^2}{4.0}$

$= 1176$ N

$\approx 1.2 \times 10^3$ N

The net force acting on the skateboarder at point B is 1200 N upwards.

**c**




$F_{net} = F_N - F_g$

Taking the upward direction as positive, $F_N = -mg$:

$F_{net} = F_N - mg$

$\Rightarrow F_N = F_{net} + mg$

$= 1.2 \times 10^3\text{ N} + 60\text{ kg} \times 9.8\text{ m s}^{-2}$

$= 1788$ N

$\approx 1.8 \times 10^3$ N

The normal force acting on the skateboarder at point B is $1.8 \times 10^3$ N upwards. This is larger than the normal force if the skateboarder was stationary. This causes the skateboarder to experience a sensation of heaviness.

### Practice problem 15

**a** $F_{net} = \frac{mv^2}{r}$

$= \frac{60 \times 13^2}{9.0}$

$\approx 1.1 \times 10^3$ N

**b** $F_{net} = F_N - mg$

$F_N = F_{net} + mg$

$= 1.1 \times 10^3 + 60 \times 9.8$

$= 1.7 \times 10^3$ N

**c** $F_g = mg$

$= 60 \times 9.8$

$= 5.9 \times 10^2$ N

The normal force is $\frac{1.7 \times 10^3}{5.9 \times 10^2} \approx 2.9$ times the force due to gravity.

### Sample problem 16

**a** $\frac{mv^2}{r} = mg - \frac{mg}{2}$

$\frac{v^2}{r} = \frac{g}{2}$

$\Rightarrow v = \sqrt{\frac{gr}{2}}$

$= \sqrt{\frac{9.8 \times 9.0}{2}}$

$= 6.6\text{ m s}^{-1}$

**b** Rearranging $\frac{mv^2}{r} = mg - F_N$ gives:

$F_N = mg - \frac{mv^2}{r}$

The force due to gravity, $mg$, is constant, so as the speed, $v$, increases, the normal force, $F_N$, gets smaller.

**c** The normal force is less than the force due to gravity, so the passenger will feel lighter.

## Practice problem 16

**a** $\boldsymbol{F}_{net} = m\boldsymbol{g} - \boldsymbol{F}_N$

$\Rightarrow \boldsymbol{F}_N = m\boldsymbol{g} - \boldsymbol{F}_{net}$

$= m\boldsymbol{g} - \dfrac{m\boldsymbol{v}^2}{r}$

$= 800 \times 9.8 - \dfrac{800 \times 4.0^2}{2.4}$

$\approx 7840 - 5333$

$= 2.5 \times 10^3$ N

**b** $\boldsymbol{F}_N = m\boldsymbol{g} - \dfrac{m\boldsymbol{v}^2}{r}$

If $\boldsymbol{F}_N = 0$, then $m\boldsymbol{g} = \dfrac{m\boldsymbol{v}^2}{r}$.

For the car to leave the road, the normal force must equal 0:

$\boldsymbol{F}_N = m\boldsymbol{g} - \dfrac{m\boldsymbol{v}^2}{r} = 0$

$\Rightarrow m\boldsymbol{g} = \dfrac{m\boldsymbol{v}^2}{r}$

$\boldsymbol{v}^2 = \boldsymbol{g}r$

$= 9.8 \times 2.4$

$= 23.52$

$\boldsymbol{v} \approx 4.8\ \text{m s}^{-1}$

For the car to leave the road at the top of the speed bump, the car would have to be travelling at a minimum of 4.8 m s$^{-1}$.

## Sample problem 17

**a** Calculate the total energy of the car at point A by adding the car's gravitational potential energy and kinetic energy:

$E_A = E_k + E_g$

$= \dfrac{1}{2}m\boldsymbol{v}^2 + m\boldsymbol{g}\Delta h$

$= 0 + (0.200 \times 9.8 \times 2.00)$

$= 3.92$ J

The total energy of the car at point B is equal to the total energy of the car at point A:

$E_B = E_k + E_g$

$= \dfrac{1}{2}m\boldsymbol{v}^2 + m\boldsymbol{g}\Delta h$

$3.92 = \left(\dfrac{1}{2} \times 0.200 \times v^2\right) + 0$

$3.92 = 0.100\boldsymbol{v}^2$

$\boldsymbol{v}^2 = \dfrac{3.92}{0.100}$

$\Rightarrow \boldsymbol{v} = \sqrt{39.2}$

$= 6.26\ \text{m s}^{-1}$

**b** At point B, the net force will be directed upwards:

$\boldsymbol{F}_{net} = \dfrac{m\boldsymbol{v}^2}{r}$

$= \dfrac{0.200 \times 6.26^2}{0.15}$

$= 78.4$ N up

**c** The net force is equal to the sum of the normal force and the force due to gravity. Take the upward direction to be positive:

$\boldsymbol{F}_{net} = \boldsymbol{F}_N + \boldsymbol{F}_g$

$\boldsymbol{F}_{net} = \boldsymbol{F}_N - m\boldsymbol{g}$

$78.4 = \boldsymbol{F}_N - (0.200 \times 9.8)$

$78.4 = \boldsymbol{F}_N - 1.96$

$\boldsymbol{F}_N = 78.4 + 1.96$

$= 80.4$ N up

**d** From part (a), it is known that the total energy of the car at all points is 3.92 J. At point C:

$E_C = E_k + E_g$

$= \dfrac{1}{2}m\boldsymbol{v}^2 + m\boldsymbol{g}\Delta h$

$3.92 = \left(\dfrac{1}{2} \times 0.200 \times \boldsymbol{v}^2\right) + (0.200 \times 9.8 \times 0.300)$

$3.92 = 0.100\boldsymbol{v}^2 + 0.588$

$0.100\boldsymbol{v}^2 = 3.332$

$\boldsymbol{v}^2 = \dfrac{3.332}{0.100}$

$\Rightarrow \boldsymbol{v} = \sqrt{33.32}$

$= 5.77\ \text{m s}^{-1}$

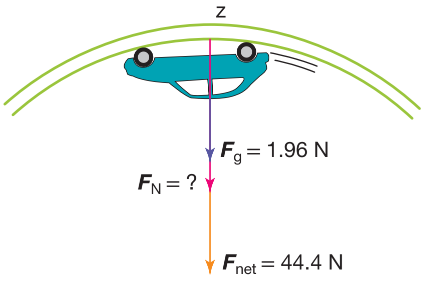
**e** The net force on the car at point C will be directed downwards towards the centre of the circle:

$\boldsymbol{F}_{net} = \dfrac{m\boldsymbol{v}^2}{r}$

$= \dfrac{0.200 \times 5.77^2}{0.15}$

$= 44.4$ N down

The net force is equal to the sum of the normal force and the force due to gravity. Take the upward direction to be positive. *Note:* In this case, both the normal force and the force due to gravity act in the downward direction:




$\boldsymbol{F}_{net} = \boldsymbol{F}_N + \boldsymbol{F}_g$

$44.4 = \boldsymbol{F}_N + (0.200 \times 9.8)$

$44.4 = \boldsymbol{F}_N + 1.96$

$\Rightarrow \boldsymbol{F}_N = 44.4 - 1.96$

$\boldsymbol{F} = 42.7$ N down

## Practice problem 17

**a** At point A, the car has only gravitational potential energy:

$E_A = mgh$

$= 0.1 \times 9.8 \times 1.3$

$= 1.274\ \text{J} \simeq 1.3\ \text{J}$

As the racetrack is frictionless, all of the energy from point A is transferred to kinetic energy at point B:

$E_B = E_A$

$\dfrac{1}{2}m\boldsymbol{v}^2 = 1.274$

$\boldsymbol{v}^2 = \dfrac{2 \times 1.274}{0.1}$

$\boldsymbol{v} = \sqrt{\dfrac{2 \times 1.274}{0.1}}$

$\approx 5.0\ \text{m s}^{-1}$

The speed of the car at the point B is 5.0 m s$^{-1}$.

**b** At point B, the car is entering a circular loop, so the net force can be calculated by the following formula:

$$\begin{aligned} F_{net} &= \frac{mv^2}{r} \\ &= \frac{0.100 \times 5.0^2}{0.20} \\ &= 12.5\text{ N up} \simeq 13\text{ N up} \end{aligned}$$

**c** The net force is equal to the normal force minus the force due to gravity, $F_{net} = F_N - F_g$. Rearranging this equation results in the formula for the normal force:

$$\begin{aligned} F_N &= F_{net} - F_g \\ &= 12.5 - 0.1 \times 9.8 \\ &= 11.7\text{ N} \simeq 12\text{ N up} \end{aligned}$$

**d** From part (a), it was found that the total energy of the car at all positions is 1.274 J:

$$E_C = mgh + \frac{1}{2}mv^2$$
$$1.274 = 0.1 \times 9.8 \times 0.2 + \frac{1}{2} \times 0.1 \times v^2$$
$$1.274 = 0.196 + 0.05v^2$$
$$0.05v^2 = 1.077$$
$$\Rightarrow v = \sqrt{\frac{1.077}{0.05}} \approx 4.6\text{ m s}^{-1}$$

The speed at point C is 4.64 m s$^{-1}$.

**e** At point C, the net force is the normal force + the force due to gravity, $F_{net} = F_N + F_g$. Rearranging this equation, the formula for the normal force is found, $F_N = F_{net} - F_g$. Therefore:

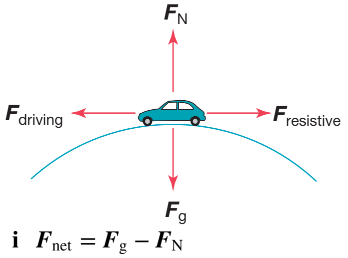
$$\begin{aligned} F_N &= \frac{mv^2}{r} - mg \\ &= \frac{0.1 \times 4.64^2}{0.2} - 0.1 \times 9.8 \\ &\approx 9.8\text{N down} \end{aligned}$$

## 1.6 Exercise

**1 a** When the ball is at the very bottom of the circle, the tension is acting directly opposite the force due to gravity, and so the tension force will be a maximum at this point.

**b** When the ball is at the very top of the circle the tension is acting in the same direction as the force due to gravity, and so the tension force will be a minimum at this point.

**2 a**




**b i** $F_{net} = F_g - F_N$

As $F_N = 0$,

$$\begin{aligned} F_{net} &= F_g \\ &= mg \\ &= 800 \times 9.8 \\ &= 7840\text{ N} \end{aligned}$$

**ii** $$\begin{aligned} F_{net} &= \frac{mv^2}{r} \\ v^2 &= \frac{F_{net} \times r}{m} \\ &= \frac{7840 \times 4}{800} \\ &= 39.2 \\ \Rightarrow v &= \sqrt{39.2} \\ &\approx 6.26\text{ m s}^{-1} \end{aligned}$$

**3 a** At point A, the car has only gravitational potential energy:

$$\begin{aligned} E_A &= mgh \\ &= 0.12 \times 9.8 \times 1 \\ &= 1.176\text{ J} \end{aligned}$$

As the racetrack is frictionless, all the energy from point A is transferred to kinetic energy at point B:

$$E_B = E_A$$
$$\frac{1}{2}mv^2 = 1.176$$
$$v^2 = \frac{2 \times 1.176}{0.12}$$
$$\Rightarrow v = \sqrt{\frac{2 \times 1.176}{0.12}} \approx 4.43\text{ m s}^{-1}$$

The speed of the car at point B is 4.43 m s$^{-1}$

**b** At point B, the car is entering a circular loop, so the net force can be calculated by the following formula:

$$\begin{aligned} F_{net} &= \frac{mv^2}{r} \\ &= \frac{0.12 \times 4.43^2}{0.1} \\ &\approx 23.5\text{ N up} \end{aligned}$$

**c** The net force is equal to the normal force minus the force due to gravity, $F_{net} = F_N - F_g$. Rearranging this equation results in the formula for the normal force:

$$\begin{aligned} F_N &= F_{net} - F_g \\ &= 23.5 - 0.12 \times 9.8 \\ &\approx 22.3\text{ N} \end{aligned}$$

**4** $$\begin{aligned} F_{net} &= \frac{mv^2}{r} \\ &= \frac{60 \times 14^2}{12} \\ &\approx 980\text{ N up} \end{aligned}$$

$$F_{net} = F_N - F_g$$
$$980 = F_N - 60 \times 9.8$$
$$980 = F_N - 588$$
$$F_N = 980 + 588$$
$$= 1568\text{ N}$$

**5 a** $$\begin{aligned} a &= \frac{v^2}{r} \\ &= \frac{7^2}{2.5} \\ &= 19.6\text{ m s}^{-2} \end{aligned}$$

**b** $\boldsymbol{F}_{\text{net}} = \frac{mv^2}{r}$

$= \frac{75 \times 7^2}{2.5}$

$= 1470\text{ N up}$

$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{N}} - \boldsymbol{F}_{\text{g}}$

$\Rightarrow \boldsymbol{F}_{\text{N}} = \boldsymbol{F}_{\text{net}} + \boldsymbol{F}_{\text{g}}$

$= 1470 + 75 \times 9.8$

$= 2205\text{ N}$

## 1.6 Exam questions

**1 a** $\text{KE} = \frac{1}{2}mv^2$

$\text{KE} = 0.5 \times 0.30 \times 6^2$ [1 mark]

$\text{KE} = 5.4\text{ J}$

**b** Find minimum velocity required to stay on track:

$v_{\text{min}} = \sqrt{rg}$

$v_{\text{min}} = \sqrt{0.4 \times 9.8}$

$v_{\text{min}} = 1.98\text{ m s}^{-1}$ [1 mark]

Find KE at top of loop:

$\text{KE}_{\text{B}} = \text{KE}_{\text{A}} - mgh$

$\text{KE}_{\text{B}} = 5.4 - 0.30 \times 9.8 \times 0.8$

$\text{KE}_{\text{B}} = 3.05\text{ J}$ [1 mark]

$\text{KE} = \frac{1}{2}mv^2$

Find velocity at top of loop:

$3.05 = 0.5 \times 0.3 \times v^2$

$v = 4.5\text{ m s}^{-1}$ [1 mark]

As the velocity at B is greater than the minimum velocity required, the ball will stay on the track [1 mark].

There were other alternative approaches that could be applied that were equally correct.

**2 a** Minimum speed occurs where $\frac{mv^2}{r} = mg$ [1 mark].

$v = \sqrt{rg}$

$v = \sqrt{9.8 \times 1.5}$

$v = 3.9\text{ m s}^{-1}$ [1 mark]

**b** $Ek_{\text{bottom}} = Ek_{\text{top}} + mgh$

$Ek_{\text{final}} = 0.5 \times 2.5 \times 6.0^2 + 2.5 \times 9.8 \times 3.0$ [1 mark]

$Ek_{\text{final}} = 118.5\text{ J}$

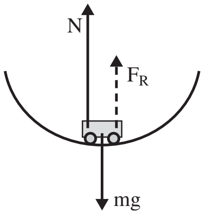
$118.5 = 0.5 \times 2.5 \times v^2$ [1 mark]

$v = 9.8\text{ m s}^{-1}$ [1 mark]

**3 a** The forces acting on the trailer are:

- The force due to gravity, vertical downward
- The normal force, perpendicular to the surface of the track, upward

The resultant force is perpendicular to the surface of the track, upward.




Allocate 1 mark per correct force represented.

**b** Students must ensure that they include the gravitational weight force as well as the centripetal force.

$F = \frac{mv^2}{r} + mg$

$F = \frac{50 \times 24^2}{12} + 50 \times 10$ [1 mark]

$F = 2900\text{ N}$ [1 mark]

**c** Students were required to explicitly state that Emily is correct. Even though gravity is still present, at the top of the loop [1 mark] if the reaction force (N) equals zero then a person would feel weightless [1 mark].

**4** $v = \sqrt{rg} = \sqrt{6.4 \times 9.8}$ [1 mark]

$= 7.9\text{ m s}^{-1}$ [1 mark]

**5** At the lowest point, the track exerts a normal reaction force N upward on the car The net force is upward.

$F_{net} = N - mg = ma = \frac{mv^2}{r}$

$\Rightarrow N = \frac{mv^2}{r} + mg = \frac{2 \times 6^2}{4} + 2 \times 10$ [1 mark]

$= 18 + 20 = 38\text{ N}$ [1 mark]

## 1.7 Review

## 1.7 Review questions

**1** The stationary car is pushed forward by the other vehicle. As a result, the seat pushes the body of an occupant forward. This happens almost instantaneously. However, without a headrest, there is nothing to push the occupant's head forward quickly. The head remains at rest until pulled forward by the spine (Newton's First Law of Motion). The head applies an equal and opposite force to the spine (Newton's Third Law of Motion), potentially causing serious injuries.

**2** To say that the passenger is thrown forward implies that a force accelerates the passenger. The car slows down rapidly in most collisions as a result of a large external force.
The passenger continues to move at the original speed of the car while the car slows down.

**3** The matching reaction to the gravitational pull of Earth on you is the gravitational pull of you on the Earth.

**4** As no forces are acting in the horizontal direction, there can be no horizontal acceleration. Therefore, the horizontal component of velocity must remain constant.

**5** The time of a projectile's flight is the time it takes to hit the ground. Therefore, the projectile cannot take longer to complete one part of its motion than the other. Time is the only useful variable that is a scalar and is the same in both the vertical and horizontal directions.

**6** When a basketball falls from rest, there is initially no air resistance. As it accelerates downwards due to the Earth's gravitational pull, the air resistance increases. The magnitude of the net force, and subsequently its acceleration, decreases. The air resistance continues to increase as the acceleration continues downwards. Because the air resistance is small compared to the basketball's weight, the basketball will not reach its terminal velocity, unless dropped from an aeroplane or helicopter in flight!

**7** Newton's first law states that an object will continue to move in a straight line with constant speed unless an unbalanced force acts on it. Therefore, the mass will continue to move forward without a propelling force, once in motion.
The centripetal force acts to change the direction of the mass, not its speed.

**8** **a** Distance is given by the area under a speed–time graph.
Braking distance = area under the graph over the last 20 s
Braking distance $= \frac{1}{2} \times 20 \times 20 = 200\,\text{m}$

**b** Total distance between stations = area under the graph

$$\text{Distance} = 200 + 50 \times 20 + \frac{1}{2} \times 50 \times 20$$
$$= 1700\,\text{m}$$
$$\Rightarrow \text{speed} = \frac{\text{distance}}{\text{time}}$$
$$= \frac{1700}{120}$$
$$\approx 14\,\text{m s}^{-1}$$

**c** $\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$
$= \frac{20}{50}$
$= 0.4\,\text{m s}^{-2}$

$\Rightarrow \boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$
$= 4.0 \times 10^4 \times 0.40$
$= 1.6 \times 10^4\,\text{N}$

Total frictional forces = 8000 N
Forward force − 8000 N $= 1.6 \times 10^4$ N
Forward force $= 2.4 \times 10^4$ N

**d** $\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$
$= \frac{-20}{20}$
$= -1.0\,\text{m s}^{-2}$

$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$
$= -4.0 \times 10^4 \times 1.0$
$= -4.0 \times 10^4\,\text{N}$

Therefore the additional frictional force
$= 4.0 \times 10^4 - 8000 = 3.2 \times 10^4\,\text{N}$

**9** **a** $v = 12\,\text{km h}^{-1} = \frac{12}{3.6}\,\text{m s}^{-1} \approx 3.33\,\text{m s}^{-1}$

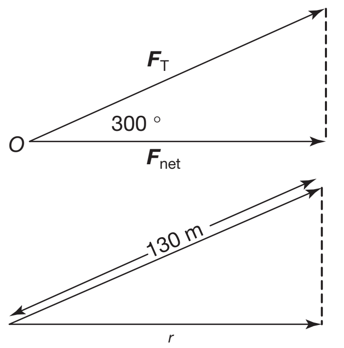
$a = \frac{v^2}{r}$
$= \frac{3.33^2}{350}$
$\approx 3.7 \times 10^{-2}\,\text{m s}^{-2}$

**b** $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$
$F_{\text{net}} = 45 \times 3.7 \times 10^{-2}$
$= 1.7$ N towards the centre of the circle

**c** $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$
$F_{\text{net}} = 1250 \times 3.7 \times 10^{-2}$
$= 46$ N towards the centre of the circle

**d** To move along the same path, the child and the train require the same *acceleration*. As the masses of the child and the train are different, different forces are needed to produce identical accelerations.

**10** **a**




$m = 0.100\,\text{kg}$
$T = 1.20\,\text{s}$
$r = 1.30 \cos 30.0°$
$v = \frac{2\pi r}{T}$
$= \frac{2\pi \times 1.3 \cos 30.0°}{1.20}$
$\approx 5.89\,\text{m s}^{-1}$

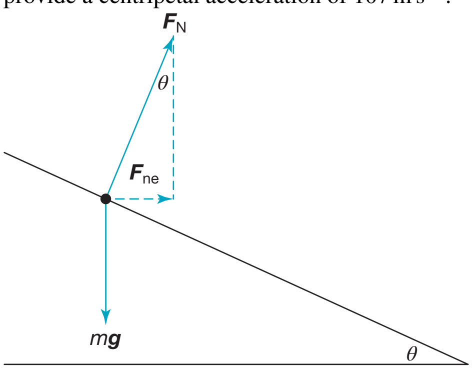
**b** $\boldsymbol{a} = \frac{4\pi^2 r}{T^2}$
$= \frac{4\pi^2 \times 1.3 \cos 30.0°}{(1.20)^2}$
$\approx 30.9\,\text{m s}^{-2}$ towards the centre of the circle

**c** $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$
$= 0.100 \times 30.9$
$= 3.09$ N towards the centre of the circle

**d** $\boldsymbol{F}_{\text{tension}} = \sqrt{\boldsymbol{F}_{\text{g}}^2 + \boldsymbol{F}_{\text{net}}^2}$
$= \sqrt{(m\boldsymbol{g})^2 + 3.09^2}$
$= \sqrt{(0.100 \times 9.8)^2 + 3.09^2}$
$\approx 3.24\,\text{N}$

**11** $\boldsymbol{v} = 40.0\,\text{m s}^{-1}$; $r = 15.0\,\text{m}$
$\boldsymbol{a} = \frac{\boldsymbol{v}^2}{r}$
$= \frac{40.0^2}{15.0}$
$\approx 107\,\text{m s}^{-2}$

The horizontal component of the normal force must therefore provide a centripetal acceleration of $107\,\text{m s}^{-2}$.




$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$
$= 107\,m$
$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{N}} \sin\theta$
$\Rightarrow \boldsymbol{F}_{\text{N}} \sin\theta = 107\,m$
$\boldsymbol{F}_{\text{N}} \cos\theta = m\boldsymbol{g}$

Dividing these equations results in:

$$\tan\theta = \frac{107\,m}{mg} = \frac{107}{9.8}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{107}{9.8}\right) \approx 84.8°$$

The road should be banked at an angle of 84.8°.

**12 a** At point A, the gymnast has gravitational potential energy only:

$$E_A = mgh = 65 \times 9.8 \times 1 = 637\text{ J}$$

All of the energy from point A is transferred to kinetic energy at point B:

$$E_B = E_A$$

$$\frac{1}{2}m\boldsymbol{v}^2 = 637$$

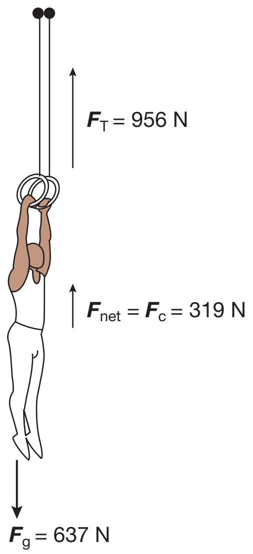
$$\boldsymbol{v}^2 = \frac{2 \times 637}{65}$$

$$\Rightarrow \boldsymbol{v} = \sqrt{\frac{2 \times 637}{65}} \approx 4.43\text{ m s}^{-1}$$

The speed of the gymnast at the point B is $4.43\text{ m s}^{-1}$.

**b** $$\boldsymbol{F}_c = \frac{m\boldsymbol{v}^2}{r} = \frac{65 \times 4.43^2}{4.0} \approx 3.2 \times 10^2\text{ N upwards}$$

**c**




(1 mark each for correctly labelling $\boldsymbol{F}_T$ and $\boldsymbol{F}_g$)

## 1.7 Exam questions

### Section A — Multiple choice questions

**1** D. There are only ever two forces acting on the ball: gravitational force, which is always straight down; and the string tension, which acts towards the centre of the rotation.

**2** A. Newton's first law states, 'An object will remain at rest or constant velocity unless acted on by an unbalanced force.'

Because the aeroplane is flying at a constant velocity, the forces acting on it are balanced, that is $F_{thrust} = F_{drag}$.

$\therefore F_{thrust} = 1500\text{N}$

**3** C

$$F = \sqrt{(200-180)^2 + (240-210)^2} = 36.1\text{ N}$$

**4** C

$$F = ma = m\frac{\Delta v}{\Delta t} = 1000 \times \frac{20}{2.5} = 8000\text{ N} = 8.0\text{ kN}$$

**5** C

$$a = \frac{F}{m} = \frac{4.0}{2.0} = 2.0\text{ m s}^{-2}$$

**6** C. Starts from rest with constant acceleration for a given time:
$v = u + at = 0 + 2.0 \times 10 = 20\text{ m s}^{-1}$

**7** C. Ignoring air resistance, the only force acting on the ball is the gravitational force.
Therefore, the acceleration of the ball as it falls is equal to $\boldsymbol{g}$ ($9.8\text{ m s}^{-2}$) and is constant.

**8** D. The kinetic energy remains constant because the magnitude of the velocity remains constant. Kinetic energy is not a vector.
The momentum changes because the direction of the velocity changes. Momentum is a vector.
***VCAA examination report*** **note:**
31% of students answered this question correctly. 48% of students incorrectly selected option B.

**9** B. Horizontal displacement is given by:
$s = vt$
Since $v = 6\text{ m s}^{-1}$, the distance is $6t$.

**10** C. The time to hit the water is found using:

$$s = \frac{1}{2}at^2$$

$$8.0 = 0.5 \times 9.8 \times t^2$$

$$t = 1.3\text{ s}$$

### Section B — Short answer questions

**11** The correct response was to demonstrate Newton's third law and state that if the action force is the gravitational force on Liesel due to the Earth, then: 'The reaction force is the gravitational force of Liesel on the Earth'. This is the force of Liesel pulling up on Earth, not Earth pushing up on Liesel.
This question was not well done. The most common error was to confuse Newton's third law with situations involving balanced forces. The upwards force of the floor on Liesel is a normal force and balances the gravitational force, which is why Liesel is not accelerating. This is not a reaction force.
Also of concern were the high number of students who stated that the reaction force was the normal force. Newton's laws, particularly the first and third and the difference between them, were poorly understood.

**12** $F_{net} = F_{th} - mg = ma$ [1 mark]

$F_{th} - (531 \times 10^3 \times 9.80) = (531 \times 10^3 \times 7.20)$

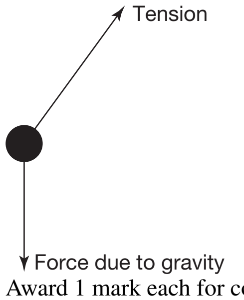
$F_{th} - 5.20 \times 10^6 = 3.82 \times 10^6$ [1 mark]

$F_{th} = 9.02 \times 10^6\text{ N}$

$= 9.02\text{ MN}$ [1 mark]

**13** The simplest way to start a consideration of forces is to state that at the top of the loop the centripetal force is provided by the gravitational force and the normal force from the track:
$\frac{mv^2}{r} = mg + F_N$. For the car to just remain in contact, $F_N = 0$.
This means that the centripetal force should be provided by gravity alone. That is: $\frac{mv^2}{r} = mg$. Cancelling the ms yields:
$v^2 = rg$, as required. The most common error was to start with a rearranged form of the required formula (e.g. $v = \sqrt{rg}$ ) and simply rearrange it. The question required an initial consideration of forces.

**14** **a**

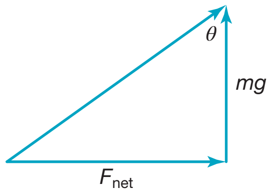



Award 1 mark each for correctly labeling each force.

***VCAA examination report* note:**

There are two forces acting on the ball. Many students identified the centripetal force as a force in addition to tension and weight, rather than being the result of the applying the tension force at an angle.

**b**




$F_g = mg \tan\theta = \frac{mv^2}{r}$ [1 mark]

$9.8 \tan 25° = \frac{v^2}{0.32}$ [1 mark]

$4.5698 = \frac{v^2}{0.32}$

$v = \sqrt{4.5698 \times 0.32}$ [1 mark]

$v = 1.2\ \text{m s}^{-1}$ [1 mark]

***VCAA examination report* note:**

There were other approaches that were also acceptable. Most students were unable to correctly identify the trigonometric components required. Very few students drew diagrams to help them with this. Students are encouraged to draw the triangle involved to help them identify the components they require.

**15** **a** Determine the vertical component of the initial velocity:

$u_{vert} = u \sin\theta$
$= 25 \sin(39°)$
$= 15.7\ \text{m s}^{-1}$

Vertically, the projectile comes to rest at the highest point of the flight ($v_{vert} = 0$).

$v_{vert} = u_{vert} + at$

$0 = 15.7 + (-9.8)t$ [1 mark]

$t = \frac{15.7}{9.8}$

$= 1.60\ \text{s}$ [1 mark]

**b** The time to the top of flight is 1.60 sec, so doubling this gives a total time of flight of 3.20 sec.
Since the horizontal velocity is constant, the horizontal range can then be found using:

$x = v_{horiz} \times t$
$= 25\cos(39°) \times 3.2$
$= 62\ \text{m}$

OR

Using the range formula:

$\text{Range} = \frac{v^2 \sin 2\theta}{g}$

$\text{Range} = \frac{v^2 \sin 2\theta}{g}$

$\text{Range} = \frac{25^2 \sin(2 \times 39)}{9.8}$ [1 mark]

$\text{Range} = 62\ \text{m}$ [1 mark]

# Topic 2 — Relationships between force, energy and mass

## 2.2 Momentum and impulse

### Sample problem 1

**a** Assign the initial direction of the car as positive:

$m = 1200$ kg; $\boldsymbol{u} = 15$ m s$^{-1}$; $\boldsymbol{v} = 0$ m s$^{-1}$; $\Delta t = 0.060$ s

$$\begin{aligned}\Delta \boldsymbol{p} &= m\boldsymbol{v} - m\boldsymbol{u}\\ &= m(\boldsymbol{v} - \boldsymbol{u})\\ &= 1200\ \times (0 - 15)\\ &= 1200 \times (-15)\\ &= -1.8 \times 10^4 \text{ kg m s}^{-1}\end{aligned}$$

The change in momentum is $1.8 \times 10^4$ kg m s$^{-1}$ in a direction opposite to the original direction of the car.

**b** Determine the impulse of the car using the change in momentum:

$$\begin{aligned}\text{Impulse on car} &= \text{change in momentum of car}\\ &= -1.8 \times 10^4 \text{ kg m s}^{-1}\end{aligned}$$

The impulse on the car is $1.8 \times 10^4$ N s in a direction opposite to the original direction of the car.

**c** Determine the magnitude of force using $\boldsymbol{F}\Delta t$:

$$\begin{aligned}\Delta \boldsymbol{p} &= \boldsymbol{F}\Delta t\\ 1.8 \times 10^4 &= \boldsymbol{F} \times 0.06\\ \Rightarrow \boldsymbol{F} &= \frac{1.8 \times 10^4}{0.060}\\ &= 3.0 \times 10^5 \text{ N}\end{aligned}$$

**d** Determine the impulse of the car:

$$\begin{aligned}\text{Impulse} &= m\Delta \boldsymbol{v}\\ &= 1200\ \ (-3 - 15)\\ &= 1200 \times (-18)\\ &= -2.16 \times 10^4 \text{ N s (or kg m s}^{-1})\end{aligned}$$

Determine the magnitude of force using $\Delta \boldsymbol{p} = \boldsymbol{F}\Delta t$:

$$\begin{aligned}\Delta \boldsymbol{p} &= \boldsymbol{F}\Delta t\\ 2.16 \times 10^4 &= \boldsymbol{F} \times 0.060\\ \Rightarrow \boldsymbol{F} &= \frac{2.16 \times 10^4}{0.060}\\ &= 3.6 \times 10^5 \text{ N}\end{aligned}$$

### Practice problem 1

**a**
$$\begin{aligned}\text{Impulse} &= \Delta \text{ momentum (positive taken as east)}\\ &= m\Delta \boldsymbol{v}\\ &= 200 \times (2 - (-8)) = 2.0 \times 10^3 \text{ N s east}\end{aligned}$$

**b**
$$\begin{aligned}\boldsymbol{F} &= \frac{\text{Impulse}}{\Delta t}\\ &= \frac{2.0 \times 10^3}{0.80}\\ &= 2.5 \times 10^3 \text{ N east}\end{aligned}$$

**c** $\boldsymbol{F}_{\text{on car}} = -\boldsymbol{F}_{\text{on barrier}} = 2.5 \times 10^3$ N west

### Sample problem 2

The magnitude of the impulse on the skater can be determined by calculating the area under the graph. This can be determined by either counting squares or by determining the shaded area:

$$\begin{aligned}\text{Magnitude of impulse} &= \text{area A} + \text{area B} + \text{area C}\\ &= \left(\frac{1}{2} \times 1.1 \times 400 + 0.9 \times 200 + \frac{1}{2} \times 0.9 \times 200\right)\\ &= (220 + 180 + 90)\\ &= 490 \text{ N s}\end{aligned}$$

Determine the magnitude of impulse using the change in momentum:

Magnitude of impulse = magnitude of change in momentum = $m\Delta \boldsymbol{v}$

$$\begin{aligned}490 \text{ N s} &= 40 \text{ kg} \times \Delta \boldsymbol{v}\\ \Rightarrow \Delta \boldsymbol{v} &= \frac{490 \text{ N s}}{40 \text{ kg}}\\ &= 12 \text{ m s}^{-1}\end{aligned}$$

Determine the speed:

As her initial speed is zero (she started from rest), her speed after 2.0 seconds is 12 m s$^{-1}$.

### Practice problem 2

$$\begin{aligned}\text{Impulse} &= \text{change in momentum}\\ &= \text{area under curve}\\ &= \frac{360 \times 1}{2}\\ &= 180 \text{ N s}\end{aligned}$$

$$\begin{aligned}\Rightarrow \Delta \boldsymbol{v} &= \frac{\Delta \boldsymbol{p}}{m}\\ &= \frac{180}{40}\\ &= 4.5 \text{ m s}^{-1}\end{aligned}$$

### Sample problem 3

**a** The 1500-kg car will have a positive velocity, and the 1200-kg car will have a negative velocity.

$m = 1500$ kg; $\boldsymbol{v} = 12.0$ m s$^{-1}$; $\boldsymbol{p} = m\boldsymbol{v}$

Determine the momentum of the first car:

$$\begin{aligned}\boldsymbol{p} &= m\boldsymbol{v}\\ &= 1500 \times 12.0\\ &= 18\,000 \text{ kg m s}^{-1}\\ &= 1.80 \times 10^4 \text{ kg m s}^{-1}\end{aligned}$$

Determine the momentum of the second car:

$m = 1200$ kg; $\boldsymbol{v} = -12.0$ m s$^{-1}$

$$\begin{aligned}\boldsymbol{p} &= m\boldsymbol{v}\\ &= 1200 \times (-12.0)\\ &= -14\,400 \text{ kg m s}^{-1}\\ &= -1.44 \times 10^4 \text{ kg m s}^{-1}\end{aligned}$$

**b**
$$\begin{aligned}\boldsymbol{p}_{\text{i}} &= 1.80 \times 10^4 + (-1.44 \times 10^4)\\ &= 3.60 \times 10^3 \text{ kg m s}^{-1}\end{aligned}$$

**c**
$$\begin{aligned}\boldsymbol{p}_{\text{f}} &= \boldsymbol{p}_{\text{i}}\\ &= 3.60 \times 10^3 \text{ kg m s}^{-1}\end{aligned}$$

**d** $p_f = mv$

$3.60 \times 10^3 = 2700 \times v$

$v = \dfrac{3.60 \times 10^3}{2700}$

$\Rightarrow v = 1.33\ \text{m s}^{-1}$

in the direction of the initial velocity of the first car

**e** The impulse on the 1200-kg car is equal to its change in momentum. The initial momentum was calculated in part a, and the final momentum can be calculated using $p = mv$, where the mass = 1200-kg, and the final velocity = 1.33 m s$^{-1}$ (as calculated in part d).

$\Delta p = p_f - p_i$

$= (1200 \times 1.33) - (-1.44 \times 10^4)$

$= 1600 + 1.44 \times 10^4$

$= 1.60 \times 10^4\ \text{kg m s}^{-1}$

in the direction of motion of the tangled wreck

**f** The impulse on the 1500-kg car is equal to the impulse on the 1200-kg car. This can be verified by calculating the change in momentum of the 1500-kg car. The initial momentum was calculated in part a, and the final momentum can be calculated using $p = mv$, where the mass = 1500 kg, and the final velocity = 1.33 m s$^{-1}$ (as calculated in part d).

$\Delta p = p_f - p_i$

$= (1500 \times 1.33) - (1.80 \times 10^4)$

$= 2000 - 1.80 \times 10^4$

$= -1.60 \times 10^4\ \text{kg m s}^{-1}$

in the direction opposite that of the 1200-kg car

## Practice problem 3

**a** Use conservation of momentum to find the velocity after impact:

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$\Rightarrow v = \dfrac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$

$= \dfrac{1000 \times 30 + 2000 \times 0}{1000 + 2000}$

$= 10\ \text{m s}^{-1}$

**b** Impulse on van = change in momentum of van

$= m\Delta v$

$= 2000 \times 10 = 2.0 \times 10^4$ N s north

**c** Impulse on car = − (impulse on van) = $-2.0 \times 10^4$ N s (south)

OR

$m\Delta v = 1000 \times (10 - 30) = -2.0 \times 10^4$ N s

**d** Use conservation of momentum to find the velocity after impact.

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow v_2 = \dfrac{m_1u_1 + m_2u_2 - m_1v_1}{m_2}$

$= \dfrac{1000 \times 30 + 2000 \times 0 - 2000 \times 12}{1000}$

$= 6.0\ \text{m s}^{-1}$

## 2.2 Exercise

**1** Impulse is equal to the change in momentum.

**2** No. The system of the two cars is not isolated. There are unbalanced frictional forces acting on the cars during and immediately after the collision.

**3 a** The system of the coal and railway cart can be considered to be isolated if only the horizontal motion is considered:

$p_i = 500 \times 3.0 + 0$

(initial horizontal momentum of the coal is zero)

$= 1500\ \text{kg m s}^{-1}$ south

$p_f = 750v$ (where $v$ = final velocity of the cart)

$p_f = p_i$

$\Rightarrow 750v = 1500$ south

$\Rightarrow v = 2.0\ \text{m s}^{-1}$ south

**b** The vertically downward momentum of the coal decreases to zero because there is an upward net force acting on it when it strikes the cart. The total momentum of the Earth–coal system has not changed.

**c** $p_i = 750 \times 2.0$

$= 1500\ \text{kg m s}^{-1}$ south

When the coal falls from the cart, the cart has a horizontal velocity of 2.0 m s$^{-1}$south:

$500v + 250 \times 2.0 = 1500\ \text{kg m s}^{-1}$ south

$500v = (1500 - 500)\ \text{kg m s}^{-1}$ south

$\Rightarrow v = \dfrac{1000}{500}\ \text{kg m s}^{-1}$ south

$= 2.0\ \text{m s}^{-1}$ south

**4** Student responses will vary. In most situations, an occupant in a larger car is safer. However, the design of the particular car, the nature of the collision and several other factors influence the likely effect of an accident on occupants. The following points should be made:

**a** The forces applied to each car by the other are equal in magnitude and opposite in direction.

**b** The change in velocity of each car is dependent on the mass of the car. Assuming that the sum of the forces other than that applied by the other car is zero, the change in velocity is inversely proportional to the mass of the car.

**c** Assuming that you are properly restrained and that the collision is head-on, your change in velocity (and therefore the deceleration you are subjected to) is less if you are in a heavier car.

**d** The body continues to move in the original direction and at the original speed of your car until an unbalanced force acts on you. If you are not restrained, the unbalanced force will be provided by the windscreen or part of the interior of the car, which has already slowed down. A smaller car will have slowed down more, so the impulse applied to you ($m\Delta v$) will be greater.

**5 a** $I = \Delta p$

$= -10$ N s

**b** $I = F_{net}\Delta t$

$\Rightarrow F_{net} = \dfrac{I}{\Delta t}$

$= \dfrac{-10}{0.10}$

$= -1.0 \times 10^2$ N

$= 1.0 \times 10^2$ N away from the wall

**c** $\boldsymbol{I} = \Delta \boldsymbol{p}$

$\boldsymbol{I} = m(\boldsymbol{v}_f - \boldsymbol{v}_i)$

$-10 = 0.400(\boldsymbol{v}_f - 15)$

$-25 = \boldsymbol{v}_f - 15$

$\Rightarrow \boldsymbol{v}_f = -10\,\text{m s}^{-1}$

$= 10\,\text{m s}^{-1}$ away from the wall

## 2.2 Exam questions

**1 a i** $F = mg\sin\theta$

$F = 2.0 \times 9.8 \times \sin 25°$ [1 mark]

$F = 8.3\,\text{N}$ [1 mark]

The most common errors were mathematical and usually involved the wrong trigonometry identity.

**ii** $F_{net} = F_g - F_f$ [1 mark]

$F_{net} = ma$

$= 2 \times 3.2$

$= 6.4\,\text{N}$

$6.4 = 8.3 - F_f$

$\Rightarrow F_f = 1.9\,\text{N}$ [1 mark]

There was no common error. Students who failed to score marks for this question generally could not demonstrate any suitable strategy to solve the problem.

**b i** The problem required 'conservation of momentum'. [1 mark]

$(m_1 v_i) + (m_2 v_i) = (m_1 + m_2)v_f$

$(2.0 \times 4.0) + (2.0 \times 0) = (4.0)v_f$ [1 mark]

$\Rightarrow v_f = 2.0\,\text{m s}^{-1}$ [1 mark]

Students who were not able to score full marks could, generally, identify the principle as conservation of momentum but were unable to express it in mathematical form.

**ii** $\text{KE}_{init} = \frac{1}{2}mv^2$

$= 0.5 \times 2 \times 4^2$

$= 16\,\text{J}$ [1 mark]

$\text{KE}_{final} = \frac{1}{2}mv^2$

$= 0.5 \times 4 \times 2^2$

$= 8\,\text{J}$ [1 mark]

The decrease in kinetic energy means the collision is inelastic. [1 mark]

The most common errors were mathematical. Students frequently forgot to square the velocity.

**2 a**
- Acceleration at W is greater than zero [1 mark] and less than $9.8\,\text{m s}^{-2}$ [1 mark].
- Acceleration at X is zero. [1 mark]
- Acceleration at Y is greater than zero and directed to the left. [1 mark]

**b** $mgh = \frac{1}{2}kx^2$ [1 mark]

$5.0 \times 9.8 \times h = 0.5 \times 100 \times 3.0^2$ [1 mark]

$h = 9.2\,\text{m}$ [1 mark]

**c** $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$0.5 \times 100 \times 3^2 = 0.5 \times 5.0 \times v^2$

$v = 13.4\,\text{m s}^{-1}$ [1 mark]

$p = mv$

$p = 5.0 \times 13.4$

$p = 67\,\text{kg m s}^{-1}$ [1 mark]

**d** The momentum is transferred to the earth.

**3 a** $f = m\frac{\Delta v}{\Delta t}$

$\Delta v = 3.3 - (-3.6)$

$= 6.9\,\text{m s}^{-1}$ [1 mark]

$F = 50 \times 10^{-3} \times \frac{6.9}{40 \times 10^{-3}}$ [1 mark]

$F = 8.6\,\text{N}$ [1 mark]

The force acts upwards. [1 mark]

**b** Some of the energy is converted to SPE in the ball. [1 mark] The rest is lost as heat/sound. [1 mark]

**c** The momentum is transferred to the earth.

**4 a** Impulse = change in momentum [1 mark]

$\Delta p = p_f - p_i$

$\Delta p = 8 - (-12)$ [1 mark]

$= 20\,\text{kg m s}^{-1}$ or $20\,\text{N s}$ [1 mark]

**b** $I = Ft$

$20 = F \times 0.01$ [1 mark]

$\therefore F = 2000\,\text{N}$ [1 mark]

**c** $Ek_{initial} = \frac{1}{2}mv^2$

$= (0.5 \times 2.0 \times 10^2) + (0.5 \times 0.2 \times 60^2)$

$= 460\,\text{J}$ [1 mark]

$Ek_{final} = \frac{1}{2}mv^2$

$= (0.5 \times 0.2 \times 40^2)$

$= 160\,\text{J}$ [1 mark]

The reduction in kinetic energy indicates that the collision is inelastic.

**5** Impulse $= F\Delta t$

$= m\Delta v$

$= 4.0 \times 2.0$

$= 8.0$ [1 mark]

$\Rightarrow$ unit is N s or $\text{kg m s}^{-1}$ [1 mark]

***VCAA Assessment Report*** **note:**

The most common error was to incorrectly quote the unit either as $\text{kg/m s}^{-1}$ or N/s.

## 2.3 Work done

### Sample problem 4

$W = Fs\cos\theta$

$= 500 \times 585\cos 30°$

$= 2.53 \times 10^5\,\text{J}$

### Practice problem 4

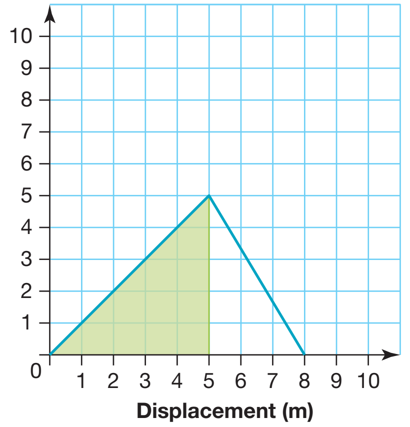
$W = Fs\cos\theta$

$= 440 \times 231\cos 52°$

$= 6.25 \times 10^4$ J

### Sample problem 5

The work done is the area under the line between 0 and 5 m:




The base of the triangle is 5 and the height is 5.

Calculate the area of the triangle to determine the amount of work done:

$A_{\text{triangle}} = \frac{1}{2} \times 5 \times 5$

$= 12.5$ J

$\simeq 1 \times 10^1$ J, to 1 s.f.

### Practice problem 5

Work done is given by the area under the graph:

$W = \frac{1}{2} \times 5 \times 3$

$= 7.5$ J

$\simeq 8$ J

### 2.3 Exercise

**1** $W = \boldsymbol{F}\Delta d$

$\Rightarrow \boldsymbol{F} = \frac{W}{\Delta d}$

$= \frac{10\,000}{50}$

$= 2.0 \times 10^2$ N

**2** $\boldsymbol{F}_{\text{T horiz}} = \boldsymbol{F}_{\text{T}} \times \cos 30°$

$= 40 \times \cos 30°$

$= 34.64$ N

$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{T horiz}} - \boldsymbol{F}_{\text{R}}$

$= 34.64 - 20$

$= 14.64$ N

$\Rightarrow W = \boldsymbol{F}_{\text{net}} \times \Delta x$

$= 14.64 \times 5$

$= 73$ J

**3** $W = \boldsymbol{F}\Delta d \times \cos\theta$

$= (125 \times 9.8) \times (1.1 \times \cos 0°)$

$= 1.3 \times 10^3$ J

**4** $W = \frac{1}{2} \times l \times h$

$= \frac{1}{2} \times 0.4 \times 20$

$= 4.0$ J

**5** $W = \boldsymbol{F}\Delta d \times \cos\theta$

$= (5.1 \times 9.8 \times 7.0) \times \cos 0°$

$= 3.5 \times 10^2$ J

### 2.3 Exam questions

**1** $E = \frac{1}{2}kx^2$

$0.9 = 0.5 \times \left(\frac{40}{0.08}\right) \times x^2$

$\Rightarrow x = 0.06$ m

**2** **a** Energy stored = work done to compress the spring

= area under the graph

$E = \frac{1}{2}$base × height [1 mark]

$= \frac{1}{2} \times 0.50 \times 72$

$= 18$ J [1 mark]

The alternative method of finding the spring constant $k$ and then calculating $\frac{1}{2}kx^2$ is more long-winded.

**b** The gain in KE = the initial stored energy.

$\frac{1}{2}mv^2 = 18$ [1 mark]

$v^2 = \frac{2 \times 18}{4}$

$= 9$

$v = 3.0$ m s$^{-1}$ [1 mark]

***VCAA Assessment Report* note:**

The most common error was not to take the square root at the end.

**3** The claim is incorrect. [1 mark]

The force is at right angles to the displacement and so zero work is done. [1 mark]

**4** $W$ = area under $F - s$ graph

$W = \frac{1}{2} \times 2 \times 500 + 2 \times 500$ [1 mark]

$W = 1500$ J [1 mark]

**5** $W$ = area under $F - s$ graph

$W = \frac{1}{2} \times 0.12 \times 300$ [1 mark]

$W = 18$ J [1 mark]

## 2.4 Kinetic and potential energy

### Sample problem 6

The net force on the car is equal to the force applied by the wall. The work done by the wall, $W$, is given by:

**a** $W = \Delta E_k$

$\Delta E_k = \frac{1}{2}m\boldsymbol{v}^2$, where $m = 600\,\text{kg}$ and $\boldsymbol{v} = 12.0\,\text{m s}^{-1}$

It is known that the car comes to a complete stop, so the final kinetic energy is 0. The change in kinetic energy is being calculated, so the focus can be kept on the initial kinetic energy.

$$W = \frac{1}{2}m\boldsymbol{v}^2$$
$$= \frac{1}{2} \times 600 \times (12.0)^2$$
$$= 4.32 \times 10^4\,\text{J}$$

**b** The magnitude is determined by:

$$W = \boldsymbol{F}_{av} \times d$$
$$4.32 \times 10^4 = \boldsymbol{F}_{net} \times 0.300 \ (\boldsymbol{F}_{av} = \boldsymbol{F}_{net} \text{ in this case})$$
$$\Rightarrow \boldsymbol{F}_{net} = 1.44 \times 10^5\,\text{N}$$

### Practice problem 6

**a** $W = \Delta E_k$

$$= \frac{1}{2}m\boldsymbol{v}^2 - \frac{1}{2}m\boldsymbol{u}^2$$
$$= \frac{1}{2} \times 800 \times 15^2 - \frac{1}{2} \times 800 \times 5.0^2$$
$$= 8.0 \times 10^4\,\text{J}$$

**b** $\boldsymbol{F} = \frac{W}{d}$

$$= \frac{8.0 \times 10^4}{20}$$
$$= 4.0 \times 10^3\,\text{N}$$

### Sample problem 7

The energy stored in the spring is equal to the amount of work done on it:

$W$ = area under graph

= area A + area B + area C

$$\Rightarrow \text{Area A} = \frac{1}{2} \times 0.15 \times 15 = 1.125\,\text{N m}$$
$$\text{Area B} = 0.1 \times 15 = 1.5\,\text{N m}$$
$$\text{Area C} = \frac{1}{2} \times 0.1 \times 5.0 = 0.25\,\text{N m}$$

$$\Rightarrow A + B + C = 1.125 + 1.5 + 0.25$$
$$= 2.9\,\text{N m}$$

The stored energy is 2.9 J.

### Practice problem 7

**a** $\Delta x = 20\,\text{cm}$

$$\Rightarrow \text{Area} = \frac{1}{2} \times 0.15 \times 15 + 15 \times 0.05 + \frac{1}{2} \times 0.05 \times 2.5$$
$$= 1.94\,\text{J}$$

**b** $\text{Area} = \frac{1}{2}kx^2$

where $k = \text{gradient} = \frac{10}{0.1} = 100\,\text{Nm}^{-1}$

$$\Rightarrow 0.5 = \frac{1}{2} \times 100x^2$$
$$\Rightarrow x = 0.1\,\text{m}$$
$$l = 0.35 - 0.1$$
$$= 0.25\,\text{m} = 25\,\text{cm}$$

### Sample problem 8

**a** $k = \frac{40}{0.20}$

$$= 2.0 \times 10^2\,\text{N m}^{-1}$$

**b** The gradient of the graph for spring A is greater than that for spring B. Therefore, spring A has a greater spring constant than spring B — in fact, it is twice the size.

**c** Strain potential energy $= \frac{1}{2}kx^2$

$$\Rightarrow k = \text{gradient}$$
$$= \frac{20}{0.20}$$
$$= 1.0 \times 10^2\,\text{N m}^{-1}$$
$$\Rightarrow \text{Strain potential energy} = \frac{1}{2} \times 1.0 \times 10^2 \times 0.20^2$$
$$= 2.0\,\text{J}$$

### Practice problem 8

**a** $k = \text{gradient} = \frac{10}{0.1} = 100\,\text{N m}^{-1}$

**b** Strain potential energy $= \frac{1}{2}k\boldsymbol{x}^2$

$$= \frac{1}{2} \times 2.0 \times 10^2 \times 0.20^2$$
$$= 4.0\,\text{J}$$

### Sample problem 9

Strain potential energy stored in the spring:

$$\frac{1}{2}kx^2 = \frac{1}{2} \times 50\,\text{N m}^{-1} \times (0.10\,\text{m})^2$$
$$= 0.25\,\text{J}$$

Determine the speed of the toy car:

$$\frac{1}{2}m\boldsymbol{v}^2 = \text{energy transformed}$$
$$\frac{1}{2} \times 0.50 \times \boldsymbol{v}^2 = 0.25$$
$$\Rightarrow \boldsymbol{v}^2 = \frac{0.25}{\frac{1}{2} \times 0.50}$$
$$\Rightarrow \boldsymbol{v} = 1.0\,\text{m s}^{-1}$$

## Practice problem 9

**a** $E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.4 \times 0.8^2$

$= 0.128$ J

**b** $\Delta E_k = \Delta E_p$

$0.128 = \frac{1}{2} \times 100x^2$

$\Rightarrow x = 0.051$ m $= 5.1$ cm

## Sample problem 10

**a** $m = 35$ kg, $\boldsymbol{g} = 9.8$ m s$^{-2}$ and $\Delta h = 9.0$ m

$\Delta E_g = m\boldsymbol{g}\Delta h$

$= 35 \times 9.8 \times 9.0$

$= 3.1 \times 10^3$ J

**b** $\frac{1}{2}m\boldsymbol{v}^2 = m\boldsymbol{g}\Delta h$

$\Rightarrow \boldsymbol{v}^2 = 2\boldsymbol{g}\Delta h$

$\boldsymbol{v} = \sqrt{2 \times 9.8 \times 9.0}$

$= 13$ m s$^{-1}$

## Practice problem 10

**a** $\Delta E_g = m\boldsymbol{g}\Delta h$

$= 60 \times 9.8 \times 25$

$= 14\,700$ J

$\simeq 1.5 \times 10^4$ J

**b** $E_{kf} = E_{ki} + \Delta E_{gp}$

$= \frac{1}{2}m\boldsymbol{v}^2 + 1.5 \times 10^4$

$= \frac{1}{2} \times 60 \times 0.5^2 + 1.5 \times 10^4$

$= 1.5 \times 10^4$ J

$\Rightarrow \boldsymbol{v} = \sqrt{\frac{2E_k}{m}}$

$= \sqrt{\frac{2 \times 1.5 \times 10^4}{60}}$

$= 22$ m s$^{-1}$

## Sample problem 11

**a** Assign the direction in which the white car is moving as positive. Assume that friction is negligible. Therefore, momentum is conserved. Momentum is calculated using the formula $\boldsymbol{p} = m\boldsymbol{v}$. The mass of the white car is 800 kg and the initial velocity was 20 m s$^{-1}$, and the mass of the blue car is 700 kg and the initial velocity was 0 m s$^{-1}$.

The initial momentum of the system, $\boldsymbol{p}_i$, is given by:

$\boldsymbol{p}_i = \boldsymbol{p}_{white} + \boldsymbol{p}_{blue}$

$= m_w\boldsymbol{v}_w + m_b\boldsymbol{v}_b$

$= (800 \times 20) + (700 \times 0)$

$\boldsymbol{p}_i = 1.6 \times 10^4$ kg m s$^{-1}$

Determine the final momentum of the system using the formula $\boldsymbol{p} = m\boldsymbol{v}$. The mass of the white car is 800 kg and the mass of the blue car is 700 kg and the final velocity was 12 m s$^{-1}$. The final momentum of the system, $\boldsymbol{p}_f$, is given by:

$\boldsymbol{p}_f = \boldsymbol{p}_{white} + \boldsymbol{p}_{blue}$

$= (800 \times \boldsymbol{v}_{white}) + (700 \times 12)$

$\boldsymbol{p}_f = 800 \times \boldsymbol{v}_{white} + 8.4 \times 10^3$

$\boldsymbol{v}_{white}$ = velocity of the white car after the collision

Since $\boldsymbol{p}_f = \boldsymbol{p}_i$:

$800\,\boldsymbol{v}_{white} + 8.4 \times 10^3 = 1.6 \times 10^4$

$800\,\boldsymbol{v}_{white} = 7.6 \times 10^3$

$\Rightarrow \boldsymbol{v}_{white} = 9.5$ m s$^{-1}$

The speed of the white car after the collision is 9.5 m s$^{-1}$.

**b** Total kinetic energy before the collision is given by:

$\frac{1}{2} \times 800 \times (20)^2 + \frac{1}{2} \times 700 \times (0)^2 = 1.6 \times 10^5$ J

Total kinetic energy after the collision is given by:

$\frac{1}{2} \times 800 \times (9.5)^2 + \frac{1}{2} \times 700 \times (12)^2 = 8.7 \times 10^4$ J

Kinetic energy is not conserved; therefore, the collision is not elastic.

## Practice problem 11

**a** Assuming friction is negligible:

**i** $m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$

$\Rightarrow \boldsymbol{v}_1 = \frac{m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 - m_2\boldsymbol{v}_2}{m_1}$

$= \frac{400 \times 2.0 + 300 \times (-2.0) - 300 \times 1.0}{400}$

$= -0.25$ m s$^{-1}$

**ii** $m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$

$\Rightarrow \boldsymbol{v}_1 = \frac{m_1\boldsymbol{u}_1 - m_2\boldsymbol{u}_2 - m_2\boldsymbol{v}_2}{m_1}$

$= \frac{400 \times 2.0 + 300 \times (-2.0) - 300 \times 2.0}{400}$

$= -1.0$ m s$^{-1}$

**b** $E_{ki} = \frac{1}{2} \times 400 \times 2.0^2 + \frac{1}{2} \times 300 \times (-2.0)^2 = 1400$ J

**i** $E_{kf} = \frac{1}{2} \times 400 \times 0.25^2 + \frac{1}{2} \times 300 \times 1.0^2 = 162.5$ J

**ii** $E_{kf} = \frac{1}{2} \times 400 \times 1.0^2 + \frac{1}{2} \times 300 \times 2.0^2 = 800$ J

As the final $E_k$ is less than the initial $E_k$, both collisions are inelastic.

## 2.4 Exercise

**1** **a** No. The total kinetic energy of the ball and the ground is less after the collision than it was before the collision.

**b** Sound, along with some heating of the ball, provides evidence that some of the ball's initial kinetic energy is transformed.

**c** Yes, momentum is conserved, assuming that the ball–ground system is an isolated system.

**2** **a** Assign east as positive.

$\boldsymbol{p}_i = m\boldsymbol{u} + m \times (-20)$

$\boldsymbol{p}_f = 2m \times 5$

$\Rightarrow m\boldsymbol{u} - 20m = 10m$

$\boldsymbol{u} - 20 = 10$

$\Rightarrow \boldsymbol{u} = 30\text{ m s}^{-1}$

$= 30\text{ m s}^{-1}$ east

**b** $E_{ki} = \frac{1}{2}m(30)^2 + \frac{1}{2}m(20)^2$

$E_{kf} = \frac{1}{2} \times 2m \times (5)^2$

$\frac{E_{kf}}{E_{ki}} = \frac{m \times 25}{\frac{1}{2}m(900+400)}$

$= 0.038$

$= \frac{1}{26}$

**3 a** $\boldsymbol{F}_{\text{spring}} + \boldsymbol{F}_g = 0$

$\Rightarrow \boldsymbol{F}_{\text{spring}} = -\boldsymbol{F}_g$

$= -mg$

$= -1.0 \times 9.8$

$= -9.8\text{ N}$ (9.8 N up)

**b** $k = \frac{\boldsymbol{F}}{x}$

= gradient of $\boldsymbol{F}$ vs $x$ graph

$= \frac{15}{0.40} = 38\text{ N m}^{-1}$

**c** A — the spring with the greatest spring constant

**d** $W$ = Area

$= 0.5 \times 5 \times 0.5$

$= 1.3\text{ J}$

**e** The greatest strain energy occurs in the spring with the $\boldsymbol{F}$ versus $x$ graph of greatest area.

At maximum extension:

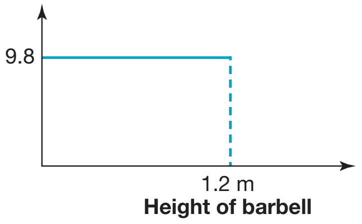
Area A $= \frac{1}{2} \times 0.20 \times 25 = 2.5\text{ J}$

Area B $= \frac{1}{2} \times 0.40 \times 15 = 3.0\text{ J}$

Area C = 1.3 J (from part d)

Therefore, spring B has the greatest strain energy.

**4 a**




**b** $\Delta E_g$ = Area under graph of force vs height

$= m \times$ area under graph (in part a.)

$= 150 \times 1.20 \times 9.8$

$= 1764\text{ J}$

$\simeq 1.8 \times 10^3\text{ J}$

**c** $W = \Delta E_g = 1.8 \times 10^3\text{ J}$

**5 a** $E_g = mgh$

$= 70 \times 9.8 \times 6.0$

$= 4116\text{ J}$

$\approx 4.1 \times 10^3\text{ J}$

**b** $E_g = mgh$

$= 80 \times 9.8 \times 7.0$

$= 5488\text{ J}$

$\approx 5.5 \times 10^3\text{ J}$

**c** $E_g = mgh$

$= 400 \times 9.8 \times (-80)$

$= 313\,600\text{ J}$

$\approx 3.1 \times 10^5\text{ J}$

**6 a** $E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 900 \times 20^2$

$= 1.8 \times 10^5\text{ J}$

**b** $1.8 \times 10^5\text{ J}$

**c** $\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$

$\Rightarrow \boldsymbol{a} = \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{s}}$

$= \frac{0 - 20^2}{2 \times 0.4}$

$= \frac{-400}{0.8}$

$= 500\text{ m s}^{-2}$

$\Rightarrow \boldsymbol{F} = m\boldsymbol{a}$

$= 900 \times 500$

$= 4.5 \times 10^5\text{ N}$

opposite to the initial direction of motion of the car

**7** The depth of penetration would double. This is because, as the height doubles, the initial gravitational energy doubles. Work is done by friction (force of the mud on the rock) as the mud brings the rock to rest. Assuming the friction force is constant, $d$ (depth) doubles, using $E_g \rightarrow W = \boldsymbol{F}d$.

## 2.4 Exam questions

**1 a** Using the conservation of energy:

$mgh = \frac{1}{2}mv^2$

$\not{m}gh = \frac{1}{2}\not{m}v^2$

The masses can be cancelled

$9.8 \times 15 = 0.5 \times v^2$

$v^2 = 300$

$v = 17.3\text{ m s}^{-1}$

**b** $mgh_A = mgh_C + \frac{1}{2}mv_C^2$ [1 mark]

Since $v^2 = rg$,

$mgh_A = mgh_C + \frac{1}{2}mrg$

Cancelling the $m$ and $g$ on both sides gives:

$h_A = h_C + \frac{1}{2}r$

Since $h_C = 2r$,

$h_A = 2.5r$

$r = \frac{15}{2.5} = 6$ [1 mark]

Therefore, the height at C will be 12 m. [1 mark]

***VCAA Assessment Report* note:**

This question was not well done. Some students substituted the velocity value from part a into the equation given in part b.

**c** The radius of the loop will have to decrease. Friction will cause the velocity to decrease [1 mark] and since the radius is related to the velocity by $r = \frac{v^2}{g}$ [1 mark], if the velocity decreases the radius will have to decrease as well [1 mark].

***VCAA Assessment Report* note:**
Of the students who were awarded marks, all could articulate that the radius needed to decrease. Most could state that this was due to a reduction in velocity, but just over 25% were able to identify the mathematical relationship between velocity and radius.

**2 a** $mg = ky$ [1 mark]

$y = \frac{2.0 \times 10}{100}$

$y = 0.20\,\text{m}$ [1 mark]

The extension of the spring from its unstretched position to its equilibrium position is 0.20 m.

**b i** By conservation of energy:

$mgx = \frac{1}{2}kx^2$ [1 mark]

$x = \frac{2mg}{k}$

$= \frac{2 \times 2.0 \times 10}{100}$

$= 0.40\,\text{m}$ [1 mark]

**ii** Total energy of the system:

$E_{\text{total}} = mgh$

$= 2.0 \times 10 \times 0.4$

$= 8\,\text{J}$ [1 mark]

The maximum velocity occurs at the mid-point where $x = 0.20$ m.

$\text{GPE}_{\text{mid}} = mgx$

$= 2.0 \times 10 \times 0.20$

$= 4.0\,\text{J}$ [1 mark]

$\text{SPE}_{\text{mid}} = \frac{1}{2}kx^2$

$= 0.5 \times 100 \times 0.20^2$

$= 2.0\,\text{J}$ [1 mark]

Therefore, there is 2.0 J of energy as kinetic energy.

$E_k = \frac{1}{2}mv^2$

$2 = 0.5 \times 2.0 \times v^2$

$\therefore v = 1.4\,\text{m s}^{-1}$ [1 mark]

**3 a** Spring constant ($k$) is given by gradient of graph. [1 mark]

$k = \frac{60}{1.5} = 40\,\text{N m}^{-1}$ [1 mark]

**b** $\text{SPE}_y = \frac{1}{2}kx^2$

$= 0.5 \times 40 \times 1^2$

$= 20\,\text{J}$ [1 mark]

$\text{SPE}_x = \frac{1}{2}kx^2$

$= 0.5 \times 40 \times 0.5^2$

$= 5\,\text{J}$ [1 mark]

$\Delta\text{SPE} = 20 - 5$

$= 15\,\text{J}$ [1 mark]

**c** The kinetic energy of the ball equals the energy stored in the spring minus the work done against gravity.

SPE = area under graph

$\text{SPE} = 0.5 \times 1 \times 40 - 0.5 \times 0.5 \times 20$

$= 15\,\text{J}$ [1 mark]

Work against gravity $= mgh$

$= 2 \times 9.8 \times 0.5$

$= 9.8\,\text{J}$ [1 mark]

$\therefore E_k = 15 - 9.8$

$= 5.2\,\text{J}$ [1 mark]

$E_k = \frac{1}{2}mv^2$

$5.2 = 0.5 \times 2 \times v^2$

$v = 2.28\,\text{m s}^{-1}$ [1 mark]

**4** $\frac{1}{2}mv_P^2 + mgh = \frac{1}{2}mv_Q^2$

$0.5 \times 4^2 + 9.8 \times 5 = 0.5 \times v_Q^2$ [1 mark]

$8 + 49 = 0.5v_Q^2$

$v_Q = 10.7\,\text{ms}^{-1}$ [1 mark]

**5** $m_1v_i + m2v_i = (m_1 + m_2)v_f$

$(4 \times 5) + (2 \times 2) = 6v_f$

$v_f = 4.0\,\text{m s}^{-1}$

$E_k(\text{initial}) = \frac{1}{2}m_1v_i^2 + \frac{1}{2}m_2v_i^2$

$= (0.5 \times 4.0 \times 5^2) + (0.5 \times 2.0 \times 2^2)$

$= 54\,\text{J}$ [1 mark]

$E_k(\text{final}) = \frac{1}{2}(m_1 + m_2)v_f^2$

$= 0.5 \times 6.0 \times 4^2$

$= 48\,\text{J}$ [1 mark]

Therefore, the collision is inelastic. [1 mark]

The most common error was to calculate the initial kinetic energy of the system but fail to calculate the final velocity, and therefore the final kinetic energy. A number of students interpreted the diagram as stating that the final velocity of the system was zero.

## 2.5 Review

## 2.5 Review questions

**1** $m_1 = m_2 = m$

Assign movement to the right as positive.

$\boldsymbol{p}_{\text{before}} = \boldsymbol{p}_{\text{after}}$

$m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_1\boldsymbol{v}_{f1} + m_2\boldsymbol{v}_{f2}$

Dividing by $m$ (because $m_1 = m_2 = m$)

$\boldsymbol{u}_1 + \boldsymbol{u}_2 = \boldsymbol{v}_{f1} + \boldsymbol{v}_{f2}$

$5 + \boldsymbol{u}_2 = 0.8 + \boldsymbol{v}_{f2}$

$\Rightarrow \boldsymbol{v}_{f2} = 4.2 + \boldsymbol{u}_2$

In an elastic collision, $E_k$ is conserved:

$E_{k_{\text{before}}} = E_{k_{\text{after}}}$

$\frac{1}{2}m_1\boldsymbol{u}_1^2 + \frac{1}{2}m_2\boldsymbol{u}_2^2 = \frac{1}{2}m_1\boldsymbol{v}_{f1}^2 + \frac{1}{2}m_2\boldsymbol{v}_{f2}^2$

$\boldsymbol{u}_1^2 + \boldsymbol{u}_2^2 = \boldsymbol{v}_{f1}^2 + \boldsymbol{v}_{f2}^2$

Substitute in $\boldsymbol{v}_{f2}$ from above:

$\boldsymbol{u}_1^2 + \boldsymbol{u}_2^2 = \boldsymbol{v}_{f1}^2 + \boldsymbol{v}_{f2}^2$

$5^2 + \boldsymbol{u}_2^2 = 0.8^2 + (4.2 + \boldsymbol{u}_2)^2$

$25 + \boldsymbol{u}_2^2 = 0.64 + (17.64 + 8.4\boldsymbol{u}_2 + \boldsymbol{u}_2^2)$

$8.4\,\boldsymbol{u}_2 = 6.72$

$\boldsymbol{u}_2 = 0.80\,\text{m s}^{-1}$ east

**2** $W = \boldsymbol{F}\Delta d\cos\theta$

$= (10 \times 9.8) \times 0.4\cos 0°$

$= 4 \times 10^1\,\text{J}$

**3 a** $W = \Delta E_g$
$= m\boldsymbol{g}\Delta h$
$= 2300 \times 9.8 \times 150$
$= 3.4 \times 10^6$ J

**b** $W = \boldsymbol{F}d$
$= m\boldsymbol{g} \sin\theta \times \frac{150}{\sin\theta}$
$= 2300 \times 9.8 \times 150$
$= 3.4 \times 10^6$ J

Note: This assumes that the only force acting is the component of the force due to gravity acting parallel to the slope.

**4 a** $E_k = \Delta E_g$
$= 60 \times 9.8 \times 30$
$= 17\,640$ J
$\approx 1.8 \times 10^4$ J

**b** At the instant that her head touches the water, her kinetic energy is zero. Therefore, all the gravitational potential energy lost has been transferred to strain potential energy in the cord:

Strain potential energy $= m\boldsymbol{g}\Delta h$
$= 60 \times 9.8 \times (50 - 1.70)$
$= 28\,400$ J
$\approx 2.8 \times 10^4$ J

**5 a** $\boldsymbol{p} = m\boldsymbol{v}$
$= 70 \times 2.0$ east
$= 1.4 \times 10^2$ kg m s$^{-1}$ east

**b** Three seconds before impact, Dean is 6.0 m from Melita because he is gliding at 2.0 m s$^{-1}$east. Taking Dean's position as the origin:

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$
$$= \frac{70 \times 0 + 50 \times 6}{120}$$
$$= 2.5 \text{ m}$$

The centre of mass is 2.5 m east of Dean.

**c** Before the collision, the centre of mass travels 3.5 m (from Dean to Melita) in 3.0 s:

$$\boldsymbol{v} = \frac{\Delta d}{\Delta t}$$
$$= \frac{3.5 \text{ m east}}{3.0 \text{ s}}$$
$$= 1.2 \text{ m s}^{-1} \text{ east}$$

**d** The momentum of the centre of mass remains constant. Therefore, the common velocity of Melita and Dean after the collision is 1.2 m s$^{-1}$east.

**e** Impulse on Melita = her change in momentum
$= \boldsymbol{p}_f - \boldsymbol{p}_i$
$= 70 \text{ kg} \times 1.2 \text{ m s}^{-1}$ east $- 0$
$= 84$ N s east

**6 a** Assume that the sum of the forces other than that of the seatbelt applied to the driver is zero.

$$W = \Delta E_k$$
$$= \frac{1}{2} \times 70 \times \left(\frac{60}{3.6}\right)^2$$
$$= 9.7 \times 10^3 \text{ J}$$

**b** $\boldsymbol{F}_{av}\, d = 9722$
$$\Rightarrow \boldsymbol{F}_{av} = \frac{9722}{0.70} = 1.4 \times 10^4 \text{ N}$$

**c** An estimate of the depression of the dashboard by the driver needs to be made. If the dashboard is depressed by 5 mm, then:

$$\boldsymbol{F}_{av} = \frac{9722}{0.005}$$
$$= 2 \times 10^6 \text{ N}$$

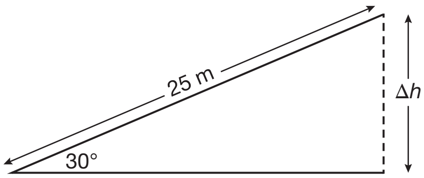
**7 a** Assume $m = 4500$ kg and $h = 10$ m: $E_g \sim 4.4 \times 10^5$ J

**b** Assume $m = 60$ kg and $h = 1.7$ m: $E_g \sim 1.0 \times 10^3$ J

**c** Assume: $m = 1.3$ kg and $h = 1.2$ m: $E_g \sim 15$ J

**d** Assume: $m = 0.058$ kg and $h = 3.0$ m: $E_g \sim 1.7$ J

**8**




**a** $W = \boldsymbol{F}d$ where $d$ = distance travelled in the direction of force:

$$W_{gravity} = m\boldsymbol{g}\Delta h$$
$$= 60 \times 9.8 \times 25 \times \sin 30°$$
$$= 7350 \text{ J}$$

**b**
$$\Delta E_k = W_{gravity}$$
$$\frac{1}{2}m\boldsymbol{v}^2 = W_{gravity} \text{ as initial } E_k \text{ is zero}$$
$$\frac{1}{2} \times 60\boldsymbol{v}^2 = 7350$$
$$\Rightarrow \boldsymbol{v} = 15.7 \text{ m s}^{-1}$$
$$\approx 16 \text{ m s}^{-1}$$

**c** $W_N = 0$ since there is no displacement in the direction of the force.

**d** Work done by net force = change in kinetic energy:

$$\boldsymbol{F}_{net}\, d = \Delta E_k$$
$$(m\boldsymbol{g} \sin\theta - \text{friction})\, d = \frac{1}{2}m\boldsymbol{v}^2$$
$$(60 \times 9.8 \sin 30° - \text{friction}) \times 25 = \frac{1}{2} \times 60 \times (7.2)^2$$
$$294 - \text{friction} = 62$$
$$\Rightarrow \text{friction} = 232 \text{ N}$$

**9 a** Assigning east as positive:

$$\boldsymbol{p}_i = 1500 \times (-20) + 2000 \times 20$$
$$= 10\,000 \text{ kg m s}^{-1}$$
$$\Rightarrow \boldsymbol{p}_f = \boldsymbol{p}_i$$
$$3500\,\boldsymbol{v} = 10\,000$$
$$\Rightarrow \boldsymbol{v} = 2.9 \text{ m s}^{-1} \text{ east}$$

**b** Impulse on truck = its change in momentum
$= \boldsymbol{p}_f - \boldsymbol{p}_i$
$= 2000 \times 2.86 - 2000 \times 20$
$= -3.4 \times 10^4$
$= 3.4 \times 10^4$ N s west

**c** $\Delta \boldsymbol{v}_{car} = \boldsymbol{v} - \boldsymbol{u}$
$= 2.9 - (-20)$
$= 23$ m s$^{-1}$

$\Delta \boldsymbol{v}_{\text{truck}} = \boldsymbol{v} - \boldsymbol{u}$
$= 2.9 - 20$
$= -17\ \text{m s}^{-1}$

The car experiences the greatest change of velocity, in magnitude.

**d** $\Delta \boldsymbol{p}_{\text{car}} = -\Delta \boldsymbol{p}_{\text{truck}}$ since the total change in momentum is zero.

This can be verified:

$\Delta \boldsymbol{p}_{\text{car}} = \boldsymbol{p}_{\text{f}} - \boldsymbol{p}_{\text{i}}$
$= 2000 \times 2.86 - 2000 \times 20$
$= -3.4 \times 10^4$
$= 3.4 \times 10^4\ \text{N s west}$

**e** Each vehicle experiences the same force (in magnitude) (Newton's Third Law of Motion).

**10** **a** $\Delta E_{\text{k}} = \frac{1}{2}m\boldsymbol{v}^2$
$= \frac{1}{2} \times 450 \times (2.0)^2$
$= 900\ \text{J}$

$\Rightarrow$ Maximum strain energy $= 900\ \text{J}$

$\frac{1}{2}kx^2 = 900\ \text{J}$

$\Rightarrow k =$ gradient of force–compression graph

$= \frac{300\ \text{kN}}{0.01\text{m}} = 3.0 \times 10^7\ \text{N m}^{-1}$

$\Rightarrow \frac{1}{2} \times 3.0 \times 10^7 \times x^2 = 900$

$\Rightarrow x = \sqrt{\frac{900 \times 2}{3.0 \times 10^7}} = 7.7 \times 10^{-3}\ \text{m}$

**b** Work done = strain energy
$= 900\ \text{J}$

**c** If the rubber bumper obeys Hooke's Law as it expands to its original shape, all the energy will be returned to the kinetic energy of the dodgem car, and the rebound speed will be $2.0\ \text{m s}^{-1}$.

## 2.5 Exam questions

### Section A — Multiple choice questions

**1** D. Spring constant is found from the gradient.

$k = \frac{\text{rise}}{\text{run}} = \frac{4 \times 10^3}{0.04}$
$k = 1.0 \times 10^5\ \text{N m}^{-1}$

**2** C. Stored potential energy is found using the area under the graph.

$E = \frac{1}{2}bh$
$E = 0.5 \times 0.02 \times \left(2.0 \times 10^3\right)$
$E = 20\ \text{J}$

**3** B

$m_{\text{i}}v_{\text{i}} = m_{\text{f}}v_{\text{f}}$
$(10\,000 \times 3.0) + (y \times 0) = (10\,000 + Y) \times 2.0$
$\Rightarrow Y = 5000\ \text{kg}$

**4** C. As both blocks have the same acceleration, they both have the same force to mass ratio.

**5** A

$W = Fd$
$W = 250 \times 20$
$W = 5\ \text{kN}$

**6** C

$Ft = mv$
$F \times 1 \times 10^{-3} = 0.040 \times 50$
$F = 2 \times 10^3\ \text{N}$

**7** C

$\sum p_{\text{initial}} = \sum p_{\text{final}}$
$10 \times 6.0 + 5.0 \times 0 = 15 \times \boldsymbol{v}$
$\boldsymbol{v} = \frac{60}{15}$
$= 4.0\ \text{m s}^{-1}$

**8** B. Momentum is conserved in all collisions, but kinetic energy is lost in this collision:

initial $E_{\text{k}} = \frac{1}{2}m\boldsymbol{v}^2$
$= \frac{1}{2} \times 10 \times 6^2$
$= 180\ \text{kJ}$

final $E_{\text{k}} = \frac{1}{2}m\boldsymbol{v}^2$
$= \frac{1}{2} \times 15 \times 4^2$
$= 120\ \text{kJ}$

**9** C. Impulse $= F\Delta t = 4.0 \times 5.0 = 20\ \text{N s}$

**10** B. Initial KE of car = work done to compress the spring = area under the F–x graph

$KE = \text{area under graph} = \frac{1}{2} \times F_{\text{max}}\, x = \frac{200 \times 0.5}{2} = 50\ \text{J}$

### Section B — Short answer questions

**11** $20\,000\ \text{km h}^{-1} = 5.56 \times 10^3\ \text{m s}^{-1}$

$E = \frac{1}{2}m\boldsymbol{v}^2$
$E = 0.5 \times 1000 \times \left(5.56 \times 10^3\right)^2$
$E = 1.54 \times 10^{10}\ \text{J}$

**12** $Ek_{\text{before}} = 0.5 \times 0.5 \times 45^2$

$Ek_{\text{before}} = 506\ \text{J}$ [1 mark]

$Ek_{\text{after}} = 0.5 \times 0.5 \times 40^2 + 0.5 \times 0.04 \times 63^2$

$Ek_{\text{after}} = 480\ \text{J}$ [1 mark]

The decrease in kinetic energy indicates an inelastic collision. [1 mark]

**13** **a** $\frac{1}{2}kx^2 = \frac{1}{2}m\boldsymbol{v}^2$ [1 mark]

$0.5 \times 1250 \times 0.15^2 = 0.5 \times 0.20 \times \boldsymbol{v}^2$
$14.06 = 0.1\boldsymbol{v}^2$

$\boldsymbol{v} = \sqrt{\frac{14.06}{0.1}}$ [1 mark]

$\boldsymbol{v} = 12\ \text{m s}^{-1}$

***VCAA examination report* note:**

Students need to be aware that where a question asks them to show that the result is a value given in the stem ($12\ \text{m s}^{-1}$ in this case), there are no marks awarded for the final value. In these cases marks are only awarded for showing working. Students who did not show adequate working (i.e. the equivalence of SPE and KE) were not awarded full marks.

**b** Vertical component:

The impact velocity is the Pythagorean sum of the horizontal and vertical velocities. [1 mark]

$\boldsymbol{v}^2 = u^2 + 2ax$

$\boldsymbol{v}^2 = 0^2 + 2(-9.8)(-2.5)$

$\boldsymbol{v} = \sqrt{784}$

$\boldsymbol{v} = 28\,\text{m s}^{-1}$ [1 mark]

This problem can also be solved by using conservation of energy.

$c^2 = a^2 + b^2$

$\boldsymbol{v}^2 = 12^2 + 7^2$

$\boldsymbol{v} = \sqrt{193}$

$\boldsymbol{v} = 14\,\text{m s}^{-1}$ [1 mark]

***VCAA examination report* note:**

The most common error was to calculate the vertical component only.

**14 a** Applying conservation of momentum:

$p_i = p_f$ [1 mark]

$1200 \times 10 = 2200 \times 6.5 + 1200 \times v_f$ [1 mark]

$12\,000 = 14\,300 + 1200v_f$

$v_f = \dfrac{-2300}{1200}$

$v_f = -1.9\,\text{m s}^{-1}$ [1 mark]

The car would be travelling at 1.9 m $\text{s}^{-1}$ to the left. [1 mark]

***VCAA examination report* note:**

Students were then required to indicate that the negative sign indicates the velocity is to the left.

Some students used compass bearings (east/west) for directions. Unless the student draws a compass rose to indicate what they mean by east and west this cannot be accepted.

**b** Before collision:

$E_k = \dfrac{1}{2}m\boldsymbol{v}^2$

$= 0.5 \times 1200 \times 10^2$

$= 6.0 \times 10^4\,\text{J}$ [1 mark]

After collision:

$E_k = \dfrac{1}{2}m_1v_1^2 + \dfrac{1}{2}m_2v_2^2$

$= \left(0.5 \times 1200 \times 1.9^2\right) + \left(0.5 \times 2200 \times 6.5^2\right)$

$= 4.9 \times 10^4\,\text{J}$ [1 mark]

The reduction in kinetic energy indicates that the collision is inelastic. [1 mark]

***VCAA examination report* note:**

Most students recognised the collision was inelastic. Common errors occurred with the calculation of the initial and final energies. Of these, it was the final that was most commonly calculated incorrectly, with students either using $-1.9\,\text{m s}^{-1}$ or omitting the energy of one of the vehicles altogether.

**c i** $F_{avg} = \dfrac{\Delta p}{\Delta t}$ [1 mark]

$= \dfrac{m\Delta v}{\Delta t}$ [1 mark]

$= \dfrac{2200 \times 6.5}{40 \times 10^{-3}}$

$= 358\,\text{kN}$ to the right [1 mark]

As the van's change in velocity is to the right, the force acts to the right.

***VCAA examination report* note:**

Many students were able to identify the direction correctly but unable to calculate the actual value. At this level, students should be able to convert between SI prefixes, but many could not convert a correctly calculated value in Newtons into kilo-Newtons.

**ii** $F_{avg} = \dfrac{\Delta p}{\Delta t}$

$= \dfrac{m\Delta v}{\Delta t}$

$= \dfrac{1200 \times (-1.9 - 10)}{40 \times 10^{-3}}$

$= -358\,\text{kN}$ [1 mark]

As the car's change in velocity is to the left, the force acts to the left.

$F_{avg} = -358\,\text{kN}$ to the left [1 mark]

***VCAA examination report* note:**

Students were not required to perform the calculation. Those who indicated that this was an example of Newton's third law and, therefore, that the force would have the same magnitude but opposite direction were also awarded full marks.

There was a small number of students who determined that a new calculation was required and then demonstrated incorrect physics.

**15** The gravitational potential energy of the car at point A on the track is equal to the gravitational potential energy and kinetic energy at point B on the track. Note that the toy car has no kinetic energy at point A because it starts from rest.

$\text{GPE}_\text{A} = \text{GPE}_\text{B} + \text{KE}_\text{B}$

$mgh_\text{A} = mgh_\text{B} + \dfrac{1}{2}mv_\text{B}^2$

$(0.25 \times 9.8 \times h_\text{A}) = (0.25 \times 9.8 \times 0.40) + \left(\dfrac{1}{2} \times 0.25 \times 3^2\right)$ [1 mark]

$2.45h_\text{A} = 0.98 + 1.125$ [1 mark]

$h_\text{A} = \dfrac{2.105}{2.45}$

$h_\text{A} = 0.86\,\text{m}$ [1 mark]

# Topic 3 — Gravitational fields and their applications

## 3.2 Newton's Universal Law of Gravitation and the inverse square law

### Sample problem 1

**a** Assume that the direction towards Earth is positive.

$$\boldsymbol{F}_{\text{on person by Earth}} = G\frac{m_{\text{Earth}} m_{\text{person}}}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 70}{\left(6.37 \times 10^6\right)^2}$$
$$= 6.9 \times 10^2 \text{ N}$$

The force due to gravity of Earth on a 70-kg person standing on the equator is $6.9 \times 10^2$ N towards the centre of Earth.

**b** $\boldsymbol{F}_{\text{on Earth by the person}} = -\boldsymbol{F}_{\text{on person by Earth}}$
$$= -6.9 \times 10^2 \text{ N}$$

The negative value means that the direction of the force is in the opposite direction to the force on the person by Earth. The force due to gravity of a 70-kg person standing on Earth's equator is $6.9 \times 10^2$ N towards the person.

### Practice problem 1

Assume that the direction towards Earth is positive.

$$\boldsymbol{F}_{\text{on Moon by Earth}} = \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 5.98 \times 10^{24}}{\left(3.844 \times 10^8\right)^2}$$
$$= 1.98 \times 10^{20} \text{ N towards Earth}$$

$$\boldsymbol{F}_{\text{on Earth by the Moon}} = -\boldsymbol{F}_{\text{on Moon by Earth}}$$
$$= -1.98 \times 10^{20} \text{ N}$$
$$= 1.98 \times 10^{20} \text{ N towards Moon}$$

(Note that students may write this as a magnitude and direction rather than using the negative force sign. It is important that the direction of the force is clearly stated.)

### 3.2 Exercise

**1** For planet Mars:

$$F_g = G\frac{m_1 m_2}{r^2}$$
$$= 6.67 \times 10^{-11} \times \frac{6.39 \times 10^{23} \times 70.0}{\left(3.39 \times 10^6\right)^2}$$
$$= 6.67 \times \frac{6.39 \times 70.0}{3.39^2}$$
$$= 2.60 \times 10^2 \text{ N} \quad \text{to 3 s.f.}$$

For planet Jupiter:

$$F_g = G\frac{m_1 m_2}{r^2}$$
$$= 6.67 \times 10^{-11} \times \frac{1.90 \times 10^{27} \times 70.0}{\left(6.69 \times 10^7\right)^2}$$
$$= 6.67 \times \frac{1.90 \times 70.0}{6.69^2} \times 10^2$$
$$= 1.98 \times 10^3 \text{ N} \quad \text{to 3 s.f.}$$

**2** The force of attraction between Earth and the Sun is the force due to gravity. Its magnitude is:

$$F_g = G\frac{m_1 m_2}{r^2}$$
$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24} \times 1.99 \times 10^{30}}{\left(1.49 \times 10^{11}\right)^2}$$
$$= 6.67 \times \frac{5.97 \times 1.99}{1.49^2} \times 10^{21}$$
$$= 3.60 \times 10^{22} \text{ N} \quad \text{to 3 s.f.}$$

**3** The force of Earth's gravitational attraction on an individual on the surface of Earth decreases with the square of the distance to the centre of Earth. Thus, if the radius of Earth is multiplied by 3 without changes to its mass, then the magnitude of the force of attraction is divided by factor of $3^2 = 9$. Therefore, the magnitude of the force due to gravity would be $\frac{1}{9}$ of the initial force.

**4** Let $m$ be the mass of the ball.

$$F_{\text{g Mercury}} = G\frac{m_{\text{Mercury}} m}{r^2_{\text{Mercury}}}$$
$$F_{\text{g Earth}} = G\frac{m_{\text{Earth}} m}{r^2_{\text{Earth}}}$$
$$\Rightarrow \frac{F_{\text{g Mercury}}}{F_{\text{Earth}}} = \frac{m_{\text{Mercury}}}{m_{\text{Earth}}}\left(\frac{r_{\text{Earth}}}{r_{\text{Mercury}}}\right)^2$$
$$= \frac{3.29 \times 10^{23}}{5.97 \times 10^{24}}\left(\frac{6.37 \times 10^6}{2.44 \times 10^6}\right)^2$$
$$= \frac{3.29}{5.97} \times \left(\frac{6.37}{2.44}\right)^2 \times 10^{-1}$$
$$= 3.76 \times 10^{-1}$$

**5** **a** $S_1$ is at $2\,r_E$ from the centre of Earth and $S_2$ is at $3\,r_E$ from the centre of Earth and $\frac{m_{S_1}}{m_{S_2}} = \frac{1}{2}$

$$F_{gs_1} = \frac{Gm_{\text{Earth}} m_{S_1}}{2r_E^2} \quad \text{and } F_{gs_2} = \frac{Gm_{\text{Earth}} m_{S_2}}{3r_E^2}$$

Thus:

$$\frac{F_{gs_1}}{F_{gs_2}} = \frac{m_{S_1}}{(2r_E)^2} \div \frac{m_{S_2}}{(3r_E)^2}$$
$$= \frac{1}{2}\left(\frac{3r_E}{2r_E}\right)^2$$
$$= \frac{9}{8}$$
$$= 1.125$$

Alternatively, the gravitational field strength decreases as the inverse square of the distance and as $S_2$ is 1.5 times further away than $S_1$, the gravitational field strength decreases by $1.5^2 = 2.25$. Thus $\frac{g_{S_1}}{g_{S_2}} = 2.25$.

In addition, the magnitude of the force due to gravity is proportional to the mass of the satellite and the gravitational field strength. Given that $\frac{m_{S_1}}{m_{S_2}} = \frac{1}{2}$ and $\frac{g_{S_1}}{g_{S_2}} = 2.25$, then $\frac{F_{gs_1}}{F_{gs_2}} = 1.125$

**b** Let $S_2$ be at $x\,r_E$ from the centre of Earth.

$$F_{gs_1} = \frac{Gm_{\text{Earth}} m_{S_1}}{2r_E^2} \quad \text{and } F_{gs_2} = \frac{Gm_{\text{Earth}} m_{S_2}}{3r_E^2}$$

Thus:

$$\frac{F_{gs_1}}{F_{gs_2}} = \frac{m_{S_1}}{(2r_E)^2} \div \frac{m_{S_2}}{(xr_E)^2}$$
$$= \frac{1}{2}\left(\frac{xr_E}{2r_E}\right)^2$$
$$= \frac{x^2}{8}$$

Therefore, for $\frac{F_{gs_1}}{F_{gs_2}}$ to equal 2, $\frac{x^2}{8}$ must equal 2.

Thus $x = 4$ and $S_2$ should orbit at 4 $r_E$ from the centre of Earth, or 3 $r_E$ above Earth's surface.

Alternatively,

$$\frac{F_{gs_1}}{F_{gs_2}} = 2$$
$$\Rightarrow \frac{m_{S_1} g_{S_1}}{m_{S_2} g_{S_2}} = 2$$
$$\frac{g_{S_1}}{g_{S_2}} = 4$$

The gravitational field strength decreases as the inverse square of the distance, so for the gravitational field strength at $S_1$ to be $4 = 2^2$ times the gravitational field strength at $S_2$, the distance from $S_2$ to the centre of Earth has to be twice the distance from $S_1$ to the centre of Earth. $S_1$ is 2 $r_E$ from the centre of Earth, thus $S_2$ is 4 $r_E$ from the centre of Earth, and thus orbits at 3 $r_E$ above Earth's surface.

**6** Let the object be at $xr_E$ from the centre of Earth.
On the Moon's surface,

$$F_{\text{on object by the Moon}} = G\frac{m_{\text{Moon}} \times m_{\text{object}}}{r^2_{\text{Moon}}}$$

At a distance $xr_E$ from the centre of Earth,

$$F_{\text{on object by Earth}} = G\frac{m_{\text{Earth}} \times m_{\text{object}}}{(xr_E)^2}.$$

Equating the magnitude of the two forces results in

$$\frac{m_{\text{Moon}}}{r^2_{\text{Moon}}} = \frac{m_{\text{Earth}}}{(xr_E)^2}.$$
$$\frac{m_{\text{Moon}}}{r^2_{\text{Moon}}} = \frac{m_{\text{Earth}}}{(xr_E)^2}$$
$$\Rightarrow x^2 = \frac{m_{\text{Earth}}}{m_{\text{Moon}}}\left(\frac{r_{\text{Moon}}}{m_{\text{Earth}}}\right)^2$$
$$= \frac{5.97 \times 10^{24}}{7.35 \times 10^{22}} \times \left(\frac{1.74 \times 10^6}{6.37 \times 10^6}\right)^2$$
$$= \frac{5.97 \times 1.74^2}{7.35 \times 6.37^2} \times 10^2$$
$$= 6.06$$
$$\Rightarrow x = 2.46 \quad \text{to 3 s.f.}$$

The object must be placed 2.46 $r_E$ from the centre of Earth.

## 3.2 Exam questions

**1** C

$$F = \frac{Gm_1m_2}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 1.0 \times 100}{0.1^2}$$
$$= 6.67 \times 10^{-7}\ \text{N}$$

**2** A. A person's perception of 'weight' is determined by the forces on their body by the surfaces that they are in contact with.

**3** D. The gravitational force is equal to $m\boldsymbol{g}$, which, in the absence of any other forces on the masses, equals $m\boldsymbol{a}$, so the gravitational force on mass A is 1000 times greater than the gravitational force on mass B, but the acceleration experienced by each mass equals $\boldsymbol{g}$.

**4** $F_g = G\frac{m_1m_2}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 300}{\left(10^7 + 6.37 \times 10^6\right)^2} \quad \text{[1 mark]}$$
$$= \frac{6.67 \times 5.97 \times 3}{16.37^2} \times 10^3$$
$$= 4.46 \times 10^2\ \text{N} \quad \text{[1 mark]}$$

**5** $F_g = G\frac{m_1m_2}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 5 \times 10^{-2}}{\left(6 \times 10^3 + 6.37 \times 10^6\right)^2} \quad \text{[1 mark]}$$
$$= \frac{6.67 \times 5.97 \times 5}{6.376^2} \times 10^{-1}$$
$$= 4.90 \times 10^{-1}\ \text{N} \quad \text{[1 mark]}$$

## 3.3 The field model

### Sample problem 2

$$g = G\frac{M}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{\left(1.74 \times 10^6 \text{m}\right)^2}$$
$$= 1.62\ \text{N kg}^{-1}$$

### Practice problem 2

$$g = \frac{GM}{r^2}$$
$$g = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{\left(149.6 \times 10^9\right)^2}$$
$$= 5.93 \times 10^{-3}\ \text{N kg}^{-1}$$

### Sample problem 3

Take towards the Moon as the positive direction.

$$g = -G\frac{M_{Earth}}{r^2} + G\frac{M_{Moon}}{r^2}$$
$$= -6.67 \times 10^{-11}\frac{5.97 \times 10^{24}}{\left(1.922 \times 10^8\right)^2} + 6.67 \times 10^{-11}\frac{7.35 \times 10^{22}}{\left(1.922 \times 10^8\right)^2}$$
$$= -1.06 \times 10^2\ \text{N kg}^{-1} \text{ to three significant figures}$$

The negative value for $g$ means that at the point halfway between Earth and the Moon, the direction of the overall gravitational field is towards Earth.

### Practice problem 3

$$g = -G\frac{M_{Sun}}{r^2_{Sun-Mercury}} + G\frac{M_{Venus}}{r^2_{Venus-Mercury}}$$
$$g = -6.67 \times 10^{-11}\frac{1.99 \times 10^{30}}{\left(5.79 \times 10^{10}\right)^2} + 6.67 \times 10^{-11}\frac{4.87 \times 10^{24}}{\left(5.01 \times 10^{10}\right)^2}$$
$$g = \underbrace{3.96 \times 10^{-2}\ \text{N kg}^{-1}}_{\text{from the Sun}} + \underbrace{1.3 \times 10^{-7}\ \text{N kg}^{-1}}_{\text{from Venus}}$$

The contribution from Venus represents $\frac{1.3 \times 10^{-7}}{3.96 \times 10^{-2}} \approx 3 \times 10^{-6}$

The effect of Venus' gravitational field on Mercury is extremely small.

## 3.3 Exercise

**1 a** $m_{ball} = 1.50\text{ kg}$;

$r_{ball} = (6.37 \times 10^6\text{ m} + 2.00\text{ m}) = 6.37 \times 10^6\text{ m}$

$$g = G\frac{m_E}{r_{ball}^2}$$
$$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6)^2}$$
$$= \frac{6.67 \times 5.97}{6.37^2} \times 10^1$$
$$= 9.81\text{ N kg}^{-1} \quad \text{to 3 s.f.}$$

**b** The only force acting on the ball is the force due to gravity. This means that $\boldsymbol{F}_{net} = \boldsymbol{F}_g$.

$$\Rightarrow F_{net} = F_g$$
$$= mg$$
$$= 1.50 \times 9.81$$
$$= 14.7\text{ N} \quad \text{to 3 s.f.}$$

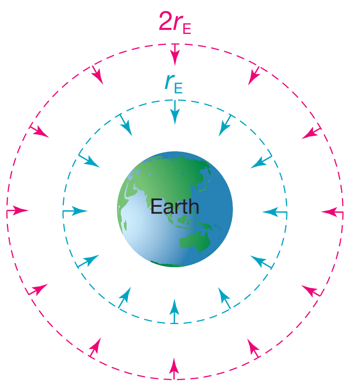
**2 a** The object $2r_E$ above the surface is $3r_E$ from the centre of Earth.

The object $1r_E$ above the surface is $2r_E$ from the centre of Earth.

The ratio of the magnitudes of the forces experienced is

$$\frac{F_{2r_E}}{F_{3r_E}} = \frac{3r_E^2}{2r_E^2} = \frac{9}{4} = 2.25$$

**b**




The direction of the force is towards the centre of Earth in both cases.

The ratio of the forces experienced is $\frac{9}{4}$, which should be reflected in the relative length of the force arrows.

**c** A uniform field is a field that has the same magnitude and same direction everywhere in a given space. As illustrated in the sketch, Earth's gravitational field is weakening as the distance to Earth increases thus it is not uniform.

**3** See table at the bottom of the page*

*3

| Planet/dwarf planet | Mass (kg) | Radius (m) | $g$ (N kg$^{-1}$) |
|---|---|---|---|
| Earth | $5.97 \times 10^{24}$ | $6.37 \times 10^6$ | $g_E = \frac{Gm_E}{r_E^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6)^2} = \frac{6.67 \times 5.97}{6.37^2} \times 10^1 = 9.81$ to 3 s.f. |
| Mars | $6.39 \times 10^{23}$ | $3.39 \times 10^6$ | $g_M = \frac{Gm_M}{r_M^2} = \frac{6.67 \times 10^{-11} \times 6.39 \times 10^{23}}{(3.39 \times 10^6)^2} = \frac{6.67 \times 6.39}{3.39^2} = 3.71$ to 3 s.f. |
| Venus | $4.87 \times 10^{24}$ | $6.05 \times 10^6$ | $g_V = \frac{Gm_V}{r_V^2} = \frac{6.67 \times 10^{-11} \times 4.87 \times 10^{24}}{(6.05 \times 10^6)^2} = \frac{6.67 \times 6.39}{3.39^2} \times 10^1 = 8.87$ to 3 s.f. |
| Pluto | $1.31 \times 10^{22}$ | $1.19 \times 10^6$ | $g_P = \frac{Gm_P}{r_P^2} = \frac{6.67 \times 10^{-11} \times 1.31 \times 10^{22}}{(1.19 \times 10^6)^2} = \frac{6.67 \times 1.31}{1.19^2} \times 10^{-1} = 0.617$ to 3 s.f. |

**4** The gravitational field lines point inwards because masses are only attracted to other masses, and gravity is always attractive, thus an isolated mass will experience an attractive force towards its own centre, as it is the source of the gravitational field.

**5 a**

| $d$ (km) | $g\ (\text{N kg}^{-1})$ |
|---|---|
| 10 000 | $g_E = \frac{Gm_E}{d^2}$ <br> $= \frac{6.67\times10^{-11}\times5.97\times10^{24}}{(10^7)^2}$ <br> $= 6.67\times5.97\times10^{-1}$ <br> $= 3.98$ to 3 s.f. |
| 20 000 | $g_E = \frac{Gm_E}{d^2}$ <br> $= \frac{6.67\times10^{-11}\times5.97\times10^{24}}{(2\times10^7)^2}$ <br> $= \frac{6.67\times5.97}{4}\times10^{-1}$ <br> $= 0.995$ to 3 s.f. |
| 30 000 | $g_E = \frac{Gm_E}{d^2}$ <br> $= \frac{6.67\times10^{-11}\times5.97\times10^{24}}{(3\times10^7)^2}$ <br> $= \frac{6.67\times5.97}{9}\times10^{-1}$ <br> $= 4.42\times10^{-2}$ to 3 s.f. |

**b i** $$\frac{\text{Field strength at 20 000 km}}{\text{Field strength at 10 000 km}} = \frac{Gm_E}{(2d)^2}\times\frac{d^2}{Gm_E} = \frac{1}{4}$$

**ii** $$\frac{\text{Field strength at 30 000 km}}{\text{Field strength at 10 000 km}} = \frac{Gm_E}{(3d)^2}\times\frac{d^2}{Gm_E} = \frac{1}{9}$$

**iii** $$\frac{\text{Field strength at 30 000 km}}{\text{Field strength at 20 000 km}} = \frac{Gm_E}{(3d)^2}\times\frac{(2d)^2}{Gm_E} = \frac{4}{9}$$

**c** As the distance from the centre of the mass is doubled (from 10 000 km to 20 000 km), the field strength is divided by $2^2 = 4$, as the distance from the centre of the mass is tripled (from 10 000 km to 30 000 km), the field strength is divided by $3^2 = 9$ and as the distance from the centre of the mass is multiplied by $\frac{3}{2}$ (from 20 000 km to 30 000 km), the field strength is divided by $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$, this demonstrates the inverse square law relationship.

**6 a** $m = 10.0$ kg and $g = 9.73\ \text{N kg}^{-1}$

$$F_g = mg = 10.0\times9.3 = 97.3\ \text{N}$$

**b** Consider up to be positive. The force due to gravity is downward. The detector to remain stationary, thus $\boldsymbol{F}_{net} = \boldsymbol{0}$.

$$\boldsymbol{F}_{net} = \boldsymbol{F}_g + \boldsymbol{F}_{up}$$
$$0 = -F_g + F_{up}$$
$$F_{up} = 97.3\ \text{N}$$

**c** Let $r$ be the distance of the detector to the centre of Earth.

$$g = 9.73\ \text{N kg}^{-1}$$
$$g = \frac{Gm_E}{r^2}$$
$$\Rightarrow r = \sqrt{\frac{Gm_E}{g}} = \sqrt{\frac{6.67\times10^{-11}\times5.97\times10^{24}}{9.73}} = \sqrt{\frac{6.67\times5.97\times10}{9.73}\times10^6}$$
$$= 6.40\times10^6\ \text{m} \quad \text{to 3 s.f.}$$

**7 a**
$$g_S = \frac{Gm_S}{r^2_{S-M}} = \frac{6.67\times10^{-11}\times1.989\times10^{30}}{(1.50\times10^{11})^2} = \frac{6.67\times1.989}{1.50^2}\times10^{-3}$$
$$= 5.90\times10^{-3}\ \text{N kg}^{-1} \quad \text{to 3 s.f.}$$
$$g_E = \frac{Gm_E}{r^2_{E-M}} = \frac{6.67\times10^{-11}\times5.97\times10^{24}}{(3.84\times10^8)^2} = \frac{6.67\times5.97}{3.84^2}\times10^{-3}$$
$$= 2.70\times10^{-3}\ \text{N kg}^{-1} \quad \text{to 3 s.f.}$$

**b** $\frac{g_S}{g_M} = 2.11$

The gravitational field strength experienced by the Moon from the Sun is approximately twice that of gravitational field strength experienced by the Moon from Earth.

It may seem surprising that the gravitational field strength of the Sun on the Moon more than doubled the gravitational field strength of Earth on the Moon, given that the Moon orbits Earth. However in fact, both the Moon and Earth orbit the Sun. The relatively large field strength of Earth and the relatively small mass of the Moon means that the Moon also orbits Earth while both bodies orbit the Sun.

**8 a** Let $x$ be the distance, in metres, between the spacecraft and the centre of Earth, and $3.84\times10^8 - x$ be the distance between the spacecraft and the centre of the Moon. The gravitational force $\boldsymbol{F}_{\text{on spacecraft by Earth}}$ is directed towards the centre of Earth while the gravitational force $\boldsymbol{F}_{\text{on spacecraft by the Moon}}$ is directed towards the centre of the Moon. The spacecraft experiences a net force of zero when $\boldsymbol{F}_{net} = \boldsymbol{0}$.

Taking towards the centre of Earth as the positive direction:

$$\boldsymbol{F}_{net} = 0$$
$$\boldsymbol{F}_{\text{on spacecraft by Earth}} + \boldsymbol{F}_{\text{on spacecraft by the moon}} = 0$$
$$F_{\text{on spacecraft by Earth}} - F_{\text{on spacecraft by the moon}} = 0$$

Thus,

$$F_{\text{on spacecraft by Earth}} = F_{\text{on spacecraft by the moon}}$$
$$\frac{Gm_E}{x^2} = \frac{Gm_M}{(3.84\times10^8 - x)^2}$$
$$\frac{5.97\times10^{24}}{x^2} = \frac{7.35\times10^{22}}{(3.84\times10^8 - x)^2}$$
$$5.97\times10^2(3.84\times10^8 - x)^2 = 7.35x^2$$

We have to solve the following quadratic equation:

$$\underbrace{(7.35 - 5.97 \times 10^2)}_{a}x^2 + \underbrace{(5.97 \times 10^2 \times 3.84 \times 10^8)}_{b}x$$

$$+\underbrace{5.97 \times 10^2 \times 3.84^2 \times 10^{16}}_{c} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = 3.46 \times 10^8$ m  to 3 s.f.

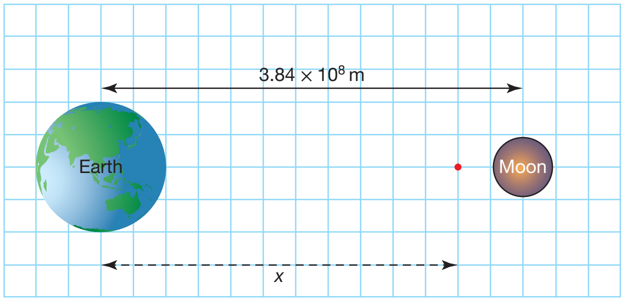
or

$x = 4.19 \times 10^8$ m  to 3 s.f.

As the spaceship must be between Earth and the Moon, the distance must be less than the distance from Earth to the Moon. Therefore the correct distance is $x = 3.46 \times 10^8$ m from the centre of Earth.

**b** $$\frac{\text{Distance spacecraft-Earth}}{\text{Distance Earth–Moon}} = \frac{3.46 \times 10^8}{3.84 \times 10^8}$$
$$\simeq 90\%$$

The spacecraft has travelled approximatively 90% of the distance Earth–Moon when the net force it experienced is zero.




**9 a** To calculate the gravitational field strength due to the Sun at the red and orange points, the distances from the Sun to these points must be determined.

The average distance from the centre of the Sun to the red dot is:

$d_{\text{S-r d}} = 1.4742 \times 10^{11} - 6.3781 \times 10^6$ km

The average distance from the centre of the Sun to the orange dot is:

$d_{\text{S-o d}} = 1.4742 \times 10^{11} + 6.3781 \times 10^6$ km

To calculate the gravitational field strength due to the Moon at the red and orange points, the distances from the Moon to these points must be determined.

The average distance from the centre of the Moon to the red dot is:

$d_{\text{M-r d}} = 3.84400 \times 10^8 - 6.3781 \times 10^6$ km

The average distance from the centre of the Moon to the orange dot is:

$d_{\text{M-o d}} = 3.84400 \times 10^8 + 6.3781 \times 10^6$ km

With these distance, and the mass of the Sun and the mass of the Moon, the gravitational field strength of the Sun and of the Moon on the different sides of Earth can then be calculated.

Taking towards the centre of the Sun be the positive direction:

- in Scenario **(a)**, the Moon and the Sun are on opposite sides of Earth, thus $\boldsymbol{g}_c = \boldsymbol{g}_S + \boldsymbol{g}_M$
  $g_c = g_S - g_M$
- in Scenario **(b)**, the Moon and the Sun are on the same side of Earth, thus $\boldsymbol{g}_c = \boldsymbol{g}_S + \boldsymbol{g}_M$
  $g_c = g_S + g_M$

See table at the bottom of the page*

***9 a**

| Scenario | dot | $g_c\ (\text{N kg}^{-1})$ |
|---|---|---|
| **(a)** | red | $g_C = \frac{Gm_S}{d^2_{\text{S-r d}}} - \frac{Gm_M}{d^2_{\text{M-r d}}}$ <br> $= 6.6743 \times 10^{-11}\left(\frac{1.98847 \times 10^{30}}{(1.4742 \times 10^{11} - 6.3781 \times 10^6)^2} - \frac{7.3460 \times 10^{22}}{(3.84400 \times 10^8 - 6.3781 \times 10^6)^2}\right)$ <br> $= 6.1104 \times 10^{-3}$ to 5 s.f. |
| **(a)** | orange | $g_C = \frac{Gm_S}{d^2_{\text{S-a d}}} - \frac{Gm_M}{d^2_{\text{M-a d}}}$ <br> $= 6.6743 \times 10^{-11}\left(\frac{1.98847 \times 10^{30}}{(1.4742 \times 10^{11} + 6.3781 \times 10^6)^2} - \frac{7.3460 \times 10^{22}}{(3.84400 \times 10^8 + 6.3781 \times 10^6)^2}\right)$ <br> $= 6.1093 \times 10^{-3}$ to 5 s.f. |
| **(b)** | red | $g_C = \frac{Gm_S}{d^2_{\text{S-r d}}} + \frac{Gm_M}{d^2_{\text{M-r d}}}$ <br> $= 6.6743 \times 10^{-11}\left(\frac{1.98847 \times 10^{30}}{(1.4742 \times 10^{11} - 6.3781 \times 10^6)^2} + \frac{7.3460 \times 10^{22}}{(3.84400 \times 10^8 - 6.3781 \times 10^6)^2}\right)$ <br> $= 6.1104 \times 10^{-3}$ to 5 s.f. |
| **(b)** | orange | $g_C = \frac{Gm_S}{d^2_{\text{S-o d}}} + \frac{Gm_M}{d^2_{\text{M-o d}}}$ <br> $= 6.6743 \times 10^{-11}\left(\frac{1.98847 \times 10^{30}}{(1.4742 \times 10^{11} + 6.3781 \times 10^6)^2} + \frac{7.3460 \times 10^{22}}{(3.84400 \times 10^8 + 6.3781 \times 10^6)^2}\right)$ <br> $= 6.1093 \times 10^{-3}$ to 5 s.f. |

**b** At the red dot location, $\dfrac{g_c \text{ in scenario (a)}}{g_c \text{ in scenario (b)}} \simeq 1$

At the orange dot location, $\dfrac{g_c \text{ in scenario (a)}}{g_c \text{ in scenario (b)}} \simeq 1$

For both locations, at the red dot or at the orange dot, having the Moon and the Sun aligned on the same side of Earth (scenario **(b)**), or on opposite sides of Earth (scenario **(a)**) does not impact significantly the combined gravitational strength of the Sun and the Moon. Thus Meredith's hypothesis is not supported by these results.

In both scenarios, the combined gravitational field strength at the red dot location is approximately the same as the combined gravitational field strength at the orange dot location (less than 1% of difference). Thus Julian's hypothesis is supported by these results.

## 3.3 Exam questions

**1** A. Let $m_S$ and $r_S$ be the mass and the radius of the Sun respectively, and $m_J$ and $r_S$ be the mass and the radius of Jupiter respectively.

$$m_J = \frac{m_S}{10^3}$$
$$r_J = \frac{r_S}{10}$$
$$r_J = \frac{r_S}{10}$$
$$g_S = G\frac{m_S}{r_S^2}$$
$$g_J = G\frac{m_J}{r_J^2}$$
$$= G\frac{m_S \times 10^{-3}}{\left(r_S \times 10^{-1}\right)^2}$$
$$= G\frac{m_S}{r_S^2} \times 10^{-1}$$
$$= \frac{g_S}{10}$$
$$\Rightarrow \frac{g_J}{g_S} = \frac{1}{10}$$

**2** $$g = \frac{GM}{r^2}$$
$$\Rightarrow M = \frac{gr^2}{G}$$
$$\frac{1.31}{\left(1.56 \times 10^6\right)^2} 6.67 \times 10^{-11} \quad \text{[1 mark]}$$
$$= \frac{1.31 \times 1.56^2}{6.67} \times 10^{23}$$
$$= 4.78 \times 10^{22}\,\text{kg} \quad \text{[1 mark]}$$

**3** $$g = \frac{GM}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 10^{12}}{1400^2} \quad \text{[1 mark]}$$
$$= \frac{6.67}{1.4^2} \times 10^{-5}$$
$$= 3.40 \times 10^{-5}\,\text{N kg}^{-1} \quad \text{[1 mark]}$$

**4** $$\frac{g_2}{g_1} = \left(\frac{r_1}{r_2}\right)^2$$
$$g_2 = g_1\left(\frac{r_1}{r_2}\right)^2$$
$$g_2 = 9.8 \times \left(\frac{r_E}{\frac{1}{2}r_E}\right)^2 \quad \text{[1 mark]}$$
$$= 9.8 \times (2)^2$$
$$= 39.2\,\text{N kg}^{-1} \quad \text{[1 mark]}$$

**5** $$F_g \propto \frac{1}{r^2}$$
$$\frac{F_{g1}}{F_{g2}} = \left(\frac{r_1}{r_2}\right)^2$$
$$F_{g2} = F_{g1}\left(\frac{r_1}{r_2}\right)^2$$
$$F_{g2} = 110\,000 \times \left(\frac{r_E}{4r_E}\right)^2$$
$$= 110\,000 \times \left(\frac{1}{4}\right)^2 \quad \text{[1 mark]}$$
$$= 110\,000 \times \frac{1}{16}$$
$$= 6875\,\text{N} \quad \text{[1 mark]}$$

## 3.4 Motion in gravitational fields, from projectiles to satellites in space

### Sample problem 4

Take upwards as the positive direction.

$\boldsymbol{a} = \boldsymbol{g} = -9.8\,\text{m s}^{-2}, \boldsymbol{u} = 20.0\,\text{m s}^{-1}, \boldsymbol{v} = 0\,\text{m s}^{-1}$

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$$
$$0^2 = 20^2 + (2 \times -9.8 \times \boldsymbol{s})$$
$$0^2 - 20^2 = 2 \times -9.8 \times \boldsymbol{s}$$
$$\boldsymbol{s} = \frac{0^2 - 20^2}{2 \times -9.8}$$
$$= 20\,\text{m}$$

The maximum height reached by the ball is 20 m.

### Practice problem 4

$\boldsymbol{a} = g = -9.8\,\text{m s}^{-2}, \boldsymbol{u} = 0\,\text{m s}^{-1}, \boldsymbol{s} = 20\,\text{m}$

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 - 2\boldsymbol{as}$$
$$= 0^2 - 2 \times 9.8 \times 20$$
$$= 392$$
$$\boldsymbol{v} = \sqrt{392}$$
$$= 20\,\text{m s}^{-1}$$

### Sample problem 5

**a** $$\boldsymbol{v} = \frac{2\pi}{T}$$
$$\boldsymbol{v} = \frac{2\pi \times 3.84 \times 10^8}{2.36 \times 10^6}$$
$$= 1020\,\text{m s}^{-1} \text{ (to 3 significant figures)}$$

**b** $a = \frac{4\pi^2 r}{T^2}$

$a = \frac{4\pi^2 \times 3.84 \times 10^8}{(2.36 \times 10^6)^2}$

$= 0.002\,72 \text{ m s}^{-2}$

or

$a = \frac{v^2}{r}$

(Ensure you use the non-rounded figure from part a.)

$a = \frac{1020^2}{3.84 \times 10^8}$

$= 0.002\,72 \text{ m s}^{-2}$

**c** $G\frac{M}{4\pi^2} = \frac{r^3}{T^2}$

$\frac{6.67 \times 10^{-11} M_{\text{Earth}}}{4\pi^2} = \frac{(3.84 \times 10^8)^3}{(2.36 \times 10^6)^2}$

$\Rightarrow M_{\text{Earth}} = \frac{4\pi^2 \times (3.84 \times 10^8)^3}{(6.67 \times 10^{-11}) \times (2.36 \times 10^6)^2}$

$= 6.02 \times 10^{24} \text{ kg}$

## Practice problem 5

$T = 2.76 \times 10^4 \text{ s}, \; r = 9.38 \times 10^6 \text{ m}$

**a** $v = \frac{2\pi r}{T}$

$= \frac{2 \times \pi \times 9.38 \times 10^6}{2.76 \times 10^4}$

$= 2.14 \times 10^3 \text{ m s}^{-1}$

**b** $a = \frac{4\pi^2 r}{T^2}$

$= \frac{2 \times (3.142)^2 \times 9.38 \times 10^6}{(2.76 \times 10^4)^2}$

$= 0.486 \text{ m s}^{-2}$

**c** $M_{\text{Mars}} = \frac{4\pi^2 r^3}{GT^2}$

$= \frac{4 \times 3.142^2 \times (9.38 \times 10^6)^3}{6.67 \times 10^{-11} \times (2.76 \times 10^4)^2}$

$= 6.41 \times 10^{23} \text{ kg}$

## Sample problem 6

**1** Mercury: $\frac{r^3}{T^2} = \frac{(5.79 \times 10^{10})^3}{(7.60 \times 10^6)^2}$

$= 3.36 \times 10^{18}$

**2** Venus: $\frac{r^3}{T^2} = \frac{(1.08 \times 10^{11})^3}{(1.94 \times 10^7)^2}$

$= 3.35 \times 10^{18}$

**3** Mars: $\frac{r^3}{T^2} = \frac{(2.28 \times 10^{11})^3}{(5.94 \times 10^7)^2}$

$= 3.36 \times 10^{18}$

The values of $\frac{r^3}{T^2}$ for the three planets are approximately the same, confirming Kepler's Third Law.

## Practice problem 6

Use values from table 2.1 in the text to evaluate $\frac{r^3}{T^2}$ for each of the planets.

Saturn:

$\frac{r^3}{T^2} = \frac{(1.43 \times 10^{12})^3}{(9.30 \times 10^8)^2}$

$= 3.38 \times 10^{18}$

Uranus:

$\frac{r^3}{T^2} = \frac{(2.87 \times 10^{12})^3}{(2.65 \times 10^9)^2}$

$= 3.37 \times 10^{18}$

Neptune:

$\frac{r^3}{T^2} = \frac{(4.50 \times 10^{12})^3}{(5.21 \times 10^9)^2}$

$= 3.36 \times 10^{18}$

## Sample problem 7

Convert all time periods to the same unit:

$T_{\text{ISS}} = 92.75 \text{ minutes} = 5565 \text{ seconds}$

$T_{\text{Intelsat}} = 24.54 \text{ hours} = 88\,344 \text{ seconds}$

Apply Kepler's Third Law:

$\frac{r^3_{\text{ISS}}}{T^2_{\text{ISS}}} = \frac{r^3_{\text{Intelsat}}}{T^2_{\text{Intelsat}}}$

$\frac{r^3_{\text{ISS}}}{5565^2} = \frac{(4.25 \times 10^6)^3}{88\,344^2}$

$\Rightarrow r^3_{\text{ISS}} = \frac{5565^2 \times (4.25 \times 10^6)^3}{88\,344^2}$

$= 3.046 \times 10^{17} \text{ m}$

$r_{\text{ISS}} = \sqrt{3.046 \times 10^{17}}$

$= 6.73 \times 10^5 \text{ m}$

## Practice problem 7

$T_{\text{Phobos}} = 7.66\text{h}, T_{\text{Deimos}} = 36.75\text{h}$

$\frac{r^3_{\text{Phobos}}}{T^2_{\text{Phobos}}} = \frac{r^3_{\text{Deimos}}}{T^2_{\text{Deimos}}}$

$r^3_{\text{Deimos}} = \frac{T^2_{\text{Deimos}} r^3_{\text{Phobos}}}{T^2_{\text{Phobos}}}$

$= \frac{(36.75)^2 \times (9.38 \times 10^6)^3}{7.66^2}$

$= 1.900 \times 10^{22}$

$r_{\text{Deimos}} = \sqrt{1.900 \times 10^{22}}$

$= 2.67 \times 10^7 \text{ m}$

### Sample problem 8

$T = 24$ hours $= 86\,400$ seconds

Apply Kepler's Third Law:

$$\frac{GM_{\text{Earth}}}{4\pi^2} = \frac{r^3}{T^2}$$

$$\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{4\pi^2} = \frac{r^3}{86\,400^2}$$

$$\Rightarrow r^3 = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 86\,400^2}{4\pi^2}$$

$$\Rightarrow r = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 86\,400^2}{4\pi^2}}$$

$$= 4.22 \times 10^7 \text{ m}$$

Subtract the radius of Earth from the radius of the satellite's orbit to determine the altitude:

altitude = radius of orbit − radius of Earth

$$= 4.22 \times 10^7 - 6.37 \times 10^6$$

$$= 3.58 \times 10^7 \text{ m}$$

### Practice problem 8

Geostationary means that the period of the satellite is 1 Martian day (or 24 hours and 37 minutes):

$$T_{\text{satellite}} = (24 \times 3600 + 37 \times 60) \text{ s} = 88\,620 \text{ s}$$

$$G\frac{M_{\text{Mars}}}{4\pi^2} = \frac{r^3_{\text{satellite}}}{T^2_{\text{satellite}}}$$

$$r^3_{\text{satellite}} = \frac{GM_{\text{Mars}} \times T^2_{\text{satellite}}}{4\pi^2}$$

$$= \frac{6.67 \times 10^{-11} \times 6.42 \times 10^{23} \times 88\,620^2}{4\pi^2}$$

$$r_{\text{satellite}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6.42 \times 10^{23} \times 88\,620^2}{4\pi^2}}$$

$$= 2.04 \times 10^7 \text{ m}$$

Subtract the radius of Earth from the radius of the satellite's orbit to determine the altitude:

altitude $= r_{\text{satellite}} - R_{\text{Mars}}$

$$= 2.04 \times 10^7 - 3.39 \times 10^6$$

$$= 1.70 \times 10^7 \text{ m}$$

### Sample problem 9

The net force on the astronaut is equal to the force due to gravity on the astronaut, because the astronaut is in circular motion around Earth with acceleration $g$. The bathroom scales are also in circular motion around Earth with acceleration $g$. This means that the normal force experienced by the astronaut from the bathroom scales is 0 N.

### Practice problem 9

The astronaut and the objects in the spacecraft are all accelerating at $g$. When the astronaut holds the 10 kg mass, the gravitational force on the (astronaut + 10 kg) is increased; however, the centripetal force increases by a proportional amount, so there is still no normal force experienced by the astronaut standing on the scales, so the scales continue to read zero.

### 3.4 Exercise

**1** $T = 27.32$ days

$$= (27.32 \times 24 \times 3600) \text{ s}$$

$$= 2\,360\,448 \text{ s};$$

$$r = 3.84 \times 10^8 \text{ m}$$

$$v = \frac{2\pi r}{T}$$

$$= \frac{2 \times \pi \times 3.84 \times 10^8}{2\,360\,448}$$

$$= 1.02 \times 10^3 \text{ m s}^{-1} \quad \text{to 3 s.f.}$$

**2**

$$\frac{r^3}{T^2} = \frac{Gm}{4\pi^2}$$

$$\Rightarrow m = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{\left(100 \times 10^3\right)^3 \times 4\pi^2}{\left(6.67 \times 10^{-11}\right) \times (27.0 \times 60 \times 60)^2}$$

$$= \frac{4\pi^2}{(6.67 \times 2.7 \times 3.6)^2} \times 10^{18}$$

$$= 6.26 \times 10^{16} \text{ kg} \quad \text{to 3 s.f.}$$

**3 a** $T = 24 \text{ h} = 8.64 \times 10^4 \text{ s}$

$$r = 6.37 \times 10^6 \text{ m}$$

$$a = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2\left(6.37 \times 10^6\right)}{\left(8.64 \times 10^4\right)^2}$$

$$= \frac{4\pi^2 \times 6.37}{8.64^2} \times 10^{-2}$$

$$= 3.37 \times 10^{-2} \text{ m s}^{-2} \quad \text{to 3 s.f.}$$

**b** In Victoria, the radius of the circular path would be smaller. If $r$ decreases, the acceleration also decreases. Therefore, in Victoria, you would experience less centripetal acceleration due to Earth's motion than people on the equator.

**4 a** $T = 92.0$ min

$$= (92.0 \times 60) \text{ s}$$

$$= 5.52 \times 10^3 \text{ s};$$

$$r = \left(6.37 \times 10^6 \text{ m} + 3.55 \times 10^5 \text{ m}\right)$$

$$= 6.725 \times 10^6 \text{ m}$$

$$v = \frac{2\pi r}{T}$$

$$= \frac{2 \times \pi \times 6.725 \times 10^6}{5.52 \times 10^3}$$

$$= \frac{2\pi \times 6.725}{5.52} \times 10^3$$

$$= 7.65 \times 10^3 \text{ m s}^{-1} \quad \text{to 3 s.f.}$$

**b** $T = 92.0$ min

$$= (92.0 \times 60) \text{ s}$$

$$= 5.52 \times 10^3 \text{ s};$$

$$r = \left(6.37 \times 10^6 \text{ m} + 3.55 \times 10^5 \text{ m}\right)$$

$$= 6.725 \times 10^6 \text{ m}$$

$$a = \frac{4\pi^2 r}{T^2}$$
$$= \frac{4\pi^2\left(6.725\times10^6\right)}{\left(5.52\times10^3\right)^2}$$
$$= \frac{4\pi^2\times6.725}{5.52^2}$$
$$= 8.71\ \text{m s}^{-2} \qquad \text{to 3 s.f.}$$

**c** $$g = \frac{Gm_E}{r^2}$$
$$= \frac{6.67\times10^{-11}\times5.97\times10^{24}}{\left(6.725\times10^6\right)^2}$$
$$= \frac{6.67\times5.97}{6.725^2}\times10^1$$
$$= 8.80\ \text{N kg}^{-1} \qquad \text{to 3 s.f.}$$

**d** The centripetal acceleration ***a*** of the space station is caused by acceleration due to gravity, ***g***. Therefore, their magnitude should be the same. Thus the answers to (**b**) and (**c**) should be the same. Discrepancies between the numbers are likely due to the rounding off of data.

**e** $m_{SS} = 1200$ tonnes
$$= 1.200\times10^6\ \text{kg}$$
$$g = 8.80\ \text{N kg}^{-1}$$
$$\Rightarrow F_g = m_{SS}g$$
$$= 1.200\times10^6\times8.80$$
$$= 1.06\times10^7\ \text{N} \qquad \text{to 3 s.f.}$$

**f** The force exerted on the astronaut by the floor of the space station is 0 N, because the space station and the astronaut both have the same acceleration ***g***.

**5**
$$G\frac{m_E}{4\pi^2} = \frac{r^3}{T^2}$$
$$\frac{6.67\times10^{-11}\times5.98\times10^{24}}{4\pi^2} = \frac{r^3}{(2.50\times3600\text{s})^2}$$
$$\frac{6.67\times5.97}{4\pi^2}\times10^{19} = \frac{r^3}{(2.50\times3.6)^2}$$
$$\Rightarrow r\sqrt{\frac{6.67\times5.97\times(2.5\times3.6)^2\times10}{4\pi^2}}\times10^6$$
$$r = 9.348\times10^6\ \text{m}$$
$$r_E + h = 9.348\times10^6\ \text{m}$$
$$\Rightarrow h = 2.98\times10^3\ \text{m} \qquad \text{to 3 s.f.}$$

**6** According to Newton's First Law, an object will move in a straight line, with constant speed, unless an unbalanced force is acting on it. As gravity acts at right angles to the satellite's velocity, it does not change the speed of the satellite; rather, it changes its direction. This causes the satellite to move around Earth. Because the force of gravity and the speed of the satellite remain constant, so must its radius as $r = \frac{mv^2}{F}$; therefore, it cannot move closer to Earth.

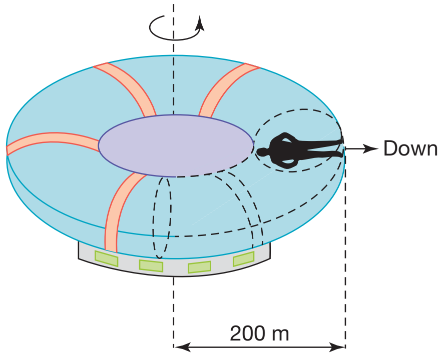
**7**
$$\frac{r_A^3}{T_A^2} = \frac{r_B^3}{T_B^2}$$
$$\frac{r_A^3}{T_A^2} = \frac{(2r_A)^3}{T_B^2}$$
$$\frac{r_A^3}{T_A^2} = \frac{8r_A^3}{T_B^2}$$
$$T_B^2 = 8T_A^2$$
$$T_B = 2\sqrt{2}T_A$$

The period would be increased by a factor of $2\sqrt{2}$.

**8**
$$\frac{r^3}{T^2} = \frac{Gm}{4\pi^2}$$
$$\Rightarrow r = \sqrt{\frac{GmT^2}{4\pi^2}}$$
$$\Rightarrow \frac{\text{distance of Saturn from the Sun}}{\text{distance of Venus from the Sun}} = \left(\frac{9.30\times10^8}{1.94\times10^7}\right)^{\frac{2}{3}}$$
$$= \left(\frac{9.30\times10}{1.94}\right)^{\frac{2}{3}}$$
$$= 13.2 \qquad \text{to 3 s.f.}$$

**9** Both the astronaut and the space station are in circular orbit around Earth, they are in free fall. Since they both have the same acceleration, the astronaut's motion is independent of the motion of the space station, so if the astronaut didn't strap herself to the chair, there wouldn't be a normal force by the chair on the astronaut and by the astronaut on the chair, and she would float around.

**10 a** One possible answer:




On Earth, a person would feel the ground pushing up on them. In the space station, the outer wall pushes the person in (centripetal force). This means that the person would feel as though the wall is the ground, and the direction opposite to the centripetal force is down.

**b**
$$a = 10\ \text{m s}^{-2}$$
$$r = 200\ \text{m}$$
$$a = \frac{4\pi^2 r}{T^2}$$
$$\Rightarrow T = \sqrt{\frac{4\pi^2 r}{a}}$$
$$= \sqrt{\frac{4\pi^2\times200}{10}}$$
$$= 28\ \text{s}$$

## 3.4 Exam questions

**1 a**
$$v = \sqrt{\frac{GM}{r}}$$
$$= \sqrt{\frac{6.67\times10^{-11}\times5.7\times10^{26}}{1.2\times10^9}} \qquad \text{[1 mark]}$$
$$= 5.6\times10^6\ \text{m s}^{-1} \qquad \text{[1 mark]}$$

**b** Using $v = \sqrt{\frac{GM}{R}}$ [1 mark], when the radius ($R$) is smaller, then the orbital speed must be faster. [1 mark]

**c** $\frac{GMm}{r^2} = \frac{4\pi^2 rm}{T^2}$

$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ = constant for all orbiting bodies [1 mark]

$\frac{(1.2\times 10^9)^3}{16^2} = \frac{(5.3\times 10^8)^3}{T^2}$ [1 mark]

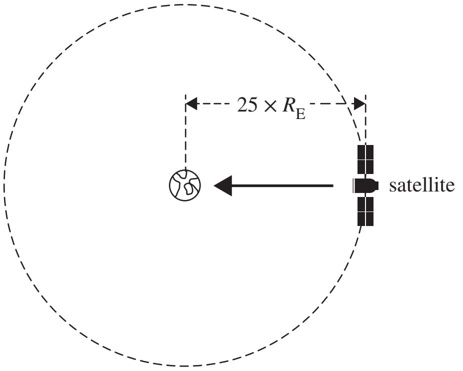
$T = 4.6$ days [1 mark]

**2 a** $a = \frac{GM}{R^2}$ [1 mark]

$a = \frac{6.67\times 10^{-11}\times 5.98\times 10^{24}}{(25\times 6.34\times 10^6)^2}$

$a = 1.6\times 10^{-2}\ \text{m s}^{-2}$ [1 mark]

**b**




[1 mark]

**c**

| | |
|---|---|
| Magnitude of acceleration | Less than |
| Kinetic energy | Less than |
| Period | More than |

Award 1 mark per correct term used.

$a = \frac{GM}{r^2}$ thus if the radius increases, the acceleration decreases.

$v = \sqrt{\frac{GM}{r}}$ thus if the radius increases, the speed decreases and so does the kinetic energy.

$\frac{r^3}{T^2}$ is a constant, thus if the radius increases, the period increases too.

**3** The correct approach involved equating Newton's law of gravitation with one of the equations for circular motion.

$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$ [1 mark]

$\therefore M = \frac{4\pi^2 r^3}{GT^2}$ [1 mark]

$M = \frac{4\pi^2 (6.9\times 10^{10})^3}{6.67\times 10^{11}\times (8.47\times 10^6)^2}$

$M = 2.71\times 10^{30}$ kg [1 mark]

***VCAA examination report* note:**

There was no common error, suggesting that students who did not immediately know what was required struggled to begin solving the problem.

There were a number of students for whom full marks were not awarded because they did not demonstrate enough working. Students are reminded that the instructions for Section B indicate the need to show working as did the question stem. The required working is a demonstration of equating Newton's law of gravitation with circular motion (e.g. either of the first two lines or some similar demonstration), the substitution shown in the second last line and the answer.

**4 a** $F = \frac{GMm}{r^2}$ [1 mark]

$F = \frac{6.67\times 10^{-11}\times 5.98\times 10^{24}\times 7.35\times 10^{22}}{(3.84\times 10^8)^2}$ [1 mark]

$F = 1.99\times 10^{20}$ N [1 mark]

**b** The orbital period will increase. [1 mark]

The ratio $\frac{R^3}{T^2}$ is constant for all satellites orbiting Earth. [1 mark]

As $R$ increases, $T$ will also increase. [1 mark]

**5 a** $r = \frac{v^2}{g}$

$= \frac{180^2}{9.8}$ [1 mark]

$= 3306$ m [1 mark]

***VCAA examination report* note:**

The most common errors were mathematical as most students who made an attempt knew the problem involved circular motion.

**b** The force of gravity is **not** zero at the top of the flight. [1 mark]

The 'zero gravity experience' is due to the lack of a contact or normal reaction force. [1 mark]

***VCAA examination report* note:**

Many students stated that the force of gravity was zero at the top and then struggled to explain how that might occur. Others stated that if the gravitational force was still present, it would be very small, which was also incorrect.

A number of students made reference to 'apparent weightlessness'. It should be pointed out that 'apparent weightlessness' is no longer part of the study design.

## 3.5 Energy changes in gravitational fields

### Sample problem 10

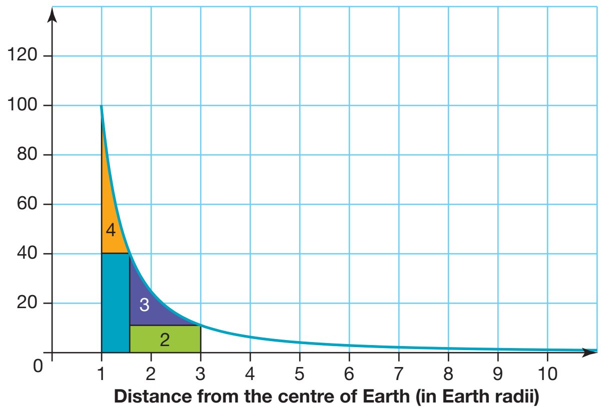
The gravitational field does the work on the mass, so the change in kinetic energy can be found from the area under the force–displacement graph.

How to find the area under a curved graph.

Method 1:

1. Divide the area under the graph into simple shapes and estimate the area in terms of the sum of the areas of the shapes.
2. If the graph is drawn on a fine grid, count the grid squares under the graph.

**3** Convert the grid areas to the correct units.




$$\text{Area 1 (blue)} = 40 \times 0.5 = 20 \text{ energy units}$$

$$\text{Area 2 (green)} = 10 \times 1.5 = 15 \text{ energy units}$$

$$\text{Area 3 (purple)} = \frac{1}{2} \times 24 \times 1.5 = 18 \text{ energy units}$$

(Note: The triangle with area $\frac{1}{2} \times 30 \times 1.5$ would be larger than the purple area, so the height of 30 was reduced to a level where the areas matched.)

$$\text{Area 4 (orange)} = \frac{1}{2} \times 53 \times 0.5 = 13.25 \text{ energy units}$$

$$\text{Total area} = 20 + 15 + 18 + 13.25 = 66.25 \text{ energy units}$$

$$1 \text{ energy unit} = 1\text{ N} \times 1 \text{ Earth radius} = 1\text{ N} \times 6.37 \times 10^6 \text{ m} = 6.37 \times 10^6 \text{ J}$$

$$\text{Therefore, the kinetic energy gained} = 66.25 \times 6.37 \times 10^6 = 4.22 \times 10^8 \text{ J}$$

Method 2:

Use this method when the graph has a relatively fine grid.

**1** Count the number of small squares between the graph and the zero-value line or horizontal axis. Tick each one as you count it to avoid counting squares twice. For partial squares, find two that add together to make one square and tick both.

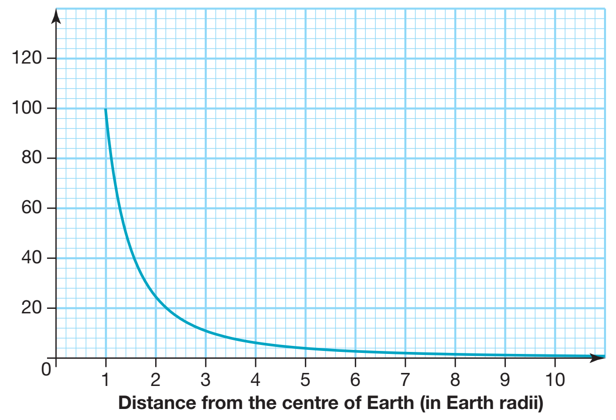
**2** Calculate the area of one small square.

**3** Multiply the area of one small square by the number of small squares.

$$\text{Number of small squares} = 80.5$$

$$\text{Area of one small square} = 4\text{ N} \times 0.2 \times 1 \text{ Earth radius} = 4\text{ N} \times 0.2 \times 6.38 \times 10^6 \text{ m} = 5.1 \times 10^6 \text{ J}$$

$$\text{Therefore, the gain in kinetic energy} = 80.5 \times 5.1 \times 10^6 \text{ J} = 4.11 \times 10^8 \text{ J}$$




## Practice problem 10

**a** Remember, there are two methods you can use for these questions. You may count the squares or split them into smaller areas. Due to the provision of small squares, the method of counting squares has been used. The area under the graph is found by counting the grid squares from $r = 2$ Earth radii to $r = 3$ Earth radii. There are ≈ 23 grid squares. (Students may have a slightly different count, so their final solution may have some variation.)

$$\text{area of each grid square} = 0.2 \times 1 \text{ Earth radius} \times (4 \times 10^6)\text{ N} = 0.2 \times 6.37 \times 10^6 \text{ m} \times 4 \times 10^6 \text{ N} = 5.096 \times 10^{12} \text{ J}$$

$$\text{total work done} = \Delta E_{\text{kinetic}} = 23 \times 5.096 \times 10^{12} \text{ J} = 1.17 \times 10^{14} \text{ J}$$

**b** Remember, there are two methods you can use for these questions. You may count the squares or split them into smaller areas. Due to the provision of small squares, the method of counting squares has been used. The area under the graph is found by counting the grid squares from the $r = 1$ Earth radius to $r = 2$ Earth radii. There are ≈ 68 grid squares.

$$\text{area of each grid square} = 0.2 \times 1 \text{ Earth radius} \times (4 \times 10^6)\text{ N} = 0.2 \times 6.37 \times 10^6 \text{ m} \times 4 \times 10^6 \text{ N} = 5.096 \times 10^{12} \text{ J}$$

$$\text{total work done} = \Delta E_{\text{kinetic}} = 68 \times 5.096 \times 10^{12} \text{ J} = 3.47 \times 10^{14} \text{ J}$$

The ratio of the two results $\left(\frac{\text{b}}{\text{a}}\right)$ is $\frac{3.47 \times 10^{14}}{1.17 \times 10^{14}} = 2.96$.

The kinetic energy gained by falling from an altitude of one Earth radius to the surface of Earth was approximately 3 times the kinetic energy gained by falling from an altitude of two Earth radii to an altitude of one Earth radius.

## Sample problem 11

Remember, there are two methods you can use for these questions. You may count the squares or split them into smaller areas. Due to the provision of small squares, the method of counting squares has been used. The area under the curve is 29 squares, with each square having an area of $0.2\text{ N kg}^{-1} \times 0.2$ Earth radii.

One Earth radius = $6.37 \times 10^6$ m

Area under curve

$= 0.2 \times 0.2 \times 29 \times 6.37 \times 10^6$ N m kg$^{-1}$
$= 7.39 \times 10^6$ N m kg$^{-1}$

Change in energy

$= 100\,\text{kg} \times 7.39 \times 10^6$ N m kg$^{-1}$
$= 7.39 \times 10^8$ N m
$= 7.39 \times 10^8$ J

## Practice problem 11

The area under the graph is found by counting the grid squares or approximating the area with geometric shapes. From a height of 3 Earth radii to 2 Earth radii, the number of squares is 20.

$$\begin{aligned}\text{area of each grid square} &= 0.2 \times 1\text{ Earth radius} \times 0.4\text{ N kg}^{-1}\\ &= 0.2 \times 6.37 \times 10^6\text{ m} \times 0.4\text{ N kg}^{-1}\\ &= 5.10 \times 10^5\text{ J kg}^{-1}\\ \text{total work done} &= m \times \text{area under graph}\\ &= \Delta E_{\text{kinetic}}\\ &= 20\text{ kg} \times 20 \times 5.10 \times 10^5\\ &= 2.04 \times 10^8\text{ J}\end{aligned}$$

## Sample problem 12

**a** $F = mg$
$= 30 \times 9.8$
$= 294$ N
$\Rightarrow W = Fs$
$= 294 \times 150$
$= 44\,100$ J

**b** $\Delta E_k$ = work done by field
$= 44\,100$ J
$= 4.4 \times 10^4$ J

## Practice problem 12

The displacement of the ball in the field is 1.0 m.

$$\begin{aligned}\text{work done by field} &= mgh\\ &= 2.0\text{ kg} \times 9.8\text{ N kg}^{-1} \times 1.0\text{ m}\\ &= 19.6\text{ J}\\ \text{total work done} &= \Delta E_{\text{kinetic}} = 20\text{ J}\end{aligned}$$

## 3.5 Exercise

**1** The change in gravitational potential energy can be obtained from the area under a force–distance graph. This loss of gravitational potential energy corresponds to a gain in kinetic energy.
There are two methods you can use for this type of question, to estimate the area under the curve:
- you may count the squares in the area under the force–displacement graph
- you may split the area under the force–displacement graph into smaller areas.

Due to the provision of small squares, the method of counting squares under the curve has been used for this question, but you may have used the other method instead.
The number of small grid squares under the curve between $3.5 \times 10^7$ m and $1.0 \times 10^7$ m is approximately 60.
$\Delta E_k \simeq 60$ energy units
$\simeq 60 \times 10^3\text{ N} \times 10^6$ m
$\simeq 6 \times 10^{10}$ J to 1 s.f.

**2** There are two methods you can use for this type of question:
- you may count the squares in the area under the force–displacement graph
- you may split the area under the force–displacement graph into smaller areas.

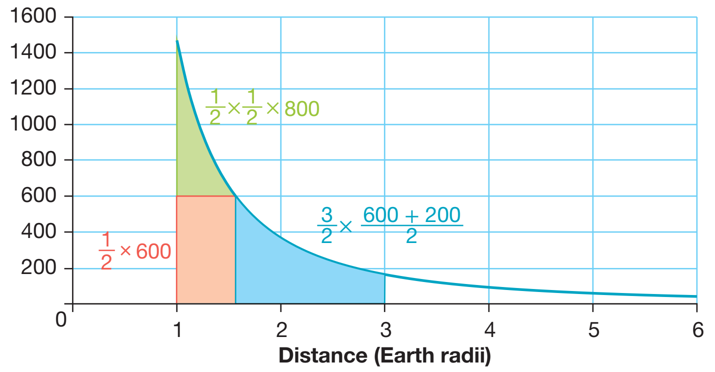
See table at the bottom of the page*

$$\text{area} \simeq \frac{1}{2} \times 600 + \frac{1}{2} \times \frac{1}{2} \times 800 + \frac{3}{2} \times \frac{600+200}{2}$$
$$\simeq 1100\text{ units}^2$$

To convert to joules, multiply the area by the radius of Earth (as the $x$-axis in the force–displacement graph is in Earth radii instead of metres)
$\Delta E_g \simeq 1100 \times 6.37 \times 10^6$
$\simeq 6 \times 10^9$ J to 1 s.f.

**3** The unit of the area under a force–distance graph is N m = J, which is a unit of energy. The area under a gravitational field strength vs. distance graph has the unit N kg$^{-1}$ = J kg$^{-1}$, which is a unit of energy per mass.

**4** As the space probe gets closer to Planet Q, the gravitational potential energy of the probe decreases, increasing its kinetic energy. As the space probe moves further away from Planet Q, towards Y, kinetic energy is converted into gravitational potential energy. The total energy of the space probe (Planet Q system) is constant because energy is conserved in a closed system.

*2


**5** Remember that the change in energy for an object that moves from one point to another can be obtained by multiplying the area under the graph of the gravitational field against distance by its mass.

Area under graph = Work per kg

Here the method of counting squares under the curve has been used, due to the provision of small squares, but you may have used the other method instead.

An altitude of $1.4 \times 10^7$ m corresponds to a distance of $2.037 \times 10^7$ m from the centre of Earth (determined by adding the altitude to the radius of Earth).

The number of small grid squares under the curve between $0.637 \times 10^7$ m and $1.4 \times 10^7$ m is approximately 100.

$$\begin{aligned}\text{Area under curve} &\simeq 100 \text{ energy units}\\ &\simeq 100 \times 10^6\,\text{m} \times 0.4\,\text{N kg}^{-1}\\ &\simeq 4 \times 10^7\,\text{J kg}^{-1}\\ \Rightarrow \Delta E_g &\simeq 4 \times 10^7\,\text{J kg}^{-1} \times 400\,\text{kg}\\ &\simeq 10^{10}\,\text{J} \quad \text{to 1 s.f.}\end{aligned}$$

**6 a** Remember that the change in energy for an object that moves from one point to another can be obtained by multiplying the area under the graph of the gravitational field against distance by its mass.

Area under graph from 400 to 600 km = Work per kg

(The area under the graph — a trapezium — has been used here. You may also choose instead to count squares.)

$$\begin{aligned}\text{Area under curve} &\approx \frac{1}{2}(8.7 + 8.3) \times 2 \times 10^5\\ &\approx 1.7 \times 10^6\,\text{J kg}^{-1}\\ \Rightarrow \text{Work} &\approx 1.7 \times 10^6\,\text{J kg}^{-1} \times 800\,\text{kg}\\ &\approx 10^9\,\text{J} \quad \text{to 1 s.f.}\end{aligned}$$

**b**

$$\begin{aligned}r &= r_E + 400\,\text{km} + 200\,\text{km}\\ &= 6.37 \times 10^6 + 6.0 \times 10^5\\ &= 6.97 \times 10^6\,\text{m}\\ T^3 &= \frac{Gm_E}{4\pi^2}\\ \Rightarrow T &= \sqrt{\frac{4\pi^2 r^3}{Gm_E}}\\ &= \sqrt{\frac{4\pi^2(6.97 \times 10^6)^3}{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}}\\ &= \sqrt{\frac{6.97^3 \times 4\pi^2}{6.67 \times 5.97} \times 10 \times 10^2}\\ &= 5.79 \times 10^3 \quad \text{to 3 s.f.}\end{aligned}$$

**c** As $\frac{r^3}{T^2}$ is constant for any satellite around Earth, the period of the 800-kg satellite could be found using the known radius of the space shuttle and the satellite's orbits and the known period of the space shuttle's orbit.

**d** If the new satellite's mass was halved, the work needed to deploy it from the shuttle would halve as $W = \boldsymbol{Fs}$ and the force is proportional to the mass. The period of the new satellite would remain unchanged as it relies only on the mass of Earth and the distance from its centre of mass.

**7 a**

$$\begin{aligned}F_g &= \frac{Gm_E m_{sat}}{r^2}\\ r &= 6.37 \times 10^6 + 2 \times 10^6\\ &= 8.37 \times 10^6\,\text{m}\\ \Rightarrow F_g &= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 2.4 \times 10^3}{(8.37 \times 10^6)^2}\\ &= \frac{6.67 \times 5.97 \times 2.4}{8.37^2} \times 10^4\\ &= 1.36 \times 10^4\,\text{N} \quad \text{to 3 s.f.}\end{aligned}$$

**b** Remember that the change in energy for an object that moves from one point to another can be obtained by multiplying the area under the graph of the gravitational field against distance by its mass.

The loss of gravitational potential energy per kg is given by the area under the graph between $r_E + 2000\,\text{km} \simeq 8.4 \times 10^6$ m and $r_E + 800\,\text{km} \simeq 7.2 \times 10^6$ m.

The area under the graph is approximately given by a trapezium shape.

$$\begin{aligned}\frac{\Delta E_g}{\text{kg}} &\simeq \frac{1}{2}(a+b)h\\ &\simeq \frac{1}{2}(5.25 + 6.75) \times 1.2 \times 10^6\\ &\simeq 7.2 \times 10^6\,\text{J kg}^{-1}\\ \Rightarrow \Delta E_g &\simeq 2.400 \times 10^3 \times 7.2 \times 10^6\\ &\simeq 2 \times 10^{10}\,\text{J} \quad \text{to 1 s.f.}\end{aligned}$$

**8**

$$\begin{aligned}\Delta E_k &= \text{work done by the gravitational field}\\ &= mgh\\ &= 8.5 \times 10^{-2}\,\text{kg} \times 9.8\,\text{N kg}^{-1} \times 3.0\,\text{m}\\ &= 2.5\,\text{J} \quad \text{to 2 s.f.}\end{aligned}$$

**9** The gravitational field only does work parallel to the displacement of the cannonball, so only the change in vertical position, or height, is important.

$$\begin{aligned}\text{Work done by gravitational field} &= mgh\\ &= 4.50 \times 9.81 \times 30.0\\ &= 1.324\,35 \times 10^3\,\text{J}\\ \Delta E_k &= 1.324\,35 \times 10^3\,\text{J}\\ \frac{1}{2}mv^2 - \frac{1}{2}mu^2 &= 1.324\,35 \times 10^3\,\text{J}\\ 2.25v^2 - 2.25 \times 10^4 &= 1.324\,35 \times 10^3\\ v^2 &= \frac{1.324\,35 \times 10^3 + 2.25 \times 10^4}{2.25}\\ v &= \sqrt{10\,588.6}\\ v &= 1.03 \times 10^2\,\text{m s}^{-1} \text{ to 3 s.f.}\end{aligned}$$

## 3.5 Exam questions

**1** A. The change in kinetic energy is equal to the work done. This is equal to the area under a force–displacement graph. As the force due to gravity is equal to the gravitational field strength multiplied by mass, $F_g = mg$, the work done can also be calculated by the area under a gravitational field strength vs. distance graph, multiplied by the mass.

**2** D. The gravitational and kinetic energy can't both be increased. As the space probe moves to a new position, if there is a change in its gravitational potential energy, it must be matched by an equal and opposite change in its kinetic energy.

**3 a** $F_{net} = F_{th} - mg = ma$ [1 mark]

$F_{th} - (531 \times 10^3 \times 9.80) = (531 \times 10^3 \times 7.20)$

$F_{th} - 5.20 \times 10^6 = 3.82 \times 10^6$

$F_{th} = 9.02 \times 10^6$ N (9.02 MN) [1 mark]

The direction of the thrust force on the rocket is **up**. [1 mark]

**b** Convert $\text{km h}^{-1}$ into $\text{m ms}^{-1}$:

$20\,000\ \text{km h}^{-1} = 5.56 \times 10^3\ \text{m s}^{-1}$ [1 mark]

$E = \frac{1}{2}mv^2$

$E = 0.5 \times 1000 \times \left(5.56 \times 10^3\right)^2$

$E = 1.54 \times 10^{10}\ \text{J}$ [1 mark]

***VCAA examination report*** **note:**

The most common errors were to fail to convert the velocity or to fail to square the velocity.

**c** The area under the graph up to 300 km is:

$\text{area} = \left(\frac{3.7 + 3.2}{2}\right) \times 300 \times 10^3$ [1 mark]

$\text{area} = 1.04 \times 10^6\ \text{J kg}^{-1}$ [1 mark]

$E = \text{mass} \times \text{area} = 1000 \times 1.04 \times 10^6$

$E = 1.04 \times 10^9\ \text{J}$ [1 mark]

***VCAA examination report*** **note:**

Some students attempted to count the small squares under the graph (there were 258.75). This generally resulted in an incorrect area calculation. Generally, students who were unable to correctly calculate the area were still aware of the need to multiply the area by the mass of the capsule to find the energy.

**d** The gravitational potential energy is converted to kinetic energy as the capsule descends [1 mark]. The kinetic energy is then converted to heat/light/sound [1 mark] due to friction between the capsule and the Martian atmosphere [1 mark].

***VCAA examination report*** **note:**

While most students correctly identified conversion of kinetic energy to other forms of energy, many did not identify what happened to the gravitational potential energy. Some students were aware that they needed to refer to the gravitational potential energy but could not adequately explain how it was reduced.

**4** Kinetic energy increases as the planet moves from a point of higher gravitational potential to lower gravitational potential. The field does work on the planet.
The work done is the area under the force versus distance graph. [1 mark]
There are approximately 57 squares under the graph. [1 mark]

The area of each square $= 0.001\ \text{N} \times 1.0 \times 10^6\ \text{m}$
$= 1.0 \times 10^3\ \text{J}$

Change in potential energy $= 57 \times 1.0 \times 10^3\ \text{J}$
$= 5.7 \times 10^4\ \text{J}$ [1 mark]

It is important to note that the number of squares is approximate, and students may still be awarded the mark if their value is close but not identical. Students may also use the method of splitting the shape into areas, rather than counting the squares themselves (however, counting squares is more precise for this question).

**5** The change in potential energy of the weather balloon can be determined by calculating the product of the mass times the area under the graph. [1 mark]
The area under the graph is trapezoidal in shape.

$\text{area} = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(9.70 + 9.53)50 \times 10^3$

$= 4.8 \times 10^5\ \text{N m kg}^{-1}$ [1 mark]

$\Delta E = 1.5 \times 4.8 \times 10^5$

$= 7.2 \times 10^5\ \text{J}$ [1 mark]

## 3.6 Review

### 3.6 Review questions

**1** At large distances from Earth, the gravitational field is non-uniform, attractive and inversely proportional to the distance from the centre of Earth.

**2** Let $m$ be the mass of the ball.

$F_{\text{g Mars}} = G\frac{m_{\text{Mars}}\,m}{r^2_{\text{Mars}}}$

$F_{\text{g Moon}} = G\frac{m_{\text{Moon}}\,m}{r^2_{\text{Moon}}}$

$\frac{F_{\text{gMars}}}{F_{\text{Moon}}} = \frac{m_{\text{Mars}}}{m_{\text{Moon}}}\left(\frac{r_{\text{Moon}}}{r_{\text{Mars}}}\right)^2$

$= \frac{6.39 \times 10^{23}}{7.35 \times 10^{22}}\left(\frac{1.74 \times 10^6}{3.39 \times 10^6}\right)^2$

$= \frac{6.39 \times 10^1}{7.35} \times \left(\frac{1.74}{3.39}\right)^2$

$= 2.29$ to 3 s.f.

**3** The rocket experiences a gravitational field that is the sum of the field from the Moon and the field from Earth.
The two fields point in opposite directions.
Initially, Earth's field dominates; however, it gets weaker as the rocket moves further away from Earth and the Moon's field becomes stronger until a point is reached where the two fields cancel each other out.
From that point on, the field of the Moon dominates the field from Earth.

**4** $G\frac{m_J}{4\pi^2} = \frac{r_C^3}{T_C^2}$

$m_J = \frac{4\pi^2\left(1.88 \times 10^9\right)^3}{6.67 \times 10^{-11} \times (400 \times 3600)^2}$

$= \frac{4\pi^2 \times 1.88^3}{6.67 \times (1.44)^2} \times 10^{26}$

$= 1.90 \times 10^{27}\ \text{kg}$ to 3 s.f.

**5** Using the method of ratios:

$\left(\frac{r_1}{r_2}\right)^3 = \left(\frac{T_1}{T_2}\right)^2$

$= \left(\frac{150}{95}\right)^2$

$= 2.493$

$= r_2^3 \times 2.493$

$= (0.535 + 6.37)^3 \times 10^{18} \times 2.493$

$= 8.21 \times 10^18$

$\Rightarrow r_1 = \sqrt{8.21} \times 10^6$

$= 9.36 \times 10^6$

The altitude $h$ of the satellite is:

$h = r - r_E$

$= 9.36 \times 10^6 - 6.37 \times 10^6$

$\simeq 3.0 \times 10^6\text{m}$ to 2 s.f.

**6** $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$

$\left(\frac{r_A}{\frac{r_A}{16}}\right)^3 = 16^3$

$\Rightarrow \frac{T_A}{T_B} = \sqrt{16^3}$

$= \sqrt{16^3}$

$= 4^3$

$= 64$

$\Rightarrow T_B = \frac{T_A}{64}$

The period would be decreased by a factor of 64.

**7** **a** Using the method of ratios:

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

$= \left(\frac{9.50 \times 10^8}{1.50 \times 10^9}\right)^3$

$= \left(\frac{9.50}{15.0}\right)^3$

$\Rightarrow T_1^2 = T_2^2\left(\frac{9.50}{15.0}\right)^3$

$= 33.844 \times 10^4$

$\Rightarrow T_1 = \sqrt{33.844} \times 10^2.$

184 days to 3 s.f.

**b** $T = 184$ days

$= 1.84 \times 2.4 \times 3.6 \times 10^6$ s

$= 1.589\,76 \times 10^7$ s

$r = 9.50 \times 10^8$ m

$v = \frac{2\pi r}{T}$

$= \frac{2\pi \times 8.00 \times 10^8}{1.226\,88 \times 10^7}$

$= \frac{2\pi \times 9.50}{1.589\,76} \times 10^1$

$= 3.75 \times 10^2$ m s$^{-1}$ to 3 s.f.

**8** **a** Using $g = \frac{Gm_E}{(r_E + h)^2}$ and rounding to 3 s.f:

| Altitude $h$(m) | $g$(N kg)$^{-1}$ |
|---|---|
| 0 | 9.813 |
| $10^3$ | 9.810 |
| $10^4$ | 9.783 |
| $10^5$ | 9.512 |
| $10^6$ | 7.331 |
| $6.37 \times 10^6$ | 0.273 |

**b** Answers will vary.

To decide whether Lakshmi's opinion is justified, consider the following:

- Between sea level and an altitude of 10 km, the variation of the gravitational field strength is approximately 0.3%, which is negligible.
- And between sea level and an altitude of 100 km, the variation of the gravitational field strength is approximately 3%, which is a small variation.
- However, above an altitude of 500 km, is not negligible anymore (greater than 10%)

Using $g = 9.8$ N kg$^{-1}$ as an approximation of Earth's gravitational field strength for altitudes lower than 10 km is acceptable.

**9** Over a distance of 10 m, the gravitational field strength of the Moon can be taken to be constant.

$g = G\frac{m_M}{r_M^2}$

$= \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{(1.74 \times 10^6)^2}$

$= \frac{6.67 \times 7.35}{1.75^2} \times 10^{-1}$

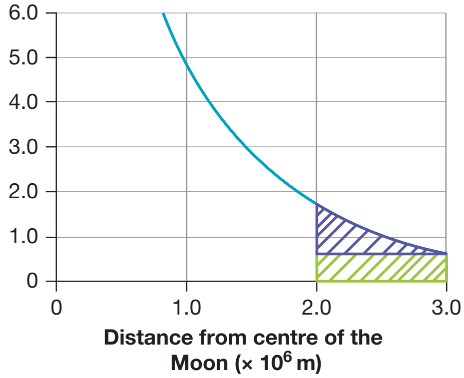
$= 1.62$ N kg$^{-1}$ to 3 s.f.

$\Delta E_g = mgh$

$= 25 \times 1.62 \times 10$

$= 4.05 \times 10^2$ J to 3 s.f.

**10** At an orbit further from the Moon, the potential energy of the satellite increases. The change in energy can be found from the product of the satellite mass and the area under the field strength vs. distance graph:




The area under the graph is approximately the area of the triangle and rectangle shown.

Area $\approx \frac{1}{2}(1.0 \times 10^6 \text{ m}) \times 1.0$ N kg$^{-1}$

$+ 1.0 \times 10^6 \text{ m} \times 0.8$ N kg$^{-1}$

$\approx 1.3 \times 10^6$ J kg$^{-1}$

$\Delta E_{potential} \approx 150 \text{ kg} \times 1.3 \times 10^6$ J kg$^{-1}$

$\approx 2 \times 10^8$ J to 1 s.f.

## 3.6 Exam questions

### Section A — Multiple choice questions

**1** B

$g = \frac{GM}{R^2}$

$\therefore R \propto \sqrt{\frac{M}{g}}$

Therefore, $R_{Phobetor}$ as a multiple of Earth's radius is:

$R \propto \sqrt{\frac{4}{1.8}} = 1.49$

Therefore, the radius of Phobetor will be ~ 1.5 times that of Earth.

**2** B

$g = \frac{GM}{r^2}$

$g = \frac{6.67 \times 10^{-11} \times 1.5 \times 10^{23}}{(2.6 \times 10^6)^2}$

$g = 1.5\,\text{m s}^{-2}$

**3** D

$g\,(\text{Earth}) = \frac{m}{r^2} = g$

If $m$ is doubled and $r$ is halved,

$g\,(\text{Planet Y}) = \frac{2m}{(0.5r)^2}$

$g\,(\text{Planet Y}) = \frac{2m}{0.25r^2}$

$\therefore g\,(\text{Planet Y}) = 8\frac{m}{r^2} = 8g$

**4** A

$g \propto \frac{1}{R^2}$

If the radius to the surface of Earth is $R$, then a height of $2R$ above the surface is $3R$ from the centre of Earth.

$g_{3R} = \frac{1}{3^2} \times g_R$

$= \frac{9.76}{9}$

$= 1.08\,\text{N kg}^{-1}$

**5** D. The kinetic energy remains constant because the magnitude of the velocity remains constant. Kinetic energy is not a vector.

The momentum changes because the direction of the velocity changes. Momentum is a vector.

***VCAA examination report* note:**

31% of students answered this question correctly. 48% of students incorrectly selected option B.

**6** C. Approximately 13 squares.

Each square is $1000 \times 3 \times 10^6 = 3 \times 10^9$ J.

Therefore, energy under the curve is approximately $4.0 \times 10^{10}$ J.

**7** D

weight $= F_g$

$= mg$

$= 60 \times 9.8$

$= 588\,\text{N}$

**8** B. The period of the orbit is given by $T = \sqrt{\frac{4\pi^2 r^3}{GM}}$.

The period is determined by the radius and the mass of the central body. It is independent of the mass of the satellite.

**9** C

$g = \frac{GM}{r^2}$

$g = \frac{6.67 \times 10^{-11} \times 3.34 \times 10^{23}}{(2.44 \times 10^6)^2}$

$g = 3.74\,\text{N kg}^{-1}$

**10** D

$\Delta E_g = mg\Delta h$

$= 10 \times 3.7 \times 2$

$= 74\,\text{J}$

## Section B — Short answer questions

**11 a** The satellite must orbit the centre of mass or the gravitational/centripetal force must be directed towards the centre of Earth [1 mark] and the satellite must orbit the same axis or be in the same plane as Earth's rotation. [1 mark]

***VCAA examination report* note:**

Most students were able to identify the same plane / axis of rotation but few referred to the gravitational force.

**b** Combining Newton's law of gravitation with circular motion.

$r^3 = \sqrt{\left(\frac{GMT^2}{4\pi^2}\right)}$ [1 mark]

$r = \sqrt{\frac{6.67 \times 10^{-11} \times 5.89 \times 10^{24} \times 86\,400^2}{4\pi^2}}$ [1 mark]

$r = 4.225 \times 10^7$ m [1 mark]

Subtracting the radius of Earth:

$4.225 \times 10^7 - 6.37 \times 10^6 = 3.59 \times 10^7$ m [1 mark]

The altitude of a geostationary satellite must be equal to $3.59 \times 10^7$ m.

***VCAA examination report* note:**

The most common errors were to incorrectly transpose the original equations and to forget to subtract the radius of Earth to get the altitude.

**c** $v = \frac{2\pi r}{T}$ [1 mark]

$v = \frac{2\pi \times 4.225 \times 10^7}{86\,400}$ [1 mark]

$v = 3.07 \times 10^3\,\text{m s}^{-1}$ [1 mark]

***VCAA examination report* note:**

A number of students used $v = \sqrt{\frac{GM}{r}}$, which was also valid. The most common error was to use the radius of Earth.

Alternative calculations:

$v = \sqrt{\frac{GM_E}{R}}$

$= \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4.225 \times 10^7}}$

$= \sqrt{\frac{6.67 \times 5.98}{4.225} \times 10^6}$

$= \sqrt{\frac{6.67 \times 5.98}{4.225}} \times 10^3$

$= 3.07 \times 10^3\,\text{m s}^{-1}$

**12 a** $6.37 \times 10^6 + 600 \times 10^3 = 6.97 \times 10^6$ m [1 mark]

**b** $T = \sqrt{\frac{4\pi^2 R^3}{GM}}$ [1 mark]

$= \sqrt{\frac{4\pi^2 (6.97 \times 10^6)^3}{(6.67 \times 10^{-11})(5.98 \times 10^{24})}}$ [1 mark]

$= 5789.159\ldots$ [1 mark]

$= 5.79 \times 10^3$ seconds [1 mark]

***VCAA examination report* note:**

The most common error was to not cube the radius. Most students who were able to find the period were able to report it to three significant figures.

**c** The ICON satellite is subject only to a gravitational force towards Earth. [1 mark]

Further, this force is constant in magnitude. This is why the satellite maintains a stable circular orbit. [1 mark]

***VCAA examination report* note:**

Some students referred to the propulsion engines by pointing out that there is no significant friction to require a propulsion force to maintain an orbit.

The most common error was to point out that the satellite is in free fall, but this does not explain the orbit.

**d** The area under the graph is given by:

$$A = b \times \left(\frac{h_1 + h_2}{2}\right)$$

$$= 600 \times 10^3 \times \left(\frac{9.80 + 8.20}{2}\right) \quad \text{[1 mark]}$$

$$= 5.40 \times 10^6\ \text{J kg}^{-1} \quad \text{[1 mark]}$$

$$\Delta E_g = A \times m$$

$$= 5.40 \times 10^6 \times 288$$

$$= 1.56 \times 10^9\ \text{J} \quad \text{[1 mark]}$$

***VCAA examination report* note:**

There were two common errors. The first was to not recognise the broken $y$-axis and to find the area of the triangle only. The second was to use the formula $E_g = mg\Delta h$ without realising that $g$ is not constant over the $\Delta h$.

**13 a** At Y, the gravitational field strength = 9.8 N kg$^{-1}$ [1 mark]

Point Y is at a distance of one earth radii $R_E$ from the centre of Earth or at the surface of Earth.

At the surface of Earth, the strength of the gravitational field strength is 9.8 N kg$^{-1}$.

**b** At the centre of Earth, the vector sum of the gravitational fields caused by all the mass of Earth is zero. [1 mark]

At the centre of Earth there are equal masses in all directions, so the gravitational attraction from one direction is balanced by an equal attraction from the opposite direction. [1 mark]

**c** The increase in potential energy per kilogram is represented by the area under the first section graph (up to $R_E$).

The increase in potential energy for the 75 kg is the area under the graph multiplied by the mass of the person.

Increase in potential energy = area under the graph × mass of person

$$E = \left(0.5 \times 6.37 \times 10^6 \times 9.8\right) \times 75 \quad \text{[1 mark]}$$

$$E = 2.3 \times 10^9\ \text{J} \quad \text{[1 mark]}$$

Note: You cannot apply the gravitational potential energy formula $\left(E_g = mgh\right)$ because the gravitational field strength is not a constant. $g$ does not equal a constant 9.8 m s$^{-2}$ from the centre of Earth to the surface of Earth.

**14 a** The only force acting on the satellite is the force of gravity. [1 mark]

This force acts towards the (centre of) Earth. [1 mark]

When an object (a satellite) is in a stable circular orbit, the object is 'falling' around the central body (in this case, Earth).

An object that is falling (ignoring any air resistance) is only under the influence of gravity.

**b** $T = \sqrt{\dfrac{4\pi^2 R^3}{GM}}$

Radius of orbit = radius of Earth + altitude of satellite above Earth's surface

$$R = 6.37 \times 10^6 + 2.00 \times 10^7$$

$$R = 2.637 \times 10^7\ \text{m} \quad \text{[1 mark]}$$

(Do not round off radius or orbit before including it in the calculation.)

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}}$$

$$T = \sqrt{\frac{4\pi^2\left(2.637 \times 10^7\right)^3}{\left(6.67 \times 11^{-11}\right) \times \left(5.98 \times 10^{24}\right)}} \quad \text{[1 mark]}$$

$$T = 4.26 \times 10^4 \text{ seconds} \quad \text{[1mark]}$$

Answer must be to three significant figures.

**15 a** From the graph, the gravitational field strength at $2.0 \times 10^8$ m is 3 N kg$^{-1}$ [1 mark]

Since *Juno* has a mass of 1500 kg, the force on Juno is $1500 \times 3 = 4500$ N or $4.5 \times 10^3$ N. [1 mark]

***VCAA examination report* note:**

Many students calculated the force using Newton's law of gravitation. Students who did this found the value to be 4752 N. While both results were considered correct, the second method was mathematically more complex and errors in this method constituted the most common error for the question.

**b** The area under the graph between $2.0 \times 10^8$ m and $1.0 \times 10^8$ m is estimated at 14 squares. [1 mark]

Each square is equal to $\left(1.0\ \text{N kg}^{-1}\right) \times \left(0.5 \times 10^8\ \text{m}\right) = 0.5 \times 10^8\ \text{J kg}^{-1}$. [1 mark]

Therefore, the change in gravitational potential energy is given by:

$$\Delta GPE = 14 \times 0.5 \times 10^8 \times 1500$$

$$= 1.05 \times 10^{12}\ \text{J} \quad \text{[1 mark]}$$

***VCAA examination report* note:**

Allowances were made for students estimating a different number of squares.

Some students also used a difference in *GPE* approach using the formula $\Delta GPE = mg_f h_f - mg_i h_i$ and substituting the different $g$ values and $h$ values. This also led to the correct answer.

A number of students approximated the area under the graph as a trapezium. These students were not awarded full marks as a trapezium is an overestimation of the area.

**c** $T = \sqrt{\dfrac{4\pi^2 r^3}{GM}}$ [1 mark]

$$= \sqrt{\frac{4\pi^2\left(6.70 \times 10^8\right)^3}{\left(6.67 \times 10^{-11}\right)\left(1.90 \times 10^{27}\right)}} \quad \text{[1 mark]}$$

$$= 3.06 \times 10^5\ \text{s} \quad \text{[1 mark]}$$

***VCAA examination report* note:**

While most students realised that this question required a combination of Newton's law of gravitation and circular motion, many could not demonstrate a suitable formula and substitution. Forgetting to cube the radius was the most common mathematical error.

# Topic 4 — Electric fields and their applications

## 4.2 Coulomb's Law and electric force

### Sample problem 1

$$F_{\text{B1 on B2}} = \frac{8.99 \times 10^9 \times 2.0 \times 10^{-9} \times 2.0 \times 10^{-9}}{(0.100)^2} = 3.6 \times 10^{-6}\,\text{N}$$

The force has a positive sign, so the two balloons repel each other. (This can also be determined by the fact that both charges are the same, and will therefore repel.)

### Practice problem 1

$$F_{\text{e on p}} = \frac{8.99 \times 10^9 \times 1.6 \times 10^{-19} \times \left(-1.6 \times 10^{-19}\right)}{\left(5.3 \times 10^{-11}\right)^2} = -8.1 \times 10^{-8}\,\text{N}$$

This force is attractive. The direction of the force is in the opposite direction from the displacement of the electron from the proton.

### 4.2 Exercise

**1 a** $F = \frac{k|q_1||q_2|}{r^2}$

$$= \frac{8.99 \times 10^9 \times 10 \times 10^{-6} \times 120 \times 10^{-6}}{0.3^2} = \frac{8.99 \times 1.2}{3^2} \times 10^2 = 1.2 \times 10^2\,\text{N}$$

**b** Both charges are positive, and like charges repel, thus the electric force on the 10-μC charge will move it away from the 120-μC charge, which is to its left, thus moving it to the right.

The direction of the electric force on the 10-μC charge is to the right.

**2 a** $F = \frac{k|q_1||q_2|}{r^2}$

$$= \frac{8.99 \times 10^9 \times 5.0 \times 10^{-6} \times 7.0 \times 10^{-6}}{0.2^2} = \frac{8.99 \times 5.0 \times 7}{2^2} \times 10^{-1} = 7.9\,\text{N}$$

**b** Unlike charges attract, thus the electric force on the 5.0-μC charge will move it towards the −7.0-μC charge, which is to its left.

The direction of the electric force on the 50-μC charge is to the left.

**3 a** As $F = k\frac{|q_1||q_2|}{r^2}$, the magnitude of the force is inversely proportional to the square of the distance between the charges $\left(F \propto \frac{1}{r^2}\right)$. By doubling the distance between the charges, the magnitude of the force would decrease by a factor of $2^2 = 4$. Thus the magnitude of the new force is $\frac{1.20 \times 10^{-3}}{4} = 3.0 \times 10^{-4}\,\text{N}$

**b** As $F = k\frac{|q_1||q_2|}{r^2}$, the magnitude of the force is proportional to the product of the size of the charges ($F \propto |q_1 q_2|$). By adding a $2q$-C charge to B, $|q_1 q_2|$ changes from $q^2$ to $3q^2$, which is an increase by a factor of 3. Thus the magnitude of the new force is $3 \times 1.20 \times 10^{-3} = 3.6 \times 10^{-3}$ N.

Note that the magnitude of the force on A by B is the same as the magnitude of the force on B by A (Newton's Third Law).

**c** As $F = k\frac{|q_1||q_2|}{r^2}$, the magnitude of the force is proportional to the product of the size of the charges ($F \propto |q_1 q_2|$). By adding a $-3q$-C, and $|q_1 q_2|$ charge to A, its total charge is now −2 C changes from $q^2$ to $2q^2$, which is an increase by a factor of 2. Thus the magnitude of the new force is $2 \times 1.20 \times 10^{-3} = 2.4 \times 10^{-3}$ N.

**d** By halving the distance between the charges, the magnitude of the force would increase by a factor of $2^2 = 4$.

In addition, the product of the charges changes from $q^2$ to $\frac{1}{2} \times 4q^2 = 2q^2$ thus the magnitude of the force would be doubled.

In total, the magnitude of the new force is greater by a factor of 8: $8 \times 1.20 \times 10^{-3} = 9.6 \times 10^{-3}$ N.

**4**

$$F = k\frac{e^2}{d^2}$$

$$2.4 \times 10^{-18} = \frac{8.99 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{r^2}$$

$$r^2 = \frac{8.99 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{2.4 \times 10^{-18}} = \frac{8.99 \times 1.6^2}{2.4} \times 10^{-11} = 9.589 \times 10^{-11}$$

$$r = \sqrt{9.589 \times 10^{-11}} = 9.8 \times 10^{-6}\,\text{m}$$

**5** Let $r$ be the distance between the test charge $q$ and point A. Therefore, the distance between the test charge and point B is $0.20 - r$.

The magnitude of the electric force exerted by the +40-nC charge on the test charge is: $\frac{kq_1 q}{r^2}$, where $q_1 = +4.0 \times 10^{-8}$ C and the magnitude of the electric force exerted by the +90-nC charge on the test charge is: $\frac{k_{q2} q}{(0.20 - r)^2}$, where $q_2 = +9.0 \times 10^{-8}$ C.

The test charge experiences a zero net force when the magnitude of the force on the test charge by A is equal to the magnitude of the force on the test charge by B.

$$\frac{kqq_1}{r^2} = \frac{kqq_2}{(0.20 - r)^2}$$

$$\frac{4}{r^2} = \frac{9}{(0.2 - r)^2}$$

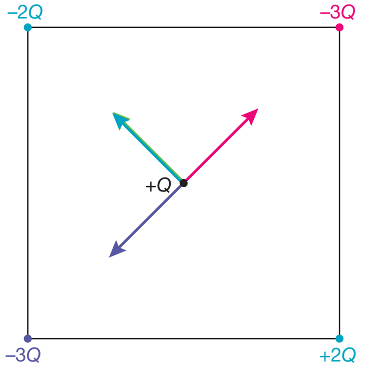
$$\Rightarrow \frac{2}{r} = \frac{3}{0.20 - r}$$

$3r = 2 \times (0.20 - r)$
$5r = 0.40$
$r = 0.08\,\text{m}$
$= 8\,\text{cm}$

**6** C. If one of the masses is negatively charged and the other is positively charged, then the electric force on the test charge by the $q_1$ charge and the electric force on the test charge by the $q_2$ charge will be in the same direction all the way along the line. Thus there will be no point on the line where the net charge will be zero.

## 4.2 Exam questions

**1** D. The electric force exerted by the top-right $-3Q$-charge on the test charge has the same magnitude as the electric force exerted by the bottom-left $-3Q$-charge but they are in opposite directions (both are attractive forces).
The force resulting from these two interactions on $Q$ is zero.
The electric force exerted by the top-left $-2Q$-charge on the test charge has the same magnitude as the electric force exerted by the bottom-right $+2Q$-charge and they are in the same direction (one is an attractive force, the other is repulsive). The force resulting from these two interactions on $Q$ is directed towards the $-2Q$-charge, as illustrated on the diagram below.




The direction of the net electric force on the test charge $+Q$ is ↖.

**2** $F = k\dfrac{q_1 q_2}{r^2}$ [1 mark]

$= \dfrac{9.0 \times 10^9 \times 2 \times 5}{1.5^2}$

$= 4.0 \times 10^{10}\,\text{N}$ [1 mark]

**3** $F = k\dfrac{q_1 q_2}{r^2}$ [1 mark]

$2.0 = \dfrac{9.0 \times 10^9 \times 1 \times 1}{r^2}$

$r^2 = 1.8 \times 10^{10}$

$\Rightarrow r = 1.3 \times 10^5\,\text{m}$ [1 mark]

**4** Let's take to the right as the positive direction and let $q$ be the value of the unknown charge.
The value of the electric force exerted by the unknown charge on the electron is:

$F = \dfrac{kqe}{r^2}$ [1 mark]

$100 = \dfrac{9.0 \times 10^9 \times \left(-1.6 \times 10^{-19}\right) \times q}{3.2^2}$

$\Rightarrow q = \dfrac{3.2^2 \times 100}{9.0 \times 10^9 \times \left(-1.6 \times 10^{-19}\right)}$

$= -7.1 \times 10^{14}\text{C}$ [1 mark]

Note that we can check the unknown force is negatively charged as like charges repel, and the force on the electron accelerates it away from the unknown charge (thus to the right).

**5** The magnitude of the gravitational force exerted by Earth on the hydrogen atom of mass $m_\text{H}$ on its surface is:
$F_\text{g} = m_\text{H} g$ [1 mark].
The magnitude of the electrical force between the proton and the electron separated by a distance $r$ is:
$F_\text{elec} = k\dfrac{e \times e}{r^2}$ [1 mark].
The magnitude of the electrical force between a proton and an electron separated by a distance $r$ is equal to the magnitude of the gravitational force exerted by Earth on a hydrogen atom on its surface when:

$m_\text{H} g = k\dfrac{e^2}{r^2}$ [1 mark]

$1.6735 \times 10^{-27} \times 9.8 = 9.0 \times 10^9 \times \dfrac{\left(1.6 \times 10^{-19}\right)^2}{r^2}$

$\Rightarrow r^2 = 9.0 \times 10^9 \times \dfrac{\left(1.6 \times 10^{-19}\right)^2}{1.6735 \times 10^{-27} \times 9.8}$

$= 1.40 \times 10^{-2}$

$\Rightarrow r = 1.2 \times 10^{-1}\text{m}$ [1 mark]

The electron and the proton should be approximately 12 cm apart.

## 4.3 The field model for point-like charges

### Sample problem 2

$E = \dfrac{kq}{r^2}$

$E = \dfrac{8.99 \times 10^9 \times 2.0 \times 10^{-7}}{(0.5)^2}$

$= 7.2 \times 10^3\,\text{N}\,\text{C}^{-1}$

### Practice problem 2

$E = \dfrac{kq}{r^2}$

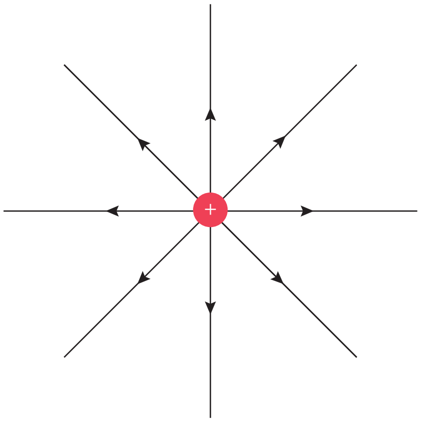
$= \dfrac{8.99 \times 10^9 \left(-3.0 \times 10^{-6}\right)}{(0.3)^2}$

$= -3.00 \times 10^5\,\text{N}\,\text{C}^{-1}$

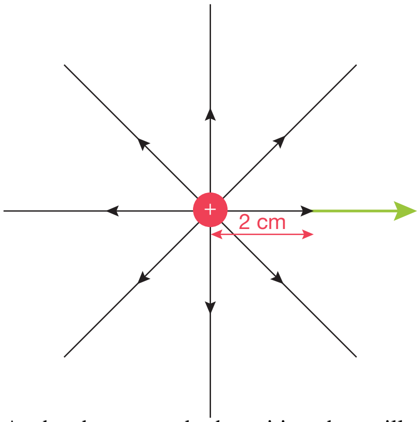
The negative sign indicates that the direction of the field is towards the point charge, that is, a positive test charge would be attracted to the field source.

## Sample problem 3

**a**

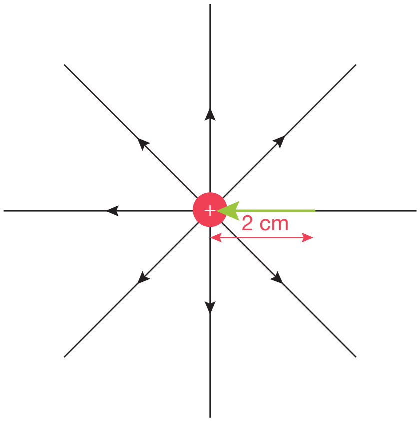


**b**



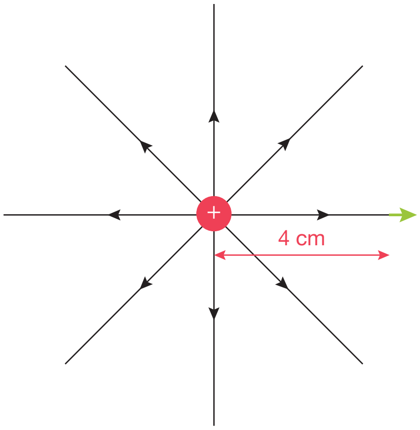

As the charges are both positive, they will repel each other.

**c**




As the charges are opposite, they will attract each other.

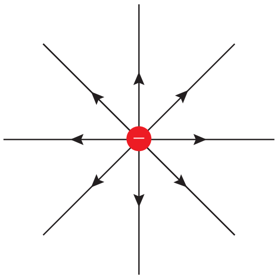
**d**



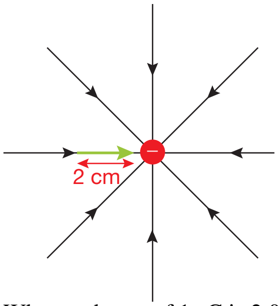

As the charges are both positive, they will repel each other. As the distance between the charges has doubled, the force between them has reduced to $\frac{1}{4}$ of the original force.

## Practice problem 3

**a**

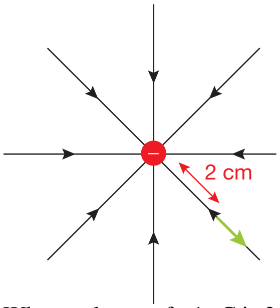


**b**



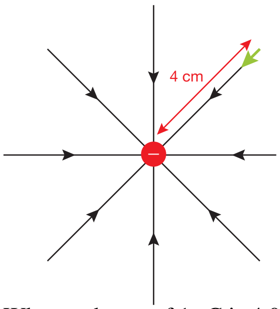

When a charge of 1 nC is 2.0 cm from the −1 μC charge, it experiences an attractive force, ***F***, towards the −1 μC charge.

**c**




When a charge of −1 nC is 2.0 cm from the −1 μC charge, it experiences a repulsive force, ***F***, of the same size, but away from the −1 μC charge.

**d**




When a charge of 1 nC is 4.0 cm from the −1 μC charge, it experiences an attractive force, $\frac{\boldsymbol{F}}{4}$, towards the −1 μC charge.

## 4.3 Exercise

**1** $E = \frac{kq}{r^2}$

$= \frac{8.99 \times 10^9 \times 120 \times 10^{-6}}{0.30^2}$

$= 1.2 \times 10^7\ \text{V m}^{-1}$

**2** $E = \frac{kq}{r^2}$

$= \frac{8.99 \times 10^9 \times 1.6 \times 10^{-19}}{\left(5.2 \times 10^{-6}\right)^2}$

$= \frac{8.99 \times 1.6}{5.2^2} \times 10^2$

$= 5.3 \times 10^1\ \text{V m}^{-1}$

**3 a** $F = qE$

$\Rightarrow E = \frac{1.5}{3.0 \times 10^{-6}}$

$= 5.0 \times 10^5\ \text{N C}^{-1}$

**b** $\boldsymbol{F} = q\boldsymbol{E}$

$\boldsymbol{F}$ is upwards and $q > 0$, thus $\boldsymbol{E}$ is upwards.

**4** $\boldsymbol{F} = q\boldsymbol{E}$

$\boldsymbol{F}$ is to the right and $q > 0$, thus $\boldsymbol{E}$ is to the left.

**5 a** $F = qE$

$\Rightarrow E = \frac{3.0}{1.5 \times 10^{-6}}$

$= 2.0 \times 10^6\ \text{N C}^{-1}$

**b** $\boldsymbol{F} = q\boldsymbol{E}$

$\boldsymbol{F}$ is downwards and $q < 0$, thus $\boldsymbol{E}$ is upwards.

**6** D. Electric field lines do not form closed loops. (Magnetic field lines do form closed loops.)

**7 a** The direction of an electric field at any point is given by the tangent to the field line through that point. If two field lines were to intersect, it would mean that the electric field at that point has two different directions (the two tangents to the field lines).

This is not possible, as a vector can only point in one direction.

**b** The direction of the force and, therefore, the acceleration will be along the field line.

Therefore, if the particle is stationary or already moving along the field line, it will move along the field line, but if the particle is crossing the field line, the direction of its initial velocity will change and it will not follow the field lines.

## 4.3 Exam questions

**1** C.

$F = \frac{kQ}{r^2}$

$F = \frac{9.0 \times 10^9 \times 5.0 \times 10^{-7}}{0.5^2}$

$F = 1.8 \times 10^4\ \text{V m}^{-1}$

**2** $\boldsymbol{F} = q\boldsymbol{E}$

$\boldsymbol{F}$ is upwards and $q < 0$, thus $\boldsymbol{E}$ is downwards.

**3** $E = \frac{kq}{r^2}$

$= \frac{8.99 \times 10^9 \times 6.0 \times 10^{-4}}{0.12^2}$ [1 mark]

$= 3.7 \times 10^8\ \text{V m}^{-1}$ [1 mark]

**4** $E = \frac{kq}{r^2}$

$80.5 = \frac{8.99.0 \times 10^9 \times q}{0.15^2}$ [1 mark]

$\Rightarrow q = 2.0 \times 10^{-10}\ \text{C}$ [1 mark]

**5** $E = \frac{kq}{r^2}$

$5.4 \frac{8.99 \times 10^9 \times 1.6 \times 10^{-19}}{r^2}$ [1 mark]

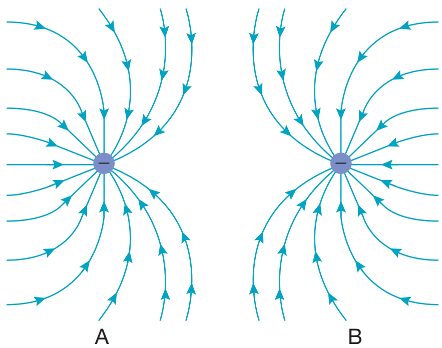
$\Rightarrow r^2 = 2.7 \times 10^{-10}$

$\Rightarrow r = 1.6 \times 10^{-5}\ \text{m}$ [1 mark]

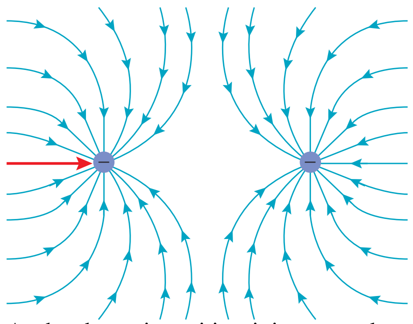
## 4.4 Electric fields from more than one point-like charge

### Sample problem 4

**a**




**b**

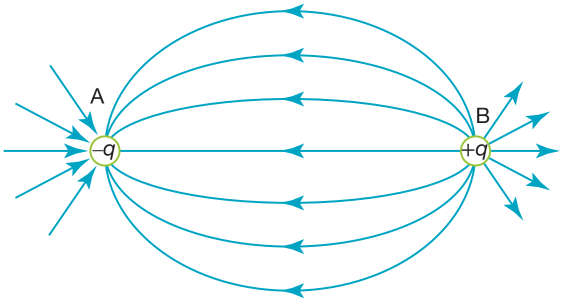


As the charge is positive, it is attracted to the negative particles A and B, which are directly to the right of the charge.

**c** When the 1.0-nC charge moves past particle A to the right-hand side, it experiences an attractive force to the left from particle A and also an attractive force to the right from particle B. Because the charge is closer to particle A than particle B, the force acting to the left is stronger than the force to the right, so the net (or overall) force on the 1.0-nC charge at this point is to the left (but it is smaller in magnitude than the force it experienced in part b).

As the charge is moved closer to particle B, the magnitude of the leftward net force decreases to zero at the point halfway between the two particles. When the charge moves closer to particle B than particle A, the net force acts to the right and becomes stronger as the charge moves closer to particle B.

### Practice problem 4




**a** A positive charge to the left of A would be attracted to the right towards the negatively charged particle (particle A).
A positive charge to the right of A would be both attracted to A by particle A and repelled towards A by particle B, so the force experienced by the charge would be larger, but to the left.

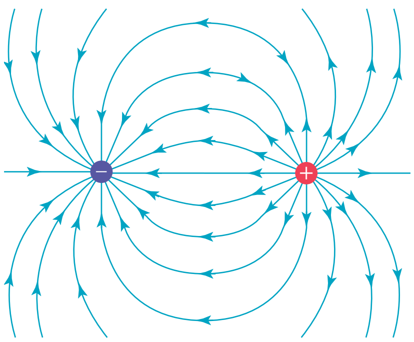
**b** As a positive charge moves along an imaginary line separating particles A and B, the direction of the force would always be to the left; however, it would be a minimum at the halfway point. Either side of the midpoint, the inverse decline of the strength of the field from one particle would be less than the increase in the strength of the other particle, causing the total field strength to increase.

## 4.4 Exercise

**1** **a** The field of A at P is to the right and strong, and the field of B at P is to the left and weaker $\left(E \propto \frac{1}{r^2}\right)$, so the resultant field is to the right towards B.

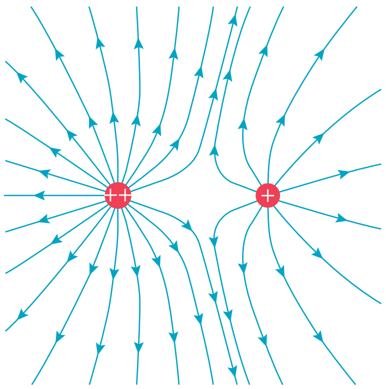

**b** The field of A at P is to the right (away from the positive charge) and strong, and the field of B at P is also to the right (towards the negative charge) but weaker, so the resultant field is to the right towards B.


**2**



The field lines point towards the negative charge, and away from the positive charge. The field lines should never cross.

**3**



**4** D. The field lines are pointing away from the charge at point A, and towards the charge at point B, thus the charge at point A is positive, and the one at point B is negative.
The density of the field lines around point A is higher than around point B, thus the size of the charge at point A is larger than at point B.

**5** **a** The electric field due to the +3.0-nC charge, at distance $x$ to the right of the charge, will point to the right (away from the +3.0-nC positive charge) and the electric field strength due to this charge has a value of: $k\frac{+3.0 \times 10^{-9}}{x^2}$.
The electric field due to the +2.0-nC charge at, at distance $(0.15 - x)$ to the left of the charge, will point to the left (away from the +2.0-nC positive charge) and the electric field strength due to this charge has a value of:
$k\frac{+2.0 \times 10^{-9}}{(0.15 - x)^2}$.
The value of the combined electric field strength from both particles, at distance $x$ from the +3.0-nC is:

$$E = k\frac{+3.0 \times 10^{-9}}{x^2} - k\frac{+2.0 \times 10^{-9}}{(0.15 - x)^2}$$

$$= 8.99 \times \left(\frac{3.0}{x^2} - \frac{2.0}{(0.15 - x)^2}\right)$$

See table at the bottom of the page*

**b** Taking to the right as the positive direction, the combined electric field strength from both particles, at distance $x$ from the +3.0-nC has a value of
$E = 8.99 \times \left(\frac{3.0}{x^2} - \frac{2.0}{(0.15 - x)^2}\right)$.
As the size of the charge on the left is larger than the size of the charge on the right, close to the +3.0-nC charge, the combined electric field will be to the right ($E \propto q$) (and thus the electric field strength will have a positive value).
When the distance to the +3.0-nC charge increases, while the distance to the +2.0-nC charge decreases, the contribution of the +2.0-nC charge to the combined electric field will increase $\left(\frac{2.0}{(0.15 - x)^2} \nearrow \text{ when } x \nearrow\right)$, while the contribution of the +3.0-nC charge will decrease $\left(\frac{3.0}{(x)^2} \searrow \text{ when } x \nearrow\right)$, and the electric field strength will

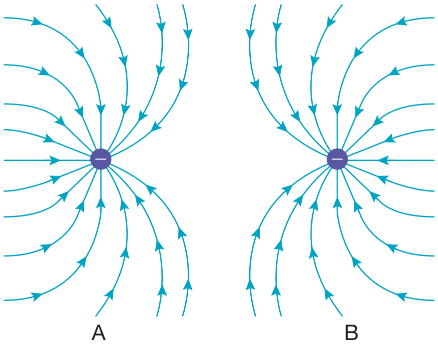
*5a

| $x$ (cm) | 1.0 | 2.5 | 5.0 | 7.5 | 8.0 | 8.5 | 9.0 | 10.0 | 12.5 | 14.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| $E\left(\text{N C}^{-1}\right)$ | $2.5 \times 10^5$ | $4.2 \times 10^4$ | $9.0 \times 10^3$ | $1.6 \times 10^3$ | $5.4 \times 10^2$ | $-5.2 \times 10^2$ | $-1.7 \times 10^3$ | $-4.5 \times 10^3$ | $-2.7 \times 10^4$ | $-1.8 \times 10^5$ |

decrease up to a point at which $\left(\frac{3.0}{x^2} = \frac{2.0}{(0.15 - x)^2}\right)$.
After that point, while still between the two particles, the contribution of the +2.0-nC charge will be larger than the contribution of the +3.0-nC charge and the electric field will point to the left (and thus the electric field strength will have a negative value).
This is confirmed by the results from part **a**, and from those values, we can estimate that the electric field changes direction at approximately 8 cm to the right of the +3.0-nC charge.
Josh is correct.
Note that solving $\left(\frac{3.0}{x^2} = \frac{2.0}{(0.15 - x)^2}\right)$ gives $x = 8.1742$ cm.

## 4.4 Exam questions

**1**



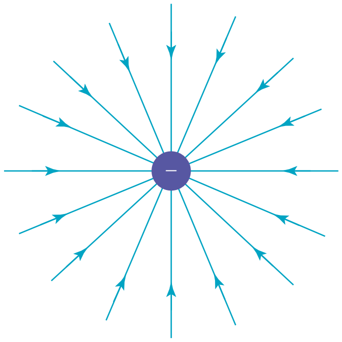

The resultant field is the vector sum of the fields from two negative charges.
The fields cancel each other out at the point halfway between the two charges. [1 mark]
Because both charges are negative, all field lines point in towards the charges. [1 mark]
Marks are deducted for field lines that cross or touch.

**2** A. Field lines show repulsion so both charges are the same. The field lines originate from the charges so they are both positive.

**3** The arrow starts at X and points to the left. [1 mark]
(*Note:* This is because the two nearest charges both exert electric forces to the left on a positive test charge at X. The negative charge on the far right exerts a force to the right but it is much smaller than the other two forces. So, the net force, and hence electric field, points left.)
***VCAA examination report*** **note:**
Students were required to draw a single, horizontal arrow pointing to the left through the point X. The most common error was to state that there was no field at point X.

**4** At X, the resultant field is the sum of the positive field to the right from the $+Q$ charge and the negative field to the left from the $-2Q$ charge.
$E \propto \frac{q}{r^2}$ thus, as the $-2Q$ charge is twice as far from X than the $+Q$ charge, and twice as large, the strength of the field from the $-2Q$ charge is only half of the strength of the field from the $+Q$ charge [1 mark] $\left(2 \times \frac{1}{2^2}\right)$, so the overall field is to the right. [1 mark]

**5 a**



The radial field lines should be distributed evenly around the centre charge. [1 mark]
The direction of the field lines should be pointing inwards (away from positive charges, towards negative charges). [1 mark]

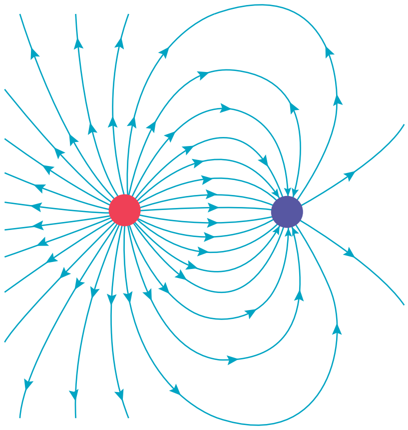
**b** $F = \frac{kq_1q_2}{r^2}$

$= \frac{8.99 \times 10^9 \times 5.0 \times 10^{-9} \times 15 \times 3.0 \times 10^{-9}}{(4 \times 10^{-2})^2}$ [1 mark]

$= 8.4 \times 10^{-5}$ N [1 mark]

**c** Unlike charges attract, thus the direction of the force on the +5.0-nC charge is towards the −3.0-nC charge, which is to its right. [1 mark]

**d** Yes.
Consider the following diaram:



Directly to the right of the −3.0-nC charge, the field lines will point to the left, back towards the charge. [1 mark]
At some point to the far right of the −3.0-nC charge, the field lines will be pointing to the right, as the total charge in the dipole is positive (−3.0 nC + 5.0 nC = +2.0 nC) [1 mark]. Therefore, somewhere in between these two points, there must be a point at which the electric field due to the 2 charges is zero [1 mark].

## 4.5 Uniform electric fields

### Sample problem 5

**a** $F = qE$
$= -1.6 \times 10^{-19}\,\text{C} \times 10\,\text{N}\,\text{C}^{-1}$
Hence, the magnitude of the force is $1.6 \times 10^{-18}$ N.

**b** The sign of the force is opposite to the sign of the field, so the electron experiences a force in the opposite direction to the electric field and will be accelerated towards the positive plate.

**c** $a = \dfrac{F_{net}}{m}$

$= \dfrac{1.6 \times 10^{-18}\,\text{N}}{9.1 \times 10^{-31}\,\text{kg}}$

$= 1.8 \times 10^{12}\,\text{m s}^{-2}$

The values of the initial velocity, acceleration and distance travelled are known, so these can be used in the equation $v^2 = u^2 + 2as$ to calculate the final velocity. For the distance, remember that the electron is halfway between the two plates, so the distance is 1.0 cm (converted to 0.01 m for use in the formula). You also need to ensure that you use your non-rounded value for acceleration during your calculations.

$v^2 = 0 + 2 \times 1.8 \times 10^{12}\,\text{m s}^{-2} \times 0.01\,\text{m}$

$\Rightarrow v = 1.9 \times 10^5\,\text{m s}^{-1}$

## Practice problem 5

**a** $F = qE$

$F = 1.6 \times 10^{-19}\,\text{C} \times 20\,\text{N C}^{-1}$

$= 3.2 \times 10^{-18}\,\text{N}$

**b** The proton experiences a force in the same direction as the electric field. The direction of the force on the proton will be towards the negative plate.

**c** $u = 0\,\text{m s}^{-1}$;

$a = \dfrac{F}{m} = \dfrac{3.2 \times 10^{-18}\,\text{N}}{1.67 \times 10^{-27}\,\text{kg}} = 1.92 \times 10^9\,\text{m s}^{-2}$;

$s = 0.01\,\text{m}$

$v^2 = u^2 + 2as$

$= 2 \times 1.92 \times 10^9\,\text{m s}^{-2} \times 0.01\,\text{m}$

$\Rightarrow v = 6.2 \times 10^3\,\text{m s}^{-1}$

## 4.5 Exercise

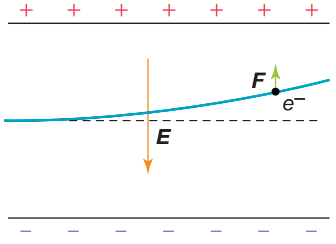
**1** The electric field points towards the negatively charged plate, and away from the positively charged plate, thus the top plate is the one positively charged.

**2** The electron is negatively charged, and it is deviated towards the top plate and away from the bottom plate, thus the top plate is positively charged and the bottom plate is negatively charged.

The electric field is thus directed towards the bottom plate (towards the negatively charged plate and away from the positively charged plate).

$\boldsymbol{F} = q\boldsymbol{E}$ and $q < 0$, thus the electric force points in the opposite direction as the electric field.

Alternatively, as the electron is accelerated towards the top plate, and $\boldsymbol{F} = m\boldsymbol{a}$, thus the electric force is upwards.




**3** **a** $a = \dfrac{F}{m}$

$= \dfrac{qE}{m}$

$= \dfrac{1.6 \times 10^{-19} \times 1.0 \times 10^6}{9.1 \times 10^{-31}}$

$= \dfrac{1.6}{9.1} \times 10^{18}$

$= 1.8 \times 10^{17}\,\text{m s}^{-2}$

**b** $u = 0$; $v = 3.0 \times 10^7\,\text{m s}^{-1}$

$v = u + at$

$\Rightarrow t = \dfrac{v}{a}$

$= \dfrac{3.0 \times 10^7}{1.8 \times 10^{17}}$

$= 1.7 \times 10^{-10}\,\text{s}$

**c** $v^2 = u^2 + 2as$

$\Rightarrow s = \dfrac{v^2 - u^2}{2a}$

$= \dfrac{\left(3.0 \times 10^7 - 0\right)^2}{2 \times 1.8 \times 10^{17}}$

$= 2.5 \times 10^{-3}\,\text{m}$

**4** **a** Time to cross the plates $= \dfrac{l}{v}$.

Acceleration towards the plate $= \dfrac{qE}{m}$.

Vertical deflection comes from:

$s = ut + \dfrac{1}{2}at^2$

$= 0 + \dfrac{1}{2} \times \dfrac{qE}{m} \times \left(\dfrac{l}{v}\right)^2$

$= \dfrac{qEl^2}{2mv^2}$

**b** Using a deflection of 1.2 mm and $s = \dfrac{qEl^2}{2mv^2}$

$1.2 \times 10^{-3} = \dfrac{1.2 \times 10^6 \times q \times 0.01^2}{2 \times 1.0 \times 10^{-10} \times 20^2}$

$q = \dfrac{1.2 \times 10^{-3} \times 2 \times 1.0 \times 10^{-10} \times 20^2}{1.2 \times 10^6 \times 0.01^2}$

$= 8.0 \times 10^{-13}\,\text{C}$

**5** A, B. The force on the charged particle is proportional to the magnitude of the electric field as is the acceleration experienced by the particle.

## 4.5 Exam questions

**1** B. $F = qE = 9.6 \times 10^{-19} \times 1 \times 10^4 = 9.6 \times 10^{-15}\,\text{N}$

**2** $F = qE$

$= 1.6 \times 10^{-19} \times 2.5 \times 10^4$ [1 mark]

$= 4.0 \times 10^{-15}\,\text{N}$ [1 mark]

**3** $F = qE$

$= ma$

$1.6 \times 10^{-19} \times 2.0 \times 10^4 = 9.1 \times 10^{-31} \times a$ [1 mark]

$\Rightarrow a = 3.5 \times 10^{15}\,\text{m s}^{-2}$ [1 mark]

**4** C

$$F = qE$$
$$= ma$$
$$1.6 \times 10^{-19} \times E = 9.1 \times 10^{-31} \times 1.8 \times 10^{14} \quad \text{[1 mark]}$$
$$\Rightarrow E = 1.0 \times 10^3\ \text{V C}^{-1} \quad \text{[1 mark]}$$

**5** **a** $a = \frac{F}{m}$

$$= \frac{qE}{m}$$
$$= \frac{2 \times 1.6 \times 10^{-19} \times 4.0 \times 10^6}{6.64 \times 10^{-27}} \quad \text{[1 mark]}$$
$$= 1.9 \times 10^{14}\ \text{m s}^{-2} \quad \text{[1 mark]}$$

**b** $u = 0$

$t = 1.0 \times 10^{-9}$ s

$a = 1.9 \times 10^{14}$ m s$^{-2}$

$$s = ut + \frac{1}{2}at^2$$
$$= 0 + \frac{1}{2} \times 1.9 \times 10^{14} \times \left(1.0 \times 10^{-9}\right)^2 \quad \text{[1 mark]}$$
$$= 9.5 \times 10^{-5}\ \text{m} \quad \text{[1 mark]}$$

**c** $v = u + at$

$$= 1.9 \times 10^{14} \times 1.0 \times 10^{-9} \quad \text{[1 mark]}$$
$$= 1.9 \times 10^5\ \text{m s}^{-1} \quad \text{[1 mark]}$$

## 4.6 Energy and motion of charges in electric fields and the linear accelerator

### Sample problem 6

$$E = \frac{V}{d}$$
$$= \frac{100}{0.05}$$
$$= 2.0 \times 10^3\ \text{V m}^{-1}$$

### Practice problem 6

$V = 30\,000\,000$ V; $d = 1.5$ km $= 1500$ m

$$E = \frac{V}{d}$$
$$E = \frac{30\,000\,000}{1500}$$
$$= 2.0 \times 10^4\ \text{V m}^{-1}$$

### Sample problem 7

**a** $\Delta E_k = qV$

$$= 1.6 \times 10^{-19} \times 100$$
$$= 1.6 \times 10^{-17}\ \text{J}$$

**b** $E_k = \frac{1}{2}mv^2$

$$\frac{1}{2} \times 9.1 \times 10^{-31} \times v^2 = 1.6 \times 10^{-17}\ \text{J}$$
$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-17}}{9.1 \times 10^{-31}}}$$
$$= 5.9 \times 10^6\ \text{m s}^{-1}$$

Note that the final speed of the electron is about 2% of the speed of light!

### Practice problem 7

**a** $\Delta E_k = qV$

$$= 1.6 \times 10^{-19} \times 1000$$
$$= 1.6 \times 10^{-16}\ \text{J}$$

**b** $E_k = \frac{1}{2}mv^2$

$$\frac{1}{2} \times 9.1 \times 10^{-31} \times v^2 = 1.6 \times 10^{-16}\ \text{J}$$
$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-16}}{9.1 \times 10^{-31}}}$$
$$= 1.9 \times 10^7\ \text{m s}^{-1}$$

### Sample problem 8

**a** $F = qE$

$$= -1.6 \times 10^{-19}\text{C} \times 10^4\ \text{N C}^{-1}$$
$$= -1.6 \times 10^{-15}\ \text{N}$$

The electric force is in the downward direction.

$$a = \frac{F}{m}$$
$$= \frac{-1.6 \times 10^{-15}\ \text{N}}{9.1 \times 10^{-31}\ \text{kg}}$$
$$= 1.758 \times 10^{15}\ \text{m s}^{-2}$$
$$= -1.8 \times 10^{15}\ \text{m s}^{-2} \text{ to 2 s. f.}$$

The acceleration is in the downward direction. Its magnitude is $1.8 \times 10^{15}$ m s$^{-2}$ to 2 s. f.

$$v^2 = u^2 + 2as$$
$$v^2 = 0 + 2 \times \left(-1.758 \times 10^{15}\ \text{m s}^{-2}\right) \times (-0.125\ \text{m})$$
$$\Rightarrow v = 2.1 \times 10^7\text{m s}^{-1}$$

The vertical velocity is downwards.

Its magnitude is $2.1 \times 10^7$m s$^{-1}$.

### Practice problem 8

**a** The magnitude of the acceleration of the electron is given by

$$a = \frac{F}{m}$$
$$= \frac{qE}{m}$$
$$= \frac{1.6 \times 10^{-19}\ \text{C} \times 2.0 \times 10^6\ \text{N C}^{-1}}{9.1 \times 10^{-31}\ \text{kg}}$$
$$= 3.516\ \text{m s}^{-2} \times 10^{17}\ \text{m s}^{-2}$$
$$= 3.6 \times 10^{17}\ \text{m s}^{-2} \text{ to 2 s. f.}$$

**b** Ignoring relativistic effects, the magnitude of the final velocity can be found using the SUVAT equation:

$v^2 = u^2 + 2as$, with $s = 25 \times 10^{-3}$ m

$$v^2 = u^2 + 2as$$
$$v^2 = 0 + 2 \times \left(3.516 \times 10^{17}\ \text{m s}^{-2}\right) \times \left(25 \times 10^{-3}\ \text{m}\right)$$
$$v^2 = 1.758 \times 10^{16}$$
$$\Rightarrow v = 1.3 \times 10^8\text{m s}^{-1} \text{ to 2 s. f.}$$

The magnitude of the final velocity is $1.3 \times 10^8$m s$^{-1}$ to 2 s. f.

## 4.6 Exercise

**1** $E = \frac{V}{d}$

$\Rightarrow d = \frac{100}{15.0}$

$= 6.7\text{ m}$

**2 a** $F = qE$, and the electric field strength is constant between the plates, thus the electric force on the charge $q$ would remain the same anywhere between the plates: $4.0 \times 10^{-3}$ N.

**b** $E = \frac{V}{d}$ and $F = qE$, thus $E \propto \frac{1}{d}$ and $F \propto \frac{1}{d}$.

If the distance between the plates changes from 20 cm to 15 cm, it is decreased by a factor of $\frac{15}{20} = \frac{3}{4}$, so the field strength and the magnitude of the force are increased by the same factor.

The magnitude of the force is now $\frac{4}{3} \times 4.0 \times 10^{-3} = 5.3 \times 10^{-3}$ N

**c** The force is increased by a factor of $\frac{6}{4} = \frac{3}{2}$, so the voltage is increased by the same factor, to $120 \times \frac{3}{2} = 180$ V.

**3** $W = qV$

$= qEd$

$= 2.0 \times 10^{-9} \times 200 \times 15 \times 10^{-3}$

$= 6.0 \times 10^{-10}$ J

**4** $\Delta E_k = qV$

$= qEd$

$= 10 \times 10^{-9} \times 300 \times 10^{-2}$

$= 3.0 \times 10^{-8}$ J

**5** $V = Ed$

$= 3.0 \times 10^{6} \times 5.0 \times 10^{-3}$

$= 1.5 \times 10^{4}$ V

**6 a** As the electron traverses the plates, it gains kinetic energy of 1000 eV.

**i** At the negative plate, $E_{PE} = 1000$ eV

$= 1.6 \times 10^{-19} \times 1000$

$= 1.6 \times 10^{-16}$ J

**ii** Halfway between, the potential difference is 500 eV.

$E_{PE} = 500$ eV

$= 1.6 \times 10^{-19} \times 500$

$= 8.0 \times 10^{-17}$ J

**iii** At the positive plate, $E_{PE} = 0$ V, and the electron has no electrical potential energy.

**b i** Halfway between the plates, 500 eV of electron potential energy has been converted into 500 eV of kinetic energy.

**ii** At the positive plate, 1000 eV of electron potential energy has been converted into 1000 eV of kinetic energy.

**7** Change in potential energy:

$V = \frac{\Delta E_K}{q}$

$= \frac{(1000\text{ eV} - 100\text{ eV})}{1}$

$E = \frac{V}{d}$

$= \frac{900}{0.01}$

$= 9.0 \times 10^{4}\text{ V m}^{-1}$

**8 a** The electrons are accelerated by the uniform electric field between the two oppositely charged conducting plates. The 100-V battery supplies the field to accelerate the electrons.

**b** $W = Vq$

$= 100 \times 1.6 \times 10^{-19}$

$= 1.6 \times 10^{-17}$ J

**c** The answer would be unchanged, because the potential difference is unchanged.

**d** The energy provided by the 6-V battery is used by the incandescent light (which glows red when the current flows). If the terminals of the 6-V battery were reversed, there would be no difference, as incandescent light bulbs are not polarised.

**e** If the terminals of the 100-V battery were reversed, the electric field between the plates would be to the right (towards the positive plate) and the force acting on the electron would be to the left (as an electron is negatively charged). Thus, the electron would not leave the left plate.

**f** $E \propto \frac{1}{d}$ and $F \propto E$, thus if $d$ is halved, then $E$ is doubled and $F$ is doubled too.

**g** When the distance between the plates is halved, the electric field strength and the magnitude of the force are both doubled, but the electric force does work over a distance that is halved, thus the work done is the same.

The stronger force in **(f)** acts over a shorter distance, achieving the same gain in kinetic energy as in **(c)** ($W = F \times d$).

**9** Halving the separation between the plates will double the electric field strength between the plates and double the acceleration of the electron. However, the overall energy per unit charge available from the field, or applied potential difference, will not change, so the energy of the emitted particles will not change.

**10** B, C

- The electric field is uniform between the plates and the strength of the electric field is $E\frac{V}{d} = \frac{500}{10^{-2}} = 5.00 \times 10^{4}\text{ V m}^{-1}$ ✗
- At a point halfway between the plates, the potential difference would be half of the 500 V across the plates. ✓
- the proton is positively charged, it will be accelerated towards the plate at the lowest potential difference. ✓
- The electric field between the plates is uniform, and $F = qE = ma$, thus the acceleration is constant, not the speed. ✗

## 4.6 Exam questions

**1 a** The principle is conservation of energy, where the work done on the charge by the field equals the change in kinetic energy of the particle. [1 mark]

$qV = \frac{1}{2}mv^2$ [1 mark]

$v = \sqrt{\frac{2qV}{m}}$

**b** $v = \sqrt{\frac{2qV}{m}}$

$= \sqrt{\frac{2\left(1.6 \times 10^{-19}\right)(200)}{9.1 \times 10^{-31}}}$ [1 mark]

$= 8.4 \times 10^{6}\ \text{m s}^{-1}$ [1 mark]

**2** $qV = \frac{1}{2}mv^2$ [1 mark]

$V = \frac{0.5 \times \left(9.1 \times 10^{-31}\right) \times \left(2.0 \times 10^{7}\right)^2}{1.6 \times 10^{-19}}$ [1 mark]

$= 1.1 \times 10^{3}\ \text{V}$ [1 mark]

**3** A. The electric field $E$ is related to the voltage difference between the plates ($V$) and the plate separation ($d$).

$E = \frac{V}{d}$

$\Rightarrow V = dE$

$= 5.0 \times 10^{-3} \times 1 \times 10^{3}$

$= 5.0\ \text{V}$

**4** A

$F = qE$

$= q\frac{V}{d}$

$= 1.6 \times 10^{-19} \times \frac{90 \times 10^{3}}{0.2}$

$= 7.2 \times 10^{-14}\ \text{N}$

**5 a** $E = \frac{V}{d}$

$= \frac{10\,000}{0.1}$ [1 mark]

$= 1.0 \times 10^{5}\ \text{V m}^{-1}$ [1 mark]

**b** $qV = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2qV}{m}}$

$= \sqrt{\frac{0.5 \times 1.6 \times 10^{-19} \times 1.0 \times 10^{5}}{9.1 \times 10^{-31}}}$ [1 mark]

$= 9.4 \times 10^{7}\ \text{m s}^{-1}$ [1 mark]

## 4.7 Review

## 4.7 Review questions

**1** $F = \frac{kq_1q_2}{r^2}$

$= \frac{8.99 \times 10^{9} \times 5.0 \times 10^{-6} \times 7.0 \times 10^{-6}}{0.2^2}$

$= \frac{8.99 \times 5.0 \times 7.0}{2^2} \times 10^{-1}$

$= 7.9\ \text{N}$

**2 a** $F = qE$

$\Rightarrow E = \frac{3.0}{-1.5 \times 10^{-6}}$

$= -2.0 \times 10^{6}\ \text{N C}^{-1}$

$= 2.0 \times 10^{6}\ \text{N C}^{-1}$

**b** $\boldsymbol{F} = q\boldsymbol{E}$ and $\boldsymbol{F}$ is downwards, while $q < 0$, thus $\boldsymbol{E}$ is upwards.

**3** $E = \frac{kq}{r^2}$

$= \frac{8.99 \times 10^{9} \times 1.6 \times 10^{-19}}{\left(1.0 \times 10^{-3}\right)^2}$

$= 8.99 \times 1.6 \times 10^{-4}$

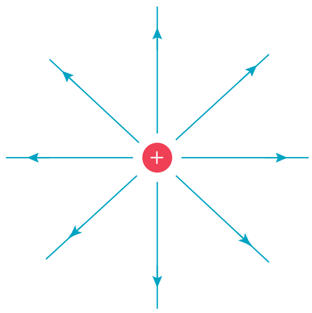
$= 1.4 \times 10^{-3}\ \text{V m}^{-1}$

**4** $\frac{50\ \text{mN}}{400\ \text{mN}} = \frac{1}{8}$

The magnitude of the force decreases by a factor $\frac{1}{8}$.

$F \propto \frac{1}{r^2}$, thus $r$ increases by a factor $\sqrt{8} = 2\sqrt{2}$

**5 a**




The radial field lines should be distributed evenly around the centre charge.

The direction of the field lines should be pointing outwards (away from positive charges, towards negative charges).

**b** $F = \frac{kq_1q_2}{r^2}$

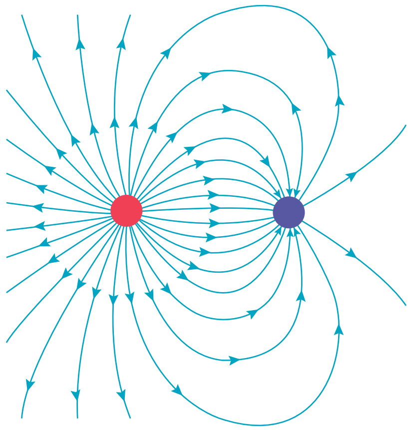
$= \frac{8.99 \times 10^{9} \times 10 \times 10^{-9} \times 15 \times 10^{-9}}{\left(2 \times 10^{-2}\right)^2}$

$= \frac{8.99 \times 1 \times 1.5}{2^2} \times 10^{-3}$

$= 3.4 \times 10^{-3}\ \text{N}$

**c** Yes.

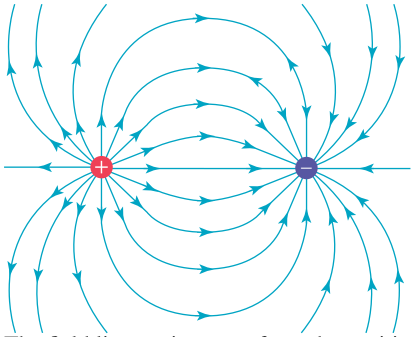
Consider the following diaram:



Directly to the right of the −10.0-nC charge, the field lines will point to the left, back towards the charge. At some point to the far right of the −10.0-nC charge, the field lines will be pointing to the right, as the total charge in the dipole is positive (−10.0 nC + 15.0 nC = +5.0 nC).

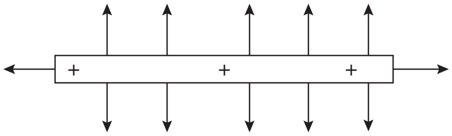
Therefore, somewhere in between these two points, there must be a point at which the electric field due to the two charges is zero.

**6**



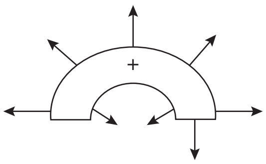
The field lines point away from the positive charge and towards the negative charge. The field lines never crosses.

**7 a**



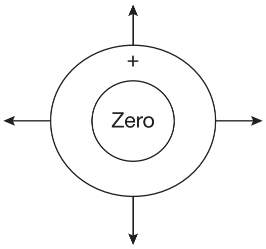
The field lines point away from the positive charges. The field lines are perpendicular to the surface.

**b**



The field lines point away from the positive charges. The field lines are perpendicular to the surface.

**c**




The strength of the electric field in the middle would be $0\,\text{V m}^{-1}$.

**8 a** The electron is charged negatively, it will be attracted to the plate charged positively, thus the direction of the electrical force is upwards: the arrow should point upwards.

**b** $V = Ed$

$= 100 \times 0.10$

$= 10\,\text{V}$

**c** The distance travelled in the field is $10 - 2.5 = 7.5\,\text{cm}$

$\Delta E_k = qE \times 0.075$

$= 1.6 \times 10^{-19} \times 100 \times 0.075$

$= 1.2 \times 10^{-18}\,\text{J}$

**9 a** The electric field is perpendicular to the surface of the plates, and is directed towards the negatively charged plate.

**b** $E = \dfrac{V}{d}$

$= \dfrac{2000}{3.0 \times 10^{-2}}$

$= \dfrac{2.000}{3.0} \times 10^5$

$= 6.7 \times 10^4\,\text{V m}^{-1}$

**c** $u = 0\,\text{m s}^{-1}; a = \dfrac{qE}{m}; s = 3.0 \times 10^{-2}\,\text{m}$

$s = ut + \dfrac{1}{2}at^2$

$\Rightarrow t^2 = \dfrac{2 \times 3.0 \times 10^{-2} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 6.7 \times 10^4}$

$= \dfrac{2 \times 3.0 \times 9.1}{1.6 \times 6.7} \times 10^{-10}$

$t = 2.3 \times 10^{-9}\,\text{s}$

**d** $W = qV$

$= 1.6 \times 10^{-19} \times 2000$

$= 3.2 \times 10^{-16}\,\text{J}$

**e** The kinetic energy change equals the work done.

$\Delta E_k = W$

$= 3.2 \times 10^{-16}\,\text{J}$

**f** The work done is proportional to the distance travelled in the field, so when the electron has travelled half the distance, it has had $1.6 \times 10^{-16}$ J of work done on it, and the kinetic energy has increased by $1.6 \times 10^{-16}$ J.

**g** If the applied potential difference is reversed, then the negative plate will become the positive plate and the electron will not leave it.

**10** Halving the separation between the plates will double the electric field strength between the plates and double the acceleration of the electron. However, the overall energy per unit charge available from the field, or applied potential difference, will not change, so the energy of the emitted particles will not change.

## 4.7 Exam questions

### Section A — Multiple choice questions

**1** B. Taking towards $Q$ as the positive direction, the electric field generated by $Q$ at a distance $x$ from $Q$ is $E_Q = -\dfrac{kQ}{x^2}$ and the the electric field generated by $4Q$ at a distance $x$ from $Q$ is $E_{4Q} = \dfrac{4kQ}{(12-x)^2}$.

The total electric field at a distance $x$ from $Q$ is

$E_T = E_Q + E_{4Q}$

$= kQ\left(-\dfrac{1}{x^2} + \dfrac{4}{(12-x)^2}\right)$.

Thus the electric field is zero when

$E_T = 0$

$\Rightarrow x = 4$

**2** D

$E = \dfrac{V}{d}$

$= \dfrac{5.0 \times 10^3}{10 \times 10^{-3}}$

$= 5.0 \times 10^5\,\text{V m}^{-1}$

**3** B. The outward field lines from P and R shoe them to be positive while the inward field lines for Q and S show them to be negative.

**4** B. Each $+2Q$ charge exerts an equal attractive force on the $-Q$ charge. The vector sum of these forces is straight down.

**5** B

$$E = \frac{V}{d}$$

$$2.0 \times 10^{-4} = \frac{V}{1.0 \times 10^{-2}}$$

$$V = \left(2.0 \times 10^{-4}\right) \times \left(1.0 \times 10^{-2}\right)$$

$$V = 2.0 \times 10^{-6}\ \text{V}$$

**6** D

$$E = \frac{kq}{r^2}$$

$$= \frac{8.99 \times 10^9 \times 2 \times 10^{-6}}{3^2}$$

$$= 2 \times 10^3\ \text{V m}^{-1}$$

**7** D

$$qV = \Delta E_k$$

$$\Rightarrow V = \frac{\frac{1}{2}mv^2}{q}$$

$$= \frac{9.1 \times 10^{-31} \times \left(8 \times 10^7\right)^2}{2 \times 1.6 \times 10^{-19}}$$

$$= 18.2\ \text{k V}$$

**8** C

$$qV = \Delta E_\text{k}$$

$$= \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = \frac{2qV}{m}$$

$$= \frac{2 \times 1.6 \times 10^{-19} \times 50 \times 10^3}{9.1 \times 10^{-31}}$$

$$= 1.76 \times 10^{10}$$

$$\Rightarrow v = 1.3 \times 10^8\ \text{m s}^{-1}$$

**9** B

$$W = qV$$

$$= 1.6 \times 10^{-19} \times 90 \times 10^3$$

$$= 1.44 \times 10^{-14}\ \text{J}$$

$$= \frac{1.44 \times 10^{-14}}{1.6 \times 10^{-19}}$$

$$= 90 \times 10^3\ \text{eV}$$

$$= 90\ \text{keV}$$

**10** A. The two point charges form an electric dipole. Field lines need to point away from the positive charge and point in towards the negative charge.

## Section B — Short answer questions

**11 a** The sphere will move up. The sphere is negatively charged, and the upper plate is positively charged.

**b** $F = \frac{q}{V}$ [1 mark]

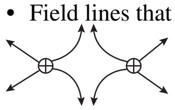
$$= \frac{2.7 \times 10^{-6} \times 15.5}{2.0 \times 10^{-3}}$$

$$= \frac{2.7 \times 1.55}{2} \times 10^{-2}$$

$= 2.1 \times 10^{-2}\text{N}$ [1 mark]

**12** Award 1 mark for each of:

- Correct field pattern with at least eight field lines
- Field lines that do not touch or cross.



**13 a** $E = \frac{V}{d}$ [1 mark]

$$= \frac{5000}{0.1}$$

$= 5 \times 10^4\ \text{V m}^{-1}$ [1 mark]

$(= 5 \times 10^{-4}\ \text{N C}^{-1})$

**b** $F = qE$ [1 mark]

$$= 5 \times 10^4 \times 1.6 \times 10^{-19}$$

$= 8 \times 10^{-15}\ \text{N}$ [1 mark]

**c** $qV = \frac{1}{2}mv^2$ [1 mark]

$$1.6 \times 10^{-19} \times 5000 = 0.5 \times 9.1 \times 10^{-1} \times v^2$$

$$v^2 = 1.8 \times 10^{15}$$

$v = 4.2 \times 10^7\ \text{m s}^{-1}$ [1 mark]

**14** The arrow starts at X and points to the left. [1 mark]
(*Note:* This is because the two nearest charges both exert electric forces to the left on a positive test charge at X. The negative charge on the far right exerts a force to the right but it is much smaller than the other two forces. So, the net force, and hence electric field, points left.)

***VCAA examination report* note:**
Students were required to draw a single, horizontal arrow pointing to the left through the point X. The most common error was to state that there was no field at point X.

**15 a** $F = \frac{kQ_pQ_e}{r^2}$

$= \frac{9.0 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{\left(53 \times 10^{-12}\right)^2}$ [1 mark] / [1 mark]

$$= 8.2 \times 10^{-8}\ \text{N}$$

Award 1 mark for the correct denominator, 1 mark for the correct numerator and award no marks if only the answer is given.

***VCAA examination report* note:**
The requirement to show all working was stated in the instructions for Section B and in the stem of the question. Students who did not show the first two lines of the above solution as their working were not awarded full marks.
The most common error was to fail to square the radius.

**b** $F_\text{net} = ma = \frac{mv^2}{r}$ [1 mark]

$\Rightarrow v = \sqrt{\frac{rF}{m}} = \sqrt{\frac{53 \times 10^{-12} \times 8.2 \times 10^{-8}}{9.1 \times 10^{-31}}}$ [1 mark]

$= 2.19 \times 10^6 = 2.2 \times 10^6\ \text{m s}^{-1}$ [1 mark]

***VCAA examination report* note:**
The most common error was to attempt to use $F = BqV$ as a starting point.

# Topic 5 — Magnetic fields and their applications

## 5.2 Magnets and magnetic fields

### Sample problem 1

**a** At X, the field lines point away from the north pole of the bar magnet, so the north point of the compass placed there would point towards the right, away from the north pole.
At Y, the field lines point towards the left, so the north point of the compass would point towards the left.
At Z, the field lines curve, so the north point of the compass would follow the tangent to the curve and would point to the right at an angle of approximately 45° upwards.

**b** The field lines are most dense at X and least dense at Y, so the points in order of increasing field strength would be Y, Z, X.

### Practice problem 1

**a** The north end of a compass placed just to the left of the north pole of the horseshoe magnet would point to the left, along the field lines.

**b** The north end of a compass placed between the north and south poles of the horseshoe magnet would point to the right, along the field lines.

**c** The field in between the poles of the horseshoe magnet would be the strongest, as can be seen from the more closely spaced field lines.

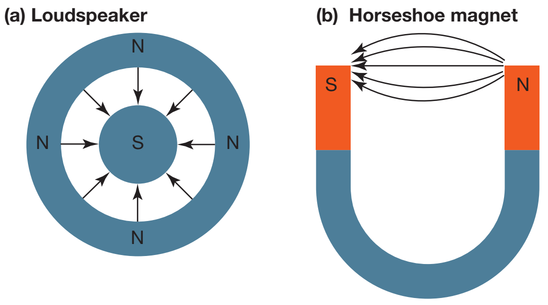
## 5.2 Exercise

**1** If a permanent magnet attracts a piece of metal, that metal must be a magnetic material. However, it is more accurate to describe that piece of metal as a ferromagnetic material, meaning that it is capable of being induced by a permanent magnet to behave like a magnet. Just being attracted to a permanent magnet does not mean that a piece of metal is a magnet; a permanent magnet must have two poles — a north pole and a south pole. Thus, a piece of metal is a permanent magnet if it also repels a known permanent magnet, when like poles are brought close together.

**2** A permanent magnet has two poles, both of which are able to induce ferromagnetic materials, such as iron nails, to behave like magnets; hence, either end of a permanent magnet is able to attract an iron nail.

**3** Since the magnetic pole of the southern hemisphere attracts the south pole of a magnet, the polarity of Earth's magnetic field in the southern hemisphere must be north.

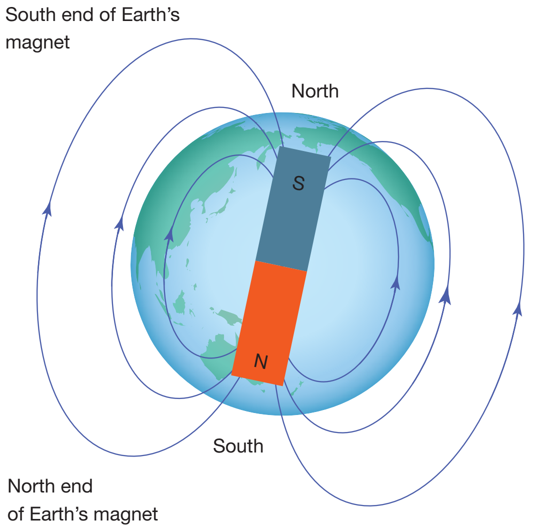
**4**



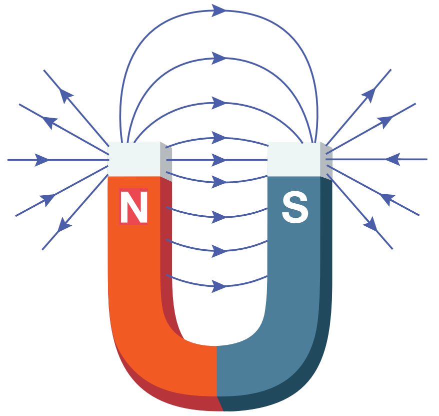

**5** As pieces of ferromagnetic material cool, the magnetic domains within the material are aligned by the Earth's magnetic field, thus forming natural permanent magnets.

## 5.2 Exam questions

**1** Magnetic field lines are always leaving the north end of the magnet and entering the south end of the magnet.



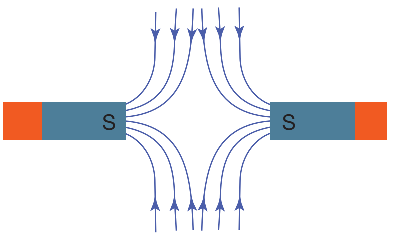

**2** A. The magnetic field between the ends of a horseshoe magnet is fairly uniform, and the field lines are oriented as shown below




thus the compass needle will be horizontal, pointing to the north end of the magnet.

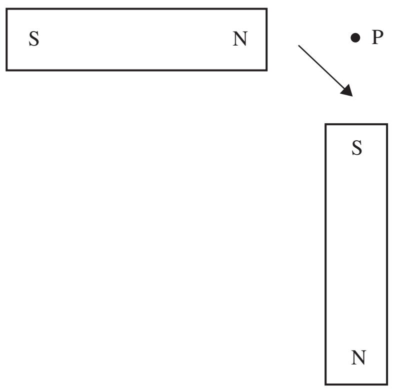
**3** **a** Like poles repel and unlike poles attract. The magnetic force between these two south poles is repulsive.

**b** For each bar magnet, the field lines loops are leaving the north pole and entering the south pole [1 mark], and the field lines should not intersect [1 mark].




**4** C. The source of a magnetic field cannot be a monopole, that is, an isolated north or south pole.

**5 a**




Award 1 mark for a single arrow as shown.

***VCAA Assessment Report* note:**

The most common error was to draw a field line from N to S.

**b** The correct answer is found by using Pythagoras's theorem.

$B = \sqrt{(10.0 \times 10^{-3})^2 + (10.0 \times 10^{-3})^2}$ [1 mark]

$B = \sqrt{2 \times 10^{-4}}$

$B = 1.41 \times 10^{-2}$ T [1 mark]

$B = 14.1$ mT

***VCAA Assessment Report* note:**

The most common error was to simply add the field strengths.

## 5.3 Magnetic fields from moving charged particles

### Sample problem 2

**a** The current points upwards, so, by applying the right-hand-grip rule, the magnetic field circulates around the wire pointing into the page at *P*.

**b** The current points downwards, so, by applying the right-hand-grip rule, the magnetic field circulates around the wire, pointing out of the page at *P*.

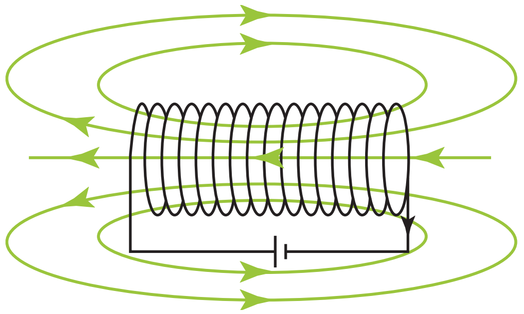
### Practice problem 2

**a** With the right-hand thumb pointing to the right, the fingers curl clockwise around the wire, so, at point Z, the field would be coming out of the page, perpendicular to the wire.

**b** With the right-hand thumb pointing into the page, the fingers would curl clockwise in the plane of the page around the wire, so that, at the point Z, the magnetic field would be pointing vertically upwards.

### Sample problem 3

Applying the right-hand-grip rule, you can see that the magnetic field passes through the centre of the coil from right to left. Because magnetic field lines must form closed loops, the field outside the coil is in the opposite direction. Inside the coil, the magnetic field is approximately uniform. Outside the coil, the strength of the magnetic field decreases away from the coil.

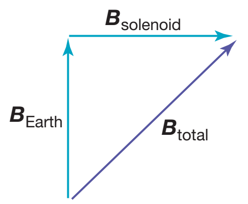


### Practice problem 3

If the current in the solenoid were reversed, then applying the right-hand-grip rule would reveal that the magnetic field through the solenoid would now be to the right. The field lines would be the same as in the sample problem, just with the directions reversed.

### Sample problem 4

**a** The combined vector sum is




The diagram shows that the resultant would point in a north-east direction.

**b** $\boldsymbol{B} = \sqrt{(\boldsymbol{B}^2_{solenoid} + \boldsymbol{B}^2_{Earth})}$

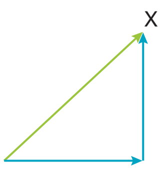
$= \sqrt{(25^2 + 25^2)}\ \mu T$

$= 35\ \mu T$

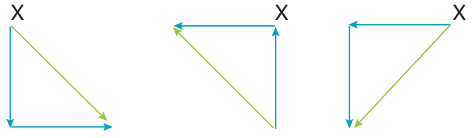
### Practice problem 4

**a** The magnetic fields add vectorially (ensure vectors are added head to tail), so the magnitude of the resultant field (shown by the green arrow in the following diagram) is

$\boldsymbol{B} = \sqrt{0.01^2 + 0.01^2} = 0.014$ T.




**b** The direction of the resultant field depends on the orientation of the two magnets and will be at 45° to the initial magnetic fields (as shown by the green arrows in the following diagrams).

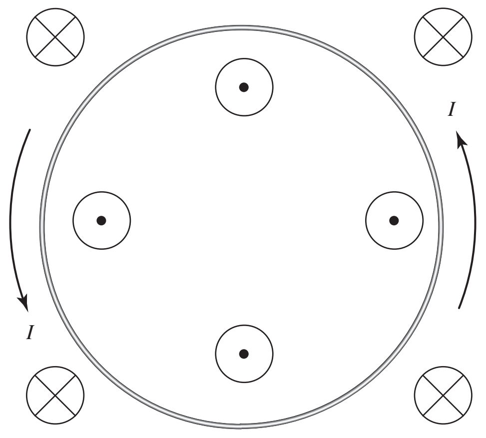


## 5.3 Exercise

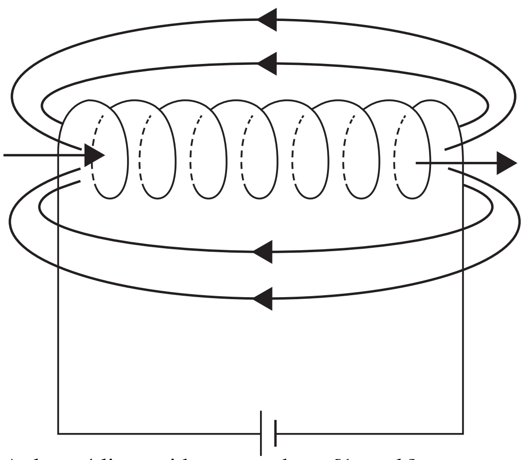
1 **a** Out of the page
  **b** Out of the page
  **c** To the right
2 **a** Up the page
  **b** To the left
  **c** Diagonally down to the right
  **d** Into the page
  **e** Out of the page
3 **a** At W, the magnetic field is coming out of the page. At X, Y and Z, the magnetic field is going into the page.
  **b** At W and X, the magnetic field is going from left to right. At Y and Z, the magnetic field is going to the left.
4 Since the iron rods are inside the solenoid, they are induced to become temporary magnets (when the solenoid has a current running through it), both with equivalent orientations of their magnetic poles. Thus, they repel each other because like poles repel.
5 **a** Using the right-hand-grip rule with the thumb pointing east in the direction of the current, the magnetic field points north. Therefore, the compass needle direction remains the same.
  **b** Using the right-hand-grip rule with the thumb pointing west in the direction of the current, the magnetic field points south. Therefore, the compass needle points in the opposite direction.

## 5.3 Exam questions

1 C. Using the right-hand-grip rule, holding the loop of wire with the thumb pointing in the direction of the current $I$, the curling of the fingers around the wire is into the page, thus the direction of the magnetic field is into the page.
2 Using the right-hand-grip rule (your fingers wrap around the wire, pointing out of the page inside the loop, and into the page outside of the loop), your thumb gives the direction of the current $I$. The direction of the current is anti-clockwise.

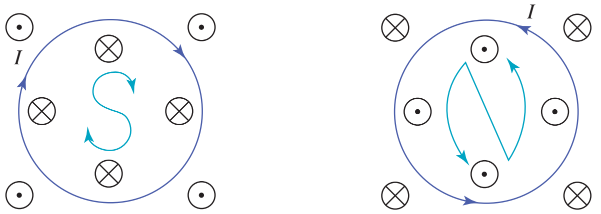



3



At least 4 lines with correct shape [1 mark]
Correct direction of arrows [1 mark]
(First mark lost if lines clearly touch or cross)
***VCAA Assessment Report*** **note:**
Many students drew their diagrams in faint pencil, which made them very hard to see.

4 C. The magnetic field inside a solenoid is approximately uniform, outside it is non-uniform.
5 Using the right-hand-grip rule, we can determine that for the face with the current going clockwise, and identified as a south face with this mnemonic tool, the direction of the magnetic field inside the loop is into the page, meaning that the field lines enter the solenoid by this face. [1 mark]
For the face with the current going anti-clockwise, and identified as a north face with this mnemonic tool, the direction of the magnetic field inside the loop is out of the page, meaning that the field lines emerged from the solenoid by this face. [1 mark]




The field lines emerge from the north face of a solenoid, and enter a solenoid from its south face, therefore this mnemonic tool can be used to correctly identify the faces of a solenoid.

## 5.4 Using magnetic fields to control charged particles, cyclotrons and mass spectrometers

### Sample problem 5

**a** $F = qvB$
$= -1.6 \times 10^{-19} \times 1.5 \times 10^{5} \times 0.060$
$= 1.4 \times 10^{-15}$ N
**b** $v$ is to the left and will be along the thumb.
$B$ is out of the page and follows the direction of the fingers.
The force on the electron is in the direction of the palm, namely pointing up the page.

## Practice problem 5

**a** $F = qvB$

$= 2 \times 1.6 \times 10^{-19} \times 1.0 \times 10^{7} \times 0.060$

$= 1.9 \times 10^{-13}$ N

**b** Apply the right-hand-slap rule ($I$ is along the thumb and $B$ is along fingers). The force is into the page.

**c** The positron is moving parallel to $B$, so there is no force on the positron.

## Sample problem 6

$$r = \frac{mv}{qB} = \frac{9.1 \times 10^{-31} \times 5.9 \times 10^{6}}{1.6 \times 10^{-19} \times 6 \times 10^{-3}} = 5.6 \times 10^{-3} \text{ m}$$

## Practice problem 6

$$qvB = \frac{mv^2}{r}$$

$$\Rightarrow \quad v = \frac{1.6 \times 10^{-19} \times 3 \times 10^{-3} \times 0.001}{9.11 \times 10^{-31} \text{ kg}} = 5.3 \times 10^{5} \text{ m s}^{-1}$$

## Sample problem 7

$$v = \frac{E}{B}$$

$$v = \frac{500}{0.0030} = 1.7 \times 10^{5} \text{ m s}^{-1} \text{ through the Wien filter}$$

## Practice problem 7

$$v = \frac{E}{B}$$

$$\Rightarrow E = 6 \times 10^{6} \times 0.02 = 1.2 \times 10^{5} \text{ N C}^{-1}$$

## 5.4 Exercise

**1** $F = q v \boldsymbol{B}$

$= 1.6 \times 10^{-19} \times 3.0 \times 10^{5} \times 4.0 \times 10^{-3}$

$= 1.9 \times 10^{-16}$ N

**2 a** $\boldsymbol{F} = 0$ N, because velocity is parallel to the magnetic field.

**b** The particle would pass through the magnetic field undeflected.

**3 a** $\boldsymbol{F} = q\boldsymbol{vB}$

$= 1.6 \times 10^{-19} \times 1.2 \times 10^{5} \times 2.4$

$= 4.6 \times 10^{-14}$ N, perpendicular to the direction of travel

**b** The electron would experience a magnetic force that is perpendicular to the direction of its travel according to the right-hand-slap rule, and thus would be deflected in that direction. As long as the electron remains in the magnetic field, it would experience a force in the perpendicular direction and would undergo uniform circular motion. If it moves out of the magnetic field, it would cease experiencing that magnetic force and would move in a straight line.

**c** $a = \dfrac{q\boldsymbol{vB}}{m}$

$$= \frac{1.6 \times 10^{-19} \times 1.2 \times 10^{5} \times 2.4}{9.1 \times 10^{-31}} = 5.1 \times 10^{16} \text{ m s}^{-2}$$

**4 a** Net force is radially directed into the middle of the arc, so, initially when the tauon enters the magnetic field, the force is downwards. Applying the right-hand-slap rule, with the thumb following the velocity to the right and the fingers following the magnetic field into the page, the force on a positive particle is upwards, which means that the charge on the tauon must be negative, reversing the direction of the force.

**b** After the tauon exits the field, there is no magnetic force acting on it, so, according to Newton's first law, it continues with the same velocity, in a straight line downwards.

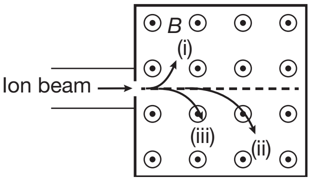
**5 a** Down the page

**b** Out of the page

**c** Down the page and right.

**6** The electron would remain undeflected by being injected parallel to the direction of the magnetic field, either in the same direction or in the opposite direction.

**7**




The magnetic field is coming out of the page. It is important to show that the electron (i) is deflected upwards, and the proton (ii) and the alpha particle (iii) are deflected downwards, with the angle of (ii) greater than that of (iii). As the mass to charge ratio is smaller for a proton, the radius is smaller, which means there is more deflection, as shown in (ii).

Note: The exact angles do not matter.

**8 a** $r = \dfrac{mv}{Bq}$

$$= \frac{9.1 \times 10^{-31} \times 3 \times 10^{7}}{4 \times 1.6 \times 10^{-19}} = 0.043 \text{ mm}$$

**b** $r = \dfrac{mv}{Bq}$

$$= \frac{1.67 \times 10^{-27} \times 3 \times 10^{7}}{4 \times 1.6 \times 10^{-19}} = 78 \text{ mm}$$

**c** $r = \dfrac{mv}{Bq}$

$$= \frac{4 \times 1.67 \times 10^{-27} \times 3 \times 10^{7}}{4 \times 2 \times 1.6 \times 10^{-19}} = 157 \text{ mm}$$

**9** Magnetic fields control the movement of particles by separating ions based on their charge and their mass. As different ions (or isotopes) have different mass to charge ratios, they have different path radii as they move. This allows a mass spectrometer to separate ions in a sample with the aid of a magnetic field.

**10** The positron has a component of its velocity parallel to the field and a component of its velocity perpendicular to the field. Applying the right-hand-slap rule, it can be seen that the component of velocity perpendicular to the field will be accelerated, causing the positron to execute anti-clockwise circles into the page. The magnitude of the velocity perpendicular to the field won't change, just the direction. In the meantime, the component of the velocity parallel to the field is unaffected, so the overall trajectory of the positron will be an anti-clockwise helix, with the circular motion perpendicular to the page.

**11** $v = \frac{E}{B}$

$8.0 \times 10^6 = \frac{5000}{B}$

$B = 0.63\ \text{m T}$

**12** $E = \frac{V}{d}$

$= \frac{1000}{0.0015}$

$= 6.7 \times 10^5\ \text{V m}^{-1}$

$\Rightarrow v = \frac{E}{B}$

$= \frac{6.7 \times 10^5}{0.5}$

$= 1.3 \times 10^6\ \text{m s}^{-1}$

## 5.4 Exam questions

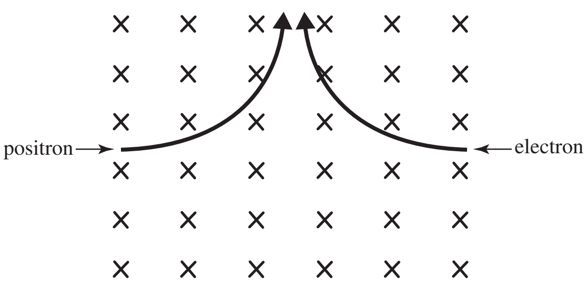
**1.** $D = 2r$

$= 2\frac{mv}{qB}$ [1 mark]

$= 2\frac{4.80 \times 10^{-27} \times 3.16 \times 10^5}{1.6 \times 10^{-19} \times 0.10}$ [1 mark]

$= 1.9 \times 10^{-1}\ \text{m}$ [1 mark]

**2**




Award 2 marks for each correct path and 2 marks for a suitable explanation.

Both arcs are circular and have the same radius, as the magnitude of their speed, mass and charge is the same and the magnetic field is uniform $\left(r = \frac{mv}{Bq}\right)$.

Using the right-hand-slap rule for the positron, with your fingers pointing into the page for $\boldsymbol{B}$ and your thumb pointing to the right for $\boldsymbol{v}$, the force $\boldsymbol{F}$ is up the page for a positively charged particle, the positron is deviated up the page.

Similarly, using the right-hand-slap rule for the electron, with your fingers pointing into the page for $\boldsymbol{B}$ and your thumb pointing to the left for $\boldsymbol{v}$, the force $\boldsymbol{F}$ is down the page for a positively charged particle, and thus up the page for a negatively charged particle. The electron is also deviated up the page.

**3** A. Using the right-hand-slap rule:

Fingers N to S, thumb clockwise ⇒ Force is out of the page.

**4 a** Using the right-hand-slap rule for a positive charge:

- your fingers point in the direction of the magnetic field, here into the page
- your thumb points in the direction of the velocity, here to the right
- your palm is upward, thus the direction of the force is up the page for a positive charge

The particle is deviated down the page, thus is it negatively charged.

Charge $q$ is negative. [1 mark]

**b** The force applied to the particle would always be at right angles to the direction of motion (right-hand-slap rule) and the force would be constant in magnitude ($F = Bqv$, where $B$, $q$ and $v$ are constant). [1 mark]

Because the force is constant and always at right angles, the motion of the particle is circular. [1 mark]

**5** $Bqv = \frac{mv^2}{r}$

$r = \frac{mv}{Bq}$

$= \frac{1.7 \times 10^{-27} \times 1.0 \times 10^6}{2.0 \times 10^{-2} \times 1.6 \times 10^{-19}}$ [1 mark]

$= 7.4 \times 10^{-1}\ \text{m}$ [1 mark]

## 5.5 Magnetic forces on current-carrying wires

### Sample problem 8

**a** $F = nIlB$

$= 1 \times 300 \times 10^{-3} \times 0.08 \times 0.25$

$= 6.0 \times 10^{-3}\ \text{N}$

**b** The force on the wire is 0 N.

### Practice problem 8

**a** $F = nIlB$

$= 1 \times 5.00 \times 10^{-5} \times 250 \times 100$

$= 1.25\ \text{N}$

**b** $B = \frac{F}{nIl}$

$= \frac{3.5}{1 \times 15 \times 0.100}$

$= 2.3\ \text{T}$

## 5.5 Exercise

**1** $F = nIl\mathbf{B}$

$= 1 \times 0.3 \times 4.5 \times 0.05$

$= 0.0675\ \text{N}$

$\approx 0.07\ \text{N}$

**2** As the component is parallel to the wire, $F = 0\ \text{N}$.

**3** $F = I\ \boldsymbol{B}$

$= 15 \times 10^{-3} \times 500 \times 2\pi \times 1.5 \times 10^{-2} \times 2$

$= 1.4\ \text{N}$

**4 a** Using the left-hand rule:
- index finger points east, like the magnetic field
- middle finger points north, like the current
- thumb points into the page, like the magnetic force

The direction of the magnetic force is into the page.

**b** Using the left-hand rule:
- index finger points south, like the magnetic field
- middle finger points east, like the current
- thumb points into the page, like the magnetic force

The direction of the magnetic force is into the page.

**c** Using the left-hand rule:
- index finger points west, like the magnetic field
- middle finger points out of the page, like the current
- thumb points south, like the magnetic force

The direction of the magnetic force is south.

**d** Using the left-hand rule:
- index finger points north, like the magnetic field
- middle finger points east, like the current
- thumb points out of the page, like the magnetic force

The direction of the magnetic force is out of the page.

**e** Using the left-hand rule:
- index finger points out of the page, like the magnetic field
- middle finger points west, like the current
- thumb points north, like the magnetic force

The direction of the magnetic force is north.

**f** Using the left-hand rule:
- index finger points into the page, like the magnetic field
- middle finger points south, like the current
- thumb points east, like the magnetic force

The direction of the magnetic force is east.

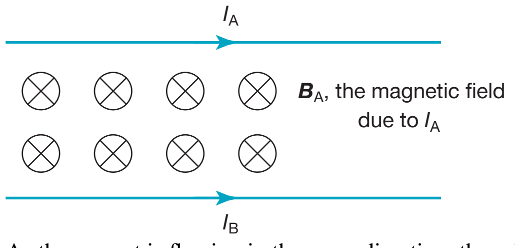
**g** Using the left-hand rule:
- index finger points out of the page, like the magnetic field
- middle finger points north-east, like the current
- thumb points south-east, like the magnetic force

The direction of the magnetic force is south-east.

**h** Using the left-hand rule:
- index finger points into the page, like the magnetic field
- middle finger points west, like the current
- thumb points south, like the magnetic force

The direction of the magnetic force is south.

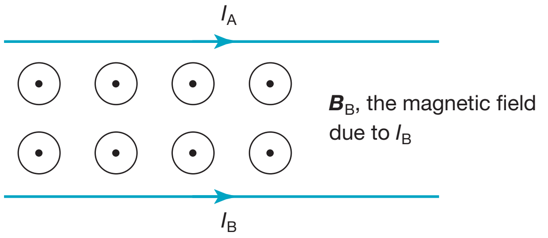
**5 a**




As the current is flowing in the same direction, there is an attractive force occurring between the wires.
The magnetic field is moving into the page, according to the right-hand-slap rule.

**b** $F_{\text{on B by A}}$ would be up the page according to the right-hand-slap rule:
- fingers point into the page, like the magnetic field
- thumb points to the right, like the current
- palm is upwards, indicating the direction of the magnetic force as north, or up the page

**c**




$\boldsymbol{F}_{\text{on B by A}}$ would be down the page according to the right-hand-slap rule

**d** Yes, because according to Newton's Third Law of Motion, $\boldsymbol{F}_{\text{on B by A}}$ is equal in magnitude but opposite in direction to $\boldsymbol{F}_{\text{on A by B}}$.

## 5.5 Exam questions

**1** D. The right-hand-slap rule should have been used, with fingers towards N, thumb towards W and palm facing downwards.

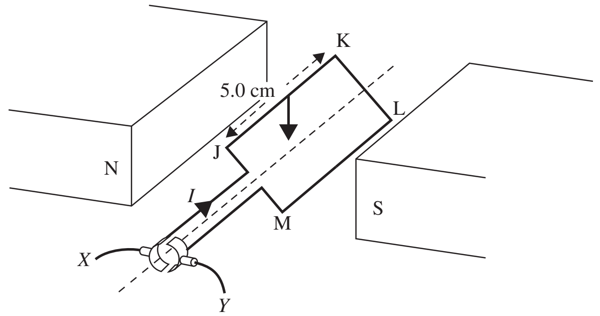
**2** C

$F = BIl$

$F = 5.0 \times 10^{-5} \times 1000 \times 1$

$F = 5.0 \times 10^{-2}$ N

**3 a**




Award 1 mark for the arrow drawn correctly as per the following diagram.
The direction of the force on side JK can be determined using the right-hand-slap rule.
The B field (fingers) is from north to south, therefore across the page from left to right.
The thumb (current) points from J to K.
The force (palm) on the side JK is therefore down.

**b** $F = nBIl$

$F = 100 \times 0.45 \times 6.0 \times 0.05$ [1 mark]

$F = 13.5$ N [1 mark]

Note: There is no mark assigned for writing the formula down. You must show correct substitution of values to receive the first mark.

**4 a** The force $\boldsymbol{F}$ on a wire of length $l$ carrying a current $I$ at right-angles to a magnetic field $\boldsymbol{B}$ is given by:

$$F = IlB$$

$$\Rightarrow B = \frac{F}{Il}$$

$$= \frac{0.32}{2000 \times 3.0} \quad \text{[1 mark]}$$

$$= 5.3 \times 10^{-5}\ \text{T} \quad \text{[1 mark]}$$

**b** The answer is C. [1 mark]
Using the right-hand-slap rule: the fingers point north; the thumb points down; the palm pushes east. [1 mark]

***VCAA Assessment Report* note:**
Many students ignored the information in the stem and tried to explain their own understanding of the flow of current in a lightning strike. Students are reminded to carefully read the question stem.

**5** $F = nIlB$
$= 10 \times 4 \times 4 \times 10^{-2} \times 2 \times 10^{-3}$ [1 mark]
$= 3.2 \times 10^{-3}$ N [1 mark]
Direction (by right-hand-slap rule) is UP [1 mark]
***VCAA Assessment Report* note:**
The most common errors were omitting the $n = 10$ value or failing to convert centimetres to metres. Correct use of formulas and the ability to convert between units are considered core skills for Physics students, and they are advised to be familiar with these skills.

## 5.6 Applying magnetic forces — the DC motor

### Sample problem 9

**a** $\boldsymbol{F} = nI\boldsymbol{B}$
**i** $\boldsymbol{F} = 1 \times 1.5 \times 0.020 \times 0.0050$
$= 1.5 \times 10^{-4}$ N, down
**ii** $\boldsymbol{F} = 0$ N,
because $I$ and $\boldsymbol{B}$ are parallel to each other
**iii** $\boldsymbol{F} = 1 \times 1.5 \times 0.020 \times 0.0050$
$= 1.5 \times 10^{-4}$N, up

**b** Torque is a maximum when the axis of rotation and the line of action of the force are perpendicular to each other, and the distance between them is at a maximum. This occurs when the coil is parallel to the field.

**c** Without the commutator, after the loop passes through the vertical position, the direction of the forces on the coil would reverse, causing the loop to slow down and then reverse the direction of rotation. Every time the loop passes through the vertical position, the direction of the forces reverse again, causing the loop to slow and reverse the direction of rotation. The loop would just oscillate around the vertical position. The commutator enables the direction of current through the loop to be reversed every time the loop passes through the vertical position, reversing the direction of the forces so that the loop can rotate continuously in the one direction.

### Practice problem 9

**a** $\boldsymbol{F} = nI\boldsymbol{B}$
**i** $\boldsymbol{F} = 1 \times 1.5 \times 0.020 \times 0.0050$
$= 1.5 \times 10^{-4}$N, up
**ii** $\boldsymbol{F} = 0$ N,
because $I$ and $\boldsymbol{B}$ are parallel to each other
**iii** $\boldsymbol{F} = 1 \times 1.5 \times 0.020 \times 0.0050$
$= 1.5 \times 10^{-4}$ N, down

**b** The only difference is that the coil rotates in the opposite direction from the coil in the sample problem.

## 5.6 Exercise

**1 a** The magnet provides a magnetic field.
**b** The brushes enable current to flow from stationary conductors, from a DC power supply to the rotating conductor coils.
**c** The commutator enables the circuit to switch the direction of the current within the course of one rotation so that the coil can rotate continuously in the same direction. The change in current direction occurs twice every rotation of the coil.
**d** Each turn of conducting wire experiences the magnetic force, so the more turns there are, the larger the force and the faster the rotation.

**2 a** If the coil is perpendicular to the magnetic field, the forces on the sides of the coil are co-linear, resulting in no turning force (or torque) on the coil and hence it would not rotate. Alternatively, in this same position, the brushes may lose contact with the commutator, resulting in current not flowing. This can be overcome by having a second coil at right angles to the first coil, each coil having its own section of the split ring commutator.
**b** Yes. This can be achieved either by connecting the motor to an AC power supply or by replacing the split ring commutator with a slip ring commutator.
**c** Yes. This can be achieved by either increasing the voltage of the DC power supply and thereby increasing the current and turning force on the coil.

**3** No. The motor would just run backwards and forwards.

**4** The two sides of the loop form a parallel circuit supplied by the battery. The current flowing through the loop interacts with the field of the button magnet to create a torque on the loop. As the loop rotates, the direction of the torque is the same, so the loop rotates continuously.

**5 a** The field coils (electromagnet) and the armature coils (rotating) may be connected in series to each other or in parallel to each other.
**b** The series-wound motor produces a large starting torque (turning force), making it ideal as motors for trains and trams. The parallel-wound motor can regulate its speed over a range of loads, making it ideal for power tools.

**6** D. Increasing the number of windings would increase the average torque. Torque is calculated using the formula $\boldsymbol{\tau} = r{\perp}\boldsymbol{F}$. The force is calculated using the formula $\boldsymbol{F} = nI\boldsymbol{B}$. Therefore, increasing the number of windings ($n$) will increase the torque.

**7** C. As the loop passes through the position of minimum torque, which would now be when the loop is horizontal, the current through the loop would not change direction, so the direction of the torque on the loop would change, stopping it from rotating continuously in the one direction.

## 5.6 Exam questions

**1 a** Applying the right-hand-slap rule (fingers to right, palm up), the current flows from K to L.
***VCAA examination report* note:**
The most common reason for not awarding full marks was to refer to a ‘right-hand rule’ with no context or explanation. As there is also a right-hand-grip rule, which

is not applicable here, failure to make the identification of the applied rule clear meant that the mark could not be awarded. Alternative names such as 'right-hand push' or 'right-hand force' were also accepted.

**b** Using the right-hand-slap rule, the current is flowing from $L$ to $K$.

**c** $F = nBIL$

$0.15 = 1 \times 0.5 \times I \times 0.10$ [1 mark]

$I = \frac{0.15}{0.05}$

$I = 3\text{ A}$ [1 mark]

***VCAA examination report*** **note:**

The most common errors were mathematical. Many students had the correct substitution but incorrect answers.

**2 a** $F = nIBl$

$F = 10 \times 2.0 \times 0.1 \times 2.0 \times 10^{-3}$ [1 mark]

$F = 4.0 \times 10^{-3}\text{ N}$ [1 mark]

**b** Students were required to indicate that the split ring commutator reverses the current in the loop every half turn to keep the motor turning in the same direction.

The purpose of the commutator is to reverse the direction of current every half-turn. [1 mark]

This ensures the motor continues rotating in the same direction. [1 mark]

**3** Clockwise (C) [1 mark]

The current flows H→G→F→E (recall that current is always flowing from the positive terminal of the battery to the negative terminal). The current on side HG is directed inside the page; hence, if we apply the right-hand-palm rule, the force on the coil HG is down. Similarly, the current on side FE is directed outside the page and hence the right-hand-palm rule shows that there will be a force up. [1 mark]

These two forces down on HG and up on FE cause the coil to start rotating clockwise. [1 mark]

***VCAA examination report*** **note:**

The most common error was not linking all three aspects: the current direction, the rule that relates current to force and the subsequent direction of rotation. Many students stated that because the current flows from H to G the side HG is forced down, without giving any reference to a link between current and force. Alternatively they would state that due to the right-hand rule, the side HG is forced down, without referencing current.

Students need to practice identifying and linking concepts within their responses rather than leaping from assertion to conclusion.

**4 a** $I = \frac{V}{R}$

$= \frac{9.0}{6.0}$

$= 1.5\text{ A}$ [1 mark]

$\Rightarrow F = nIlB$

$= 10 \times 1.5 \times 12 \times 10^{-2} \times 0.50$

$= 0.90\text{ N}$ [1 mark]

Right-hand-palm rule gives direction as $D$ (vertically down). [1 mark]

***VCAA examination report*** **note:**

The most common errors were omitting the 10 loops, failing to convert 12 cm to 0.12 m or stating the wrong direction.

**b** The current inside KL is **parallel** to the magnetic field. [1 mark]

Hence, the force on that side is 0 N. [1 mark]

***VCAA examination report*** **note:**

Students were required to identify that the current inside KL runs parallel to the magnetic field or magnetic flux. A number of students stated that the reason was because the current was not perpendicular to the magnetic field. Students should be aware that the current does not have to be perpendicular for a force to exist, and that any angle between the current and the field greater than zero degrees will result in a force on the wire. Students are reminded to think carefully about their wording.

**5 a** Both A and B were correct. The magnetic force on the side XY will be zero when it is horizontal with the current as shown in Figure 18 AND horizontal with the current in the opposite direction to that shown in Figure 18.

**b** When the coil is horizontal [1 mark]

the turning effect (torque) is a maximum

OR the force is at right-angles to the plane of the coil. [1 mark]

***VCAA Assessment Report*** **note:**

Many students struggled with this question and were not able to identify horizontal as the correct position. Some students who were able to identify horizontal as the correct position were often unable to clearly articulate why.

**c** Both A and B. [1 mark]

The size of turning force is $F = nIlB$. Increasing voltage increases current $I$; Increasing number of turns increases $n$. [1 mark]

***VCAA Assessment Report*** **note:**

The most common error was students not being able to identify both improvements.

## 5.7 Similarities and differences between gravitational, electric and magnetic fields

### Sample problem 10

Similarities:

1. The gravitational field generated from a point mass and the electric field generated from a positive charge both obey an inverse square law.
2. The gravitational field and electric field are both non-uniform and static.
3. The point mass and positive charge can both be described as monopoles.

Differences:

1. The direction of the gravitational field is inwards towards the point mass, whereas the electric field generated from a positive charge is outwards, away from the point charge.
2. The gravitational field is attractive for all other masses, whereas the electric field generated from a positive charge repels other positive charges and attracts negative charges.

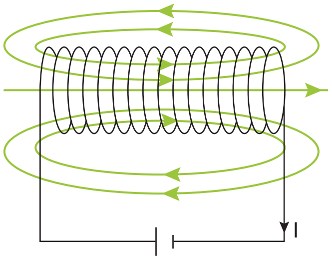
## Practice problem 10

The electric field from an electric dipole and the magnetic field from a bar magnet or solenoid have very similar overall shapes. The key differences are as follows:

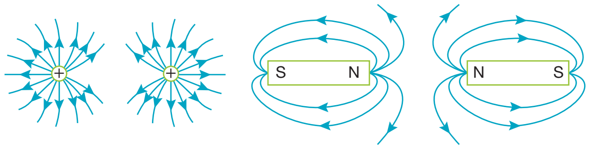
1. For a bar magnet, there are no field lines inside the magnet between the poles.
2. All the magnetic field lines must form closed loops so that all field lines must eventually loop back to pass through the solenoid from the other side.
3. In the electric dipole, the direction of the field lines change sign as they pass through the poles. This is not the case for magnetic field lines, and it is not possible to isolate points in space as 'the north pole' or 'the south pole'.

## 5.7 Exercise

1. Repulsion and attraction were both observed.
2. Similarities: the inverse square law exists, proportional to the product of a property of the objects (a consequence of Newton's third law).
   Difference: mass cannot be positive or negative, whereas charge can.
3. A uniform magnetic field is found inside a solenoid whereas the field outside the solenoid is non-uniform, as illustrated below.




4. The overall field shapes are similar; however, the magnetic field lines all form closed loops, and, unlike the positive charges, the north poles cannot be isolated to single points.




5. $F_g = G\dfrac{M_1M_2}{r_2}$

$$= \frac{6.67\times10^{-11}\times9.11\times10^{-31}\times1.67\times10^{-27}}{(5.3\times10^{-11})^2}$$
$$= 3.61\times10^{-47}\ \text{N}$$

$F_e = k\dfrac{q_1q_2}{r_2}$

$$= \frac{8.99\times10^{9}\times1.6\times10^{-19}\times1.6\times10^{-19}}{(5.3\times10^{-11})^2}$$
$$= 8.1\times10^{-8}\ \text{N}$$

From these calculations, it can be seen that the electric force is 39 orders of magnitude greater than the gravitational force; therefore, the gravitational force of attraction has virtually no influence on the motion of the electron and proton in a hydrogen atom.

6. $qE = mg$

$$\Rightarrow E = \frac{mg}{q}$$
$$= \frac{1.67\times10^{-27}\times10}{1.6\times10^{-19}}$$
$$= 1.0\times10^{-7}\ \text{N C}^{-1}$$

7. $F_e = F_g$

$$10\times1.6\times10^{-19}\times E = 1.0\times10^{-8}\times9.8$$
$$\Rightarrow E = 6.1\times10^{10}\ \text{N m}^{-1}$$

Note that this value for the electric field is far too high. This is an oversimplified version of the experiment. The experiment is not conducted in a vacuum, so there are also drag forces acting on the drop, reducing the size of the electric field needed for the forces on the drop to be balanced and for the oil drop to move at a constant speed.

8. $\dfrac{GM_1M_2}{r^2} = \dfrac{kq^2}{r^2}$

$$\Rightarrow q^2 = \frac{GM_1M_2}{k}$$
$$= \frac{6.67\times10^{-11}\times5.98\times10^{24}\times7.35\times10^{22}}{8.99\times10^{9}}$$
$$= 3.26\times10^{27}\ \text{C}$$
$$\Rightarrow q = 5.7\times10^{13}\ \text{C}$$

Students don't need to know the separation because it cancels, as both forces follow inverse square laws.

9. $50\times10^{-6} = \dfrac{4\pi\times10^{-7}\times I}{2\pi\times0.02}$

$$\Rightarrow I = 5.0\ \text{A}$$

## 5.7 Exam questions

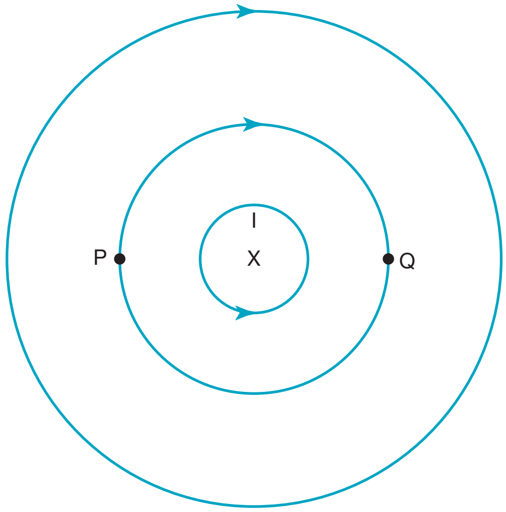
1. C. A static magnetic field exists around magnetised materials and around a wire carrying a constant current.
2. D. This field shape is not possible for a gravitational field, nor for a magnetic field (it looks similar to the field shape for a bar magnet but remember that magnetic field lines form continuous loops, leaving the north end of the magnet and entering the south end of the magnet).
   It is possible for an electric field, and given the orientation of the field lines, the point charge on the left would be negatively charged, while the point charge on the right would be positively charged. Thus among those options, an electric dipole is the only possible source.
3. The field lines are directed towards the source. This diagram could correspond to the gravitational field of a point mass [1mark], or the electric field of a negatively charged point charge [1 mark] (as the electric field lines point away from positive charges and towards negative charges).
4. Answers will vary.
   This field shape represents a uniform field [1 mark]. The following are example sources of gravitational, electric and magnetic uniform fields, respectively:
   - near the surface of a planet or star or other large mass
   - between charged conducting plates
   - inside a large solenoid (or between the arms of a large horseshoe magnet)

**5** Within the scope of this course, Lara is correct in noting that masses are always the sources of gravitational fields and electric charges are always the source of electrical fields. [1 mark]
However, magnetic fields can also be created by moving charges, not just by magnetic materials. [1 mark]
Note that electric fields can also be created by changing magnetic fields (and thus by time-varying electric currents), this is, however, outside the scope of this course.

## 5.8 Review

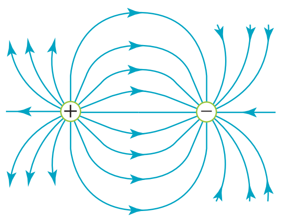
## 5.8 Review questions

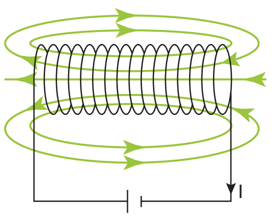
**1 a**




**b** The field from the wire at P is 20 μT, in the same direction as the Earth's magnetic field, so the total magnetic field strength is 30 μT pointing up the page. The field from the wire at Q is 20 μT in the opposite direction to the Earth's magnetic field, so the total magnetic field strength is 10 μT pointing down the page.

**2** The field from an electric dipole and the magnetic field from a solenoid have a very similar overall shape. The key differences are (i) all the magnetic field lines must form closed loops so that all field lines must eventually loop back to pass through the solenoid from the other side. (ii) In the electric dipole, the direction of the field lines change sign as they pass through the poles. This is not the case for magnetic field lines, and it is not possible to isolate a point in space as 'the north pole' or 'the south pole'.




**3**

| Data Set | Gravitational | Electric | Magnetic |
|---|---|---|---|
| **Field A** | Surface of Earth | Between +ve and −ve charged conducting plate | Inside large solenoid |
| **Field B** | Earth | −ve point charge | Not possible |
| **Field C** | Not possible | −ve and +ve charge, electric dipole | Not possible (similar to electric dipole, but not identical) |

Award 1 mark per correct response.

**4 a** $F = qv\boldsymbol{B}$

$= 1.6 \times 10^{-19} \times 5.0 \times 10^{6} \times 250 \times 10^{-3}$

$= 2.0 \times 10^{-13}$ N

**b** $\boldsymbol{a} = \dfrac{qv\boldsymbol{B}}{m}$

$= \dfrac{1.6 \times 10^{-19} \times 5.0 \times 10^{6} \times 250 \times 10^{-3}}{9.1 \times 10^{-31}}$

$= 2.2 \times 10^{17}$ m s$^{-2}$

**c** $\boldsymbol{a} = \dfrac{qv\boldsymbol{B}}{m}$

$= \dfrac{1.6 \times 10^{-19} \times 5.0 \times 10^{6} \times 250 \times 10^{-3}}{1.7 \times 10^{-27}}$

$= 1.2 \times 10^{14}$ m s$^{-2}$

**5** When the ionisation track appears, the net force on the particle, which is a negative particle, is upwards, so the magnetic field experienced is out of the page.

**6** $\boldsymbol{B} = \dfrac{\boldsymbol{p}}{qr}$

$= \dfrac{1.0 \times 10^{-18}}{1.6 \times 10^{-19} \times 1000}$

$= 6.3$ m T, perpendicular to the momentum of the electrons

**7 a** $\Delta E_k = qV$

$= 2 \times 1.6 \times 10^{-19} \times 2000$

$= 6.4 \times 10^{-16}$ J [1 mark ]

$E = \dfrac{1}{2}mv^2$

$6.4 \times 10^{-16}\text{ J} = \dfrac{1}{2} \times 1.2 \times 10^{-26}\text{ kg} \times \boldsymbol{v}^2$

$\Rightarrow \boldsymbol{v}^2 = \dfrac{2 \times 6.4 \times 10^{-16}}{1.2 \times 10^{-26}\text{ kg}}$

$\Rightarrow \boldsymbol{v} = 3.3 \times 10^{5}$ m s$^{-1}$ [1 mark]

**b** $F = qv\boldsymbol{B}$

$= 2 \times 1.6 \times 10^{-19} \times 3.3 \times 10^{5}\text{ m s}^{-1} \times 0.50\text{ T}$ [1 mark]

$= 5.3 \times 10^{-14}$ N [1 mark]

**c** $qvB = \dfrac{mv^2}{r}$ [1 mark]

$r = \dfrac{1.2 \times 10^{-26} \times 3.3 \times 10^{5}}{2 \times 1.6 \times 10^{-19} \times 0.50}$ [1 mark]

$= 0.025$ m [1 mark]

**8 a** At P, the magnetic field from Q is out of the page, which is indicated by a dot. [1 mark]

**b** At Q, the force on the right-hand wire is to the right. [1 mark]

**c** The two wires repel each other. [1 mark]
The magnetic field from the left-hand wire at Q is coming out of the page so that, applying the right-hand-slap rule, the force on the current in the right-hand wire is to the right. [1 mark]
The magnetic field from the right-hand wire at P is coming out of the page so that, applying the right-hand-slap rule, the force on the current in the left-hand wire is to the left. [1 mark]
Forces are in opposite directions, so the wires repel each other.

**9** **a** $\boldsymbol{F} = nI\boldsymbol{B}$
$= 50 \times 3.0 \times 0.15 \times 0.030$ [1 mark]
$= 0.68\text{ N}$ [1 mark]

**b** The arrow should be pointing vertically upwards. [1 mark]

**c** If the polarity of the DC supply is reversed, then current initially flows in the opposite direction through the loop [1 mark], so the direction of rotation of the loop is reversed. [1 mark]

**d** For the motor to spin continuously, the split in the ring of the commutator needs to be perpendicular to the direction of the magnetic field. [1 mark]
This means that the change in current direction in the loop occurs when the loop is perpendicular to the field, or vertical in the diagram shown. [1 mark]
If the change in current direction does not occur, then as the loop moves past the vertical, the direction of the torque on the loop changes so that the loop is slowed, and it starts to rotate in the opposite direction. This will be the case if the commutator is attached with the split in the ring significantly altered from being perpendicular to the vertical. [1 mark]

**10** Yes. Since the direction of the conventional current for electrons is opposite to that for positively charged particles, the force of the magnetic field is directed upwards. This force is balanced by the electric force on the electrons, which is directed downwards.

## 5.8 Exam questions

### Section A — Multiple choice questions

**1** A. L is not deflected, it has no charge.
Using the left-hand rule, it can be determined that K is positively charged and M is negatively charged.

**2** A. The current is parallel to the field. The formula $F = BIl$ only applies if the angle between the current and the field is 90°. Students are not required to be able to find the force if the angle is between 0° and 90°.

**3** C. The force is always acting at right angles to the velocity so there is no force acting to change the magnitude of the velocity.

**4** B. Using $Bqv = \frac{mv^2}{r}$, it can be shown that the radius is directly proportional to velocity. If the velocity is halved, the radius is halved.

**5** A. Magnetic forces can be attractive or repulsive, whereas gravitational forces can only be attractive.

**6** B. The field from N to S should be identified by an arrow pointing to the right.

**7** A. Use the right-hand-slap rule: Fingers to the left (N to S), thumb up, palm out of the page.

**8** A. Use the right-hand-grip rule: Thumb up, fingers indicate direction of field.

**9** B
$F = BIl$
$1.0 = B \times 2000 \times 10$
$B = 5 \times 10^{-5}\text{ T}$

**10** D. There is no experimental evidence for the existence of magnetic monopoles.

### Section B — Short answer questions

**11** **a** The magnetic force acting on the electron is always perpendicular to the electron's velocity [1 mark] and has a constant magnitude [1mark] ($F = qvB$).
Thus the path is circular.

**b** $Bqv = \frac{mv^2}{r}$ [1 mark]
$= \frac{9.1 \times 10^{-31} \times 2.0 \times 10^{7}}{1.6 \times 10^{-19} \times 2.5 \times 10^{-3}}$ [1 mark]
$= 4.6 \times 10^{-2}\text{ m}$ [1 mark]

**12** **a**

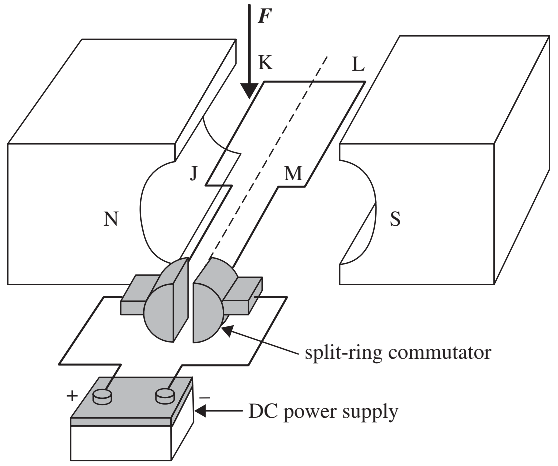
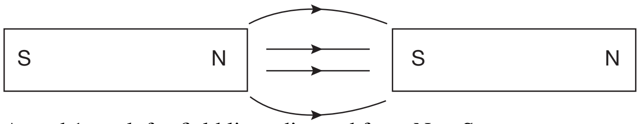



**Figure 4**

**b** The role of the split ring commutator is to reverse the direction of the current every half turn to maintain a constant direction of rotation.

**13**




Award 1 mark for field lines directed from N to S.
Award 1 mark for the correct shape (curved on the outside, straight directly in between the poles).

**14**

| Field type | Monopoles only | Dipoles only | Both monopoles and dipoles |
|---|---|---|---|
| Gravitation | ✓ | | |
| Magnetism | | ✓ | |
| Electricity | | | ✓ |

Award 1 mark for each row.
***VCAA examination report* note:**
Most students understood that magnets only came in dipoles but were unsure of the other two.

**15 a** The correct approach is to equate the force due to the electric field to the force due to the magnetic field [1 mark], thus:

$E_q = Bqv_0$

$$v_0 = \frac{Eq}{Bq}$$
$$= \frac{E}{B}$$

**b** $v_0 = \frac{E}{B}$

$$= \frac{500 \times 10^3}{0.25}$$
$$= 2.0 \times 10^6 \text{ m s}^{-1}$$

***VCAA examination report* note:**

The most common error was to not convert the voltage from kV to V. Students at this level are expected to be able to convert between SI prefixes.

**c i** They would arrive at point Z.

**ii** The direction of the force due to the electric field is upwards as the electron is attracted to the positively charged side of the field.

Using the right-hand-slap rule with your fingers pointing into the page like ***B***, your thumb pointing to the right like ***v***, the direction of the magnetic force is upwards for a positively charged particle and downwards for a negatively charged particle. The force due to the electric field will remain unchanged as it is independent of velocity.

The force due to the magnetic field will increase as it is dependent on velocity. [1 mark] This will result in an unbalanced force down the page towards Z. [1 mark]

Thus the electron is more deviated downwards.

# Unit 3 — Area of Study 2 review

## Practice examination

### Section A — Multiple choice questions

**1** A

The gravitational potential is given by $g = \frac{GM}{r^2}$.

$$g = \frac{GM}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 1.9 \times 10^{27}}{(7.0 \times 10^7)^2}$$
$$\approx 26\,\text{N kg}^{-1}$$

**2** A

The gravitational potential varies with distance according to the inverse square law, that is: $\boldsymbol{g} \propto \frac{1}{r^2}$. Therefore, at an altitude of $2r_{\text{Moon}}$, the distance from the centre of the Moon would be $3r_{\text{Moon}}$. The gravitational potential would decrease by a factor of 9, since $\frac{1}{3^2} = \frac{1}{9}$.

$$\frac{1.6}{9} \approx 0.18\,\text{N kg}^{-1}$$

**3** B

The gravitational force between the two spherical bodies is given by $F_g = \frac{Gm_1m_2}{r^2}$.

The mass of each sphere is 4.5 kg, and their centres of masses are 1 diameter apart: 9 m.

$$F_g = \frac{Gm_1m_2}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 4.5 \times 4.5}{9^2}$$
$$= 1.7 \times 10^{-11}\,\text{N}$$

**4** D

The gravitational field strength around Earth varies according to the inverse square law; therefore, it is non-uniform. It does not change in time; therefore, it is static.

**5** C

According to the satellite equation $\frac{R^3}{T^2} = \frac{GM}{4\pi^2}$, the orbital periods of satellites orbiting at the same altitude are the same. The gravitational force on a satellite is given by $F = \frac{4\pi^2 mr}{T^2}$; hence, for satellites orbiting at the same altitude, the heavier satellite will experience a greater amount of gravitational force.

**6** B

According to the satellite equation $\frac{R^3}{T^2} = \frac{GM}{4\pi^2}$, the orbital period of the satellite increases with the orbital radius of the satellite. Since the orbital period increases at a higher rate than the orbital radius, the orbital speed given by $v = \frac{2\pi R}{T}$ will decrease.

**7** A

$$\boldsymbol{F} = q\boldsymbol{E}$$
$$F = 9.6 \times 10^{-19} \times 250$$
$$= 2.4 \times 10^{-16}\,\text{N}$$

**8** C

The electric force acting on the fixed sphere is given by

$$\boldsymbol{F} = \frac{kq_1q_2}{r^2}$$
$$F = \frac{8.99 \times 10^9 \times 4.5 \times 10^{-6} \times 3.2 \times 10^{-6}}{(0.075)^2}$$
$$= 23.0\,\text{N}$$

**9** B

The electric field strength due to the fixed sphere is given by

$$E = \frac{kq_2}{r^2}$$
$$= \frac{8.99 \times 10^9 \times 4.5 \times 10^{-6}}{(0.075)^2}$$
$$= 7.2 \times 10^6\,\text{N C}^{-1}$$

**10** C

The electric field strength between the two plates is given by

$$E = \frac{V}{d}$$
$$= \frac{120}{32 \times 10^{-3}}$$
$$= 3.8 \times 10^3\,\text{N C}^{-1}$$

**11** C

An electron has electric charge and so may be deflected by an electric field. When it is moving, it is a current and so it may be deflected by a magnetic field. An electron also has mass and so may be deflected by a gravitational field.

**12** D

The gain in kinetic energy is equal to the work done.

$$E_k = W = qEd$$
$$= 1.6 \times 10^{-19} \times 2300 \times 0.03$$
$$= 1.1 \times 10^{-17}\,\text{J}$$

**13** C

Electrical field lines point away from positive charges, and towards negative charges, thus the right-hand charge is negative and the left-hand charge is positive.

**14** A

Since the conductor is parallel to the magnetic field lines, the magnetic force on the conductor is zero.

**15** B

The magnetic field runs from south to north, the current runs from east to west. Using the right-hand-slap rule, the direction of the force is 'down'.

**16** B

Using the right-hand-grip rule, the magnetic field direction is determined to be represented by option B (thumb points into the page, in the direction of the conventional current; the fingers curl around the wire in the direction of the magnetic field).

**17** B

The magnetic force on a current-carrying conductor is $F = BIl$, which rearranges to

$$B = \frac{F}{Il}$$
$$= \frac{0.35}{3.5 \times 0.15}$$
$$\approx 0.67\,\text{T}$$

**18** B

Using the right-hand-slap rule, the side of the coil PQ will be pushed upwards by the magnetic force. Hence, the coil will rotate anticlockwise.

**19** B

The radius of the electron path is given by

$$r = \frac{mv}{qB}$$
$$= \frac{9.1 \times 10^{-31} \times 8.0 \times 10^6}{1.6 \times 10^{-19} \times 0.008}$$
$$= 5.7 \times 10^{-3}\ \text{m}$$
$$= 5.7\ \text{mm}$$

**20** C

Since the magnetic force is always acting perpendicularly to the direction of electron motion, the electrons accelerate centripetally and travel in a uniform circular motion. The electric force acts in one direction and accelerates the electrons in one direction, similar to gravitational force in projectile motion, resulting in a parabolic path.

## Section B — Short answer questions

**21 a** $g = \frac{GM}{r^2}$

$$= \frac{1.67 \times 10^{-11} \times 1.35 \times 10^{23}}{(2.57 \times 10^6)^2} \quad \text{[1 mark]}$$
$$= 1.36\ \text{N kg}^{-1} \quad \text{[1 mark]}$$

**b** For satellites orbiting the same planet, the ratio $\frac{R^3}{T^2}$ is a constant.

Therefore, $\frac{R^3_{\text{Titan}}}{T^2_{\text{Titan}}} = \frac{R^3_{\text{Tethys}}}{T^2_{\text{Titan}}}$ [1 mark]

Thus

$$T_{\text{Tethys}} = \sqrt{\frac{R^3_{\text{Tethys}} T^2_{\text{Titan}}}{R^3_{\text{Titan}}}} \quad \text{[1 mark]}$$
$$= \sqrt{\frac{(2.95 \times 10^8)^3 (1.38 \times 10^6)^2}{(1.22 \times 10^9)^3}}$$
$$= 1.64 \times 10^5\ \text{s} \quad \text{[1 mark]}$$

**c** Using the satellite equation $\frac{R^3}{T^2} = \frac{GM}{4\pi^2}$ [1 mark], together with Titan's orbital data, the following is obtained:

$$M = \frac{4\pi^2 R^3}{GT^2} \quad \text{[1 mark]}$$
$$= \frac{4\pi^2 \times (1.22 \times 10^9)^3}{6.67 \times 10^{-11} \times (1.38 \times 10^6)^2}$$
$$= 5.64 \times 10^{26}\ \text{kg} \quad \text{[1 mark]}$$

**d** $v = \frac{2\pi r}{T}$ [1 mark]

$$= \frac{2\pi \times 1.22 \times 10^9}{1.38 \times 10^6}$$
$$= 5.55 \times 10^3\ \text{m s}^{-1} \quad \text{[1 mark]}$$

**22 a** $E = \frac{V}{d}$ [1 mark]

$$= \frac{120}{0.15}$$
$$= 800\ \text{V m}^{-1} \quad \text{[1 mark]}$$

**b** The electric force on an electron is also the net force acting on the electron.

Thus,

$$F = Eq = ma$$
$$a = \frac{Eq}{m} \quad \text{[1 mark]}$$
$$= \frac{800 \times 1.6 \times 10^{-19}}{9.31 \times 10^{-31}}$$
$$= 1.37 \times 10^{14}\ \text{m s}^{-2} \quad \text{[1 mark]}$$

**c** The gain in kinetic energy is equal to the work done.

$$W = Vq = E_k \quad \text{[1 mark]}$$
$$E_k = 120 \times 1.6 \times 10^{-19}$$
$$= 1.92 \times 10^{-17}\ \text{J} \quad \text{[1 mark]}$$

**d** The final velocity may be calculated from the gain in kinetic energy.

$$E_k = \frac{1}{2}mv^2$$
$$1.92 \times 10^{-17} = \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2 \quad \text{[1 mark]}$$
$$v = \sqrt{\frac{2 \times 1.92 \times 10^{-17}}{9.1 \times 10^{-31}}}$$
$$= 6.50 \times 10^6\ \text{m s}^{-1} \quad \text{[1 mark]}$$

**23 a** Using the right-hand-push rule, with the magnetic field directed to the right and the magnetic force directed upwards (so that the coil rotates clockwise), the current must be travelling out of the page, or from K to J. [1 mark]

**b** $F = nBIl$

$$B = \frac{F}{nIl} \quad \text{[1 mark]}$$
$$= \frac{1.2}{40 \times 3.2 \times 0.029}$$
$$= 0.32\ \text{T} \quad \text{[1 mark]}$$

**c** The coil will rotate clockwise to a vertical position and stop moving [1 mark]. This is because the magnetic force on the side JK is upward, and the magnetic force on the side LM is downward, and the forces do not impart a twisting motion to the coil to cause it to rotate. [1 mark]

**d** $F = nFIl$

$$12 = 17 \times 0.32 \times 32 \times l \quad \text{[1 mark]}$$
$$l = \frac{12}{17 \times 0.32 \times 32}$$
$$= 0.06893$$
$$= 6.9\ \text{cm} \quad \text{[1 mark]}$$

# Topic 6 — Generation of electricity

## 6.2 BACKGROUND KNOWLEDGE Generating voltage and current with a magnetic field

### Sample problem 1

$$\begin{aligned}\varepsilon &= \boldsymbol{B}l\boldsymbol{v}\\ &= 0.25 \times 5.0 \times 10^{-2} \times 0.4\\ &= 5.0 \times 10^{-3}\text{ V}\\ &= 5.0\text{ mV}\end{aligned}$$

### Practice problem 1

$$\begin{aligned}\varepsilon &= \boldsymbol{B}l\boldsymbol{v}\\ \Rightarrow \boldsymbol{v} &= \frac{\varepsilon}{\boldsymbol{B}l}\\ &= \frac{1.0}{0.25 \times 0.05}\\ &= 80\text{ m s}^{-1}\end{aligned}$$

### 6.2 Exercise

**1** $\boldsymbol{B} = 0.70$ T, $l = 10$ cm $= 0.10$ m, $\boldsymbol{v} = 0.20$ m s$^{-1}$

$$\begin{aligned}\varepsilon &= \boldsymbol{B}l\boldsymbol{v}\\ &= 0.70 \times 0.10 \times 0.20\\ &= 0.0140\text{ V}\\ &= 14\text{ mV}\end{aligned}$$

**2** $\varepsilon = 100$ mV $= 0.100$ V, $\boldsymbol{B} = 0.850$ T, $l = 14.3$ cm $= 0.143$ m

$$\begin{aligned}\varepsilon &= \boldsymbol{B}l\boldsymbol{v}\\ \Rightarrow \boldsymbol{v} &= \frac{\varepsilon}{\boldsymbol{B}l}\\ &= \frac{0.100}{0.850 \times 0.143}\\ &\approx 0.823\text{ m s}^{-1}\end{aligned}$$

**3** $\varepsilon = 90$ mV $= 0.09$ V, $\boldsymbol{B} = 0.60$ T, $\boldsymbol{v} = 0.18$ m s$^{-1}$

$$\begin{aligned}\varepsilon &= \boldsymbol{B}l\boldsymbol{v}\\ \Rightarrow l &= \frac{\varepsilon}{\boldsymbol{B}\boldsymbol{v}}\\ &= \frac{0.090}{0.60 \times 0.18}\\ &\approx 0.83\text{ m}\end{aligned}$$

**4** As the rod falls downwards (which is the same direction as the magnetic field) its velocity perpendicular to the field is 0 m s$^{-1}$. The induced voltage is therefore 0 V.

**5** $\varepsilon = Blv$

Thus

$$\begin{aligned}B &= \frac{\varepsilon}{lv}\\ &= \frac{35 \times 10^{-3}}{7.0 \times 10^{-2} \times 0.05}\\ &= 10\text{ T}\end{aligned}$$

**6** **a** At a speed of 6000 m s$^{-1}$, the pair would travel 6000 m in 1.0 s.

**b** The area swept by the 5.0-km long cable as it travels 6000 m in 1 second is:

$A = 5000 \times 6000 = 3 \times 10^7$ m$^2$

### 6.2 Exam questions

**1** The velocity of the aeroplane perpendicular to the magnetic field is:

$$\begin{aligned}2200\text{ km h}^{-1} &= 2\,200\,000\text{ m h}^{-1}\\ &= \frac{2\,200\,000}{3600}\text{ m s}^{-1}\\ &\approx 611\text{ m s}^{-1}\end{aligned}$$

$$\begin{aligned}\Rightarrow \varepsilon &= \boldsymbol{B}l\boldsymbol{v}\\ &= 5 \times 10^{-5} \times 30 \times 611\\ &\approx 0.92\text{ V}\end{aligned}$$

**2** The energy came from the loss of gravitational potential energy as the metal rod was falling. Some of the gravitational potential energy was converted to kinetic energy, and some was converted to electrical potential energy of separated charges.

**3** As the rod falls through the magnetic field, the electrons inside the metal rod experience a force from the magnetic field and are pushed to the left end of the rod, thus creating a voltage difference between the left end and the right end of the rod. The number of electrons accumulating on the left is limited by the electrostatic repulsion among the electrons, thus limiting the voltage difference between the ends of the rod. If the rod falls faster, the force due to the magnetic field increases, thereby allowing more electrons to overcome the electrostatic repulsion and gather at the left, thus creating a higher voltage difference with the right end.

**4**

$$\begin{aligned}120\text{ km h}^{-1} &= \frac{120\,000}{3600}\\ &= 33.33\text{ m s}^{-1}\end{aligned}$$

$$\begin{aligned}\varepsilon &= \boldsymbol{B}lv\\ &= 4.0 \times 10^{-6} \times 1.5 \times 33.33\\ &= 0.002\text{ V}\end{aligned}$$

**5**

$$\begin{aligned}\varepsilon &= Blv\\ &= 5.0 \times 10^{-5} \times 4.0 \times 2.5\\ &= 5 \times 10^{-4}\text{ V}\end{aligned}$$

## 6.3 Magnetic flux

### Sample problem 2

**a**

$$\begin{aligned}\Phi_B &= B_\perp \times A\\ &= 0.05\text{ T} \times 0.3\text{ m}^2\\ &= 0.015\text{ Wb}\end{aligned}$$

**b** First calculate area $A$.

$$\begin{aligned}\Rightarrow A &= \pi r^2\\ &= \pi \times 0.08^2\\ &= 0.02\text{ m}^2\end{aligned}$$

The magnetic field is perpendicular to the area, so use the formula $\Phi_B = B_\perp A$ to determine the flux.

$$\begin{aligned}\Rightarrow \Phi_B &= B_\perp \times A\\ &= 0.2\text{ T} \times 0.02\text{ m}^2\\ &= 0.004\text{ Wb}\end{aligned}$$

**c** The plane of the loop is parallel to the magnetic field, hence there are no field lines passing through the area.

$$\Rightarrow B_\perp = 0$$
$$\Phi_B = B_\perp \times A = 0 \times A = 0 \text{ Wb}$$

## Practice problem 2

The side length of the square is 6.0 cm, or 0.06 m. The area of the loop is $0.06 \times 0.06 = 0.0036$ m$^2$.

The magnetic field is perpendicular to the area of the loop, so the flux can be found using the formula,

$$\Phi_B = \boldsymbol{B}_\perp A = 0.15 \times 0.0036 = 0.000\,54 \text{ Wb}$$

## 6.3 Exercise

1 Magnetic flux is the amount of magnetic field across an area, and is the product of the magnetic field strength and the perpendicular cross-sectional area.

2 Flux is equal to the perpendicular area multiplied by the magnetic field strength.

**a** $\Phi_B = B_\perp A = 3.0 \times 0.05 = 0.15$ Wb

**b** $\Phi_B = B_\perp A = 0.4 \times 4.5 \times 10^{-4} = 1.8 \times 10^{-4}$ Wb

**c** $A = \pi r^2 = \pi \times 0.30^2 = 0.28$ m$^2$

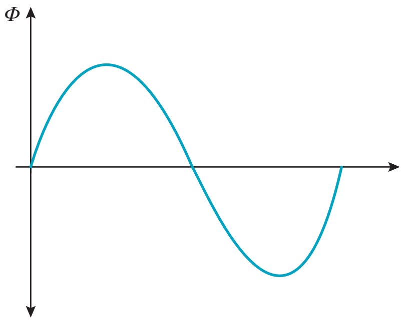
$\Phi_B = B_\perp A = 0.025 \times 0.28 = 7.1 \times 10^{-3}$ Wb

3 **a** In the position shown in the diagram, the area is parallel to the magnetic field, so the perpendicular area is 0 m$^2$. The magnetic flux passing through is 0 Wb.

**b** The area of the loop is 0.05 m × 0.05 m = 0.0025 m$^2$.

$\Rightarrow \Phi_B = B_\perp A = 0.030 \times 0.0025 = 0.000\,075$ Wb

**c** The flux graph is sinusoidal in shape, starting at 0, then rising to a maximum after a quarter rotation, falling back to 0 at a half rotaion, down to a minimum at 3 quarters of a rotation and then back to 0 after a fullrotation.




4 $\Phi = BA$, where B is in T, and the area is in m$^2$ and the flux in Wb.
$= 7.0 \times 2.5$
$= 17.5$ Wb

5 Apply the formula $\Phi_B = BA$ with $B = 0.09$ T and $A = 38$ cm$^2 = 3.8 \times 10^{-3}$ m$^2$.
Thus $\Phi_B = 9 \times 10^{-2} \times 3.8 \times 10^{-3} = 3.4 \times 10^{-4}$ Wb

6 **a** $\Phi = BA$
thus
$$B = \frac{\Phi}{A} = \frac{6.0}{2.0} = 3.0 \text{ T}$$

**b** $\Phi = BA$
thus
$$A = \frac{\Phi}{B} = \frac{7.2 \times 10^{-3}}{9.0 \times 10^{-3}} = 0.8 \text{ m}^2$$

7 $\Phi = BA = 5 \times 10^{-2} \times (2 \times 10^{-2})^2 = 2.0 \times 10^{-5}$ Wb

8 **a** First determine the area:
$A = 20.0 \times 10^{-2} \times 10.0 \times 10^{-2} = 2.0 \times 10^{-2}$ m$^2$
$\Phi_B = B_\perp \times A = 6 \times 10^{-3} \times 2 \times 10^{-2} = 1.2 \times 10^{-4}$ Wb

**b** First determine the area:
$A = \pi r^2 = \pi \times \left(1.20 \times 10^{-2}\right)^2 = 1.44 \times 10^{-4} \times \pi$ m$^2$
$\Phi_B = B_\perp \times A = 1.5 \times \pi \times 1.44 \times 10^{-4} = 6.8 \times 10^{-4}$ Wb

**c** The plane of the loop is parallel to the magnetic field, thus $B_\perp = 0$ and $\Phi_B = 0$ Wb.

## 6.3 Exam questions

1 B
$\phi = BA = 0.03 \times 10 \times 10^{-4} = 3.0 \times 10^{-5}$ Wb

2 $B = \dfrac{\phi}{A} = \dfrac{0.20}{0.3 \times 0.4}$ [1 mark]
$= 1.7$ T [1 mark for the correct magnitude]
[1 mark for the correct unit]

***VCAA examination report*** **note:**
The most common error was to include $n = 10$ in the formula. Students are reminded that the number of loops does not affect the flux calculation.

3 Flux $\phi = BA$
$= 0.0050 \times 0.0060$ [1 mark]
$= 3.0 \times 10^{-5}$ Wb or T m$^2$ [1 mark]

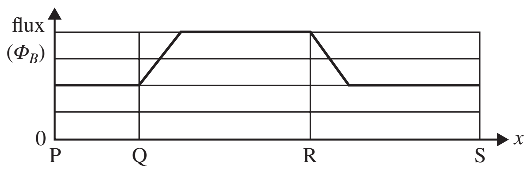
**4** 


**Figure 23b**

Award 2 marks for a correct sketch of the magnitude of the magnetic flux: flux is constant before the loop passes through the magnets, and after it has passed.
The magnitude of the flux increases when the leading edge of the loop reaches Q, and reaches a maximum when the whole loop is between the magnets. It decreases back to its initial value once the leading ledge of the loop reaches R.

**5** B. Magnetic flux is $\Phi = BA_{\text{perpendicular}}$.
In orientation A, the flux is 0; in orientation B, the flux is 0.04 Wb. [1 mark]
In orientation C, the flux is between these two extreme values; hence, it must be 0.03 Wb, i.e. option B. [1 mark]

## 6.4 Generating emf from a changing magnetic flux

### Sample problem 3

**a** $A = L \times W$
$= 0.25\text{ m} \times 0.30\text{ m}$
$= 0.075\text{ m}^2$
$\Delta\Phi_B = \Phi_{B\text{ final}} - \Phi_{B\text{ initial}}$
$= (BA)_{final} - (BA)_{initial}$
$= (0.66\text{ T} \times 0.075\text{ m}^2) - (0\text{ T} \times 0.075\text{ m}^2)$
(The initial field strength through the coil is zero.)
$= (0.050\text{ T m}^2) - (0\text{ T m}^2)$
$= 0.050$ Wb into the page
Use Faraday's Law to determine the induced emf:
$\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$
$= -1 \times \dfrac{0.050\text{ Wb}}{2.0\text{ s}}$
$= -0.025\text{ V}$
So the magnitude of the induced voltage is 0.025 V.
(The minus sign indicates that the induced emf opposes the change in magnetic flux.)

**b** Determine the direction of the flux, and whether the flux is increasing or decreasing:
The flux increases from 0 Wb to 0.050 Wb into the page as the loop moves into the field.
Use Lenz's Law to determine the direction of the induced magnetic field:
Lenz's Law states that the induced magnetic field opposes the change in flux. Because the flux is increasing into the page, the direction of the induced magnetic field will be in the opposite direction to the flux. Hence the direction of the induced field will be out of the page.
Use the right-hand-grip rule to determine the direction of the induced current:
In order for the induced magnetic field to be directed out of the page, the induced current must flow in an anticlockwise direction around the loop.

**c** $V = IR$
$0.025\text{ V} = I \times 0.5\ \Omega$
$I = \dfrac{0.025\text{ V}}{0.5\ \Omega}$
$= 0.05\text{ A}$

### Practice problem 3

**a** $A_{\text{initial}} = \pi r^2$
$= \pi \times 0.05^2$
$\approx 0.007\,85\text{ m}^2$
$A_{\text{final}} = \pi r^2$
$= \pi \times 0.03^2$
$\approx 0.002\,83\text{ m}^2$
$\Delta\Phi_B = \Phi_{B\text{ final}} - \Phi_{B\text{ initial}}$
$= (BA)_{\text{final}} - (BA)_{\text{initial}}$
$= (0.55 \times 0.002\,83) - (0.55 \times 0.007\,85)$
$= -0.002\,76\text{ Wb}$
$\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$
$= -1 \times \dfrac{-0.002\,76}{0.15}$
$= 0.0184\text{ V}$

**b** According to Lenz's Law, the current will induce a magnetic field that acts in the opposite direction to the change in flux. The magnetic field is coming out of the page, and the change in flux is negative, so it is into the page. Therefore, the current will induce a magnetic field out of the page. By the right-hand rule, the current must be travelling in the anticlockwise direction.

**c** Using Faraday's Law, the induced emf can be found:
$\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$
$= \dfrac{-1 \times -0.002\,761}{0.15}$
$\approx 0.184\text{ V}$
The current can then be found using Ohm's Law:
$\Rightarrow V = IR$
$I = \dfrac{V}{R}$
$= \dfrac{0.184}{0.4}$
$= 0.46\text{ A}$

## 6.4 Exercise

1. Increasing the number of turns increases the magnetic flux through the coils by increasing the cross-sectional area.
2. The initial flux is into the page, while the final flux is zero. The direction of the change in flux is out of the page. Hence, the flux from the induced current would be into the page, opposite to the direction of the change in flux. Using the right-hand-grip rule, the induced current would be clockwise around the loop.
3. **a** The initial flux through the top coil is zero. A clockwise current in the bottom coil would produce a final flux into the page as seen from above. Hence, the direction of the change in flux is into the page. The flux from the induced

current would be out of the page and would be anticlockwise around the top loop.

**b** No, the effect would not be different, because the flux directions in part a are also applicable if the coils are swapped.

**c** Yes, the effect would be different: the opposite direction, clockwise, would result.

**4** **a** An anticlockwise current is produced.

**b** Yes, the effect is different: a clockwise current is produced.

**5** **a** Yes, there is an induced current as there is a change in flux. The initial flux is out of the page, and the final flux is zero. The change in flux is into the page. The flux from the induced current is out of the page and hence the current is anticlockwise around the loop.

**b** Yes, there is an induced current. The reasoning from part a applies equally in this situation.

**c** There is no induced current as there is no change in flux.

**d** There is no induced current as there is no change in flux.

**6** **a** Area of loop $= \pi \times 0.05^2$

$\approx 0.007\ 85\ \text{m}^2$

$\Rightarrow \Delta\Phi_B = (BA)_{\text{final}} - (BA)_{\text{initial}}$

$= (0.40 \times 0) - (0.40 \times 0.007\ 85)$

$= -0.003\ 14\ \text{Wb}$

$\Rightarrow \varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$

$= \dfrac{-1 \times -0.003\ 14}{0.2}$

$= 0.0157\ \text{V}$

**b** $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$

$= \dfrac{-1 \times -25}{1.5}$

$\approx 17\ \text{V}$

**c** $\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$

$= \dfrac{-1 \times (-95)}{2.5}$

$\approx 38\ \text{V}$

**7** **a** $l = 2\pi r = 10\text{cm}$

$\Rightarrow r = \dfrac{10}{2\pi}$

$= 1.5\,91...\ \text{cm}$

Area of loop $= \pi \times 0.015\ 91...^2$

$\approx 0.000\ 795...\ \text{m}^2$

$\Delta\Phi_B = (BA)_{\text{final}} - (BA)_{\text{initial}}$

$= (0.60 \times 0) - (0.60 \times 0.000\ 795...)$

$= -0.000\ 477...\ \text{Wb}$

$\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$

$= \dfrac{-1 \times -0.000\ 477...}{0.3}$

$= 0.001\ 591...\ \text{V}$

$V = IR$

$\Rightarrow I = \dfrac{V}{R}$

$= \dfrac{0.001\ 591...}{0.4}$

$= 0.003\,97...\ \text{A}$

$= 0.0040\ \text{A}$

**b** Area of loop $= 0.26^2$

$= 0.0676\ \text{m}^2$

$\Delta\Phi_B = (BA)_{\text{final}} - (BA)_{\text{initial}}$

$= (1.2 \times 0.0676) - (0.2 \times 0.0676)$

$= 0.0676\ \text{Wb}$

$\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$

$= \dfrac{-1 \times 0.0676}{0.5}$

$= -0.1352\ \text{V}$

$\Rightarrow$ magnitude of voltage $= 0.1352\ \text{V}$

$V = IR$

$\Rightarrow I = \dfrac{V}{R}$

$= \dfrac{0.1352}{0.25}$

$= 0.5408\ \text{A}$

**c** $\text{Area}_{\text{initial}} = \pi \times 0.08^2$

$\approx 0.02$

$\text{Area}_{\text{final}} = \pi \times 0.04^2$

$\approx 0.005$

$\Delta\Phi_B = (\boldsymbol{B}A)_{\text{final}} - (\boldsymbol{B}A)_{\text{initial}}$

$= (2 \times 0.005) - (2 \times 0.02)$

$= -0.03\ \text{Wb}$

$\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$

$= \dfrac{-1 \times -0.03}{0.8}$

$= 0.04\ \text{V}$

$V = IR$

$\Rightarrow I = \dfrac{V}{R}$

$= \dfrac{0.04}{0.2}$

$= 0.2\ \text{A}$

**8** Using the formula for the induced emf:

$\varepsilon = \boldsymbol{B}lv$

$= 0.0001 \times 5000 \times 6000$

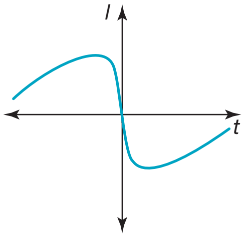
$= 3000\ \text{V}$

**9** **a** **i** As the magnet's north end approaches the loop, the initial flux is zero and the final flux is pointing down; hence, the flux change is pointing down and the direction of the induced flux is pointing up, resulting in an anticlockwise current viewed from above.

**ii** As the magnet passes through the loop, there is no flux change and no current in the loop.

**iii** As the magnet's south end recedes away from the loop, the initial flux is pointing down and the final flux is zero; hence, the flux change is pointing up and the direction of the induced flux is pointing down, resulting in a clockwise current viewed from above.

**b** The induced current–time graph has the following shape.




**c** The electrical energy comes from the loss of gravitational potential energy as the magnet drops. This means that not all of the gravitational potential energy is converted into kinetic energy, as in the case of a non-magnet; hence, the falling magnet would fall more slowly than a non-magnet.

**d** The overall change in magnetic flux through the loop is zero; therefore, the two areas under the graph — one above the *x*-axis and the other below the *x*-axis — would cancel each other out.

**10** $\Delta\Phi_B = \Phi_{\text{B final}} - \Phi_{\text{B initial}}$

$= (BA)_{\text{final}} - (BA)_{\text{initial}}$

$= (0 \times 1.6 \times 10^{-3}) - (3.0 \times 1.6 \times 10^{-3})$

$= -0.0048\ \text{Wb}$

$\varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$

$= \dfrac{-1 \times -0.0048}{1}$

$= 0.0048\ \text{V}$

$V = IR$

$\Rightarrow I = \dfrac{V}{R}$

$= \dfrac{0.0048}{0.2}$

$= 0.024\ \text{A}$

$\Rightarrow Q = It$

$= 0.024\ \text{C}$

**11** The passing magnet would induce an emf in both loops; that is, the electrons inside the metal loop as well as the plastic loop would experience a magnetic force from the passing magnet. However, since plastic is an insulator, no current would flow in the plastic loop, in contrast to the metal loop.

**12** As the north end of the magnet approaches the loop of conducting wire, it would induce a current in the loop. If Lenz's Law is reversed, the induced current would have a magnetic field that attracts the north end of the magnet. The magnet would accelerate and gain kinetic energy from nothing, hence violating the principle of the conservation of energy.

**13 a** A current will flow to induce a N pole at the left end of the coil. Right-hand rule: thumb points left, fingers show direction of current in the coil. Current flows in direction of X to Y.

**b** There is no change in magnetic flux through the coil so no current is induced.

**c** Any two answers from the following:
- Move the magnet faster
- Have more turns on the coil
- Use a stronger magnet
- Use a smaller resistance

## 6.4 Exam questions

**1 a** Decrease. [1 mark]

Since the area of the coil decreases when the students pull on it, the magnetic flux through the coil will decrease.

**b** The loop experiences a decrease in flux into the page [1 mark]. Lenz's Law states the induced current will produce an increasing flux into the page [1 mark]. Using the right-hand-grip rule the induced current will flow clockwise around the loop. [1 mark]

***VCAA examination report* note:**

Most students were able to identify there would be a change in flux, although many were unable to describe it as decreasing. Many students were also unable to link the decreasing flux to the clockwise direction of the induced current. There were a significant number of responses that stated 'anticlockwise' with no explanation of why, suggesting students do not clearly understand the application of Lenz's Law.

**c** From Figure 6a to Figure 6b, the loop is experiencing an increasing flux into the page. Applying the right-hand-grip rule, the induced current will flow anticlockwise around the loop. [1 mark]

From Figure 6b to Figure 6c, the loop is experiencing a decreasing flux into the page. Applying the right-hand-grip rule, the induced current will flow clockwise around the loop. [1 mark]

***VCAA examination report* note:**

There were two common incorrect responses. The first was to get the two directions reversed. This was generally accompanied by erroneous or absent reasoning. The second was to attempt to describe the current flow in each figure (rather than between figures). This was also accompanied by erroneous or absent reasoning.

Students and teachers are encouraged to spend more time discussing different scenarios for applying Lenz's Law.

**2** See table at the bottom of the page*
Award 1 mark for correct sketch and correct current for each movement.

***VCAA examination report* note:**
There was a range of errors indicating that induction is only understood at a superficial level and questions like this, which are not covered in textbooks, were difficult for students to answer. Of note, there were a number of responses in which no graph was drawn for B. As it was not possible to determine what the student intended, these were scored as zero. While the direction of the field around a current-carrying wire is in the study design, a quantitative understanding of its shape is not in the study design and so a range of graphs for C was accepted.

**3 a** Using the right-hand-slap rule:
- your fingers (out straight) represent the magnetic field, from the north pole to the south pole
- the palm of the hand represents the force acting in the coil (the direction of rotation is given)
- your thumb represents the current perpendicular to the magnetic field.

The current is clockwise when viewed from above. [1 mark]
Note that you can use different methods to determine the direction of the current. For instance, using the right-hand-grip rule: grip the wire carrying the current with your right hand, with your fingers wrapped around the wire in the direction of the magnetic field. Your thumb is pointing in the direction the current. The current is clockwise.

**b** One quarter turn:

$$t = 0.25 \times \frac{1}{60}$$

$$t = 0.0042\ \text{s} \qquad \text{[1 mark]}$$

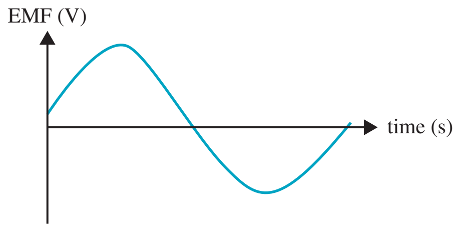
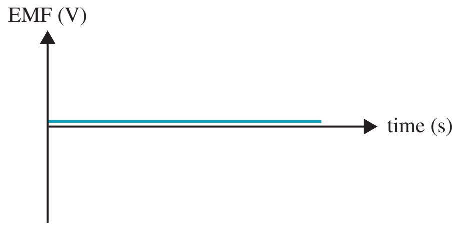
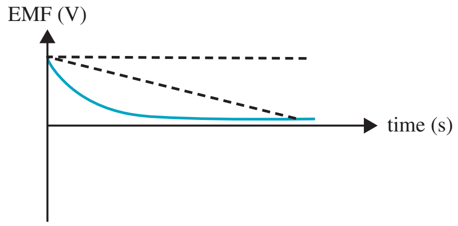
Now area:

$$\epsilon = nB\frac{\Delta A}{\Delta t}$$

$$3.5 \times 10^{-3} = 200 \times 5.0 \times 10^{-3} \times \frac{\Delta A}{0.0042} \qquad \text{[1 mark]}$$

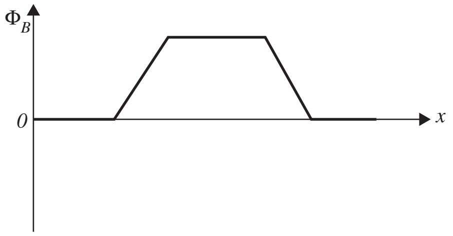
$$\Delta A = 1.5 \times 10^{-4}\ \text{m}^2 \qquad \text{[1 mark]}$$

***2**

| **Movement** | **Possible EMF** | **Direction of any induced current (alternating/clockwise/anticlockwise/no current)** |
|---|---|---|
| **A**<br>rotation about<br>$x$-axis |  | Alternating |
| **B**<br>moving from<br>Position 1 to<br>Position 2 |  | No current |
| **C**<br>moving from<br>Position 2 to<br>Position 3 |  | Clockwise |

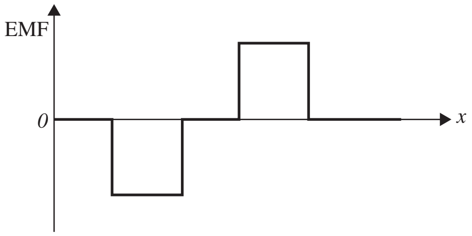
**c** See figure at the bottom of the page*
Award 2 marks for a correct sketch of the output EMF.
***VCAA examination report* note:**
The VCAA is aware that the graph shows the EMF displayed on the CRO over two full turns. There was no evidence that students did not understand what was required.

**4 a**




Award 2 marks for correct sketch of magnetic flux.
***VCAA examination report* note:**
The position on the $x$-axis was not important.

**b**




Award 2 marks for correct sketch of EMF.
***VCAA examination report* note:**
The position on the $x$-axis was not important and the graph did not have to line up with the graph in Question 4a.

**c** $\varepsilon = n\dfrac{\Delta\Phi}{\Delta t}$

$\varepsilon = 100 \times \dfrac{2.0 \times 10^{-3} \times 4.0 \times 10^{-4}}{2}$ [1 mark]

$\varepsilon = 4.0 \times 10^{-5}$ V [1 mark]

**d** The loop is experiencing an increasing downwards flux [1 mark].
According to Lenz's Law the induced flux will be upwards [1 mark].
Using the right-hand-grip rule, the current will flow in an anticlockwise direction [1 mark].

**5 a** $\varepsilon = N\dfrac{\Delta\Phi}{\Delta t}$ [1 mark]

$\varepsilon = 10 \times \dfrac{2 \times 10^{-2} \times 1.6 \times 10^{-3}}{0.5}$ [1 mark]

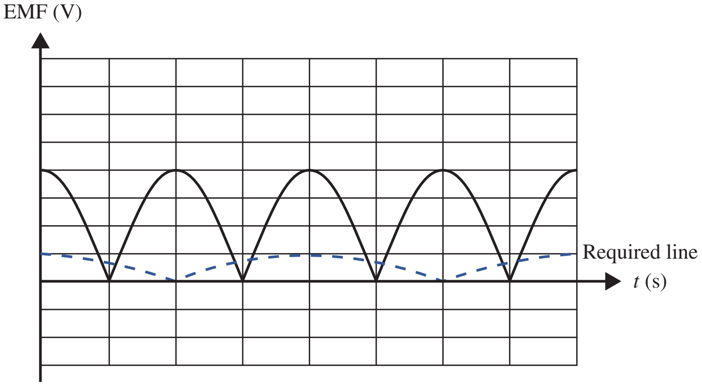
$\varepsilon = 6.4 \times 10^{-4}$ V [1 mark]

***VCAA examination report* note:**
The most common error was to calculate the correct change in flux over time but not to multiply by 10, the number of loops.

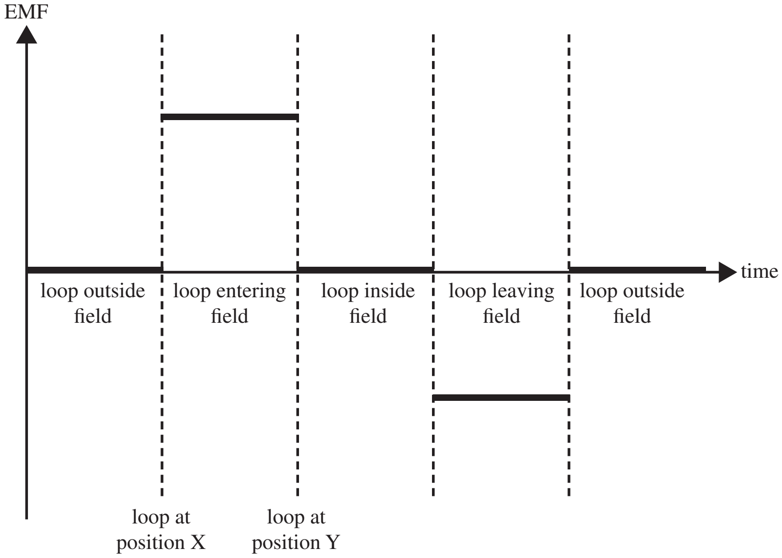
**b** See figure at the bottom of the page**
Award 3 marks for correct sketch of the EMF.

---

***3 c**




****5 b**



***VCAA examination report* note:**
An inverted version of the graph was acceptable. This question was not answered well, with the most common error being to draw a flux graph. Other responses included sine waves and a range of biphasic waveforms, which suggested that few students have an adequate understanding of Faraday's law and how to apply it to find EMF.

## 6.5 Generators and alternators

## 6.5 Exercise

**1 a** $\Phi = B \times A$
$= 0.25 \times 0.6 \times 0.5$
$= 7.5 \times 10^{-2}$ Wb

**b** $\varepsilon = -N\dfrac{\Delta\Phi}{\Delta t}$
$= -1 \times \dfrac{0 - 7.5 \times 10^{-2}}{0.5}$
$= 0.15$ V

**c** The frequency is halved (from 6 Hz to 3 Hz), so the period should be twice the original value and the amplitude should be reduced to half the original value.
See figure at the bottom of the page*

**2 a** $\Phi = BA$
$= 0.15 \times 6.0 \times 10^{-3}$
$= 9.0 \times 10^{-4}$ Wb

**b** $\varepsilon = 4.0 \times 10^{-2}$ V
$N = 50$
$\Delta\Phi = 0 - 15$
$= -0.15$ T
$\Delta t = ?$

Transposing the formula $\varepsilon = -N\dfrac{\Delta\Phi}{\Delta t}$
$\Delta t = -N\dfrac{\Delta\Phi}{\varepsilon}$
$= -50 \times \dfrac{-0.15}{4.0 \times 10^{-2}}$
$= 187.5$ s
$= 1.9 \times 10^{2}$ s

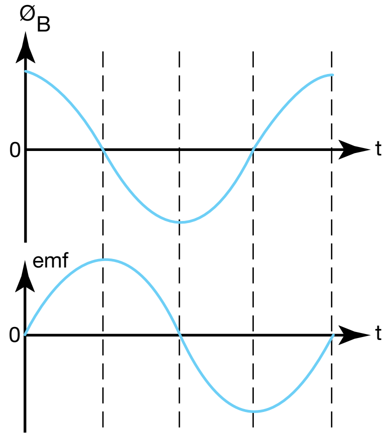
**3 a** $\varepsilon \propto N$. Increasing the number $N$ of loops of the coil increases the average emf and thus the current.

**b** $\varepsilon \propto \dfrac{1}{\Delta t}$. Decreasing the rate of rotation of the coil (increasing $\Delta t$) decreases the average emf and thus the current.

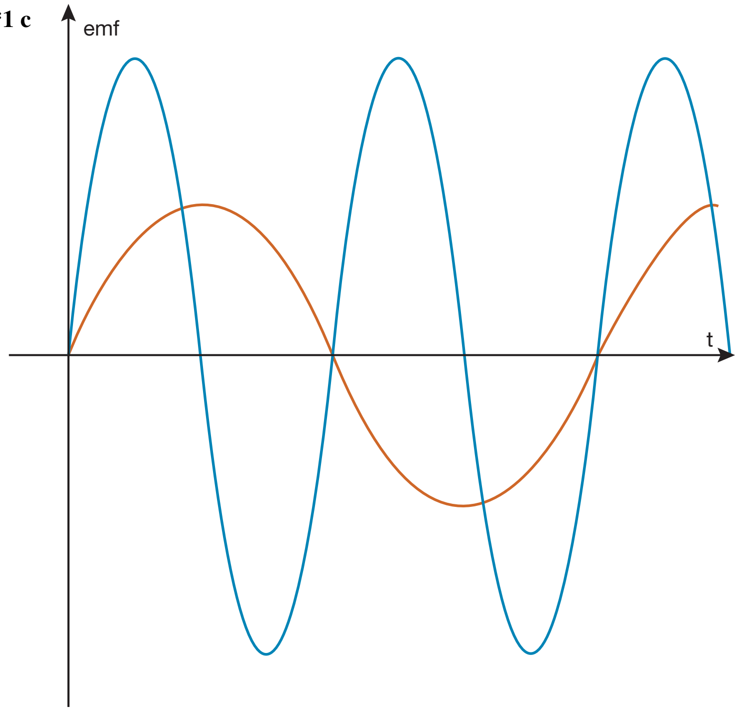
**c** $\varepsilon \propto \Delta\Phi$. Increasing the strength of the magnetic field increases the average emf and thus the current.

**4 a** The current flows from Q to P (using the right-hand grip rule for coils).

**b** Flux starts at a maximum. Emf graph versus time is given by the negative of the gradient of the flux versus time graph.




***1 c**



## 6.5 Exam questions

**1 a** $\Phi = BA$

$1.2 \times 10^{-3} = B \times 0.060$ [1 mark]

$B = 2.0 \times 10^{-2}$ T [1 mark]

***VCAA examination report* note:**

This question was generally well done with mathematical errors being most common. Of concern were the number of students who included the number of turns in the equation. Students need to remember that the number of turns is not a part of this equation.

**b** $\varepsilon = n\dfrac{\Delta\Phi}{\Delta t}$

$\Delta t = \dfrac{1}{4} \times \dfrac{1}{2.5}$

$\Delta t = 0.1$ s [1 mark]

$\varepsilon = 200 \times \dfrac{1.2 \times 10^{-3}}{0.1}$ [1 mark]

$\varepsilon = 2.4$ V [1 mark]

***VCAA examination report* note:**

The most common error was to incorrectly calculate the $\Delta t$ value. Most commonly this was left at 0.4 s.

**2 a**

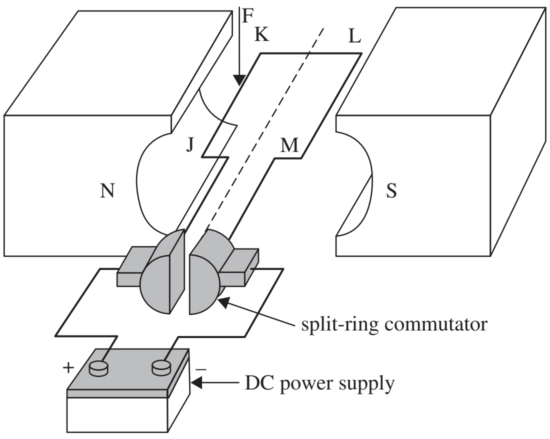



**Figure 4**

**b** The role of the split ring commutator is to reverse the direction of the current every half revolution to maintain a constant direction of rotation.

**3 a** The electrical equipment shown is an alternator. [1 mark]

An alternator is effectively an AC generator.

This is an alternator (AC generator) because it has slip rings (not a split ring commutator) connected to an output and the loop is rotated by an external source.

**b i** The flux through the loop as shown is zero (0 Wb). [1 mark]

**ii** The plane of the loop is parallel to the magnetic field. Therefore, no magnetic field lines pass through the loop. [1 mark]

**c** There are 20 revolutions per second. Frequency = 20 Hz

Period $(T) = \dfrac{1}{f}$

$T = \dfrac{1}{f}$

$T = \dfrac{1}{20}$

$T = 0.05$ seconds [1 mark]

**d** $\Phi_{max} = BA$

$\Phi_{max} = 0.40 \times (0.50 \times 0.25)$ [1 mark]

$\Phi_{max} = 0.05$ Wb [1 mark]

**e** The time for a quarter of a turn is $\dfrac{1}{4}$ of the period.

$\dfrac{1}{4} \times 0.05 = 0.0125$ seconds

$\varepsilon = -N \times \dfrac{\Delta\Phi}{\Delta t}$

$= -1 \times \dfrac{0 - 0.05}{0.0125}$ [1 mark]

$= 4$ V [1 mark]

**f** The amplitude of the output voltage can be increased by:

- increasing the rate of rotation of the loop
- increasing the number of loops
- increasing the strength of the magnetic field
- increasing the area of the loop.

Award 1 mark for any two of the above.

**g** Changing the slip rings to a split ring commutator rectifies the output, making the EMF output only one sign (e.g. positive).

See figure at the bottom of the page.*

**4 a** $F = nIlB$

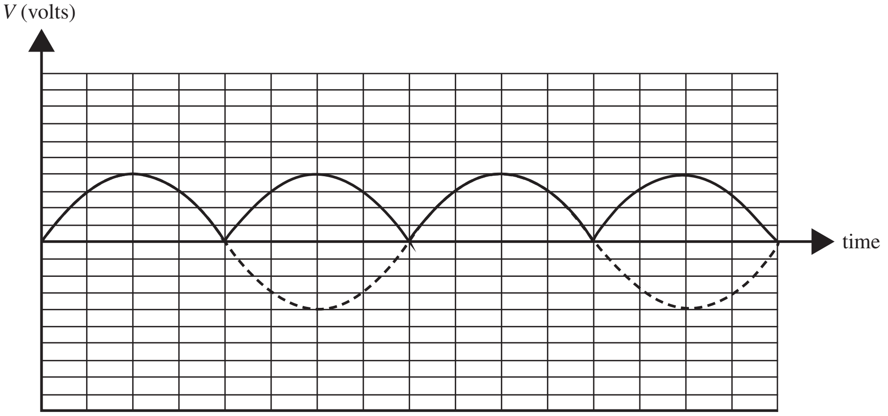
$F = 10 \times 2.0 \times 0.1 \times 2.0 \times 10^{-3}$ [1 mark]

$F = 4.0 \times 10^{-3}$ N [1 mark]

**b** Answers will vary.

A split ring commutator is a device used in a DC motor to reverse the coil's current direction every half revolution of the loop [1 mark] so that he direction of the current

***3 g**



flowing through the external circuit remains the same [1 mark]. The alternating current in the loop is thus converted into direct current [1 mark].

***VCAA examination report* note:**

Students were required to indicate that the split ring commutator reverses the current in the loop every half turn to keep the motor turning in the same direction.

**5 a** Answers will vary.

To determine the direction of rotation, let's determine whether the force acting on HG (or on FE) is upwards or downwards. Using the right-hand-slap rule for instance:

- your fingers (out straight) represent the magnetic field, from the north pole to the south pole, thus from left to right [1 mark]
- your thumb represents the current H → G → F → E, so it is pointing into the page for branch HG [1 mark]
- the palm of the hand represents the force, here the palm of your hand is downwards, thus the force acting on HG is downwards and the motor will rotate in a clockwise direction. [1 mark]

***VCAA examination report* note:**

Students were required to explain how they determined the direction. They were required to identify the direction of the current in the loop as H to G or similar, then relate this to the direction of rotation. The most appropriate way to do this was to refer to the right-hand rule or similar. The most common error was not linking all three aspects: the current direction, the rule that relates current to force and the subsequent direction of rotation. Many students stated that because the current flows from H to G the side HG is forced down, without giving any reference to a link between current and force. Alternatively they would state that due to the right-hand rule, the side HG is forced down, without referencing current. Students need to practise identifying and linking concepts within their responses rather than leaping from assertion to conclusion.

**b** The motor would not function anymore [1 mark] as the loop would only rotate $\frac{1}{4}$ of a turn before stopping [1 mark].

***VCAA examination report* note:**

Students were required to identify that the motor would no longer function. Further, they were required to point out that the loop would rotate by one-quarter of a revolution to the vertical position but would not rotate further. The most common errors were to provide statements regarding the purpose of slip rings and/or split ring commutators, or to describe how the motor would no longer work as a generator. A number of students stated that the loop would rotate 180 degrees before stopping, and a few who stated that the loop would oscillate between the horizontal and vertical positions.

## 6.6 Photovoltaic cells

### 6.6 Exercise

**1** A photovoltaic cell is a device that transforms electromagnetic energy, such as light from the Sun, directly into electrical energy.

**2** PV panels produce DC voltage. It must be transformed into 230 V AC for use in buildings.

**3** If **photons** have enough energy, they cause **electrons** to be removed from atoms in the cell.
If the photons striking a solar cell **do not** have enough energy, their energy is transformed into **thermal** energy and the solar cell heats up.

**4** The n-type layer releases an electron when it absorbs energy from a photon that has enough energy. The electron drifts across the junction to fill in the 'positive holes' of the p-type layer, generating an electric current.

**5** B. False. Photovoltaic cells are only able to convert a fraction of the energy into electricity. Some photons are reflected or some turn into thermal energy.

### 6.6 Exam questions

**1** Photons from the Sun strike the solar cell. If a photon has enough energy, it will remove an electron from the n-type layer. This electron will then travel across the junction (boundary layer) to fill in the positive holes of the p-type layer, resulting in the generation of an electric current, which can then turn on the light bulb.

**2** This doping process is important because it is what allows electricity to be generated between the n-type and p-type layer when the cell is struck by the photons.

**3** B. While option C is a valid response, option B is more accurate. The more people that use solar energy, the less carbon emissions are generated from fossil fuels. For each kWh of electricity generated by solar energy, up to 1.18 kg of carbon dioxide is not released.

**4** In series, this will provide a total voltage output of 80 V, while keeping the amperage low, given current is shared in a series circuit.

**5** No, you would not be able to use electrical appliances in your home. The reason is that these appliances require an alternating current (AC) operating on a 230 V supply voltage.

## 6.7 Review

### 6.7 Review questions

**1** The current that is induced by the change in flux as the magnet approaches the tube will create a field to oppose this change. Therefore, the tube will have a south pole at the end near A and a north pole at the end near B.

**a** At Point A there will be a repulsive force.

**b** At Point B there will be an attractive force.

**2** $\varepsilon = -N \times \frac{\Delta\Phi}{\Delta t}$

At $N = 1$:

$\Delta\Phi = 3.2 \times 10^{-2} \times 8$
$= 0.256$

Since there are 10 rotations per second, and only $\frac{1}{40}$ s is required to get into position: $\Delta t = 0.25 \times 10^{-1}$.

$$\Rightarrow \varepsilon = -N \times \frac{\Delta\Phi}{\Delta t} = -1 \times \frac{0.256}{0.25 \times 10^{-1}} = 10.24\ \text{V}$$

**3** $\Phi = B \times A$

$\Rightarrow A = 0.02 \times 0.02$

$= 0.0004\ \text{m}^2$

$B = 5 \times 10^{-2}\ \text{T}$

$\Rightarrow \Phi = 0.0004 \times 0.005$

$= 2 \times 10^{-6}\ \text{Wb}$

**4** $\varepsilon = -N \times \frac{\Delta\Phi}{\Delta t}$

$= -10 \times \frac{\left(\frac{B}{3} - B\right)}{t}$

$= \frac{\frac{20B}{3}}{t}$

$= \frac{20B}{3t}\ \text{V}$

**5** $\varepsilon = Blv$

$= 5 \times 10^{-5} \times 5 \times 3$

$= 5 \times 10^{-4} \times 3$

$= 1.5 \times 10^{-3}\ \text{V}$

**6** An advantage of using photovoltaic cells as an energy source is that:

– sunlight is free
– excess energy can be stored in a battery or sold back to the grid
– no carbon emissions and quiet to run
– it is not reliant on fossil fuels [1 mark for an advantage listed]

A disadvantage of using photovoltaic cells is that:

– they can only generate electricity on sunny days
– to generate the maximum amount of electricity, they must face a certain direction
– they can be easily damaged
– they can be costly to install
– they have low efficiency. [1 mark for a disadvantage listed]

**7** **a** $\Phi = B \times A$

$= 9 \times 10^{-5} \times 0.25 \times 0.25$

$= 5.625 \times 10^{-6}\ \text{Wb}$

**b** Ninety degrees about the horizontal axis will render the coil parallel to the magnetic field and thus there will be zero flux threading the coil.

**c** $\varepsilon = -N \times \frac{\Delta\Phi}{\Delta t}$

$= -1 \times \frac{0 - 5.625 \times 10^{-6}}{0.5}$

$= 1.125 \times 10^{-5}\ \text{V}$

**8** **a** $\Phi = B \times A$

$A = \frac{\Phi}{B}$

$= \frac{4 \times 10^{-3}}{0.7}$

$= 5.71 \times 10^{-3}\ \text{m}^2$

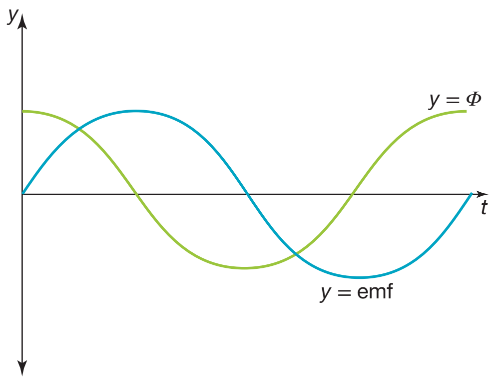
**b** $\varepsilon = -N \times \frac{\Delta\Phi}{\Delta t}$

The loop is rotated once every 0.1 s. This means that a quarter rotation occurs in 0.025 seconds:

$\varepsilon = -1 \times \frac{0 - 4 \times 10^{-3}}{0.025}$

$= 0.16\ \text{V}$

**c**




Award (1 mark) for the graph for flux.
Award (1 mark) for the graph for emf.

**d** Any two of the responses below are acceptable for the:

- increase the rate of rotation.
- increase the number of coils that create the loop.
- increase the magnetic field strength.
- increase the size of the coil.

**9** **a** $\Phi = B \times A$

$= 0.82 \times 0.3$

$= 0.246\ \text{Wb}$

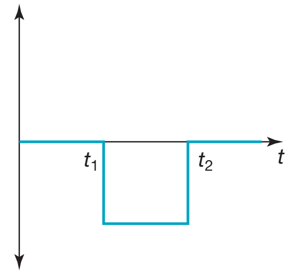
**b** $\Phi_{\text{final}} = 0.82 \times 0.1$

$= 0.082\ \text{Wb}$

$\Rightarrow \varepsilon = -1 \times \frac{0.082 - 0.246}{0.1}$

$= 1.6\ \text{V}$

**c**




Award (1 mark) for correct labels.
Award (1 mark) for correct shape.

**10** **a** The induced voltage can be thought of as the negative rate of change of flux.

Given the flux is being reduced by a constant amount until it reaches zero, it is expected that the voltage will be constant and positive as is seen in the graph in the question.

**b** $\varepsilon = -N \times \frac{\Delta\Phi}{\Delta t}$

$= \frac{A\left(\Phi_{\text{final}} - \Phi_{\text{initial}}\right)}{\Delta t}$

$= -1 \times \frac{A \times (0 - 1.7)}{5 \times 10^{-3}} = 1.2\ \text{V}$

$\Rightarrow A = 3.53 \times 10^{-3}\ \text{m}^2$

## 6.7 Exam questions

### Section A — Multiple choice questions

**1** C

$\varepsilon = B\frac{\Delta A}{\Delta t}$

$= 3.5 \times 10^{-4} \times \frac{0.05}{0.20}$

$= 8.8 \times 10^{-5}$ V

**2** C

$\varepsilon = \frac{\Delta \Phi}{\Delta t}$

$1.2 = 6 \times \frac{0.05}{\Delta t}$

$\Delta t = 0.25$ s

**3** D. The induced emf is equal to the negative gradient of the graph of magnetic flux versus time.
For the first half, the gradient is positive and constant; hence, the emf is constant and negative.
For the second half, the gradient is negative and constant; hence, the emf is constant and positive.

**4** A. This is a DC generator. The commutator allows a direct current to flow in the external circuit.
A split ring commutator is a device that reverses the direction of a current from a generator so that it flows in one direction through an electric circuit.

**5** D. According to Faraday's Law, the relationship between induced EMF and the frequency of a generator is:

$\varepsilon = -N\frac{\Delta \Phi}{\Delta t}$

That is, the rate of change of flux is directly related to the induced EMF OR the time taken to change the flux is inversely related to the induced EMF.
Therefore, if the rate of rotation is halved (the time taken to change the flux increases) then the EMF will decrease.
If the rate of rotation decreases by a factor of two, the time taken to change the flux increases by a factor of two, and therefore the induced EMF decreases by a factor of two (halved).
At 25 revolutions per second the period is doubled; therefore, the voltage will be halved.

**6** D

$f = \frac{1}{T}$

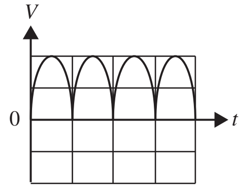
$f = \frac{1}{0.04}$

$f = 25$ Hz

**7** C. The halving of the period and the doubling of the voltage is consistent with doubling the speed of rotation according to the formula: $\varepsilon = \frac{\Delta \Phi}{\Delta t}$.

**8** C. The display should show double the period and half the amplitude of the original display.

**9** D. The split-ring commutator will reverse the voltage every half turn.




**10** A. Faraday's Law states $\varepsilon = -N\frac{\Delta \Phi}{\Delta t}$
The induced EMF is equal to the negative rate of change of flux or the negative gradient of the flux graph.
The answer is graph A because the voltage is shown as a sine graph and the gradient is a cosine graph.
The answer could be expected to be a negative cosine graph (based on Faraday's Law) but the question asks 'which of the following graphs best shows the relationships for this electrical generator', therefore, a positive cosine graph is the best response.

### Section B — Short answer questions

**11** **a** Anticlockwise. [1 mark]
The loop is travelling down near the north pole of the magnet and the magnetic force is to the left. Using the right-hand-slap rule, the induced current is into the page, which means the induced current will continue anticlockwise around the loop.

**b** $\varepsilon = -N\frac{\Delta \phi}{\Delta t}$ [1 mark]

$= -1 \times \frac{0.2 \times 10^{-3} \times 0.050 \times 0.035}{0.02}$ [1 mark]

$= -7 \times 10^{-5}$ V [1 mark]

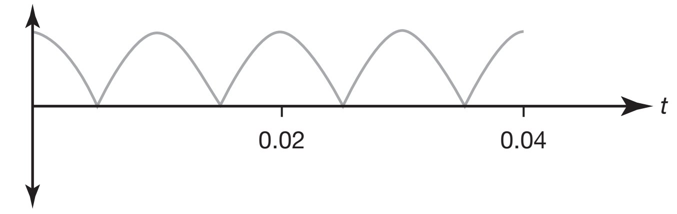
***VCAA examination report* note:**
The most common errors were forgetting to convert the mT to T or incorrectly calculating the period.

**c** See figure at the bottom of the page*
Award 1 mark for the correct shape, 1 mark for the correct period and 1 mark for sketching 2 rotations.

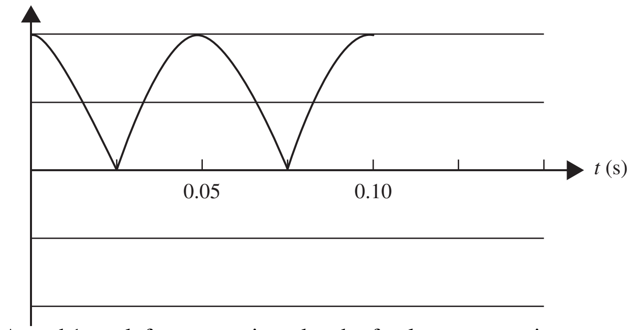
***VCAA examination report* note:**
The most common errors were drawing an unrectified sine or cosine wave. Most students got the period of the wave correct.

**d** Award 1 mark each for any two of the following:
- increase field strength
- increase the number of coils
- increase the area of coil
- increase the rotation rate or decrease period of rotation.

***11 c**



**12**




Award 1 mark for appropriate sketch of voltage versus time, and 1 mark for appropriate scales.

**13 a** The slip rings maintain a continuous electrical connection with the spinning loop [1 mark] and are used when an AC output is required [1 mark].

**b** $\Phi = BA$

$\Phi = 0.20 \times 0.02 \times 0.03$ [1 mark]

$\Phi = 1.2 \times 10^{-4}$ Wb [1 mark]

**c** $f = \frac{1}{T}$

$f = \frac{1}{0.2}$

$f = 5$ Hz [1 mark]

**d** See figure at the bottom of the page*

Award 2 marks for correct sketch, same frequency, emf doubled as the number of turns is doubled.

**14 a** $\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$ [1 mark]

$\varepsilon = 20 \times 4.0 \times 10^{-2} \times \frac{0.010}{0.10}$ [1 mark]

$\varepsilon = 0.080$ V [1 mark]

***VCAA examination report* note:**

The question asked for the magnitude of the EMF, so the minus sign has been omitted from the working.

**b** See figure at the bottom of the page**

Award 2 marks for appropriate sketch.

***VCAA examination report* note:**

Students were required to draw a sine wave of 2.5 cycles. An inverted graph was also acceptable.

**15 a** The force on the wire can be calculated thus:

$F = BIl$

$F = 0.02 \times 0.5 \times 0.05$ [1 mark]

$F = 5 \times 10^{-4}$ N [1 mark]

**b** Using the right-hand-slap rule:

- your fingers (out straight) represent the magnetic field, from the north pole to the south pole, thus from left to right [1 mark]
- your thumb represents the current W → X → Y → Z, so it is pointing out of the page for branch XW [1 mark]
- the palm of the hand represents the force, here the palm of your hand is upwards, thus the force acting on XW is upwards and the motor will rotate in a anticlockwise direction [1 mark]

***VCAA examination report* note:**

Students were required to identify that the loop would rotate anticlockwise. The reason is that the current flows from W to X and from Y to Z. Using a rule such as the right-hand slap rule shows that the side WX will be forced down while the side YZ will be forced upwards resulting in anticlockwise rotation.

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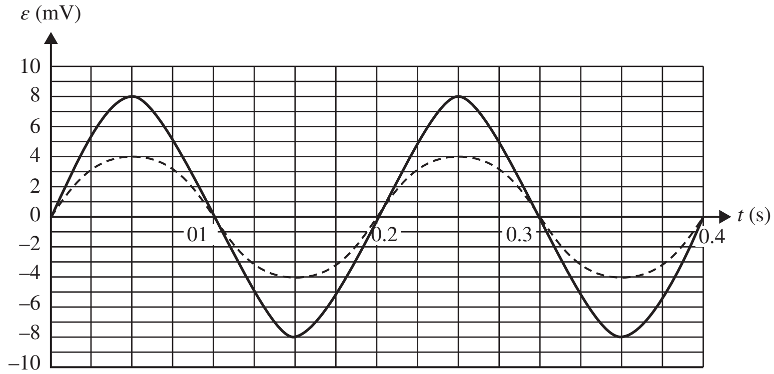
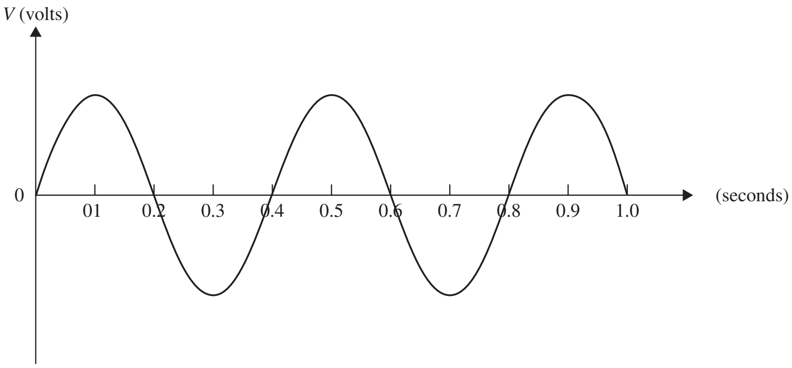
***13 d**




****14 b**



**c** The role of a split ring commutator is to reverse the coil's current direction every half revolution of the loop so that the direction of the current flowing through the external circuit remains the same [1 mark].

The alternating current in the loop is thus converted into direct current. [1 mark]

***VCAA examination report*** **note:**

Students were required to identify that the role of the commutator is to reverse the direction of the current every half turn to keep the motor rotating in the same direction.

# Topic 7 — Transmission of electricity

## 7.2 Peak, RMS and peak-to-peak voltages

### Sample problem 1

$$V_{\text{RMS}} = \frac{V_{\text{peak}}}{\sqrt{2}}$$

$$6.3 = \frac{V_{\text{peak}}}{\sqrt{2}}$$

$$V_{\text{peak}} = 6.3\sqrt{2}$$

$$\Rightarrow V_{\text{p-p}} = 2V_{\text{peak}}$$

$$= 2 \times 6.3 \times \sqrt{2}\ \text{V}$$

$$= 18\ \text{V}$$

### Practice problem 1

$$P = I_{\text{RMS}} V_{\text{RMS}}$$

$$\Rightarrow I_{\text{RMS}} = \frac{P}{V_{\text{RMS}}}$$

$$= \frac{1800}{230} \approx 7.826\ \text{A}$$

$$\Rightarrow I_{\text{peak}} = \sqrt{2} \times I_{\text{RMS}}$$

$$= \sqrt{2} \times 7.826 \approx 11.07\ \text{A}$$

### 7.2 Exercise

1 $V_{\text{peak}} = V_{\text{RMS}}\sqrt{2}$

$= 6.3 \times \sqrt{2} = 8.9\ \text{V}$

2 a There are 6 divisions in the $x$-axis. The period is T = 5 × 6 = 30 ms.

b $f = \frac{1}{T} = \frac{1}{30 \times 10^{-3}} = 33.3\ \text{Hz}$

c There are 5 divisions in the $y$-axis. The peak voltage is $V_{\text{peak}} = 10 \times 5 = 50\ \text{mV}$.

d $V_{\text{peak-to-peak}} = 2V_{\text{peak}} = 100\ \text{mV}$

e $V_{\text{RMS}} = \frac{V_{\text{peak}}}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35\ \text{mV}$

3 $V_{\text{RMS}} = \frac{V_{\text{peak}}}{\sqrt{2}} = \frac{9}{\sqrt{2}} = 6.4\ \text{V}$

### 7.2 Exam questions

1 $V_{\text{RMS}} = \frac{V_P}{\sqrt{2}} = \frac{3.5}{\sqrt{2}} = 2.5\ \text{V}$ [1 mark]

2 $f = \frac{1}{T}$

$= \frac{1}{40 \times 10^{-3}}$ [1 mark]

$= 25\ \text{Hz}$ [1 mark]

3 $f = \frac{1}{T} = \frac{1}{40 \times 10^{-3}}$ [1 mark]

$= 25\ \text{Hz}$ [1 mark]

4 A

$V_{\text{RMS}} = \frac{V_{\text{peak}}}{\sqrt{2}}$

$= \frac{V_{\text{p-p}}}{2\sqrt{2}}$

$= 8.5\ \text{V}$

5 The period of rotation is 80 ms. [1 mark]

$f = \frac{1}{T}$

$= \frac{1}{0.08}$

$= 12.5\ \text{Hz}$ [1 mark]

## 7.3 Transformers

### Sample problem 2

a $P = VI$

$100 = 230I$

$I = \frac{100}{230}$

$= 0.435\ \text{A}$

b $V = IR$

$230 = 0.435\,R$

$R = \frac{230}{0.435}$

$= 529\ \Omega$

c i $R_{\text{total}} = R_{\text{copper}} + R_{\text{globe}}$

$= \left(2.00\ \text{m} \times 0.0220\ \Omega\,\text{m}^{-1}\right) + 529\ \Omega$

$\approx 529\ \Omega$

The resistance in the wire is negligible.

ii $V = IR$

$I = \frac{V}{R}$

$= \frac{230\ \text{V}}{529\ \Omega}$

$= 0.435\ \text{A}$

$V = IR$

$= 0.435\ \text{A} \times 529\ \Omega$

$= 230\ \text{V}$

The voltage drop across the globe is 230 V.

d i $R_{\text{total}} = R_{\text{copper}} + R_{\text{globe}}$

$= \left(100 \times 10^3\ \text{m} \times 0.022\ \Omega\,\text{m}^{-1}\right) + 529\ \Omega$

$\approx 2729\ \Omega$

The total resistance of the circuit is 2730 Ω.
The resistance of the copper wire is not negligible.

ii $V = IR$

$I = \frac{V}{R}$

$= \frac{230\ \text{V}}{2729\ \Omega}$

$= 0.0842\ \text{A}$

$V = IR$

$= 0.0842\ \text{A} \times 529\ \Omega$

$= 44.6\ \text{V}$

e The globe would not light up.

## Practice problem 2

**a** $P = VI$

$$I = \frac{P}{V} = \frac{800}{230} = 3.48\ \text{A}$$

**b** $V = IR$

$$R = \frac{V}{I} = \frac{230}{3.48} = 66.1\ \Omega$$

**c i** $R_{\text{total}} = R_{\text{copper}} + R_{\text{toaster}}$

$$= \left(1.00\ \text{m} \times 0.0350\ \Omega\ \text{m}^{-1}\right) + 66.1\ \Omega = 66.1\ \Omega$$

**ii** The current through the circuit can be found using Ohm's Law:

$V = IR$

$$I = \frac{V}{R} = \frac{230}{66.1} \approx 3.48\ \text{A}$$

The voltage drop can then be found by Ohm's Law:

$V = IR$

$$= 3.48 \times 66.1 \approx 230\ \text{V}$$

**d i** $R_{\text{total}} = R_{\text{copper}} + R_{\text{toaster}}$

$$= \left(20\,000\ \text{m} \times 0.0350\ \Omega\ \text{m}^{-1}\right) + 66.1\ \Omega = 766\ \Omega$$

**ii** The current through the circuit can be found using Ohm's Law:

$V = IR$

$$I = \frac{V}{R} = \frac{230\ \text{V}}{766\ \Omega} \approx 0.300\ \text{A}$$

The voltage drop can then be found by Ohm's Law:

$V = IR$

$$= 0.300 \times 66.1 \approx 19.8\ \text{V}$$

**e** Most of the 230 V is dissipated in the copper wire, leaving only a small amount for the toaster. The toaster would not operate properly.

## Sample problem 3

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{230}{12} = \frac{190}{N_2}$$

$$N_2 = \frac{12 \times 190}{230} = 9.9$$

The secondary coil consists of approximately 10 turns.

## Practice problem 3

Use the formula $P_1 = V_1 I_1$ to find the current:

$$P_1 = V_1 I_1$$

$$10\,000 = 1000 \times I_1$$

$$I_1 = 10\ \text{A}$$

$$\Rightarrow \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

$$\frac{N_1}{N_2} = \frac{0.5}{10}$$

$$\frac{N_1}{N_2} = \frac{1}{20}$$

The turns ratio is 1:20, making this transformer a step-up transformer.

## 7.3 Exercise

**1** Use the formula for the voltage and number of turns in a transformer:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\Rightarrow V_2 = \frac{V_1 \times N_2}{N_1}$$

$$V_2 = \frac{230 \times 2000}{100}$$

$$V_2 = 4600\ \text{V}$$

**2 a** $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$\Rightarrow V_2 = \frac{V_1 \times N_2}{N_1} = \frac{230 \times 6}{300} = 4.6\ \text{V}$$

**b** $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$\Rightarrow V_1 = \frac{N_1 \times V_2}{N_2} = \frac{300 \times 9.0}{6} = 450\ \text{V}$$

**3 a** $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$\Rightarrow N_1 = \frac{V_1 \times N_2}{V_2} = \frac{230 \times 50}{12} \approx 960\ \text{turns}$$

**b** Current in each globe:

$$I = \frac{P}{V} = \frac{5}{12} = 0.4167\ \text{A}$$

Twenty globes would draw a total of:

$20 \times 0.4167 = 8.33 \approx 8\ \text{A}$

**c** Use conservation of energy to obtain the current in the primary coil:

$$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$

$$\Rightarrow I_1 = \frac{I_2 \times V_2}{V_1}$$
$$= \frac{8.33 \times 12}{230}$$
$$\approx 0.4\text{ A}$$

**4** A transformer requires a changing magnetic flux in order to work; thus, a transformer will not work with a DC input voltage, which produces a constant magnetic flux.

**5** The core of transformers is made of an iron alloy that is easy to magnetise, allowing the magnetic field to change direction quickly as the current changes direction.

**6** **a** As the primary voltage is higher than the secondary voltage, this is a step-down transformer.

**b** $$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$
$$\Rightarrow N_2 = \frac{N_1 \times V_2}{V_1}$$
$$= \frac{230 \times 2000}{10\,000}$$
$$= 46\text{ turns}$$

**7** **a** $$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$
$$\Rightarrow V_2 = \frac{V_1 \times N_2}{N_1}$$
$$= \frac{60 \times 900}{400}$$
$$= 135\text{ V}$$

**b** $P = IV$
$$= 0.30 \times 135$$
$$= 41\text{ W}$$

**c** $$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$
$$\Rightarrow I_1 = \frac{I_2 \times V_2}{V_1}$$
$$= \frac{P_2}{V_1}$$
$$= \frac{40.5}{60}$$
$$= 0.68\text{ A}$$

## 7.3 Exam questions

**1** $$\frac{V_p}{V_s} = \frac{I_s}{I_p}$$
$$\frac{240}{5} = \frac{3}{I_p}$$
$$I_p = 0.06\text{ A}$$

**2** **a** $V_p = V_{\text{RMS}} \times \sqrt{2}$ [1 mark]
$$= 12 \times \sqrt{2}$$
$= 17\text{ V}$ [1 mark]

***VCAA examination report* note:**

Students are reminded that fractions and surds are seen as part of the working process and not the final answer.

Students who responded with $12\sqrt{2}$ were recognised as having demonstrated the correct working but not having demonstrated the correct answer.

**b** $$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$
$$= \frac{240}{12}$$
$= 20$ or $20 : 1$ [1 mark]

**c** In an ideal transformer, $P_p = P_s$.

$P = VI$ [1 mark]

$$0.9 = 240I$$
$$I = \frac{0.9}{240}$$
$$= 0.00375\text{ A}$$
$= 3.8\text{ mA}$ [1 mark]

***VCAA examination report* note:**

The most common error was to use the 12 V from the secondary side of the transformer.

**3** X: Primary coil (connected to power supply)

Y: Secondary coil (connected to load)

Function: This is a step-up transformer because there are more secondary coils (turns) than primary coils (turns).

$N_{\text{secondary}} > N_{\text{primary}}$

**4** Find the current in the secondary coil using Ohms Law:

$V = IR$

$$I_s = \frac{V_{RMS}}{R}$$
$$I_s = \frac{240}{1200}$$

Apply the relationship between current in a transformer and the number of primary and secondary coils.

$$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$
$$I_p = I_s \times \frac{N_s}{N_p}$$
$$I_p = 0.2 \times \frac{6000}{1000}$$

$I_p = 1.20$ A (RMS current)

**5** input $V_{\text{peak}} = 45 \times$ output $V_{\text{peak}}$

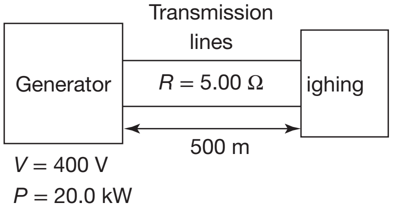
$= 45 \times 300 = 13\,500\text{ V}$ [1 mark]

$$\Rightarrow \text{input } V_{RMS} = \frac{V_{peak}}{\sqrt{2}}$$
$= \frac{13\,500}{\sqrt{2}} = 9.55 \times 10^3\text{ V}$ [1 mark]

## 7.4 Power distribution and transmission line losses

### Sample problem 4

**a** **i**



$P = VI$

$I = \frac{P}{V}$

$= \frac{20\,000}{400}$

$= 50.0\text{ A}$

The current through the cables is 50.0 A.

(Note: Using $V = IR$ with $V = 400$ V and $R = 5.00\,\Omega$ is incorrect because 400 V is the voltage supplied by the generator, it is not the voltage drop across the cables.)

**ii** $V = IR$

$= 50.0 \times 5.00$

$= 250\text{ V}$

**iii** $P_{\text{loss}} = I^2R$

$= 50.0 \times 50.0 \times 5.00$

$= 12\,500\text{ W}$

(Note: This answer could have been obtained by using $P = VI$, with $V = 250$ V from solution ii; however, there is a risk that 400 V may be used by mistake, so it is better to use $I^2R$.)

$\% P_{\text{loss}} = \frac{12\,500}{20\,000} \times \frac{100}{1}$

$= 62.5\%$

**iv** $V_{\text{generator}} = V_{\text{cables}} + V_{\text{lights}}$

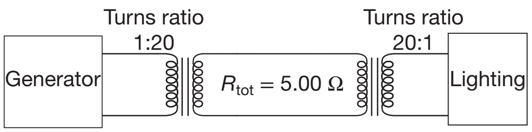
$V_{\text{lights}} = V_{\text{generator}} - V_{\text{cables}}$

$= 400\text{ V} - 250\text{ V}$

$= 150\text{ V}$

At this distance, the voltage drop across the cables is too much to leave sufficient voltage to operate the lights at their designated voltage. Given the noise of the generators, they cannot be moved closer. Therefore, step-up and step-down transformers with turns ratios of 20 are used to reduce the power loss in the cables and increase the voltage at the lights.

**b** **i**




$V = 400$ V

$P = 20.0$ kW

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$\frac{400\text{ V}}{V_2} = \frac{1}{20}$

$V_2 = 20 \times 400$

$= 8000\text{ V}$

$P_1 = P_2 = 20\,000\text{ W};\ V_2 = 8000\text{ V}$

$\Rightarrow P_1 = P_2 = V_2 I_2$

$I_2 = \frac{P_1}{V_2}$

$= \frac{20\,000}{8000}$

$= 2.50\text{ A}$

**ii** $V = IR$

$= 2.50 \times 5.00$

$= 12.5\text{ V}$

**iii** $P_{\text{loss}} = I^2 R$

$= 2.50 \times 2.50 \times 5.00$

$= 31.3\text{ W}$

$\% P_{\text{loss}} = \frac{31.25}{20\,000} \times \frac{100}{1}$

$= 0.156\%$

This is $\left(\frac{1}{20}\right)^2$ or $\frac{1}{400}$ of the original power loss! This is an impressive reduction.

**iv** Voltage supplied to the step-down transformer is therefore: $8000\text{ V} - 12.5\text{ V} = 7988\text{ V}$.

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$\frac{7988}{V_2} = \frac{20}{1}$

$V_2 = \frac{7988}{20}$

$\approx 400\text{ V}$

---

### Practice problem 4

**a** **i** $I_{\text{cable}} = \frac{P}{V}$

$= \frac{50.0 \times 10^3}{250}$

$= 200\text{ A}$

**ii** $V_{\text{cable}} = I_{\text{cable}} R$

$= 200 \times 0.200$

$= 40.0\text{ V}$

**iii** $P_{\text{loss}} = I^2_{\text{cable}} R$

$= 200^2 \times 0.200$

$= 8000\text{ W}$

$= 8.00\text{ kW}$

$\Rightarrow \%P_{\text{loss}} = \frac{8000}{50.0 \times 10^3} \times 100 = 16.0\%$

**iv** $V = 250 - 40.0$

$= 210\text{ V}$

**b** **i** The voltage in the transmission line is:

$V_2 = 10 \times V_1$

$= 10 \times 250$

$= 2500\text{ V}$

The current in the transmission line is:

$I_2 = \frac{P}{V_2}$

$= \frac{50.0 \times 10^3}{2500}$

$= 20.0\text{ A}$

**ii** $V_{\text{cable}} = I_{\text{cable}} R$

$= 20.0 \times 0.200$

$= 4.00\text{ V}$

**iii** $P_{\text{loss}} = I^2_{\text{cable}} R$

$= 20.0^2 \times 0.200$

$= 80.0\text{ W}$

$\Rightarrow \%P_{\text{loss}} = \frac{80.0}{50.0 \times 10^3} \times 100 = 0.160\%$

**iv** Employing a step-down transformer with the same turns ratio, the voltage supplied to the step-down transformer is 2500 − 4 = 2496 V.

The voltage supplied on the secondary side of the step-down transformer is:

$$V_2 = \frac{V_1}{10} = \frac{2496}{10} = 250\text{ V}$$

---

## 7.4 Exercise

**1 a** $I = \frac{P}{V} = \frac{50\,000}{250} = 200\text{ A}$

**b** $P_{loss} = I_{cable}^2 R = 200^2 \times 0.30 = 12\,000\text{ W} = 12\text{ kW}$

**c** $V_{cable} = I_{cable} R = 200 \times 0.30 = 60\text{ V}$

**d** $V = 250 - 60 = 190\text{ V}$

**e i** $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$\Rightarrow V_2 = \frac{V_1 \times N_2}{N_1} = \frac{250 \times 20}{1} = 5000\text{ V}$$

By conservation of energy,

$$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$

$$\Rightarrow I_2 = \frac{V_1 \times I_1}{V_2} = \frac{250 \times 200}{5000} = 10.0\text{ A}$$

**ii** $P_{loss} = I_{cable}^2 R = 10.0^2 \times 0.30 = 30\text{ W}$

**iii** $V_{cable} = I_{cable} R = 10.0 \times 0.30 = 3.0\text{ V}$

**iv** The voltage in the transmission line on the secondary side of the step-up transformer is 5000 V. Employing a step-down transformer with the same turns ratio, the voltage supplied to the step-down transformer is 5000 − 3 = 4997 V. The voltage supplied on the secondary side of the step-down transformer is:

$$V_2 = \frac{V_1}{20.0} = \frac{4997}{20.0} = 250\text{ V}$$

**2 a** $V = RI$

$$230 = R \times 40$$

$$R = \frac{230}{40} = 5.75\ \Omega$$

$R = 5.8\ \Omega$ to 2 s.f.

**b** The total resistance of the circuit is $R = R_{appliance} + R_{line}$, with $R = 5.8\ \Omega$ and $R_{line} = 0.2\ \Omega$

Thus $R_{appliance} = 5.6\ \Omega$

Alternatively, without using the answer to part a.:

$$V = (R_{appliance} + R_{line})I$$

$$230 = (R_{appliance} + 0.2) \times 40$$

$$222 = R_{appliance} \times 40$$

$$R_{appliance} = \frac{222}{40} = 5.6\ \Omega \quad \text{to 2 s. f.}$$

**c** The voltage drop across the transmission lines is:

$$V_{line} = I_{line} R = 40 \times 0.20 = 8.0\text{ V}$$

Therefore, the voltage at the house is 230 − 8.0 = 222 V

**d i** The 20-kW workshop operating off the 230-V supply has the resistance:

$$R_{workshop} = \frac{V}{I}$$

Recall the formula $P = IV$

$$R_{workshop} = \frac{V}{\left(\frac{P}{V}\right)} = \frac{V^2}{P} = \frac{230^2}{20\,000} = 2.645\ \Omega \approx 2.7\ \Omega$$

**ii** The equivalent resistance of the circuit with the house and workshop in parallel is

$$\Rightarrow R_{eff} = \frac{R_{house} \times R_{workshop}}{R_{house} + R_{workshop}} = \frac{5.75 \times 2.645}{5.75 + 2.645} = 1.8\ \Omega$$

**iii** Current in the workshop:

$$I = \frac{V}{R} = \frac{230}{2.65} = 87\text{ A}$$

**iv** Total current in the transmission line

= 40 + 87 = 127 A

**e i** Voltage drop across transmission lines:

$$V_{line} = I_{line}R = 127 \times 0.20 = 25\text{ V}$$

**ii** Voltage at the house and workshop:

230 − 25 = 205 V

**f** Employing a step-up transformer with a turns ratio of 1:10, the current in the transmission line is:

$$I_2 = \frac{I_1}{10} = \frac{126.8}{10} = 12.68\text{ A}$$

The voltage drop across the transmission lines is:

$$V_{line} = I_{line} \times R = 12.68 \times 0.20 = 2.5\text{ V}$$

The voltage across the transmission line on the secondary side of the step-up transformer is:
$V_2 = V_1 \times 10 = 230 \times 10 = 2300\,\text{V}$
At the step-down transformer, after the voltage drop across the transmission line, the voltage on the primary side is:
$2300 - 2.5 = 2297.5\,\text{V}$
On the secondary side of the step-down transformer, the voltage is: $V_{sec} = \frac{V_{prim}}{10} = \frac{2297.5}{10} = 229.8\,\text{V}$, which is within 1% of 230 V for the appliances to work properly.

**g** When the workshop is turned off, it does not draw any current on the transmission line, resulting in a lower transmission line current and therefore a lower voltage drop across the transmission line. This increases the voltage at the primary side of the step-down transformer, and therefore a higher voltage at the house.

**3 a** Conservation of energy means that the power in the secondary coil is the same as that supplied by the generator. Therefore, the current in the transmission lines is:

$$I_2 = \frac{P}{V_2} = \frac{220 \times 10^6}{330 \times 10^3} = 667\,\text{A}$$

**b** $P_{loss} = I^2_{line}R$
$= 667^2 \times 0.40$
$= 177.78 \times 10^3 \approx 180\,\text{kW}$

**c** $V_{line} = I_{line}R$
$= 667 \times 0.40$
$= 266.7 \approx 270\,\text{V}$

**d** Voltage available: $330\,000 - 270 = 329\,730 \approx 330\,\text{kV}$
Power available: $329\,733 \times 667 = 219\,932 \approx 220\,\text{MW}$

**4 a** The current in the transmission line is:

$$I_{line} = \frac{P}{V} = \frac{500 \times 10^6}{40 \times 10^3} = 12\,500\,\text{A}$$

The power loss in the transmission lines is:
$P_{loss} = I^2_{line}R$
$= 12\,500^2 \times 0.80$
$= 125 \times 10^6\,\text{W}$
$= 125\,\text{MW}$

$$\Rightarrow \%P_{loss} = \frac{125}{500} \times 100 = 25\%$$

**b** If the voltage is stepped up to 400 kV using a transformer with a turns ratio of 10:1, the current in the transmission line would be:

$$I_{line} = \frac{12\,500}{10} = 1250\,\text{A}$$

The power loss in the transmission lines would be:
$P_{loss} = I^2_{line}R$
$= 1250^2 \times 0.80$
$= 1.25 \times 10^6\,\text{W}$
$= 1.3\,\text{MW}$

$$\Rightarrow \%P_{loss} = \frac{1.25}{500} \times 100 = 0.25\%$$

The power loss in transmission would decrease from 25% to 0.25%.

## 7.4 Exam questions

**1 a** $\frac{N_P}{N_S} = \frac{V_P}{V_S}$

$\frac{N_P}{N_S} = \frac{240}{8}$ [1 mark]

$\frac{N_P}{N_S} = 30 : 1$ [1 mark]

**b** $\frac{V_P}{V_S} = \frac{I_S}{I_P}$

$\frac{240}{8} = \frac{I_S}{0.35}$ [1 mark]

$I_S = 10.5\,\text{A}$ [1 mark]

***VCAA examination report* note:**
The VCAA is aware of the error in the wording for this question. The RMS current delivered by the power point while the battery is charging is given in the question stem as 0.35 A. However, most students recognised that the question was asking them to calculate the RMS current delivered to the battery, which is calculated by:

$\frac{V_P}{V_S} = \frac{I_S}{I_P}$

$\frac{240}{8} = \frac{I_S}{0.35}$

$I_S = 10.5\,\text{A}$

Students were awarded full marks for either 0.35 A or 10.5 A.

**2 a** $I_{line} = \frac{P_{globe}}{V_{globe}}$

$I_{line} = \frac{480}{240}$

$I_{line} = 2\,\text{A}$ [1 mark]
$P_{loss} = I^2R$
$P_{loss} = 2^2 \times 40$
$P_{loss} = 160\,\text{W}$ [1 mark]

**b** $V_{line} = IR$
$V_{line} = 2.0 \times 40$ [1 mark]
$V_{line} = 80\,\text{V}$ [1 mark]
$V_{in} = 240 + 80$
$V_{in} = 320\,\text{V}_{RMS}$ [1 mark]

**c** $I_{wires} = \frac{1}{8} \times 2.0$

$I_{wires} = 0.25\,\text{A}$ [1 mark]
$P_{loss} = I^2R$
$P_{loss} = 0.25 \times 40$ [1 mark]
$P_{loss} = 2.5\,\text{W}$ [1 mark]

**3 a** As the transformer ratio is 4:1 and the current in the transmission lines is 3 A, this means that the current through the globe is $3.0 \times 4.0 = 12$ A. [1 mark]
$P = VI$
$= 4.0 \times 12$
$= 48\,\text{W}$ [1 mark]

***VCAA examination report* note:**
The most common error was to calculate the power using the current in the transmission lines as the current through the globe. Another common error was to calculate the current through the globe as one-quarter of the transmission line current rather than four times the transmission line current.

Further improvement is needed on basic transformer behaviour.

**b** **Method 1:**

The voltage drop across the transmission lines is:

$V = IR$

$= 8.0 \times 3.0$

$= 24\text{ V}$ [1 mark]

The input voltage to the transformer is four times the globe voltage or 16 V. [1 mark]

The output voltage of the power supply must be the sum of these two voltages:

$V = 24 + 16 = 40\text{ V}$ [1 mark]

**Method 2:**

The power dissipated in the light globe is 48 W.

The total power lost in the transmission line is:

$P_{\text{loss}} = I^2R$

$= 3.0^2 \times 8$

$= 72\text{ W}$ [1 mark]

The total power supplied is $48\text{ W} + 72\text{ W} = 120\text{ W}$ [1 mark]

$P = VI$

$120 = 3V$

$V = 40\text{ V}$ [1 mark]

***VCAA examination report*** **note:**

The most common error was to use 4 Ω for the resistance of the lines rather than 8 Ω. Other common errors were to find either the voltage drop across the lines or the voltage at the primary coil of the transformer and assume that this was the answer.

**c** $P_{\text{loss}} = I^2R$

$= 3.0^2 \times 8$ [1 mark]

$= 72\text{ W}$ [1 mark]

***VCAA examination report*** **note:**

Students who knew to use the correct formula generally substituted an erroneous current.

**d** The stem stated that 'the light globe operates correctly, with $4.0\text{ V}_{\text{RMS}}$ across it'. From this it can be determined that the current through the globe is still 12 A. [1 mark]

As the transformer ratio is now 8:1, the line current is one-eighth of this:

$I_{\text{line}} = \dfrac{12}{8} = 1.5\text{ A}$ [1 mark]

The power loss can now be calculated:

$P_{\text{loss}} = I^2R$

$= 1.5^2 \times 8$

$= 18\text{ W}$ [1 mark]

***VCAA examination report*** **note:**

Many students struggled with how to apply the new 8:1 ratio.

**e** Some valid reasons are:

- reducing the transmission current
- reducing the power lost in the lines
- reducing the size or cost of the wires.

Award 1 mark for each valid reason, maximum 2 marks.

***VCAA examination report*** **note:**

A common error was to state one of the factors above and explain that action in detail. While the response was often correct, the question required two reasons.

A significant number of students did not seem to have any knowledge of why high voltages are used.

**4 a** $P_{\text{loss}} = I^2R = (200)^2 \times 3.0$ [1 mark]

$= 120\,000 = 120\text{ kW}$ [1 mark]

***VCAA examination report*** **note:**

The most common error was incorrectly converting to kW.

**b** $P_{\text{loss}} = I^2R = 400^2 = 4.8 \times 10^5\text{ W}$ [1 mark]

$\dfrac{4.8 \times 10^5}{120\,000} = 4$

$\Rightarrow$ a factor of 4 increase [1 mark]

Alternatively, the following method may be determine the factor the power loss would increase.

If the line voltage is halved, then the line current is doubled (for the same power).

Original power loss: $P_1 = I_1{}^2R$

New power loss: $P_2 = (2I_1)^2 R = 4I_1{}^2R = 4P_1$

***VCAA examination report*** **note:**

Many students were unable to find the new current. A number of students who found the new current reported the power loss ($4.8 \times 10^5$ W) rather than the factor-of-four increase.

**5 a** $P_A = \dfrac{V^2}{R}$

$= \dfrac{18^2}{9}$ [1 mark]

$= 36\text{ W}$ [1 mark]

**b** Total resistance of lines and globe B is:

$R_r = 1.5 + 1.5 + 9$

$= 12\,\Omega$

$\Rightarrow I_{\text{line}} = \dfrac{V}{R_r}$

$= \dfrac{18}{12}$

$= 1.5\text{ A}$ [1 mark]

$\Rightarrow V_{\text{line}} = I_{\text{line}}R_{\text{line}}$

$= 1.5 \times 3.0$

$= 4.5\text{ V}$ [1 mark]

OR students could use a voltage divider approach sharing the 18 V between the line and globe B.

***VCAA Assessment Report*** **note:**

This question was not answered well. The range of responses indicated that students struggled to model this. Many students simply added all the resistances in the circuit. It is recommended that students spend more time learning to analyse line loss.

**c** Using the line current found earlier, the simplest method is to use:

$P_B = I^2R_B$ [1 mark]

$= 1.5^2 \times 9$ [1 mark]

$= 20\text{ W}$ [1 mark]

An alternative method is:

$V_{\text{globe }B} = V_{\text{supply}} - V_{\text{line}}$

$= 18 - 4.5$

$= 13.5\text{ V}$ [1 mark]

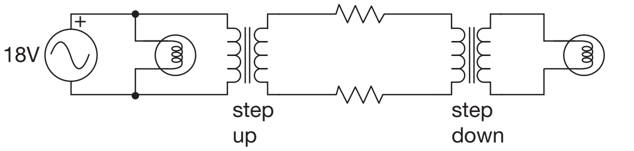
$\Rightarrow P_B = \dfrac{V^2}{R_B}$

$= \dfrac{13.5^2}{9}$ [1 mark]

$= 20\text{ W}$ [1 mark]

***VCAA Assessment Report* note:**
The most common error in this question was not being able to identify the voltage across the globe.

**d** A suitable diagram is shown below, which includes:
- 18 V AC supply, the line resistors and globe B correct [1 mark]
- Both transformers correctly placed and labelled [1 mark]




***VCAA Assessment Report* note:**
The most common error was to draw the circuit with a DC supply. Another common error was not knowing how to draw or place transformers in the circuit.

**e** Transmission power line loss is proportional to $I^2$. [1 mark]
The output power of a transformer is fixed, equal to the input power. [1 mark]
The step-up transformer increases the output voltage and so decreases the line current for the same power.
This reduces the power loss in the line. [1 mark]

***VCAA Assessment Report* note:**
While most students were able to articulate one or two of these points, few were able to articulate all three. The most common omission was the concept of fixed or constant power delivery.

## 7.5 Review

## 7.5 Review questions

**1 a** $P = \frac{V^2}{R}$

$$R = \frac{V^2}{P} = \frac{6.0^2}{24} = 1.5\,\Omega$$

**b** $V_{peak} = V_{RMS} \times \sqrt{2}$
$$= 6.0 \times \sqrt{2} = 8.5\,\text{V}$$

**2 a** $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$\frac{240}{18} = \frac{N_1}{30}$$
$$N_1 = 30 \times \frac{240}{18} = 400$$

**b** Since the power input is the same as the power output, the input power must also be 40 W. The primary voltage is 240 V.

$$P = VI$$
$$I = \frac{P}{V} = \frac{40}{240} = 0.17\,\text{A}$$

**3** $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$\frac{110}{V_{out}} = \frac{30}{1150}$$
$$V_{out} = \frac{110 \times 1150}{30} = 4217\,\text{V} \approx 4200\,\text{V}$$

**4** $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$\frac{240}{1200} = \frac{N_1}{N_2}$$
$$\frac{N_2}{N_1} = \frac{5}{1}$$

**5 a** This is a step-down transformer, so the secondary coil will have fewer turns.
The turn ratio is 5:1. Therefore the secondary coil has $\frac{2000}{5} = 400$ turns.

**b** The voltage required by the globe is 2.0 V; therefore, the voltage on the primary side of the transformer is $2.0 \times 5 = 10$ V (using the 5:1 ratio).
The current out of the transformer is:

$$P = VI$$
$$6.0 = 2I$$
$$I = 3.0\,\text{A}$$

Therefore, the current into the transformer must be $\frac{3.0}{5} = 0.60$ A.
Finally, the power loss in the transmission cables is given by:

$$P = I^2R = 0.60^2 \times (2.0 + 2.0) = 1.4\,\text{W}$$

**c i** The transformer is assumed to be ideal, so the power at the input to the primary coil is the same as the power at the globe: 6.0 W.

**ii** A total of 1.4 W is lost in transmission, so $6.0 + 1.4 = 7.4$ W is delivered at the output of the power supply.

**d** The globe would not glow.
A transformer requires a changing current in the primary coil to produce a changing magnetic field. The resulting changing flux in the secondary coil induces a voltage.
Since the 12.4 V battery supplies a constant voltage to the input of the transformer, the magnetic field in the transformer core remains constant and results in a constant magnetic flux in the secondary coil. Because there is no change in flux in the secondary coil, no voltage is induced and hence the globe will not glow.

**6** The definition of an ideal transformer involves having $P_{in} = P_{out}$.

$$V_{in}I_{in} = V_{out}I_{out}$$
$$240 \times I_{primary} = 13 \times 2.7$$
$$I_{primary} = 0.15\text{ A}$$

**7** **a**
$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$
$$\frac{240}{8.0 \times 10^3} = \frac{60}{N_p}$$
$$N_p = \frac{60 \times 8.0 \times 10^3}{240}$$
$$= 2000\text{ turns}$$

**b** Since the transformer is ideal, power in = power out.

$\Rightarrow$ power in $= 4.0 \times 10^5$
$$P = VI$$
$$4.0 \times 10^5 = 8.0 \times 10^3 \times I$$
$$\Rightarrow I = 50\text{ A}$$

**c** The power loss in the cables is given by:
$$P = I^2R$$
$$= 50^2 \times 4.0$$
$$\Rightarrow P_{loss} = 1.0 \times 10^4\text{ W}$$

**d**
$$V = IR$$
$$V_{loss} = 50 \times 4.0$$
$$= 200\text{ V}$$

Therefore, the substation is supplying the electricity at:
$8.0 \times 10^3 + 200 = 8.2 \times 10^3$ V

## 7.5 Exam questions

### Section A — Multiple choice questions

**1** B

***VCAA examination report*** **note:**

The question is unclear as to whether the transformer is a step-up or step-down. As a result, both the following solutions were accepted.

$30.0\text{ mA} \times 350 = 1.1 \times 10^4\text{ mA}$

$30.0\text{ mA} \div 350 = 0.086\text{ mA}$

**2** D
$$V_{p\text{-}p} = 2 \times V_{RMS} \times \sqrt{2}$$
$$V_{p\text{-}p} = 2 \times 240 \times \sqrt{2}$$
$$V_{p\text{-}p} = 680\text{ V}$$

**3** B. High voltages reduce the energy losses in the transmission lines. Power loss is proportional to $I^2$. Transformers allow power to be transmitted at high voltages and low currents.

**4** D. The right hand slap rule should have been used, with fingers towards N, thumb towards W and palm facing downwards.

**5** C
$$F = BIl$$
$$F = 5 \times 10^{-5} \times 1000 \times 1$$
$$F = 5 \times 10^{-2}\text{ N}$$

**6** A
$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$
$$\frac{V_p}{V_s} = \frac{240}{12}$$
$$\frac{V_p}{V_s} = \frac{20}{1}$$

**7** A. 12 $V_{DC}$ is equivalent to 12 $V_{RMS}$.

**8** C
$$V_p = \sqrt{2} \times V_{RMS}$$
$$V_p = \sqrt{2} \times 240$$
$$V_p = 339\text{ V}$$

**9** A
$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$
$$= \frac{200}{16}$$
$$\frac{N_p}{N_s} = \frac{15}{1}$$

**10** B
$$P = VI$$
$$30 = 16 \times I$$
$$I = 1.9\text{ A}$$

### Section B — Short answer questions

**11** **a** Stepping up the voltage allows the current to be reduced while maintaining constant power. The reason for reducing the current is that the power lost is related to the transmission current by: $P = I^2R$.

**b**
$$P = VI$$
$$I = \frac{500 \times 10^6}{500 \times 10^3}$$
$$I = 1000\text{ A or } 1.0\text{ kA}$$

**c** First calculate the power lost:
$$P = I^2R$$
$$= 1000^2 \times 30$$
$$= 30 \times 10^6\text{ W (30 MW)}$$

Then subtract this from the power delivered by the generator:
$$P_{avail} = 500 \times 10^6 - 30 \times 10^6$$
$$= 470\text{ MW}$$

**12** **a** The brightness of the globe will be decreased [1 mark]. Students could then refer to reduced current, increased voltage drop in the cables or increased power lost in the cables [1 mark].

**b** The independent variable is the resistance of the cables [1 mark]. The dependent variable is the current in the cables [1 mark].

***VCAA examination report*** **note:**

The most common errors were to state the resistance of the cables is the dependent variable and vice versa or to make a generic statement about variables.

There were a number of students who simply stated that the independent variable is the resistance, but this was not sufficient as it does not discriminate between the resistance of the cables and the resistance of the globe.

**c**
$$V = RI$$
$$V = (R + r)I \quad \text{[1 mark]}$$
$$V = RI + rI$$
$$rI = V - RI$$
$$r = \frac{V}{I} - R$$
$$r = \frac{24}{I} - R \quad \text{[1 mark]}$$

***VCAA examination report*** **note:**

Students struggled to demonstrate what was required. Many attempted to substitute values from the table into the equation (which results in different values for $R$) and then claimed the results were close enough to support their demonstration. Demonstrating that an equation works for a set of values is not a proof.

**d**

| Resistance of cables, r (Ω) | Current in cables, i (A) | $\frac{1}{i}\left(A^{-1}\right)$ |
|---|---|---|
| 2.4 | 2.4 | 0.42 |
| 3.6 | 2.0 | 0.50 |
| 6.4 | 1.7 | 0.59 |
| 7.6 | 1.5 | 0.67 |
| 10.4 | 1.3 | 0.77 |

Award 1 mark for 3 or more correct values.

Award another mark for all 6 correct values.

***VCAA examination report* note:**

There were a number of students who rounded the values to one decimal place. While this was not penalised, it did affect their ability to accurately plot their data and analyse it. Students are warned against rounding. The data in the tables had at least two significant figures and students should retain this level of detail in their working.

There were also students who responded with fractions ($\frac{1}{2.4}, \frac{1}{2.0}, \frac{1}{1.7}$ etc). This was not accepted as it is restating information provided in the question. The column header gives the form of the fraction as 1/i and the column header for the preceding column is i.

**e** See figure at the bottom of the page*

Award 1 mark for correctly plotting all points.

Award 1 mark for correct scale on vertical axis.

Award 1 mark for correct scale on horizontal axis.

Award 1 mark for correct uncertainty bars.

Award 1 mark for correctly labelled axes.

Award 1 mark for correct line of best fit.

***VCAA examination report* note:**

Most students were able to demonstrate the skills required. Students who did not score well generally did so for all parts of the question, suggesting they had little understanding of experimental process or data visualisation.

A number of students truncated the *x*-axis, which prevents using the *y*-intercept for analysis. Unless students are sure they will not need the *y*-intercept, they should not break the *x*-axis when plotting data.

**f** The value for R is found from the *y*-axis intercept. [1 mark]

The correct value is 7 Ω [1 mark] and allowance was made for students' plotting of their data.

***VCAA examination report* note:**

The most common error was to find the gradient of the graph.

Many students attempted to use the equation from Part c as though this equation was, in some way, the conclusion to the experiment and could be relied upon to provide answers that should have been found from the graph.

Students are advised to put more effort into developing and practising the skills associated with experimental design and data visualisation.

**13 a** $P = V \times I$

$V = I \times R$

Combining these two equations:

$$P = \frac{V^2}{R}$$

$$P = \frac{12^2}{12}$$

Power = 12 W [1 mark]

Remember that when calculating power ratings $V_{RMS}$ values must be used.

**b** Award 1 mark for any one of the three options below.

The lights are dimmer than expected because:

**Option A:** The power delivered to the lights is less than 12 W.

OR

**Option B:** The voltage delivered to the lights is less than 12 V.

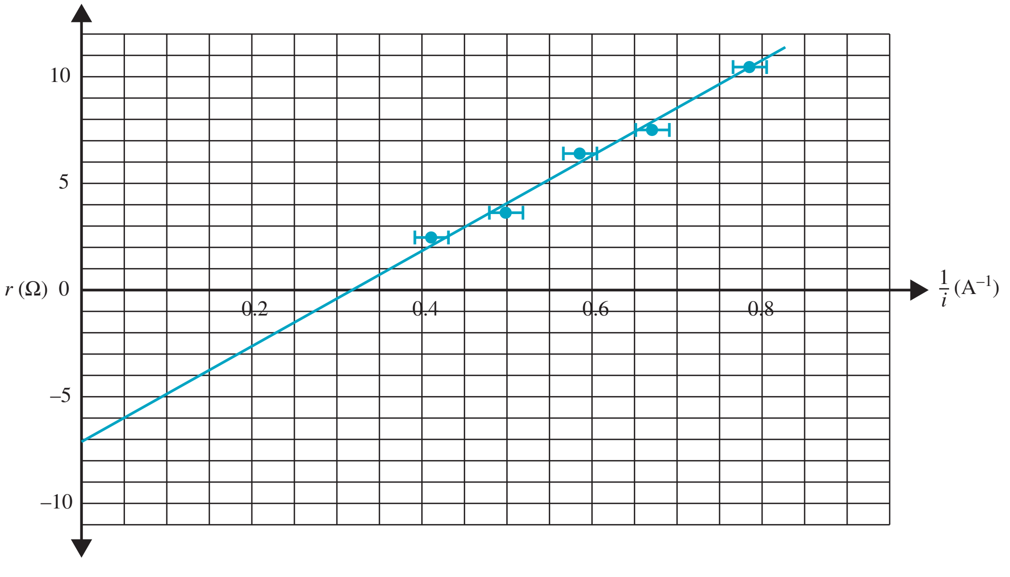
OR

**Option C:** The current delivered to the lights is less than 1 A.

The total resistance of the system: $R = 12\,\Omega + 3\,\Omega$

$= 15\,\Omega$ [1 mark]

***12 e**



The current in the system: $I = \frac{V}{R} = \frac{12}{15} = 0.8$ A [1 mark]

Award 1 mark for any one of the three supporting calculations below.

Supporting calculations:

**Option A:**

$P_{loss} = I^2R$

$= 0.8^2 \times 3$

$= 1.92$ W lost in the lead.

**Option B:**

$V_{loss} = IR$

$= 0.8 \times 3$

$= 2.4$ V lost over the lead.

**Option C:**

$I_{required} = 1$ A.

$I_{delivered} = 0.8$ A

**c** The home owner must move the transformer from the house end of the lead to the light end of the lead. [1 mark]

This will reduce the current in the lead. Therefore, there will be less power loss over the lead. [1 mark]

$P_{loss} = I^2R$, so less current will mean less power loss.

**14 a** An ideal transformer is one where no power is lost; that is, $P_{primary} = P_{secondary}$ [1 mark].

**b** $\frac{N_p}{N_s} = \frac{V_p}{V_s}$

$\frac{N_p}{N_s} = \frac{240}{12}$

$\frac{N_p}{N_s} = \frac{20}{1}$ [1 mark]

**c** $P = \frac{V^2}{R}$

$P = \frac{12^2}{6}$ [1 mark]

$P = 24$ W [1 mark]

**d** $R_{wire} = 12 \times 2 \times 0.05$

$R_{wire} = 1.2\,\Omega$

Total resistance $= 6 + 1.2$

Total resistance $= 7.2\,\Omega$ [1 mark]

$I = \frac{V}{R}$

$I = \frac{12}{7.2}$

$I = 1.67$ A [1 mark]

$V_{globe} = 1.67 \times 6.0$

$V_{globe} = 10$ V [1 mark]

**e**
- There is less power delivered to Light 1 compared to Light 2. [1 mark]
- Power is lost along the transmission lines. [1 mark]
- The observed brightness is proportional to the delivered power. [1 mark]

**15 a** $P = VI$

$P = 415 \times 100$

$P = 41\,500$ W

$P = 41.5$ kW [1 mark]

***VCAA examination report* note:**

There was no common error for this question.

**b** $P_{supply} = P_{generated} - P_{loss}$

$P_{loss} = I_{line}^2 \times R_{line}$

$P_{loss} = 100^2 \times 2$

$P_{loss} = 20\,000$ W [1 mark]

$P_{supply} = 41\,500 - 20\,000$

$P_{supply} = 21\,500$ W [1 mark]

No, the power supplied, 21.5 kW, is less than the 40 kW required [1 mark].

***VCAA examination report* note:**

The most common errors were mathematical and involved miscalculating the powers of 10. For example: $P_{loss} = I^2R = 100^2 \times 2 = 2000$ W.

**c** $\frac{I_{line}}{I_{gen}} = \frac{N_p}{N_s}$

$\frac{I_{line}}{100} = \frac{1}{10}$

$\Rightarrow I_{line} = 10$ A [1 mark]

$P_{loss} = I^2R$

$P_{loss} = 10^2 \times 2$

$P_{loss} = 200$ W [1 mark]

$P_{delivered} = P_{generated} - P_{lost}$

$P_{delivered} = 41\,500 - 200$

$P_{delivered} = 41\,300$ W

$P_{delivered} = 41.3$ kW [1 mark]

***VCAA examination report* note:**

The most common errors were mathematical.

# Unit 3 — Area of Study 3 review

## Practice examination

### Section A — Multiple choice questions

**1.** A

$38\,\text{cm}^2 = 0.0038\,\text{m}^2$

$$\begin{aligned}\Phi &= BA \\ &= 0.09 \times 0.0038 \\ &\approx 3.4 \times 10^{-4}\,\text{Wb}\end{aligned}$$

**2.** C

The induced EMF in the loop is calculated from $\varepsilon = -N\dfrac{\Delta\Phi}{\Delta t}$.

$$\begin{aligned}\varepsilon &= -N\frac{\Delta\Phi}{\Delta t} \\ &= -80 \times \frac{-3.42 \times 10^{-4}}{0.35} \\ &= 7.8 \times 10^{-2}\,\text{V}\end{aligned}$$

**3.** D

The induced EMF in the loop is given by $\varepsilon = -N\dfrac{\Delta\Phi}{\Delta t}$ and is therefore dependent on the gradient of the magnetic flux. When the gradient of the saw-tooth graph is positive, as in the given graph, the EMF induced is a negative value. The gradient is positive, and equal in magnitude for the 2 time periods. This corresponds to 2 periods of negative EMF, as in option D.

**4.** C

The resistance of the metal has no influence on the EMF generated, though it may affect the size of the current flowing in the ring while an EMF is generated.

**5.** A

According to Lenz's Law, the induced flux will oppose the change in magnetic flux. Since the change in flux is upward through the ring, the induced flux is downward through the ring. Using the right-hand- grip rule for the induced current, downward flux is the result of a clockwise current.

**6.** B

- Slip rings maintain a continuous electrical connection with the spinning loop and are used when an **AC** output is required.
- Motors convert **electrical energy into mechanical energy**.
- A split ring commutator reverse the direction of the coil's current every **half** revolution of the loop.
- 'A split ring commutator is a device used to convert alternating current into direct current.' is a correct statement

**7.** B

First, the pole of the solenoid closest to the metal ring is determined to be 'south' when the solenoid circuit is active. The initial flux through the ring will be to the left. When the solenoid circuit is switched off, the change of flux will be to the right through the ring. According to Lenz's Law, the induced flux will oppose the change in magnetic flux. Since the change in flux is right through the ring, the induced flux is left through the ring. Using the right-hand-grip rule for the induced current, leftward flux is the result of an anticlockwise current.

**8.** D

An inactive solenoid circuit is unable to generate a change in magnetic flux through the metal ring.

**9.** B

As the rate of rotation is lower, the period of the new rate of rotation is longer. This means that the period for a change in flux is higher, resulting in a lower EMF produced, according to $\varepsilon \propto \dfrac{1}{\Delta t}$.

**10.** D

Lenz's Law states that the magnetic field of the induced current will oppose the change in magnetic flux through the ring.

**11.** C

Increasing the resistance of the ring will decrease the current according to Ohm's Law.

**12.** D

The voltage ratio is equal to the turns ratio,

$$\begin{aligned}\frac{V_{\text{primary}}}{V_{\text{secondary}}} &= \frac{N_{\text{primary}}}{N_{\text{secondary}}} \\ \frac{240}{18} &= \frac{N_{\text{primary}}}{360} \\ N_{\text{primary}} &= \frac{360 \times 240}{18} \\ N_{\text{primary}} &= 4800\end{aligned}$$

**13.** C

The RMS current is calculated from

$$\begin{aligned}P_{\text{RMS}} &= V_{\text{RMS}} I_{\text{RMS}} \\ I_{\text{RMS}} &= \frac{P_{\text{RMS}}}{V_{\text{RMS}}} \\ &= \frac{54}{18} \\ &= 3.0\,\text{A}\end{aligned}$$

**14.** B

The peak current is calculated from $I_{\text{peak}} = \sqrt{2} I_{\text{RMS}}$, where

$$\begin{aligned}I_{\text{RMS}} &= \frac{P}{V_{\text{RMS}}} \\ &= \frac{54}{240} \\ &= 0.225\,\text{A}.\end{aligned}$$

$$\begin{aligned}I_{\text{peak}} &= \sqrt{2} I_{\text{RMS}} \\ &= \sqrt{2} \times 0.225 \\ &\approx 0.32\,\text{A}\end{aligned}$$

**15.** B

Transformers require changing magnetic flux in order to generate an EMF. The alternating current on the primary side creates an alternating magnetic flux through the secondary coils, which generates an alternating EMF.

**16.** B

Transformers work when the magnetic flux created by the primary coil generates EMF in the secondary coil. The iron core channels the magnetic flux between the two coils.

**17.** B

Current on the primary side of the step-up transformer is calculated from $P = IV$.

$$\begin{aligned}P &= IV \\ I &= \frac{P}{V} \\ &= \frac{84\,000}{230} \\ &= 365\,\text{A}\end{aligned}$$

The current in the transmission line is given by the formula $\frac{I_{primary}}{I_{secondary}} = \frac{N_{secondary}}{N_{primary}}$.

$$\frac{I_{primary}}{I_{secondary}} = \frac{N_{secondary}}{N_{primary}}$$

$$I_{secondary} = \frac{I_{primary} \times N_{primary}}{N_{secondary}} = \frac{365 \times 1}{20} \approx 18\text{ A}$$

**18.** B

Due to the voltage loss in the transmission line, plus the fact that the turns ratio for the step-up transformer is the same as that for the step-down transformer, the voltage available after step-down will be lower than the voltage before step-up.

**19.** D

A higher turns ratio will reduce the transmission line current as the secondary voltage will be higher. Since the reduced transmission line current will result in a lower voltage loss, the voltage available will be higher.

**20.** B

Reducing the turns ratio of the step-up transformer will increase the transmission line current, leading to a higher transmission line loss according to the equation $P_{loss} = I^2R$.

## Section B — Short answer questions

**21. a.** As the magnet moves closer to the coil, the change in flux is to the right [1 mark].

According to Lenz's Law, the induced current in the coil will oppose the change in flux, with an induced magnetic flux to the left [1 mark].

Using the right-hand-grip rule, the current travels from X to Y through the resistor. [1 mark]

**b.** There is no current because an EMF is generated in the coil only when there is a change of flux through the coil [1 mark].

As the magnet is stationary, there is no change in flux through the coil. [1 mark]

**c.** Any two of the following:

- Use a stronger magnet
- Move the magnet faster
- Use a coil with more turns of the conductor

Award 1 mark for each of the above dot points, maximum 2 marks.

**22. a.** The flux is given by:

$$\Phi = BA = 0.15 \times 6.0 \times 10^{-3} = 9.0 \times 10^{-4}\text{ Wb} \quad [1 mark]$$

**b.** As the loop rotates, the magnetic flux, which is directed to the right, is decreasing. According to Lenz's Law, the induced current in the coil will oppose the change in flux by increasing the induced magnetic flux to the right.

Using the right-hand-grip rule, the current travels out of the page, along the top side of the loop; this current flows along the conductor, which contacts the slip ring connected to P. Therefore the current flows from P to Q. [1 mark]

**c.** The EMF is given by $\epsilon = -N\frac{\Delta\Phi}{\Delta t}$.

$$\epsilon = -N\frac{\Delta\Phi}{\Delta t}$$

$$\Delta t = -N\frac{\Delta\Phi}{\epsilon} \quad [1 mark]$$

$$= -50 \times \frac{-9.0 \times 10^{-4}}{40 \times 10^{-3}} = 1.1\text{ s} \quad [1 mark]$$

**23. a.** $\frac{V_{primary}}{V_{secondary}} = \frac{N_{primary}}{N_{secondary}}$

$$N_{secondary} = \frac{N_{primary} \times V_{secondary}}{V_{primary}} \quad [1 mark]$$

$$= \frac{75 \times 1200}{240} = 375\text{ turns} \quad [1 mark]$$

**b.** $\frac{V_{primary}}{V_{secondary}} = \frac{I_{secondary}}{I_{primary}}$

$$I_{secondary} = \frac{I_{primary} \times V_{primary}}{V_{secondary}} \quad [1 mark]$$

$$= \frac{4 \times 240}{1200} = 0.8\text{ A} \quad [1 mark]$$

**c.** The power loss is given by:

$$P = I^2R = 4^2 \times 3.5 = 56\text{ W} \quad [1 mark]$$

56 W as a percentage of the supplied 1000 W is:

$\frac{56}{1000} \times 100 = 5.6\%$ [1 mark]

**24. a.** $\frac{N_{primary}}{N_{secondary}} = \frac{V_{primary}}{V_{secondary}} = \frac{6}{240} = 1 : 40$ [1 mark]

**b.** Since the fuse on the primary side limits the current to a maximum of 1.0 A, the maximum power is:

$$P = IV = 240 \times 1.0 = 240\text{ W} \quad [1 mark]$$

**c.** Using the maximum power delivered to the primary coil, $P = 240$ W:

$$P = VI$$

$$I = \frac{P}{V} = \frac{240}{6} = 40\text{ A} \quad [1 mark]$$

# Topic 8 — Light as a wave

## 8.2 Light as a wave

### Sample problem 1

$T = 2.0\,\text{m s}$
$= 2.0 \times 10^{-3}\,\text{s}$
$\lambda = 68\,\text{cm}$
$= 0.68\,\text{m}$

$$v = f\lambda$$
$$\Rightarrow v = \frac{\lambda}{T}$$
$$v = \frac{0.68\,\text{m}}{2.0 \times 10^{-3}\,\text{s}}$$
$$= 3.4 \times 10^{2}\,\text{m s}^{-1}$$

### Practice problem 1

A wave will travel a distance equal to one wavelength in a time equal to its period. The speed of the wave is equal to

$$v = \frac{\lambda}{T}$$
$$= \frac{0.510}{1.50 \times 10^{-3}}$$
$$= 340\,\text{m s}^{-1}$$

### Sample problem 2

$f = 550\,\text{Hz}$, $v = 335\,\text{m s}^{-1}$

$$v = f\lambda$$
$$\Rightarrow \lambda = \frac{v}{f}$$
$$= \frac{335\,\text{m s}^{-1}}{550\,\text{Hz}}$$
$$= 0.609\,\text{m}$$

### Practice problem 2

Use the wave equation to find the speed. Thus:

$$v = f\lambda$$
$$= 587 \times 0.571$$
$$= 335\,\text{m s}^{-1}$$

### Sample problem 3

**a** $T = \frac{1}{f}$

$$= \frac{1}{5.60 \times 10^{14}}$$
$$= 1.79 \times 10^{-15}\,\text{s}$$

**b** $\lambda = \frac{c}{f}$

$$= \frac{3.00 \times 10^{8}}{5.60 \times 10^{14}}$$
$$= 5.36 \times 10^{-7}\,\text{m}$$
$$= 5.36 \times 10^{2}\,\text{nm}$$
$$= 536\,\text{nm}$$

### Practice problem 3

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}}{4.50 \times 10^{-7}}$$
$$= 6.70 \times 10^{14}\,\text{Hz}$$
$$T = \frac{1}{f} = \frac{1}{6.70 \times 10^{14}}$$
$$= 1.50 \times 10^{-15}\,\text{s}$$

## 8.2 Exercise

**1 a** $\lambda = \frac{v}{f}$

$$= \frac{340}{256}$$
$$= 1.33\,\text{m}$$

**b** $T = \frac{1}{f} = 3.91 \times 10^{-3}\,\text{s}$

**c** The speed of sound in air is $340\,\text{m s}^{-1}$, thus it takes $\frac{1000}{340} = 2.94$ s for a sound wave to travel 1 km.

**2**

| $v\,(\text{ms}^{-1})$ | $f(\text{Hz})$ | $\lambda(\text{m})$ |
|---|---|---|
| 335 | 500 | 0.67 |
| 300 | 12 | 25 |
| 1500 | 5000 | 0.30 |
| 60 | 24 | 2.5 |
| 340 | 1000 | 0.34 |
| 260 | 440 | 0.59 |

**3 a** A rate of 5 each half second is 10 each second, thus the frequency is 10 Hz. The period $T$ is $T = \frac{1}{f} = 0.1$ s.

**b** Using $v = \lambda f$, the speed is $v = 2.6 \times 10^{-2} \times 10 = 0.26\,\text{m s}^{-1}$.

**c** The speed of the wave depends upon the medium. Without modifying the medium, the speed of the wave remains the same, but, if the frequency is doubled, then the wavelength is halved.

**4** $T = \frac{1}{f}$

$$= \frac{1}{4.8 \times 10^{14}}$$
$$= 2.08 \times 10^{-15}$$
$$= 2.1 \times 10^{-15}\,\text{s}$$

**5** $\lambda = \frac{c}{f}$

$= \frac{3.0 \times 10^8}{1.0 \times 10^{10}}$

$= 3.0 \times 10^{-2}$ m

$= 3.0$ cm

$\lambda = \frac{c}{f}$

$= \frac{3.0 \times 10^8}{1.0 \times 10^{12}}$

$= 3.0 \times 10^{-4}$

$= 0.30$ mm

Microwaves have wavelengths that range from fractions of a millimetre to a few centimetres.

**6** $f = \frac{c}{\lambda}$

$= \frac{3.0 \times 10^8}{2.7 \times 10^{-11}}$

$= 1.1 \times 10^{19}$ Hz

$T = \frac{1}{f}$

$= \frac{1}{1.1 \times 10^{19}}$

$= 9.0 \times 10^{-20}$ s

**7** **a** $f = \frac{1}{T}$

$= \frac{1}{20 \times 10^{-3}}$

$= 50$ Hz

**b** $\lambda = \frac{c}{f}$

$= \frac{3.0 \times 10^8}{50}$

$= 6.0 \times 10^6$ m

**8** **a** $\lambda = \frac{c}{f}$

$= \frac{3.0 \times 10^8}{6.5 \times 10^{14}}$

$= 4.6 \times 10^{-7}$ m

**b** $\lambda = \frac{c}{f}$

$= \frac{2.0 \times 10^8}{6.5 \times 10^{14}}$

$= 3.1 \times 10^{-7}$ m

**c** $f = \frac{v}{\lambda}$

$= \frac{3.1 \times 10^{-7}}{6.5 \times 10^{14}}$

$= 4.8 \times 10^7$ Hz

**9** Both electric and magnetic fields are static about the charge. If the charge changes direction or accelerates in a straight line, a ripple is created in both fields, which is associated with an electromagnetic wave: all accelerated charged particles produce electromagnetic radiation consistent with Maxwell's model for electromagnetic waves.

## 8.2 Exam questions

**1** A

$v = f\lambda$

$= \frac{\lambda}{T}$

$= \frac{1.2}{1.6}$

$= 0.75$ m s$^{-1}$

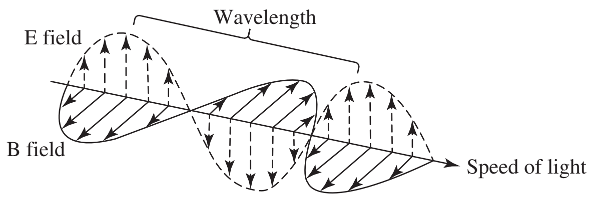
**2** C. Amplitude is found from the displacement axis (8 cm). Frequency is found using:

$c = f\lambda$

$18 = f \times \left(6 \times 10^{-2}\right)$

$f = 300$ Hz

**3**




Award 1 mark for correctly labelling the electric and magnetic fields, 1 mark for correctly labelling the wavelength and 1 mark for correctly labelling the speed of light.

***VCAA examination report* note:**

The most common error was to indicate the wavelength incorrectly. Typically, students indicated the distance from one of the E field peaks to the next B field peak or vice versa.

**4** The speed of a wave can be calculated using the frequency of the wave and the wavelength of the wave.

The wavelength ($\lambda$) is given as 1.40 m.

The period (and hence the frequency) of the wave can be determined given the time take for point P to move from maximum displacement to zero. This is equivalent to $\frac{1}{4}$ of the period. Therefore the period $T$ is:

$T = 4 \times 0.120$

$= 0.480$ s [1 mark]

$f = \frac{1}{T}$

$f = \frac{1}{0.480}$

$f = 2.08$ Hz [1 mark]

Then

$v = f \times \lambda$

$v = 2.08 \times 1.40$

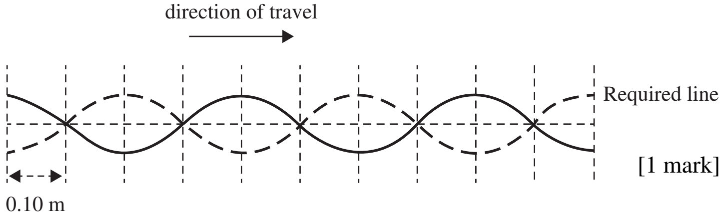
$v = 2.92$ m s$^{-1}$ [1 mark]

**5** **a** $v = f\lambda$

$8.0 = f \times 0.4$ [1 mark]

$f = 20$ Hz [1 mark]

**b**




[1 mark]

$v = \dfrac{\Delta x}{\Delta t}$

$\Delta x = 8.0 \times (25 \times 10^{-3})$

$\Delta x = 0.02\text{ m}$ [1 mark]

This means that all points on the wave move two segments to the right.

**c** Assuming that the velocity will remain the same [1 mark], the wavelength will get shorter [1 mark].

## 8.3 Interference, resonance and standing waves

### Sample problem 4

**a** There are four nodes for this standing wave.

**b** There are three antinodes for this standing wave.

**c** The wave is the 3rd harmonic for this wire.

**d** $L = \dfrac{3}{2}\lambda$ for the 3rd harmonic.

Thus:

$1.5 = 1.5\lambda$

$\Rightarrow \lambda = 1.0\text{ m}$

**e** $T = \dfrac{1}{f_3}$

$= \dfrac{1}{4.2}$

$= 0.2381 \approx 0.24\text{ s}$

**f** $v = f_3\lambda_3$

$= 4.2 \times 1.0$

$= 4.2\text{ m s}^{-1}$

**g** $f_3 = 4.2 = 3 \times f_1$

$f_1 = \dfrac{4.2}{3} = 1.4\text{ Hz}$

The fundamental frequency for the wire is 1.4 Hz.

$\lambda_1 = 2L = 2 \times 1.5 = 3.0\text{ m}$

The fundamental wavelength is 3.0 m.

### Practice problem 4

**a** The fundamental frequency will still have a wavelength of $2L = 2 \times 1.5 = 3.0\text{ m}$. The speed of the travelling wave is now $6.0\text{ m s}^{-1}$. Thus:

$f_1 = \dfrac{v}{\lambda_1} = \dfrac{6.0}{3.0} = 2.0\text{ Hz}$

$f_2 = 2 \times 2.0 = 4.0\text{ Hz}$

$f_3 = 3 \times 2.0 = 6.0\text{ Hz}$

The above are in accordance with the general rule for the $n$th harmonic $f_n = nf_1$.

**b** The wavelength of the first harmonic is always twice the length of the string fixed at both ends, in this case $2 \times 1.5 = 3.0\text{ m}$. The wavelength of the various harmonics is related to the wavelength of the first harmonic $\lambda_1$. Thus $\lambda_n = \dfrac{\lambda_1}{n}$ gives the first harmonic's wavelength as 3.0 m, the second harmonic's wavelength as $\dfrac{3.0}{2} = 1.5\text{ m}$ and the third harmonic's wavelength as $\dfrac{3.0}{3} = 1.0\text{ m}$.

## 8.3 Exercise

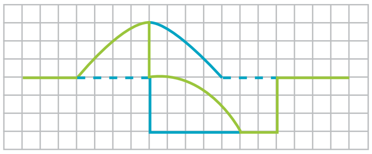
**1 a** Diagram C. Destructive interference occurs when a wave with a positive amplitude passes through a wave with a negative amplitude.

**b** Diagrams A and B. Constructive interference occurs when waves with amplitudes of the same sign pass through each other.

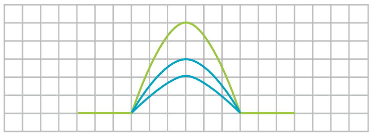
**2** Superposition is the addition of the amplitudes of two or more wave disturbances at a point in space and time. It occurs whenever there is a disturbance arising from two or more sources of waves.

**3** Constructive interference occurs when two or more wave disturbances superimpose (add together) to give a resultant amplitude larger than the amplitudes of either wave.

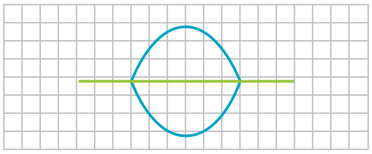
**4 a**



**b**

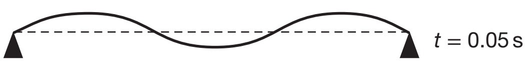


**c**



**5** Adjacent nodes in a standing wave are separated by a distance of $\dfrac{\lambda}{2}$. Therefore, $\lambda = 2 \times 0.75 = 1.5\text{ m}$.

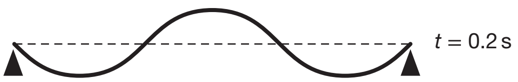
**6 a**



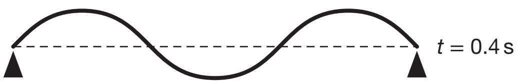

The graph should be similar in shape, but have a smaller amplitude than the graph at $t = 0$.

**b**


**c**




**d**




**7 a** At these points, constructive interference is occurring. At any instant in time, a wave from one loudspeaker is adding to a synchronised wave from the other speaker. A compression from one speaker will add to a compression from the other. Half a period later, a rarefaction will add to a rarefaction from the other speaker.

**b** 1.0 m, since the distance between adjacent antinodes is 0.50 m, that being half a wavelength.

**c** $v = f\lambda$
$= 330 \times 1.0$
$= 330\,\text{m s}^{-1}$

**8** **a** $v = f\lambda$
$= 4.0 \times 1.2$
$= 4.8\,\text{m s}^{-1}$

**b** $d = \dfrac{\lambda}{2}$
$= \dfrac{1.2}{2}$
$= 0.60\,\text{m}$

**c** The maximum displacement is equal to twice the amplitude of each of the waves: 20 cm = 0.20 m. In this situation, maximum constructive interference will give a maximum displacement of twice the amplitude of the individual waves used to create the standing wave.

**d** During each second, four complete cycles are made. The maximum number of times that the string is instantaneously straight is 9 if, at $t = 0$, the string is straight; otherwise, the number of times is one less: 8.

**9** **a** $v = f\lambda$
$= 2.5 \times 2.4$
$= 6.0\,\text{m s}^{-1}$

**b** $d = \dfrac{\lambda}{2}$
$= \dfrac{2.4}{2}$
$= 1.2\,\text{m}$

**c** The maximum displacement is equal to the amplitude twice: $2 \times 0.04\,\text{m} = 0.08\,\text{m}$

**d** During each second, two and a half complete cycles are made; that is, five complete cycles are made in two seconds. The maximum number of times that the string is instantaneously straight is five in any given one-second interval.

**10** The distance between the fixed ends of the guitar string is 0.80 m and thus the wavelength of the first harmonic is $2 \times 0.80\,\text{m} = 1.60\,\text{m}$. The speed of the travelling waves that make the standing waves is $650\,\text{m s}^{-1}$. The fundamental frequency is:

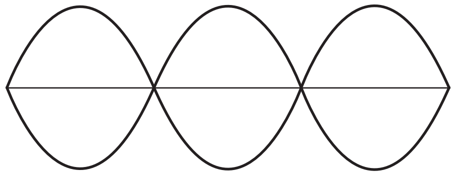
$f_1 = \dfrac{v}{2L} = \dfrac{v}{\lambda_1}$
$= \dfrac{650}{1.60}$
$= 406\,\text{Hz}$

The next two harmonics, namely the second and the third are:
$f_2 = 406.25 \times 2 = 813\,\text{Hz}$
$f_3 = 406.25 \times 3 = 1219\,\text{Hz}$

## 8.3 Exam questions

**1** **a** $v = f\lambda$
$= 250 \times 1.6$
$= 400\,\text{m s}^{-1}$

***VCAA examination report*** **note**
The most common error was o use 0.8 as thewavelength.



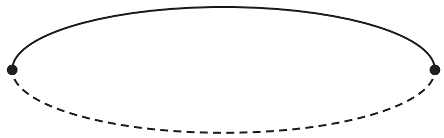
**b** Award 1 mark for the correct shape and award 1 mark for the waveform taking up the entire length of the string.

***VCAA examination report*** **note:**
The most common error was to draw two solid sine waves rather than the waveform envelope. The figure was described as 'the standing wave envelope' in the question stem so students had a model of what was required.

**2** **a** $v = f \times \lambda$
$\lambda = \dfrac{v}{f}$
$\lambda = \dfrac{40}{7.5}$
$\lambda = 5.3\,\text{m}$

**b** No, a standing wave will not form in the string.
For a standing wave in a string, $\lambda = \dfrac{2l}{n}$, or $l = n\dfrac{\lambda}{2}$.
This is because the string length is not a multiple of $\dfrac{\lambda}{2}$.

**3** **a**



The simplest standing wave occurs when the wavelength is twice the length of the string $\left(L = \dfrac{\lambda}{2}\right)$.
Award 1 mark for each correctly drawn standing wave, with a node at each support.

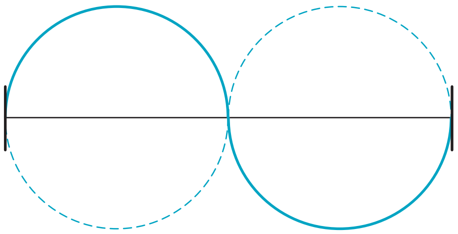
**b** $\lambda = 2L$
$\lambda = 2 \times 0.92$ [1 mark]
$\lambda = 1.84\,\text{m}$ [1 mark]

**c** $v = f\lambda$
$224 = f \times 1.8$ [1 mark]
$\therefore f = 122\,\text{Hz}$ [1 mark]

**4** **a** Wavelength of the lowest frequency resonance (fundamental) is twice the length of the string.
Hence, $\lambda = 8.0\,\text{m}$. [2 marks]

**b** For the second-lowest frequency resonance, the wavelength is equal to the string length.
Now, $f = \dfrac{v}{\lambda} = \dfrac{240}{4} = 60\,\text{Hz}$. [2 marks]

**5**
- Waves travelling along the string are reflected at each end. [1 mark]
- These waves, travelling in opposite directions, combine constructively and destructively to produce an interference pattern of antinodes and nodes. [1 mark]
- The diagram below shows two waves at a moment when they destructively interfere to produce a straight horizontal line. [1 mark]



Note: Award 1 mark for each of two key points in a valid explanation.
Award 1 mark for drawing a valid diagram.

***VCAA examination report* note:**
Many students were able to identify the interference aspect but were unable to link this to travelling waves. There were also a number of students who drew diagrams of pulses travelling on strings, which made it difficult for them to demonstrate the production of the standing wave.

## 8.4 Diffraction of light

### Sample problem 5

For radio waves

$\lambda = \frac{v}{f} = \frac{3.0 \times 10^8}{10^6} = 300$ m and ratio $\frac{\lambda}{w} = \frac{300}{30} = 10$ is much greater than 1, thus large diffraction spread.

For microwaves

$\lambda = \frac{v}{f} = \frac{3.0 \times 10^8}{10^{11}} = 3$ mm and ratio $\frac{\lambda}{w} = \frac{0.003}{30} = 1.0 \times 10^{-4}$ is much less than 1 thus small diffraction spread.

There will be little shadow behind the building using radio waves, but a shadow behind the building when using microwaves.

### Practice problem 5

**a** Use $\lambda = \frac{v}{f}$ to find each of the wavelengths for the microwaves and infrared. Thus

$\lambda = \frac{3.0 \times 10^8}{6.0 \times 10^{11}} = 5.0 \times 10^{-4}$ m for the microwaves and

$\lambda = \frac{3.0 \times 10^8}{1.5 \times 10^{11}} = 2.0 \times 10^{-5}$ m for the infrared.

The microwaves have a much larger wavelength compared to the infrared and would be expected to diffract more readily. If the narrow slit was $5.0 \times 10^{-4}$ m then $\frac{\lambda}{w} = 1$ for the microwaves whereas $\frac{\lambda}{w} = 0.04$ for the infrared source indicating little significant diffraction.

**b** To observe diffraction effects using the infrared source the slit should be narrowed so that $w$ is comparable to the wavelength $2.0 \times 10^{-5}$ m.

## 8.4 Exercise

1 The diffraction of light when passed through a narrow opening is strong evidence to support that light is a type of wave

2 **a** Using the factor $\frac{\lambda}{w}$, as $\lambda$ decreases, $\frac{\lambda}{w}$ decreases and thus the amount of diffraction decreases.

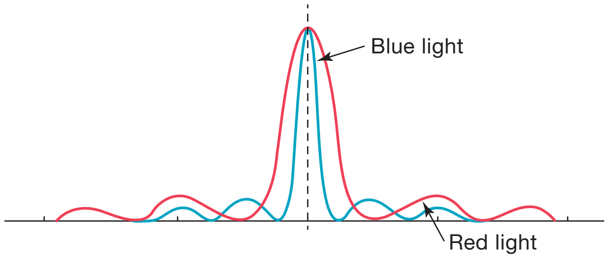
**b** Using the factor $\frac{\lambda}{w}$, as $\lambda$ increases, $\frac{\lambda}{w}$ increases and thus the amount of diffraction increases.

**c** Using the factor $\frac{\lambda}{w}$, as $w$ decreases, $\frac{\lambda}{w}$ increases and thus the amount of diffraction increases.

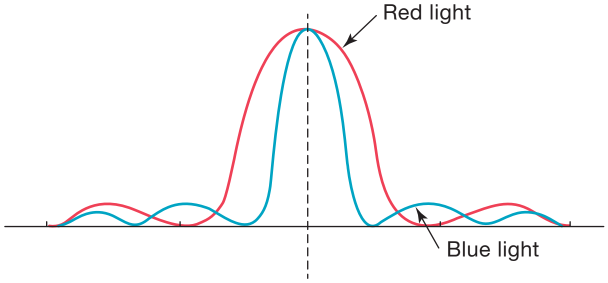
**d** Using the factor $\frac{\lambda}{w}$, as $w$ increases, $\frac{\lambda}{w}$ decreases and thus the amount of diffraction decreases.

3 **a** The first minimum occurs where the path difference for light rays travelling from different places on the opening reaches $\frac{\lambda}{2}$. This will be a region where the intensity of the light will be diminished due to destructive interference.

**b**




**c**




4 **a** Diffraction depends on the wavelength or colour of light. White light is a mixture of the different colours in the spectrum, so when it diffracts while passing through a small slit, the colours that make up the white light form slightly different diffraction patterns.

**b** The positions of the minima are given by $\sin\theta = \frac{\lambda}{w}$. The red end of the spectrum has the longest wavelengths; therefore, $\theta$ for any minimum occurs at a greater angle than for blue light and other parts of the visible spectrum.

5 **a** A blurred edge is evidence of diffraction and hence the wavelike nature of light.

**b** Red light has a longer wavelength compared to green light and so the pattern would reveal more significant diffraction; the edge of the shadow would be more blurry (less sharp).

**c** With a smaller object, the degree of diffraction observed would be more significant than before; the edge of the shadow would be less sharp.

## 8.4 Exam questions

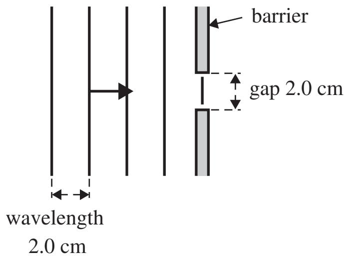
1 **a** To form a diffraction pattern the mesh must have tiny gaps for the light to pass through. [1 mark]

***VCAA examination report* note:**
There was no quantitative data given in the question and students are not required to know the $\frac{\lambda}{w}$ ratio that yields the optimum diffraction.

**b** The window represents a $w$ value that is much greater than the $w$ value associated with the mesh. Since the $\frac{\lambda}{w}$ ratio is now much smaller [1 mark], there will be much less diffraction [1 mark].

**2** Diffraction depends on the ratio: $\frac{\lambda}{w}$. If this ratio is close to 1 then diffraction is most obvious.
At 10 000 Hz, $\frac{\lambda}{w} = \frac{0.03}{0.8} = 3.75 \times 10^{-2}$ so diffraction is minimal. [1 mark]
At 100 Hz, $\frac{\lambda}{w} = \frac{3.34}{0.8} = 4.18$ so diffraction is much more significant. [1 mark]

**3** Diffraction is determined by the ratio $\frac{\lambda}{w}$




**4** $y \propto \frac{\lambda}{w}$ [1 mark]
Therefore, if the wavelength ($\lambda$) increases, the width of the pattern ($y$) also increases. [1 mark]

**5** The correct response is D.
Width of pattern is given by width $\propto \frac{\lambda}{d}$
So, increasing d will decrease the pattern width

***VCAA examination report* note:**
A large number of students referred to Young's double-slit experiment, which was not appropriate. Other students could not refer to factors that affect diffraction. It is recommended that students familiarise themselves with diffraction as well as interference.

## 8.5 Interference of light

### Sample problem 6

**a** $d(PS_1) - d(PS_2) = \lambda$

$$d(PS_2) = d(PS_1) - \lambda = 10.00\text{ m} - 1.00\text{ m} = 9.00\text{ m}$$

**b** $d(PS_1) - d(PS_2) = \frac{1}{2}\lambda$

$$d(PS_2) = d(PS_1) - \frac{1}{2}\lambda = 10.00\text{ m} - 0.50\text{ m} = 9.50\text{ m}$$

### Practice problem 6

**a** For the second antinodal line, the path difference is given by: $|d(PS_1) - d(PS_2)| = 2\lambda = 2.0$ and $d(PS_1) = 10.0$ m.
Thus: $10.0 - d(PS_2) = \pm 2.0$, and hence $d(PS_2) = 10.0 \pm 2.0 = 8.0$ m as P is closer to $S_1$ than $S_2$, so select the smaller of the two possibilities.

**b** For the second nodal line, the path difference is given by: $|d(PS_1) - d(PS_2)| = \frac{3\lambda}{2} = 1.5$ and $d(PS_1) = 10.0$ m. Thus: $10.0 - d(PS_2) = \pm 1.5$, and hence $d(PS_2) = 10.0 \pm 1.5 = 8.5$ m as P is closer to $S_1$ than $S_2$, so select the smaller of the two possibilities.

### Sample problem 7

**a** $3\lambda = 3 \times 640\text{ nm} = 1920\text{ nm}$

**b** The second dark band arises because of destructive interference where the path difference is $\frac{3\lambda}{2}$. $S_2$ is further away from this dark band than $S_1$ by the following distance:

$$\frac{3\lambda}{2} = \frac{3 \times 640}{2} = 910\text{ nm}$$

**c** The pattern is now more compact or compressed due to the smaller wavelength of the purple light.

### Practice problem 7

**a** Point A is on the second nodal line from the central maximum.
The path difference is:

$$\frac{3\lambda}{2} = \frac{3 \times 530}{2} = 795\text{ nm}$$

Point B is on the second antinodal line from the central maximum.
The path difference is:

$$2\lambda = 2 \times 530 = 1060\text{ nm}$$

**b** The pattern will compress; adjacent maxima and minima will be closer together.

**c** The pattern will be red and, since the wavelength for red light is greater than for green light, the pattern will dilate.

**d** The pattern is indicative of the superposition of waves combining from two sources that are close to each other and in phase. Only wavelike sources produce such a pattern.

### Sample problem 8

$$\Delta x = \frac{\lambda L}{d} = \frac{589 \times 10^{-9} \times 1.50}{0.500 \times 10^{-3}} = 0.00177\text{ m} = 1.77\text{ mm}$$

### Practice problem 8

**a** $\Delta x = \frac{\lambda L}{d}$

$$\Rightarrow \lambda = \frac{\Delta x d}{L} = \frac{1.3 \times 10^{-3} \times 1.0 \times 10^{-3}}{2.0} = 6.5 \times 10^{-7}\text{ m} = 650\text{ nm}$$

**b** Light with wavelength 650 nm corresponds to red light.

**c** If blue light was used, the pattern would be blue and compressed relative to the pattern produced by red light.

**d** The line spacing could be made easier to measure if the screen was moved further from the pair of slits, since $\Delta x \propto L$.

## 8.5 Exercise

**1 a** Young's experiment clearly demonstrates the interference of light passing through two narrow closely spaced slits. This is strong evidence for the wavelike nature of light.

**b** An interference pattern consists of evenly spaced bright and dark fringes. Bright fringes are produced by the constructive interference of light passing through each slit. This constructive interference occurs when the path difference is 0, $\lambda$, $2\lambda$, $3\lambda$, and so on. Dark fringes are produced by the destructive interference of light passing through each slit. This destructive interference occurs when the path difference is $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$, and so on.

**2 a** The pattern on the screen is evidence for the wave nature of light as the pattern is fully explained by the process of interference. Interference is a property associated with waves but not particles.

**b** There is bright light at point W so it is a point where constructive interference is taking place.

**c** $|S_1C - S_2C| = 0$ as C is located on the central maximum.

**d** $|S_1W - S_2W| = 2\lambda = 2 \times 530 = 1060\,\text{nm}$

**e** $\frac{\lambda}{2} = \frac{530}{2} = 265\,\text{nm}$

$\frac{3\lambda}{2} = \frac{3 \times 530}{2} = 795\,\text{nm}$

$\frac{5\lambda}{2} = \frac{5 \times 530}{2} = 1325\,\text{nm}$

**f** There are two dark fringes between C and W.

**3 a i** The bright band corresponds to constructive interference, where crests from the light waves coming through the two slits arrive together, and troughs in the light waves arrive together.

**ii** The dark band is where destructive interference occurs. At all times, the sum of the waves from the two slits is zero, with the crests from one slit coinciding with the troughs from the other.

**b i** 0. The light rays are equidistant from the slits.

**ii** $\frac{\lambda}{2} = 317\,\text{nm}$

**iii** $2\lambda = 1266\,\text{nm}$

**c** The wavelength is smaller, so the bright fringes in the interference pattern would be closer together. As the path difference for the bright fringes equals $n\lambda$, a smaller wavelength means that each bright fringe would be closer to the central bright band.

**d** The increase in distance between the slits would result in the bright bands being closer together.

**e** Moving the screen further away would result in the interference pattern spreading out, increasing the distance between the bright bands.

**4 a** Constructive interference: path difference = $n\lambda$, $n$ = 0, 1, 2, and so on, so possible path differences are: 0, 1.06 μm, 2 × 1.06 = 2.12 μm, 3 × 1.06 = 3.18 μm, and so on.

**b** Destructive interference: path difference = $\left(n - \frac{1}{2}\right)\lambda$, $n$ = 1, 2, 3, and so on, so possible path differences are $\frac{1.06}{2} = 0.53$ μm, $\frac{3 \times 1.06}{2} = 1.59$ μm, $\frac{5 \times 1.06}{2} = 2.65$ μm, and so on.

**5** To find the distance between any two adjacent bright bands in an interference pattern, use $\Delta x = \frac{\lambda L}{d}$:

$$\Delta x = \frac{430 \times 10^{-9} \times 1.00}{0.500 \times 10^{-3}}$$
$$= 8.60 \times 10^{-4}\,\text{m}$$
$$= 0.860\,\text{mm}$$

Therefore, the distance from the central maximum to the third bright band is:

$3 \times \Delta x = 2.58$ mm or $2.58 \times 10^{-3}$ m

**6 a** Use $\Delta x = \frac{\lambda L}{d}$ rearranged:

$$\lambda = \frac{\Delta x \times d}{L}$$
$$= \frac{1.90 \times 10^{-2} \times 1.34 \times 10^{-4}}{3.80}$$
$$= 6.70 \times 10^{-7}\,\text{m}$$
$$= 670\,\text{nm}$$

**b** With slits that are more closely spaced, the distance between adjacent fringes, $\Delta x$, would increase. In this case:

$$\Delta x = \frac{\lambda L}{d}$$
$$= \frac{6.7 \times 10^{-7} \times 3.8}{1.0 \times 10^{-4}}$$
$$= 0.025\,46$$
$$= 2.5\,\text{cm}$$

**7** In each situation, the distance from the central maximum to the point in question is the same. Therefore:

$$3 \times \frac{600 \times 10^{-9} L}{d} = 4 \times \frac{\lambda L}{d}$$
$$\Rightarrow 4\lambda = 3 \times 600 \times 10^{-9}$$
$$\Rightarrow \lambda = 450 \times 10^{-9}\,\text{m}$$
$$= 450\,\text{nm}$$

## 8.5 Exam questions

**1** B. The spacing of the bands is given by the formula: $w = \frac{\lambda L}{d}$. Therefore, if $\lambda$ is decreased (green light has a shorter wavelength than red line), then the width, $w$, will also decrease.

**2 a** The point C is bright because the path difference is zero resulting in constructive interference. [1 mark] The dark band to the left of C has a path difference of $\frac{\lambda}{2}$, which results in destructive interference. [1 mark]

***VCAA examination report*** **note:**

Many students referred only to the process that gives rise to constructive and destructive interference. This does not explain the two specific regions identified in the question stem. Students are reminded that copying generic information from their A3 sheet will not score marks as it will not be specific enough to respond to the question.

**b** The experiment demonstrates interference. [1 mark] Interference is a wave phenomenon. [1 mark]

***VCAA examination report* note:**
The study design (p.44) states that students must be able to 'explain the results of Young's double slit experiment with reference to constructive and destructive interference of coherent light'. Many students made reference to diffraction and while Young's original apparatus made use of diffraction to create his coherent light source, diffraction is not a finding of the experiment. As such, these responses were not awarded marks. Students are expected to be familiar with the keystone experiments in physics and their findings especially when they are articulated in the study design.

**c** $w = \frac{\lambda L}{d}$

$w = \frac{cL}{fd}$

$w = \frac{3 \times 10^8 \times 2.00}{6.00 \times 10^{14} \times 1.0 \times 10^{-4}}$ [1 mark]

$w = 0.01$ m [1 mark]

***VCAA examination report* note:**
The most common errors were mathematical and involved incorrectly converting frequency to wavelength or using frequency in place of wavelength.

**3 a** The bright fringe in question is the fourth bright fringe so the path difference is four wavelengths. The fact that the fringe is a bright fringe indicates that constructive interference is occurring.

***VCAA examination report* note:**
The most common error was to simply refer to constructive interference in general and not discuss the path difference of four wavelengths. Students are reminded that questions are rarely asking for general information and they should read carefully to identify the specific aspects of the question being asked.

**b** $\Delta x = \frac{\lambda L}{d}$

$\frac{1.26 \times 10^{-2}}{4} = \frac{\lambda \times 2.00}{4.0 \times 10^{-4}}$

$\lambda = 630\,\text{nm}$

***VCAA examination report* note:**
The most common error was to use $1.26 \times 10^{-2}$ m as the actual $\Delta x$ value rather than divide it by four first.

**4 a** At point C, the path difference is zero [1 mark], therefore there will be large waves (constructive interference) [1 mark].

***VCAA examination report* note:**
Students were required to state that at point C there would be 'large waves' due to constructive interference because the path difference is zero. It was not enough to simply state 'due to constructive interference' as this explains all maxima and the question referred specifically to the cause of the central maximum at point C.

**b** $\Delta x = \frac{\lambda L}{d}$

$42 = \frac{420 \times \lambda}{60}$ [1 mark]

$\therefore \lambda = 6\,\text{m}$ [1 mark]

**5 a** The second dark region is due to a 1.5 wavelength difference. [1 mark]

$S_1X - S_2X = 1.5\lambda$
$= 1.5 \times 3.0$
$= 4.5\,\text{cm}$ [1 mark]

**b** Young's experiment demonstrated interference [1 mark], interference is a property of waves [1 mark], therefore, Young's experiment supports the wave model of light. [1 mark]

***VCAA examination report* note:**
Students were required to identify that Young's experiment demonstrated interference and that interference is a property of waves. Therefore, Young's experiment supports the wave model of light.

## 8.6 Review

## 8.6 Review questions

**1 a** The period, $T = \frac{1}{f}$
$= \frac{1}{2.0}$
$= 0.50\,\text{s}$

**b** Use $v = f\lambda$ rearranged: $\lambda = \frac{v}{f}$
$= \frac{2.5}{2.0}$
$= 1.3\,\text{m}$

**c** The frequency increases and hence the wavelength decreases as $v = f\lambda$ where $v$ is a constant; adjacent pulses are closer together. The speed of the wave is a property of the medium, which has not altered, and hence speed is unchanged.

**2 a** The period $T = \frac{1}{f}$
$= \frac{1}{926}$
$= 1.08 \times 10^{-3}\,\text{s}$

**b** Use $\lambda = \frac{v}{f}$
$= \frac{340}{926}$
$= 0.370\,\text{m}$

**3 a** Blue light: $\lambda = \frac{c}{f}$
$= \frac{3.0 \times 10^8}{6.5 \times 10^{14}}$
$= 4.6 \times 10^{-7}\,\text{m}$

Yellow light: $\lambda = \frac{c}{f}$
$= \frac{3.0 \times 10^8}{5.2 \times 10^{14}}$
$= 5.8 \times 10^{-7}\,\text{m}$

**b** Using $T = \frac{1}{f}$
Blue light: $T = 1.5 \times 10^{-15}\,\text{s}$
Yellow light: $T = 1.9 \times 10^{-15}\,\text{s}$

**4** First, calculate the time required to produce $1.0 \times 10^6$ cycles. The time required to produce one cycle is:

$$T = \frac{1}{f}$$
$$= \frac{1}{6.5 \times 10^{14}}$$
$$= 1.54 \times 10^{-15}\ \text{s}$$

Thus, the time required to produce $1.0 \times 10^6$ cycles is:
$1.0 \times 10^6 \times 1.54 \times 10^{-15} = 1.54 \times 10^{-9}$ s
The distance between the start and finish of the pulse is:
$d = ct$
$= 3.0 \times 10^8 \times 1.54 \times 10^{-9}$
$= 0.46$ m
The pulse of light is therefore approximately 46 cm long.

**5** **a** $f_1 = 82.41$ Hz
$f_2 = 2 \times 82.41 = 164.8$ Hz
$f_3 = 3 \times 82.41 = 247.2$ Hz
$f_4 = 4 \times 82.41 = 329.6$ Hz

**b** The wavelength of the first harmonic $\lambda_1$, sometimes known as the fundamental, is equal to twice the length of the string: $2L = 2 \times 0.75 = 1.50$ m.

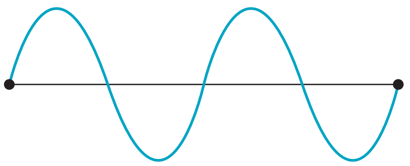
**c** Use $v = f_1\lambda_1 = 82.41 \times 1.50 = 124\ \text{m s}^{-1}$.

**d** Standing waves on a guitar string are composed of transverse travelling waves as the vibration is perpendicular to the direction of propagation of the travelling wave.

**e** In the third harmonic there are 4 nodes spanning 1.5 wavelengths. The distance between adjacent nodes is:
$\frac{L}{3} = \frac{0.75}{3}$
$= 0.25$ m
$= 25$ cm

**6** **a** The fourth harmonic $f_4 = 2460$ Hz. Use $f_n = nf_1$, thus:
$2460 = 4 \times f_1$
$\Rightarrow f_1 = \frac{2460}{4}$
$= 615$ Hz

**b** The fourth harmonic is a standing wave consisting of four antinodes as shown blow:



**c** $v = f\lambda$
$= f \times 2L$
$= 615 \times 2 \times 0.58$
$= 713\ \text{m s}^{-1}$

**d** In the fourth harmonic there are 5 nodes spanning 2 wavelengths. The distance between adjacent nodes is:
$\frac{L}{4} = \frac{0.58}{4}$
$= 0.145$ m
$= 14.5$ cm

**7** When light passes around an obstacle or through a sufficiently narrow opening, it displays wavelike behaviour in the form of diffraction. Not only does the direction of light change but, in addition, interference effects, such as those shown in the diagram, produce bright and dark fringes around edges that are well explained by constructive and destructive interference.

**8** Diffraction describes the bending or change in the direction of the propagation of waves when they pass through narrow openings in which they spread out or around objects, blurring otherwise sharp shadows.

**a** Diffraction is less evident with smaller wavelengths.

**b** Diffraction is more evident when the width of an opening is decreased.

**9** The wavelength of light $\lambda$ used can be increased, or the width of the opening $w$ can be decreased. In general, the amount of diffraction is related to the ratio $\frac{\lambda}{w}$. When this ratio is large, typically greater than $\frac{1}{2}$ (there is no precise value), diffraction effects are readily observed. For sufficiently small values of this ratio, diffraction effects become insignificant.

**10** Each colour in white light will produce its own diffraction pattern. Each colour will have a slightly different spread, with red spread the most and purple spread the least, due to variations in wavelength. Hence, the pattern has a spectrum of coloured fringes that are consistent with a typical diffraction pattern for waves passing through a narrow opening.

**11** **a** The path difference is 0, thus $S_2A = 12.0$ m.
For this problem, $\lambda = 0.80$ m.

**b** Since B lies on the first nodal line, the path difference is
$\frac{\lambda}{2} = \frac{0.800}{2} = 0.400$ m

**c** $|S_1C - S_2C| = 1.60$ m
$= 2 \times 0.800$
$= 2\lambda$.
Since the path difference is a whole number of wavelengths, the point C is a point where constructive interference is occurring. Waves from $S_1$ and $S_2$ arrive at C in phase and thus add to give a resultant amplitude larger than either wave.

**d** $|S_1D - S_2D| = 3.00$ m
$S_1D = 14.8$ m
$|14.8 - S_2D| = 3.00$ m
$S_2D = 11.8$ m or $17.8$ m
However, it can be seen that $S_2$ is closer to point D than $S_1$ is; therefore, $S_2D$ is 11.8 m.

**12** **a** $|S_1X - S_2X| = \frac{\lambda}{2}$ because X lies on the first nodal line.

**b** $|S_1X - S_2X| = |14.8 - 15.8| = 1.0$ m, thus $\frac{\lambda}{2} = 1.0$ and so $\lambda = 2.0$ m.

**c** Use $v = f\lambda = 8.0 \times 2.0 = 16\ \text{m s}^{-1}$.

**13** **a** The third dark fringe is associated with a path difference of
$\frac{5\lambda}{2} = 1250$ nm.
Thus, $\lambda = \frac{2 \times 1250}{5} = 500$ nm.

**b** The second bright fringe is associated with a path difference of:
$2\lambda = 2 \times 500 = 1000$ nm or 1.0 μm

**14** The spacing between adjacent bright fringes $\Delta x$ can be increased by a factor of 2:

**i** by increasing the distance of the screen from the slits by a factor of 2

**ii** by increasing the wavelength of the light used by a factor of 2

**iii** by decreasing the distance between the slits, the slit separation, by a factor of 2.

**15** Use the relationship $\Delta x = \frac{\lambda L}{d}$ rearranged so that:

$$L = \frac{\Delta x \times d}{\lambda}$$
$$= \frac{2.0 \times 10^{-2} \times 1.0 \times 10^{-4}}{6.5 \times 10^{-7}}$$
$$= 3.1\ \text{m}$$

## 8.6 Exam questions

### Section A — Multiple choice questions

**1** C. Transverse waves travel in a direction perpendicular to their vibrations; longitudinal waves travel parallel to their vibrations.

**2** D

$v = f\lambda$

$v = 5.0 \times 2.0$

$v = 10\ \text{m s}^{-1}$

**3** B. When mechanical waves travel through a medium, only energy is transferred. There is no net movement of the medium.

**4** D. Sound waves cannot travel through a vacuum but electromagnetic waves can travel through a vacuum.

**5** B. Electromagnetic waves have velocities through media that are a function of their wavelength. This gives rise to refraction.

**6** D. microwaves, infra-red, visible light, ultraviolet

**7** C

$\Delta x = \frac{\lambda L}{d}$

Decreasing the slit separation ($d$) will increase the width ($\Delta x$).

**8** A. Point P is a node. Nodes are the result of destructive interference.

**9** B

$\lambda = \frac{v}{f} = \frac{330}{30} = 11\ \text{m}$

**10** B

$w \propto \frac{1}{a}$

As the slit widens, the width of the peak will narrow.

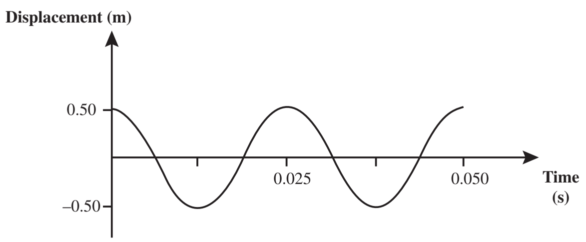
### Section B — Short answer questions

**11 a** $T = \frac{1}{f}$

$T = \frac{1}{40}$

$T = 25 \times 10^{-3}\ \text{s}$ [1 mark]

**b**




The unit for the displacement is *m*, and the unit for time is *s*. [1 mark]

The amplitude of the wave 0.50 m, thus the range should vary between −0.50 and 0.50. [1 mark]

The wave should show at least two complete cycles. [1 mark]

**12 a** $f = \frac{c}{\lambda}$

$f = \frac{3 \times 10^8}{3 \times 10^{-2}}$

$f = 10^{10}\ \text{Hz}$ [1 mark]

**b** $S_2X - S_1X = \left(n + \frac{1}{2}\right)\lambda$ [1 mark]

$= 1.5 \times 3$

$= 4.5\ \text{cm}$ [1 mark]

**c** The pattern will widen.

The width of the pattern is proportional to the ratio $\frac{\lambda}{w}$, where $w$ is the width of the gap. If the wavelength increases, the width of the pattern will also increase.

**13 a** $\lambda = \frac{v}{f}$

$\lambda = \frac{340}{340}$

$\lambda = 1\ \text{m}$

**b** At the centre, constructive interference occurs (path difference is zero), this is a loud region. [1 mark]

The first quiet region (destructive interference) occurs at $\frac{\lambda}{4}$, the next loud region (constructive interference) occurs at $\frac{\lambda}{2}$ and the next loud region occurs at $\frac{3\lambda}{4}$. [1 mark]

Thus the student has moved 0.75 m from the centre. [1 mark]

***VCAA examination report* note:**

Students were required to identify that at the centre there will be a region of relative loudness. They were not required to articulate why this would be so, but many correctly referred to the path difference being zero or another correct aspect of physics. The first node or quiet region will be one-quarter wavelength from this position. (0.25 m) and the second node or quiet region will be one-half wavelength further on (0.50 m). Thus, the total distance from the centre to the second quiet region will be (0.75 m). This question was not answered well and students had great difficulty explaining the physics of the situation. Many tried to apply a form of mathematical analysis but were unable to find a way to do this.

**14 a** The path difference is 0.806 − 0.723 = 0.083 m. This is also $3\lambda$.

Therefore, $\lambda = \frac{0.083}{3} = 2.77 \times 10^{-2}\ \text{m}$. [1 mark]

$f = \frac{c}{\lambda}$

$f = \frac{3.00 \times 10^8}{2.77 \times 10^{-2}}$

$f = 1.08 \times 10^{10}\ \text{Hz}$ [1 mark]

***VCAA examination report* note:**

The most common error was to fail to convert cm to m. Students must ensure they are working in the correct units.

**b** The signal strength between $P_0$ and $P_1$ is a minimum because the path difference is $\frac{\lambda}{2}$. [1 mark]

This results in destructive interference. [1 mark]

***VCAA examination report* note:**

The most common error was to provide a generic explanation for destructive interference and state the

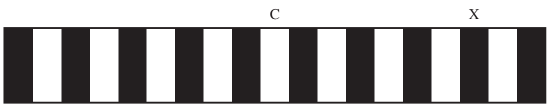
$\left(n - \frac{1}{2}\right)$ formula. This did not address the question, which was specifically about the node between $P_0$ and $P_1$. Students are advised against copying responses from their A3 reference sheet, as these responses will not be awarded marks if they do not address the question.

**15 a** The path difference to point C is zero, [1 mark] and this results in constructive interference at C. [1 mark]

***VCAA examination report* note:**

While most students were able to identify constructive interference as the mechanism for the bright reason, they then went on to explain interference in detail. Simply identifying constructive interference explains all bright regions and not the specific region. Students need to remember to focus on responding to the question asked.

**b**




Award 1 mark for correct placement of point X.

The path difference in terms of lambda is:

$$\frac{2.14 \times 10^{-6}}{610 \times 10^{-9}} = 3.5\,\lambda \quad [1\ \text{mark}]$$

3.5 $\lambda$ will place the X above the fourth dark band from the centre.

***VCAA examination report* note:**

Some students who did not receive any marks showed no understanding of how to solve the problem.

# Topic 9 — Light as a particle

## 9.2 Could light have particle-like properties as well?

### Sample problem 1

**a** **i** $E = hf$
$= 6.63 \times 10^{-34} \times 6.7 \times 10^{14}$
$= 4.4 \times 10^{-19}\text{ J}$

**ii** $p = \dfrac{E}{c}$
$= \dfrac{4.4 \times 10^{-19}}{3.0 \times 10^{8}}$
$= 1.5 \times 10^{-27}\text{ N s}$

**b** $f = \dfrac{c}{\lambda} \Rightarrow E = hf \Rightarrow E = \dfrac{hc}{\lambda}$

$p = \dfrac{E}{c} \Rightarrow p = \dfrac{hc}{\lambda c} \Rightarrow p = \dfrac{h}{\lambda}$

$p = \dfrac{h}{\lambda}$
$= \dfrac{6.63 \times 10^{-34}}{6.50 \times 10^{-7}}$
$= 1.02 \times 10^{-27}\text{ N s}$

### Practice problem 1

$E = pc$ and $E = hf$:
$f = \dfrac{pc}{h}$
$= \dfrac{9.8 \times 10^{-28} \times 3.0 \times 10^{8}}{6.63 \times 10^{-34}}$
$= 4.4 \times 10^{14}\text{ Hz}$

### Sample problem 2

**a** $E_{\text{photon}} = \dfrac{hc}{\lambda}$
$= \dfrac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{515 \times 10^{-9}}$
$= 3.86 \times 10^{-19}\text{ J}$

**b** $\text{power} = \dfrac{\text{energy emitted}}{\text{time interval}}$
$= \dfrac{E}{\Delta t}$
$= \dfrac{N\,E_{\text{photon}}}{\Delta t}$

where: $N$ is the number of photons emitted in the time interval $\Delta t$.

$N = \dfrac{\text{power} \times \Delta t}{E_{\text{photon}}}$

$N = \dfrac{0.3\text{W} \times 1\text{s}}{3.86 \times 10^{-19}\text{ J}}$
$= 8 \times 10^{17}\text{ photons s}^{-1}$

Since each photon carries a tiny amount of energy, huge numbers of photons are emitted from quite ordinary light sources in each second.

### Practice problem 2

$N = \dfrac{\text{power} \times \Delta t}{E_{\text{photon}}}$
$= \dfrac{1000 \times 1}{6.63 \times 10^{-34} \times 104.6 \times 10^{6}} = 1.4 \times 10^{28}\text{ photons s}^{-1}$

### Sample problem 3

**a** $W = Vq$
$= 500 \times 1.6 \times 10^{-19}$
$= 8.0 \times 10^{-17}\text{ J (or 500 eV)}$

**b** kinetic energy

**c** $E_\text{k} = W$
$= 8.0 \times 10^{-17}\text{ J (or 500 eV)}$

**d** $E_\text{k} = \dfrac{1}{2}mv^2 = 8.0 \times 10^{-17}\text{ J}$
$v = \sqrt{\dfrac{2E_\text{k}}{m}}$
$= \sqrt{\dfrac{2 \times 8.0 \times 10^{-17}}{9.1 \times 10^{-31}}}$
$= 1.33 \times 10^{7}\text{ m s}^{-1}$

This is substantially slower than the speed of light; therefore, relativistic effects may be ignored.

**e** $p = mv$
$= 9.1 \times 10^{-31} \times 1.33 \times 10^{7}$
$= 1.2 \times 10^{-23}\text{ N s}$

### Practice problem 3

**a** In electron volts,
$E = \dfrac{1.26 \times 10^{-17}}{1.6 \times 10^{-19}}\text{ eV}$
$= 78.75\text{ eV}$
$= 79\text{ eV}$

**b** A stopping potential of 79 V will bring these electrons to rest.

**c** Use $E = \dfrac{1}{2}mv^2 = 1.26 \times 10^{-17}$ with electron mass $m = 9.1 \times 10^{-31}$ kg.

Transpose to get $v = \sqrt{\dfrac{2 \times 1.26 \times 10^{-17}}{9.1 \times 10^{-31}}}$.
$= 5.3 \times 10^{6}\text{ m s}^{-1}$

**d** $p = mv$
$= 9.1 \times 10^{-31} \times 5.3 \times 10^{6}$
$= 4.8 \times 10^{-24}\text{ N s}$

## 9.2 Exercise

**1** **a** $\lambda = \dfrac{c}{f}$

$= \dfrac{3.0 \times 10^8}{4.59 \times 10^{14}} = 6.5 \times 10^{-7}$ m

**b** $T = \dfrac{1}{f}$

$= \dfrac{1}{4.59 \times 10^{14}} = 2.18 \times 10^{-15}$ s

**2** For green light at, say, 515 nm:

$E_{\text{photon}} = \dfrac{hc}{\lambda}$

$= \dfrac{6.6262 \times 10^{-34} \times 3.0 \times 10^8}{515 \times 10^{-9}}$

$= 3.86 \times 10^{-19}$ J

So, the detection limit of $2.0 \times 10^{-17}$ J is equivalent to:

$\dfrac{2.0 \times 10^{-17}\,\text{J}}{3.86 \times 10^{-19}\,\text{J}} = 52$ photons.

**3** Electromagnetic radiation travels at the speed of light, $3.0 \times 10^8\,\text{m s}^{-1}$.
To complete the table, the value given and the speed of light will have to be substituted into the wave equations:

$c = f\lambda$

$E = hf$

$p = \dfrac{h}{\lambda}$

**a** $c = f\lambda$

$f = \dfrac{c}{\lambda}$

$= \dfrac{3.0 \times 10^8}{10.6 \times 10^{-6}}$

$= 2.8 \times 10^{13}$ Hz

$E = hf$

$= 6.63 \times 10^{-34} \times 2.83 \times 10^{13}$

$= 1.88 \times 10^{-20}$ J

$p = \dfrac{h}{\lambda}$

$= \dfrac{6.63 \times 10^{-34}}{10.6 \times 10^{-6}}$

$= 6.25 \times 10^{-29}$ N s

**b** $E = hf$

$f = \dfrac{E}{h}$

$= \dfrac{3.14 \times 10^{-19}}{6.63 \times 10^{-34}}$

$= 4.74 \times 10^{14}$ Hz

$c = f\lambda$

$\lambda = \dfrac{c}{f}$

$= \dfrac{3.0 \times 10^8}{4.74 \times 10^{14}}$

$= 6.33 \times 10^{-7}$ m

$= 633$ nm

$p = \dfrac{h}{\lambda}$

$= \dfrac{6.63 \times 10^{-34}}{6.33 \times 10^{-7}}$

$= 1.05 \times 10^{-27}$ N s

**c** $p = \dfrac{h}{\lambda}$

$\lambda = \dfrac{h}{p}$

$= \dfrac{6.63 \times 10^{-34}}{1.125 \times 10^{-27}}$

$= 5.89 \times 10^7$ m

$= 589$ nm

$c = f\lambda$

$f = \dfrac{c}{\lambda}$

$= \dfrac{3.0 \times 10^8}{5.89 \times 10^{-7}}$

$= 5.09 \times 10^{14}$ Hz

$E = hf$

$= 6.63 \times 10^{-34} \times 5.09 \times 10^{14}$

$= 3.37 \times 10^{-19}$ J

**d** $c = f\lambda$

$\lambda = \dfrac{c}{f}$

$= \dfrac{3.0 \times 10^8}{1.55 \times 10^{15}}$

$= 1.94 \times 10^{-7}$ m

$= 194$ nm

$E = hf$

$= 6.63 \times 10^{-34} \times 1.55 \times 10^{15}$

$= 1.03 \times 10^{-18}$ J

$p = \dfrac{h}{\lambda}$

$= \dfrac{6.63 \times 10^{-34}}{1.94 \times 10^{-7}}$

$= 3.42 \times 10^{-27}$ N s

**e** $E = hf$

$f = \dfrac{E}{h}$

$= \dfrac{2.01 \times 10^{-16}}{6.63 \times 10^{-34}}$

$= 3.03 \times 10^{17}$ Hz

$c = f\lambda$

$\lambda = \dfrac{c}{f}$

$= \dfrac{3.0 \times 10^8}{3.03 \times 10^{17}}$

$= 9.90 \times 10^{-10}$ m

$= 0.990$ nm

$p = \dfrac{h}{\lambda}$

$= \dfrac{6.63 \times 10^{-34}}{9.90 \times 10^{-10}}$

$= 6.69 \times 10^{-25}$ N s

| Source | Wavelength | Frequency (Hz) | Energy | Momentum (N s) |
|---|---|---|---|---|
| Infrared from $CO_2$ laser | 10.6 μm | $2.83 \times 10^{13}$ | $1.88 \times 10^{-20}$ J, 0.117 eV | $6.25 \times 10^{-29}$ |
| Red helium–neon laser | 633 nm | $4.74 \times 10^{14}$ | $3.14 \times 10^{-19}$ J, 1.96 eV | $1.05 \times 10^{-27}$ |
| Yellow sodium lamp | 589 nm | $5.09 \times 10^{14}$ | $3.37 \times 10^{-19}$ J, 2.11 eV | $1.125 \times 10^{-27}$ |
| UV from excimer laser | 194 nm | $1.55 \times 10^{15}$ | $1.03 \times 10^{-18}$ J, 6.42 eV | $3.43 \times 10^{-27}$ |
| X-rays from aluminium | 0.990 nm | $3.03 \times 10^{17}$ | $2.01 \times 10^{-16}$ J, 1.25 keV | $6.69 \times 10^{-25}$ |

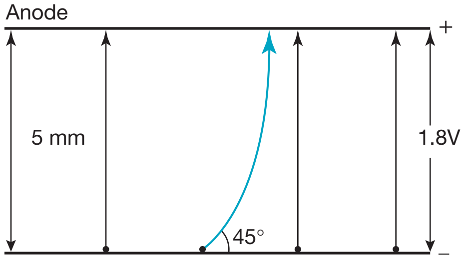
**4** Electrons have energy $3.2 \times 10^{-20}\text{ J} = \dfrac{3.2 \times 10^{-20}}{1.6 \times 10^{-19}} = 20\text{ eV}$.

Thus a potential difference of 20 V will be required to stop electrons with kinetic energy 20 eV.

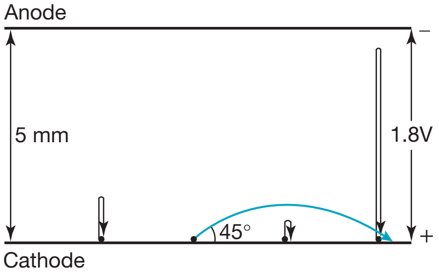
**5 a** $E_k = \dfrac{1}{2}mv^2$

$$\Rightarrow v = \sqrt{\frac{2E_k}{m}}$$
$$= \sqrt{\frac{2 \times 0.8 \times 1.60 \times 10^{-19}}{9.1 \times 10^{-31}}}$$
$$= 5.3 \times 10^5\text{ m s}^{-1}$$

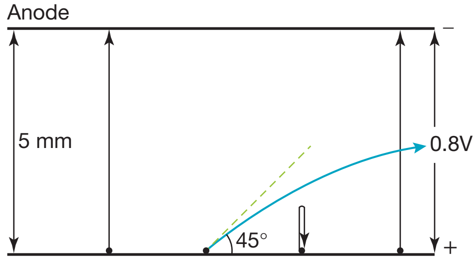
**b i**




**ii**




**iii**




## 9.2 Exam questions

**1** B

$$E = hf$$
$$1.33 \times 10^6 = 4.14 \times 10^{-15} \times f$$
$$f = 3.21 \times 10^{20}\text{ Hz}$$

**2** $E = \dfrac{hc}{\lambda}$ [1 mark]

$E = \dfrac{4.14 \times 10^{-15} \times 3 \times 10^8}{2.0 \times 10^{-9}}$ [1 mark]

$E = 6.21\text{ eV}$ [1 mark]

**3** Use $E = hf = \dfrac{hc}{\lambda}$ thus $\lambda = \dfrac{hc}{E} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{4.52 \times 10^{-34}}$

$= 4.40 \times 10^{-7}\text{ m} = 440\text{ nm}$

**4** Use $E = pc$ thus $p = \dfrac{E}{c} = \dfrac{4.52 \times 10^{-19}}{3.00 \times 10^8} = 1.50 \times 10^{-27}\text{ N s}$

**5 a** Use $E_k = \dfrac{1}{2}mv^2 = 9.22 \times 10^{-18}\text{ J}$

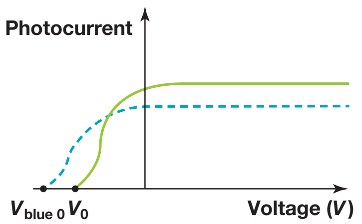
**b** $E_k = 3.2\text{ eV} = 3.2 \times 1.60 \times 10^{-19}\text{ J} = 5.12 \times 10^{-19}\text{ J}$

Use $E_k = \dfrac{1}{2}mv^2$ thus

$v = \sqrt{\dfrac{2E_k}{m}} = 1.06 \times 10^6\text{ m s}^{-1}$ [2 marks]

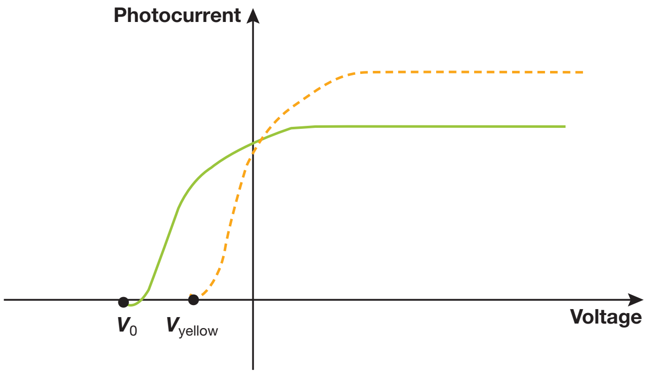
## 9.3 The photoelectric effect and experimental data

### Sample problem 4




### Practice problem 4

Yellow light consists of photons with lower energy, so the stopping voltage is smaller. The light has greater intensity and so consists of a greater number of photons incident on the photocell per second and so the photocurrent is larger.




## Sample problem 5

**a** $E_k = qV_0$

$2.6 \times 10^{-19}$ J $= 1.6 \times 10^{-19}$ C $\times V_0$

$V_0 = \frac{2.6 \times 10^{-19}\text{ J}}{1.6 \times 10^{-19}\text{ C}}$

$= 1.6$ V

A stopping voltage of 1.6 V will stop the electrons emitted from the surface.

**b** $E_k = qV_0$

$= 1.6 \times 10^{-19} \times 4.2$

$= 6.7 \times 10^{-19}$ J

A stopping voltage of 4.2 V will stop electrons with energy 4.2 eV, which is $4.2 \times 1.6 \times 10^{-19} = 6.7 \times 10^{-19}$ J.

## Practice problem 5

Convert electron energy from joules to eV:

$E = \frac{4.8 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.0$ eV

A stopping voltage of 3.0 V will remove all the kinetic energy from the electrons.

## Sample problem 6

**a** $f = \frac{c}{\lambda}$

$= \frac{3.0 \times 10^8}{4.25 \times 10^{-7}}$

$= 7.06 \times 10^{14}$ Hz

$= 7.1 \times 10^{14}$ Hz

**b** $E = hf$

$= 6.63 \times 10^{-34} \times 7.1 \times 10^{14}$

$= 4.7 \times 10^{-19}$ J

$= \frac{4.7 \times 10^{-19}}{1.6 \times 10^{-19}}$

$= 2.9$ eV

**c** $1.25 \times 1.6 \times 10^{-19} = 2.0 \times 10^{-19}$ J

**d** $\phi = 2.9 - 1.25$

$= 1.7$ eV

$= 2.7 \times 10^{-19}$ J

**e** $f_0 = \frac{\phi}{h}$

$= \frac{2.7 \times 10^{-19}}{6.63 \times 10^{-34}}$

$= 4.0 \times 10^{14}$ Hz

$\lambda = \frac{c}{f_0}$

$= \frac{3.0 \times 10^8}{4.0 \times 10^{14}}$

$= 7.4 \times 10^{-7}$ m or 740 nm

**f** $E_{k\,max} = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^8}{3.90 \times 10^{-7}}$ J $- 1.7$ eV

$= 5.1 \times 10^{-19}$ J $- 1.7$ eV

$= \frac{5.1 \times 10^{-19}}{1.6 \times 10^{-19}} - 1.7$ eV

$= 3.19 - 1.7$ eV

$= 1.5$ eV

A stopping voltage of 1.5 V is required to stop the emitted electrons.

## Practice problem 6

**a** Since the stopping voltage is 0.87 V, electrons with maximum energy will have kinetic energy 0.87 eV. In joules, this is equal to

$0.87 \times 1.6 \times 10^{-19} = 1.392 \times 10^{-19}$

$= 1.4 \times 10^{-19}$

**b** Use $E_e = E_{photon} - \phi$ and solve for $\phi$ where

$E_e = 1.392 \times 10^{-19}$ and

$E_{photon} = \frac{hc}{\lambda}$

$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{425 \times 10^{-9}} = 4.68 \times 10^{-19}$ J

Thus, $\phi = 4.68 \times 10^{-19} - 1.392 \times 10^{-19}$

$= 3.29 \times 10^{-19}$ J

$= 2.1$ eV

**c** Use $h = \frac{\phi}{f_0}$ to find the threshold frequency $f_0$.

Thus, $f_0 = \frac{\phi}{h}$

$= \frac{3.29 \times 10^{-19}}{6.63 \times 10^{-34}} = 4.96 \times 10^{14}$ Hz.

And, $\lambda = \frac{c}{f_0}$

$= \frac{3 \times 10^8}{4.96 \times 10^{14}} = 6.05 \times 10^{-7} = 605$ nm

**d** Light with wavelength 650 nm will consist of photons with energy:

$E = hf$

$= \frac{hc}{\lambda}$

$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{650 \times 10^{-9}} = 3.06 \times 10^{-19}$ J.

The energy is insufficient to overcome the work function $\phi$ ($3.29 \times 10^{-19}$ J) and thus the photoelectric effect will not occur. Alternatively, light of wavelength 650 nm has a frequency, in this case only $4.6 \times 10^{14}$ Hz, which is below the threshold frequency ($4.96 \times 10^{14}$ Hz) for the photoelectric effect.

## Sample problem 7

**a** Row 1: $c = f\lambda$

$$f = \frac{c}{\lambda}$$
$$= \frac{3.0 \times 10^{8}}{6.63 \times 10^{-7}}$$
$$= 4.52 \times 10^{14}$$
$$E = hf$$
$$= 6.63 \times 10^{-34} \times 4.52 \times 10^{14}$$
$$= 3.00 \times 10^{-19}\ \text{J}$$
$$= \frac{3.00 \times 10^{-19}}{1.60 \times 10^{-19}}\ \text{eV}$$
$$= 1.88\ \text{eV}$$
$$E_e = V \times q_e$$
$$= 0.45 \times 1.6 \times 10^{-19}$$
$$= 7.20 \times 10^{-20}\ \text{J}$$

Row 2: $c = f\lambda$

$$\lambda = \frac{c}{f}$$
$$= \frac{3.0 \times 10^{8}}{6.14 \times 10^{14}}$$
$$= 4.89 \times 10^{-7}\ \text{m}$$
$$= 489\ \text{nm}$$
$$E = hf$$
$$= 6.63 \times 10^{-34} \times 6.14 \times 10^{14}$$
$$= 4.07 \times 10^{-19}\ \text{J}$$
$$= \frac{4.07 \times 10^{-19}}{1.60 \times 10^{-19}}\ \text{eV}$$
$$= 2.54\ \text{eV}$$
$$E_e = V \times q_e$$
$$V = \frac{E_e}{q_e}$$
$$= \frac{1.84 \times 10^{-19}}{1.60 \times 10^{-19}}$$
$$= 1.15\ \text{V}$$

| Wavelength of light used (nm) | Frequency of light used × $10^{14}$ (Hz) | Photon energy of light used (eV) | Stopping voltage readings (V) | Maximum photo-electron energy (J) |
|---|---|---|---|---|
| 663 | **4.52** | **1.88** | 0.450 | **7.20 × $10^{-20}$** |
| **489** | 6.14 | **2.54** | **1.15** | 1.84 × $10^{-19}$ |

**b** The gradient of the line can be calculated by the rise over run of the data points from the table in part **a**.

$$\text{gradient} = \frac{1.15 - 0.45}{6.14 - 4.52}$$
$$= 0.43$$
$$y = 0.43x + c$$

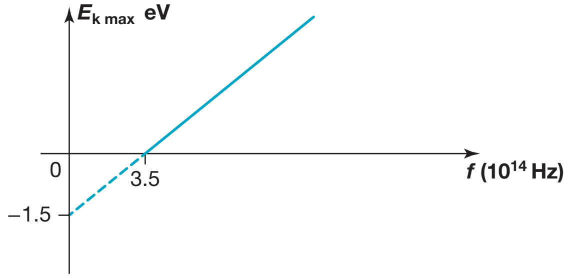
When $x = 4.52,\ y = 0.45$:

$$0.45 = 0.43 \times 4.52 + c$$
$$c = 0.45 - 0.43 \times 4.52$$
$$= -1.5$$

So the line has gradient 0.43 and $y$-intercept −1.5

The $x$-intercept can be found by letting $y = 0$.

$$y = 0.43x - 1.5$$
$$0 = 0.43x - 1.5$$
$$x = \frac{1.5}{0.43}$$
$$= 3.5$$



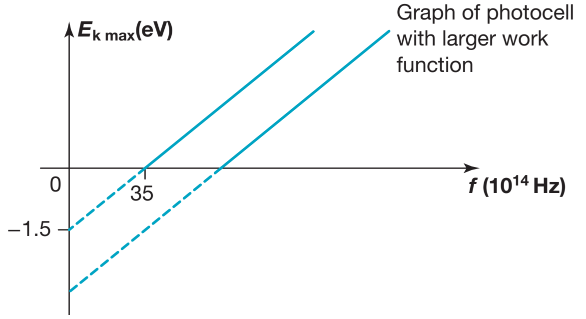

**c** **i** Planck's constant is given by the gradient of the graph:

$$\text{gradient} = \frac{0.43\ \text{eV}}{10^{-14}\ \text{Hz}} = 4.3 \times 10^{-15}\ \text{eV s}$$
$$\Rightarrow 4.3 \times 10^{-15} \times 1.6 \times 10^{-19} = 6.9 \times 10^{-34}\ \text{J s}$$

**ii** threshold frequency = 3.5 × $10^{14}$ Hz.

**iii** work function = 2.4 × $10^{-19}$ J = 1.5 eV.

**d** The graph for a photocell with a larger work function will have the same gradient, but a different $y$-intercept (lower value for $E_{k\ max}$).




**e** $1.7 \times 1.6 \times 10^{-19} = 6.63 \times 10^{-34} \times 9.12 \times 10^{14} - \phi$

$$\phi = 6.05 \times 10^{-19} - 2.72 \times 10^{-19}$$
$$= 3.33 \times 10^{-19}\ \text{J}$$
$$= 2.07\ \text{eV}$$

**f** With the light intensity doubled, the photocurrent would double.

**g** The stopping voltage would remain the same, 1.70 V, as the colour and hence the frequency of the light source is unchanged.

## Practice problem 7

**a** Row 1: $E = hf$

$$f = \frac{E}{h}$$
$$= \frac{3.19 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$
$$= 7.70 \times 10^{14}\ \text{Hz}$$
$$c = f\lambda$$
$$\lambda = \frac{c}{f}$$
$$= \frac{3.0 \times 10^{8}}{7.70 \times 10^{14}}$$
$$= 3.90 \times 10^{-7}\ \text{m}$$
$$= 390\ \text{nm}$$

$E_e = V \times q_e$

$V = \frac{E_e}{q_e}$

$= \frac{3.78 \times 10^{-19}}{1.60 \times 10^{-19}}$

$= 2.36\ \text{V}$

Row 2: $c = f\lambda$

$f = \frac{c}{\lambda}$

$= \frac{3.0 \times 10^{8}}{5.24 \times 10^{-7}}$

$= 5.73 \times 10^{14}\ \text{Hz}$

$E = hf$

$= 6.63 \times 10^{-34} \times 5.73 \times 10^{14}$

$= 3.80 \times 10^{-19}\ \text{J}$

$= \frac{3.80 \times 10^{-19}}{1.60 \times 10^{-19}}\ \text{eV}$

$= 2.37\ \text{eV}$

$E_e = V \times q_e$

$= 1.54 \times 1.6 \times 10^{-19}$

$= 2.46 \times 10^{-19}\ \text{J}$

Answers are in bold.

| Wavelength of light used (nm) | Frequency of light used × $10^{14}$ (Hz) | Photon energy of light used (eV) | Stopping voltage (V) | Maximum photo-electron energy (J) |
|---|---|---|---|---|
| **390** | **7.70** | 3.19 | **2.36** | $3.78 \times 10^{-19}$ |
| 524 | **5.73** | **2.37** | 1.54 | $\mathbf{2.46 \times 10^{-19}}$ |

**b** The gradient of the line can be calculated by the rise over run of the data points from the table in part **a**.

gradient $= \frac{2.36 - 1.54}{7.70 - 5.73}$

$= 0.42$

$y = 0.42x + c$

When $x = 5.73,\ y = 1.54$:

$1.54 = 0.42 \times 5.73 + c$

$c = 1.54 - 0.42 \times 5.73$

$= -0.85$

So the line has gradient 0.92 and $y$-intercept $-0.85$.

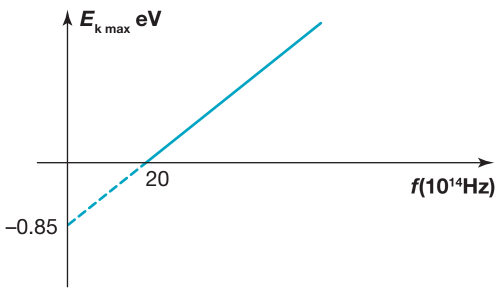
The $x$-intercept can be found by letting $y = 0$.

$y = 0.42x - 0.85$

$0 = 0.42x - 0.85$

$x = \frac{0.85}{0.42}$

$= 2.0$




**c** **i** Planck's constant = gradient of graph

$= \frac{3.78 \times 10^{-19} - 2.46 \times 10^{-19}}{(7.69 - 5.73) \times 10^{14}}$

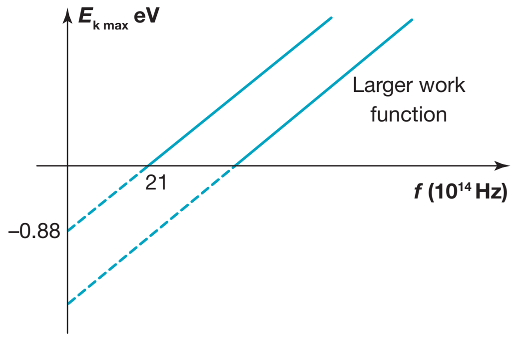
$= 6.73 \times 10^{-34}\ \text{J s}$

which is close to the accepted value. It also has the value $4.2 \times 10^{-15}$ eV s.

**ii** From the line of best fit in the graph (b), the threshold frequency = $x$-axis intercept = $2.0 \times 10^{14}$ Hz.

**iii** From the line of best fit in the graph (b), the work function = $y$-axis intercept = 0.85 eV.

**d** The graph for a photocell with a larger work function will have the same gradient, but a different $y$-intercept.




**e** Use $E_{k\,max} = hf - \phi$ to calculate work function $\phi$.

$1.59 \times 1.6 \times 10^{-19} = 6.63 \times 10^{-34} \times 8.25 \times 10^{14} - \phi$

$\phi = 5.47 \times 10^{-19} - 2.54 \times 10^{-19}$

$= 2.93 \times 10^{-19}\ \text{J}$

$= 1.8\ \text{eV}$

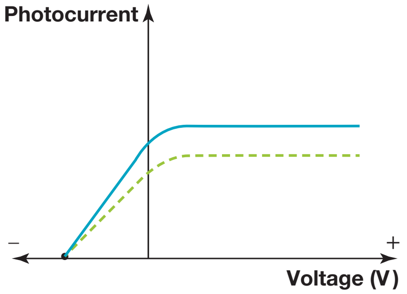
**f** With the light intensity now halved, the photocurrent would also halve to 19 μA.

**g** The stopping voltage would remain the same, 1.59 V, as the colour and, hence, frequency and photon energy of the light source remains unchanged.

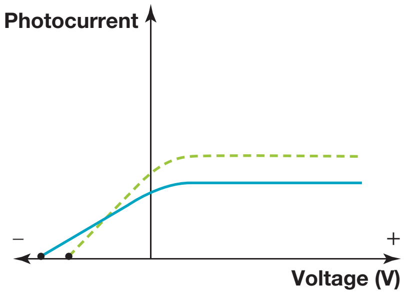
## 9.3 Exercise

**1** **a** The maximum current occurs when the accelerating voltage causes all ejected electrons to be collected at the anode. The voltage required for this is greater than zero because some electrons leave at an angle and their parabolic path may miss the anode at lower voltages. When the voltage opposes the motion towards the cathode, electrons travelling towards the anode slow down. When the magnitude of the voltage is large enough, the electrons reverse direction and so do not contribute to the current. At a high enough retarding voltage, *all* electrons turn around before reaching the anode, so the current is zero.

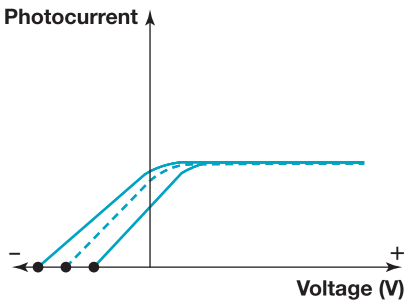
**b** Increasing the intensity without changing the frequency would increase the number of photons per second reaching the cathode but not change their energy. As a result, the photoelectrons will have the same energy spread and, therefore, the same stopping voltage, but there will be more electrons ejected per second, resulting in a higher photocurrent.




**c** Increasing the frequency of the light would increase the energy of each photon, resulting in higher energy photoelectrons and a greater stopping voltage. If the intensity is unchanged, the energy per second reaching the cathode is not changed, but since each photon has a greater energy, this means there are fewer photons per second reaching the cathode and the photocurrent will be reduced.




**d** There will be the same number of photons per second and they have the same energy, but a different energy is required to eject an electron. This will change the maximum $E_k$ of the electrons and therefore the stopping voltage, but since the same number of photons reaches the cathode each second, the maximum photocurrent will be unchanged. Two answers are shown. One is for a material with a greater work function (lower maximum $E_k$ and therefore lower stopping voltage) and the other is for a material with a smaller work function (higher maximum $E_k$ and therefore higher stopping voltage).




**2 a** Use the graph to read off the current when voltage = 0, that is, 0.85 μA.

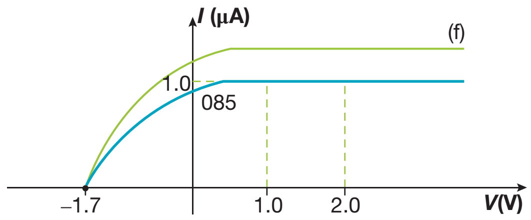
**b** Use the graph to read off the current when voltage = 1.0 V, that is, 1.0 μA.

**c** Increasing the voltage from 1.0 to 2.0 V has no effect on the current: the current is 1.0 μA.

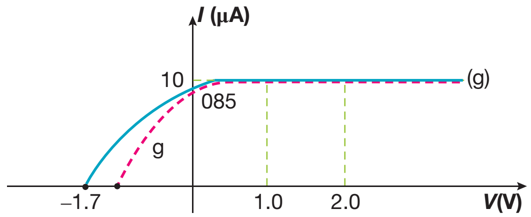
**d** Once the voltage has reached the point where all electrons are reaching the plate, the current in the photocell is determined solely by the intensity of the light source. There is no increase in light intensity; therefore, there is no increase in the photocurrent.

**e** Use the graph, this time to find the stopping voltage when the current = 0 A, that is, a stopping voltage of 1.7 V or 1.7 eV = $2.7 \times 10^{-19}$ J.

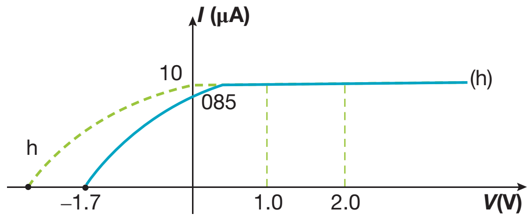
**f** Note, on the same graph, that the numbers on the *y*-axis have increased in value, the stopping voltage remains the same, and the frequency of the light used is the same (see curve (f) in the following graph).




**g** The graph has the same shape but the stopping voltage is closer to the origin — less than 1.7 V (see curve (g) in the following graph).




**h** The graph has the same shape but the stopping voltage is now greater than 1.7 V (see curve (h) in the following graph).




**3 a** Since electrons are emitted with a maximum energy of 0.67 eV, a stopping voltage of 0.67 V will be required to stop these electrons.

**b** Use $E_e = E_{photon} - \phi$

Thus: $0.67 = E_{photon} - 3.80$

$\Rightarrow E_{photon} = 4.47\text{ eV}$

$$E_{photon} = 4.47 \times 1.6 \times 10^{-19}\text{ J}$$
$$= 7.15 \times 10^{-19}\text{ J}$$

$$E_{photon} = hf$$

$$f = \frac{E_{photon}}{h}$$
$$= \frac{7.15 \times 10^{-19}}{6.63 \times 10^{-34}}$$
$$= 1.08 \times 10^{15}\text{ Hz}$$

**c** $$f_0 = \frac{\phi}{h}$$
$$= \frac{3.80 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}}$$
$$= 9.17 \times 10^{14}\text{ Hz}$$

**4 a** $\phi = hf_0$

$= 4.14 \times 10^{-15} \times 6.2 \times 10^{14}$

$= 2.6\,\text{eV} = 4.1 \times 10^{-19}\,\text{J}$

**b** $3.4 \times 10^{-19}\,\text{J} = \dfrac{3.4 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.1\,\text{eV}$

Therefore the stopping voltage is 2.1 V.

**c** Firstly, use $E_e = E_{photon} - \phi$

$3.4 \times 10^{-19} = hf - 4.1 \times 10^{-19}$

$hf = 7.5 \times 10^{-19}$

$f = \dfrac{7.5 \times 10^{-19}}{6.63 \times 10^{-34}}$

$= 1.1 \times 10^{15}\,\text{Hz}$

$\lambda = \dfrac{c}{f} = \dfrac{3.0 \times 10^{8}}{1.13 \times 10^{15}} = 2.65 \times 10^{-7} = 265\,\text{nm}$

$p = \dfrac{h}{\lambda}$

$= \dfrac{6.63 \times 10^{-34}}{2.65 \times 10^{-7}}$

$= 2.50 \times 10^{-27}\,\text{N s}$

**5 a** Electron energy $= 4.0 \times 10^{-19}\,\text{J}$

$= \dfrac{4.0 \times 10^{-19}}{1.60 \times 10^{-19}}$

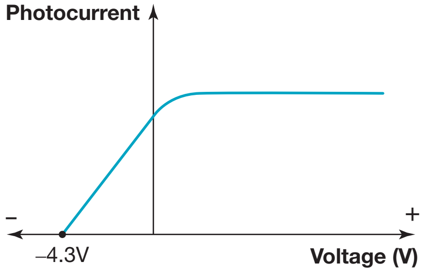
$= 2.5\,\text{eV}$

A retarding voltage of 1.0 V would reduce the kinetic energy by 1.0 eV to 1.5 eV, or $2.4 \times 10^{-19}$ J.

**b** A retarding voltage of 2.5 V is required to completely transform the 2.5 eV of kinetic energy into electric potential energy, that is, to stop the electron.

**c** A stopping voltage of 4.3 V means that the highest kinetic energy of electrons is 4.3 eV, or $6.9 \times 10^{-19}$ J.

**d**




**6** $E_{photon} = \dfrac{hc}{\lambda}$

$= \dfrac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{200 \times 10^{-9}}$

$= 9.945 \times 10^{-19}\,\text{J}$

$= \dfrac{9.945 \times 10^{-19}}{1.60 \times 10^{-19}}$

$= 6.22\,\text{eV}$

$E_{k\ \text{max photoelectron}} = E_{photon} - \phi$

$= 6.22\,\text{eV} - 5.10\,\text{eV}$

$= 1.12\,\text{eV}$

To stop an electron with 1.12 eV of kinetic energy requires a stopping voltage of 1.12 V.

**7 a** Maximum $E_k = E_{photon} - \phi$

$E_{photon} = \dfrac{hc}{\lambda}$

$= \dfrac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{254 \times 10^{-9}}$

$= 7.83 \times 10^{-19}\,\text{J}$

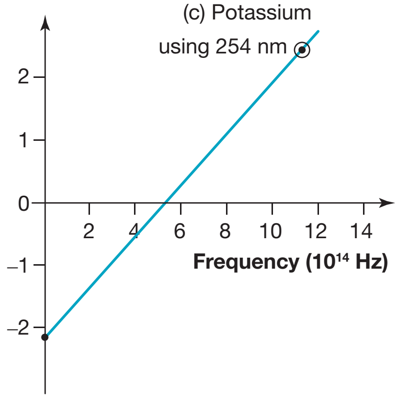
$= 4.89\,\text{eV}$

Maximum $E_k = 4.89\,\text{eV} - 2.30\,\text{eV}$

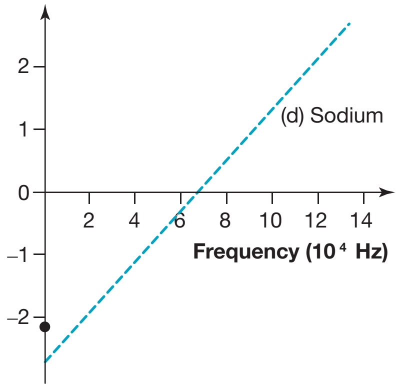
$= 2.59\,\text{eV}$, or $4.14 \times 10^{-19}\,\text{J}$

**b** To transform this kinetic energy into electric potential energy would require 2.59 V.

**c**




**d**




**8** The frequency of each wavelength of light can be calculated by the equation $f = \dfrac{c}{\lambda}$:

$f = \dfrac{3.0 \times 10^{8}}{3.66 \times 10^{-7}} = 8.20 \times 10^{14}\,\text{Hz}$

$f = \dfrac{3.0 \times 10^{8}}{4.05 \times 10^{-7}} = 7.41 \times 10^{14}\,\text{Hz}$

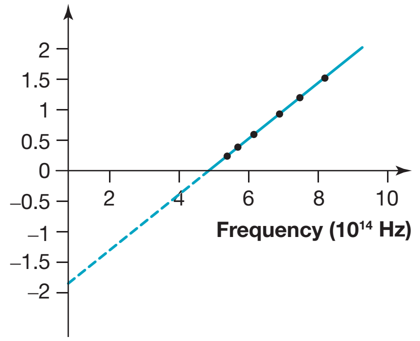
$f = \dfrac{3.0 \times 10^{8}}{4.36 \times 10^{-7}} = 6.88 \times 10^{14}\,\text{Hz}$

$f = \dfrac{3.0 \times 10^{8}}{4.92 \times 10^{-7}} = 6.10 \times 10^{14}\,\text{Hz}$

$f = \dfrac{3.0 \times 10^{8}}{5.46 \times 10^{-7}} = 5.49 \times 10^{14}\,\text{Hz}$

$f = \dfrac{3.0 \times 10^{8}}{5.79 \times 10^{-7}} = 5.18 \times 10^{14}\,\text{Hz}$

Using these values, the graph can be plotted as shown below.




**a** Threshold frequency is the point where the graph crosses the frequency axis, that is, where the maximum $E_k$ of the electrons, and the stopping voltage, is zero:

$f_0 \approx 4.6 \times 10^{14}$ Hz.

**b** $\lambda = \frac{c}{f}$

$= \frac{3.0 \times 10^8}{4.6 \times 10^{14}}$

$= 6.5 \times 10^{-7}$ m

**c** $\phi = hf_0$

$= 6.63 \times 10^{-34}$ J s $\times 4.6 \times 10^{14}$ Hz

$= 3.0 \times 10^{-19}$ J $= 1.9$ eV

or, take the value of the $y$-axis intercept from the stopping voltage versus frequency graph.

**d** Planck's constant is the gradient of the graph:

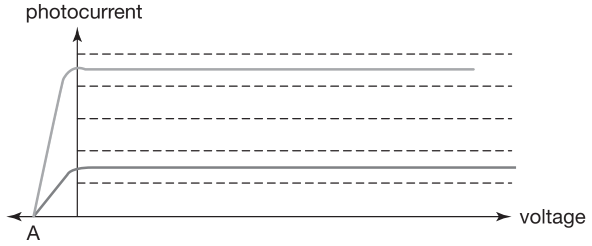
$$\text{gradient} = \frac{1.48 - 0.24}{8.20 \times 10^{14} - 5.18 \times 10^{14}} = \frac{1.24}{3.02 \times 10^{14}}$$

$= 4.1 \times 10^{-15}$ eV s

$= 6.56 \times 10^{-34}$ J s

## 9.3 Exam questions

**1 a**




Award 1 mark for a correct shape, starting from point A.

Award 1 mark for the correct maximum value of the photocurrent.

**b** Point A is the stopping voltage. [1 mark]

***VCAA examination report*** **note:**

The most common error was to identify it as 'threshold voltage', which implied students were confusing this graph with a graph of kinetic energy versus frequency.

**c** The stopping voltage is sufficient to turn back even the most energetic photoelectrons. [1 mark]

***VCAA examination report*** **note:**

The most common error was to refer to work function in some way. Students wrote about the energy required to be emitted from the metal.

**2 a i** Planck's constant, $h$, is equal to the gradient of the graph.

$\text{Planck's constant} = \frac{\text{rise}}{\text{run}}$

$h = \frac{2.0}{3.8 \times 10^{14}}$

$h = 5.3 \times 10^{-15}$ eV s

Award 1 mark for the numerical answer and 1 mark for the units.

Note that the units are not eV $s^{-1}$.

The units can be found by looking at the units of the gradient calculation:

$h = \frac{\text{rise}}{\text{run}}$

$h = \frac{E_{\text{h max}}\ (\text{in eV})}{\text{frequency}\ (\text{in Hz})}$

and Hertz is per second or $s^{-1}$.

The units of the gradient are therefore

$\frac{\text{eV}}{\text{Hz}} = \frac{\text{eV}}{\text{s}^{-1}} = \text{eVs}.$

**ii** The maximum wavelength of light that would cause the emission of photoelectrons is equivalent to the light with the minimum or threshold frequency ($f_0$), which is the horizontal intercept.

$f_0 = 3.7 \times 10^{14}$ Hz

$v = f \times \lambda$

$\lambda_{max} = \frac{v}{f_0}$

$\lambda_{max} = \frac{3 \times 10^8}{3.7 \times 10^{14}}$

$\lambda_{max} = 8.1 \times 10^{-7}$ m

$\lambda_{max} = 810$ nm [1 mark]

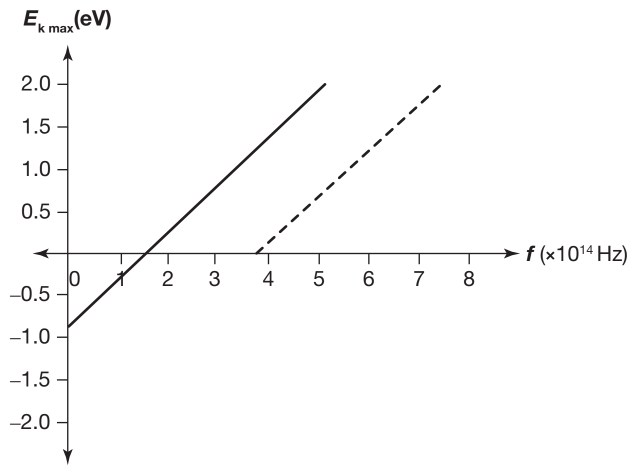
**iii** The work function of the metal is the vertical intercept of the graph.

Extending the line of best fit back to $f = 0$ gives an intercept value of approximately 1.9 eV.

The value should **not** be stated as a negative value because the work function represents the minimum ionisation energy (the minimum energy) required by an electron to escape the attraction of the metal to which it is bound. [1 mark]

**b** The new graph will have a work function that is half of the previous value (approximately 0.95 eV).

The gradient of these lines is Planck's constant, a constant value for all experimental results. Therefore, the new line is parallel to the first. [2 marks]




**3** **a** The cut-off potential is equal to the maximum KE of the photoelectron (in eV).

$V_0 = h(f - f_0) = 4.14 \times 10^{-15} \times (6.25 - 5.50) \times 10^{14}$ [1 mark]

$= 0.31$ V [1 mark]

***VCAA examination report* note:**

Some students did not understand the need to find the difference between the two energies.

Some used the incorrect Planck's constant.

**b** See the figure at the bottom of the page*

Award 1 mark for showing the new graph with a larger photocurrent.

Award 1 mark for showing that the new graph is moved left, demonstrating a larger cut-off potential.

***VCAA examination report* note:**

The most common error was to increase the photocurrent but keep the same stopping voltage.

**4** **a** $h = \dfrac{3}{6 \times 10^{14}}$ [1mark]

$h = 5.0 \times 10^{-15}$ eVs [1mark]

***VCAA examination report* note:**

Students were required to find the gradient of the graph.

**b** From the graph, $2.0 \times 10^{14}$ Hz

**c** From $y$-intercept on the graph, $\phi = 1.0$ eV

**d** See the figure at the bottom of the page**

The slope will not change [1 mark], however the cut-off frequency increases with the work function ($hf_0 = \phi$) and the $y$-intercept becomes $-2.5$ eV instead of $-1$ eV. [1 mark]

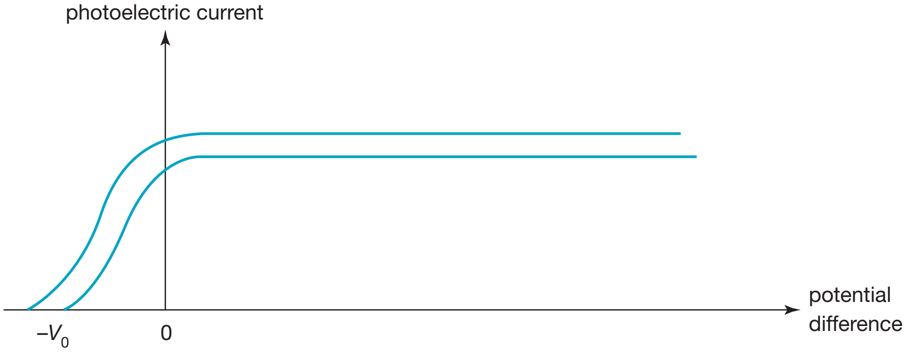
**5** The most appropriate method was to use:

$W = hf_0$ [1 mark]

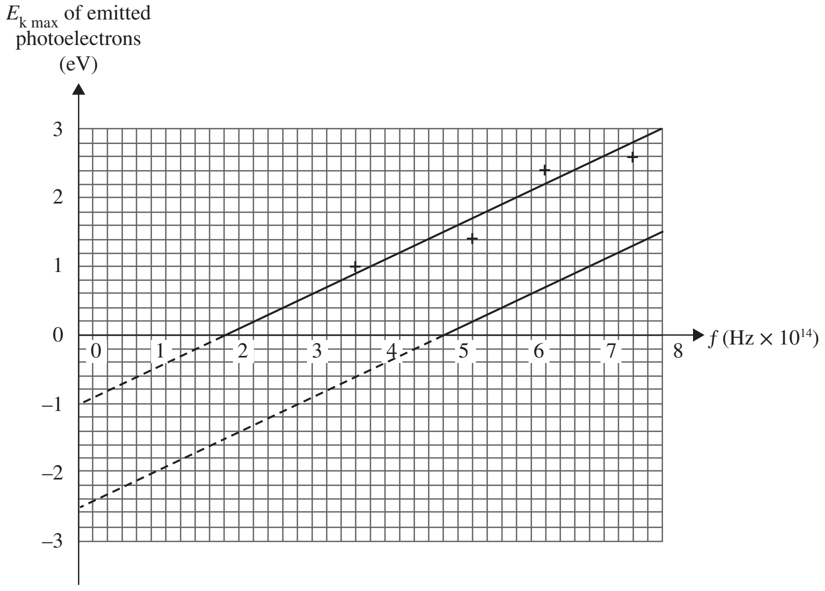
$3.2 \times 106 - 19 = h \times 6.5 \times 10^{14}$ [1 mark]

$h = 4.9 \times 10^{-34}$ J s [1 mark]

***3 b**




****4 d**



## 9.4 Limitations of the wave model

### 9.4 Exercise

**1** If light behaved like a wave, when the intensity of light is increased, the stopping voltage would also increase, as energy is being delivered at a faster rate (and electrons would have higher kinetic energy).

**2** Some examples of observations that support the particle model include:
- For a given frequency of light, the photocurrent is dependent in a linear fashion on the brightness or intensity of light.
- The energy of photoelectrons is independent of intensity of light and only linearly dependent on frequency.
- There is no significant time delay between incident light striking a photocell and the subsequent emission of electrons, and this observation is independent of intensity.
- A threshold frequency exists below which the photoelectric effect does not occur, and this threshold is independent of intensity.

**3** When treating light as a wave, there is no threshold frequency as energy transfer to electrons from a light source is cumulative and, eventually, emission will occur. However, when light is treated as a particle, there is a threshold frequency, as photons with energy less than the work function cannot free electrons from the photocell, and they will not be emitted.

**4** Light is thought to behave in two different ways — as both a particle and a wave. Many believe that these are two different models, so it seems strange to use the wave model to complete calculations involving the particle model. However, these two models are not competing models — they exist in duality and work together to explain the behaviour of light. Light behaves as both a 'particle' and a 'wave', so using the frequency from one model to calculate the energy of another model is quite reasonable. Using the wave model, you can relate the frequency and wavelength with the speed of light and, using the particle model, you can relate Planck's constant and frequency with the speed of light. The frequency is in fact useful in both models so, while it seems contradictory, it is incredibly useful for the calculations.

### 9.4 Exam questions

**1** C. To find the maximum kinetic energy, one must have the stopping voltage. This will be the reading on the voltmeter as the current reaches zero.

**2** The model supported is the particle model. The reasons that the student could give included, but were not limited to:
- one photon per electron interaction
- the first photon causes the release of the first photoelectron
- the wave model predicts an accumulation of energy over time.

***VCAA examination report* note:**
A lot of students correctly identified the particle model but then copied generic information regarding the photoelectric effect experiment. Of concern were the number of these responses that did not make any reference to the time delay finding. It was not clear whether this was because students did not understand the question or the results of the photoelectric effect experiment.

**3**
- **Absence of time delay**
  - The wave model predicts that no photoelectrons will be emitted until sufficient energy has been delivered to the metal.
  - Experimentally, there is no measurable time delay before electrons are ejected.
- **The existence of a threshold frequency**
  - The wave model predicts that light of any frequency should be able to eject photoelectrons, since the energy delivered by a wave depends only on the intensity of light.
  - Experiment shows that light below a certain frequency **does not** eject photoelectrons, regardless of its intensity.
- **The absence of any effect of light intensity on stopping potential**
  - The wave model predicts that greater intensity will deliver more energy to the metal and so increase the maximum KE of the photoelectrons. This should lead to an increase in the stopping potential.
  - Experiment shows that the stopping potential is **not affected** by changing the light intensity.

*Note:* Award 1 mark each for clearly stating two aspects of the photoelectric effect. [2 marks]
Award 1 mark for each reasonable explanation of how the wave model fails to explain the two aspects chosen; the explanation may lack some clarity or completeness. [2 marks]
Award an additional mark if either or both of the explanations are complete and clearly expressed. [1 mark]

***VCAA examination report* note:**
Many students spent a great deal of their response discussing either the results of the photoelectric effect that did not refer to the wave model or discussing the particle explanation for the phenomena described above, rather than the wave predictions. While the first sentence made the general statement that 'The results of photoelectric effect experiments in general provide strong evidence for the particle-like nature of light.', the question posed to the students related to the wave model and its failure to explain certain phenomena. References to the particle model were off-topic and were not awarded marks.

**4** ***VCAA examination report* note:**
Students were required to identify one result that they wished to discuss. Options included the existence of a threshold frequency, the lack of delay in the release of photoelectrons at low intensities, or the fact that increasing the intensity of the light source does not increase the kinetic energy of the photo electrons. They were then required to identify the wave model prediction and how the results contradicted the prediction. Finally, they were required to identify how a particle model would support the prediction.
The following is an example of a possible response.
There should be no threshold frequency. The wave model predicts that all light carries energy proportional to the amplitude of the wave. Therefore, all light should be able to produce photoelectrons. The results show that only light with frequencies above a threshold frequency can produce photoelectrons. The particle model of light predicts that photons have an energy proportional to their frequency and only photons with a high enough frequency will have sufficient energy to release photoelectrons.

Award one mark for identifying one result, one mark for identifying the corresponding wave model prediction and how the result contradict the prediction, one mark for identifying how the particle model support the prediction.

**5** The evidence is the photoelectric effect. [1 mark]
The particle model is supported by (any one of):
- no time delay in electron emission
- KE of electrons depends on light frequency not intensity
- the existence of a threshold frequency for light. [1 mark]

## 9.5 Review

## 9.5 Review questions

**1 a** Since 1.0 nm = $1.0 \times 10^{-9}$ m,

$$650\,\text{nm} = 650 \times 1.0 \times 10^{-9}\,\text{m} = 6.5 \times 10^{-7}\,\text{m}$$

**b** Use: $f = \dfrac{c}{\lambda}$

$$= \frac{3.0 \times 10^{8}}{6.5 \times 10^{-7}} = 4.6 \times 10^{14}\,\text{Hz}$$

**c** $E = hf$

$$= 6.63 \times 10^{-34} \times 4.6 \times 10^{14}$$
$$= 3.1 \times 10^{-19}\,\text{J (or 1.9 eV)}$$

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{6.5 \times 10^{-7}} = 1.0 \times 10^{-27}\,\text{N s}$$

**2** The energy of a single emitted photon is:

$$E = hf = 6.63 \times 10^{-34} \times 1.059 \times 10^{8} = 7.02 \times 10^{-26}\,\text{J photon}^{-1}$$

$$\Rightarrow \text{photons emitted per second} = \frac{\text{power of transmitter}}{7.02 \times 10^{-26}} = \frac{1000}{7.02 \times 10^{-26}} = 1.42 \times 10^{28}\,\text{photons s}^{-1}$$

**3 a** $E = hf$

$$= 6.63 \times 10^{-34} \times 6.8 \times 10^{14}$$
$$= 4.51 \times 10^{-19}\,\text{J} = 2.8\,\text{eV}$$

(If you use $h = 4.14 \times 10^{-15}$ eV s, then the photon energy has units of eV and the answer will be 2.8 eV directly.)

**b** Use: $E_{\text{electron}} = E_{\text{photon}} - \phi$

$$\Rightarrow \phi = E_{\text{photon}} - E_{\text{electron}}$$
$$= 4.51 \times 10^{-19} - 2.4 \times 10^{-19}$$
$$= 2.11 \times 10^{-19}\,\text{J} = 1.3\,\text{eV}$$

**c** Since the maximum electron energy = $2.4 \times 10^{-19}$ J = 1.5 eV, a stopping voltage of 1.5 V is required. Remember: the photoelectron energy in the unit eV is numerically equal to the stopping voltage when expressed in the unit V.

**4 a** Photons with a frequency below the threshold frequency will not have sufficient energy for electrons on the surface, or below the surface, to overcome the work function $\phi$. The photoelectric effect will not occur.

**b** Use: $\phi = hf_0$

$$= 6.63 \times 10^{-34} \times 3.50 \times 10^{14}$$
$$= 2.32 \times 10^{-19}\,\text{J} = 1.45\,\text{eV}$$

**c** Use $E_{\text{electron}} = E_{\text{photon}} - \phi$

$$= hf - hf_0$$
$$= 6.63 \times 10^{-34} \times 5.10 \times 10^{15} - 2.32 \times 10^{-19}$$
$$= 3.15 \times 10^{-18}\,\text{J}$$
$$= 19.7\,\text{eV}$$

**5 a** The data given is sufficient to determine their value for Planck's constant, which would be equal to the gradient of a graph of electron energy versus frequency using their two pieces of data.

Thus: $h = \dfrac{(1.60 - 0.72)}{(6.2 - 4.1) \times 10^{14}}\,\text{eV s}$

$$= 4.2 \times 10^{-15}\,\text{eV s}$$
$$= 6.7 \times 10^{-34}\,\text{J s}$$

**b** Use the first piece of information given, $f = 4.1 \times 10^{14}$ Hz and $E_{\text{electron}} = 0.72\,\text{eV}$, since the stopping voltage = 0.72 V.

Thus: $E_{\text{electron}} = E_{\text{photon}} - \phi$

$$\Rightarrow \phi = E_{\text{photon}} - E_{\text{electron}}$$
$$= 4.14 \times 10^{-15} \times 4.1 \times 10^{14} - 0.72$$
$$= 1.70 - 0.72$$
$$= 0.98\,\text{eV}$$
$$= 1.6 \times 10^{-19}\,\text{J}$$

**6 a** The stopping voltage will stay the same because the same frequency and hence the same energy photons will be incident on the photocell. Emitted electrons will have the same energy and hence will be stopped by the same stopping voltage. The photoelectron current will be increased due to an increased intensity of the light used. More photons strike the surface per second and hence more electrons will be emitted per second.

**b** In this case, the stopping voltage will be greater as the photon energy increases and hence the energy of emitted electrons increases. The photocurrent will be the same as the photon intensity is the same.

**c** A smaller work function $W$ will mean that emitted electrons will be emitted with greater energy when photons of the same frequency are used. This will accordingly require a larger stopping voltage. The photocurrent will be unchanged.

**7 Reason 1:**
Since the photocurrent is proportional to the brightness of the light, it lends support to the idea that light is a stream of particles. The greater the number of particles striking a surface per second, the greater the number of electrons emitted per second. A wave model does not make any obvious predictions concerning brightness and current, but incorrectly makes a prediction concerning the brightness of a light source and electron energy.

**Reason 2:**
The energy of the photoelectrons is dependent on the frequency of the light, consistent with the Planck model and consolidated by Einstein; the light consists of individual photons of energy proportional to frequency. A wave model makes no such assertion.

**Reason 3:**
The lack of delay between light striking a surface and the subsequent emission of electrons, both independent of

brightness and frequency, indicates that the light arriving as particle-like packets. A wave model would predict a variable delay time due to brightness, which is not observed.

**Reason 4:**

The existence of a threshold frequency below which the photoelectric effect does not occur, regardless of the intensity of light, is not a prediction made using a wave model. It is consistent with a particle model for light where the photons do not have sufficient energy to cause electrons to be emitted regardless of how much time is given for the process. Also implicit is that energy transfers from photons to electrons are in a one-to-one ratio.

## 9.5 Exam questions

### Section A – Multiple choice questions

**1** A. Light demonstrates particle behaviour through the photoelectric effect.

**2** A. If the intensity is increased, then the number of photons hitting the metal will increase, leading to more photoelectrons. Since the frequency of the photons has not changed, their energy has not changed and as a result the maximum kinetic energy of the photoelectrons will remain unchanged.

**3** C. 'The maximum kinetic energy of the photoelectrons depended only on the light intensity' is the only option that is not a finding of the photoelectric effect experiment.

**4** C. The results of the photoelectric effect experiment show that, if light intensity is increased, it will increase the number of photons striking the surface. This will result in more photoelectrons being released. However, each photon will have the same energy, so the maximum kinetic energy of the photoelectrons remains unchanged.

**5** A

$E_{(J)} = E_{(eV)} \times 1.6 \times 10^{-19}$

$E_{(J)} = 2.0 \times 1.6 \times 10^{-19}$

$E_{(J)} = 3.2 \times 10^{-19}$ J

**6** D. Work function is directly proportional to cut-off frequency. [1 mark]

**7** B

$E_{k\ max} = \frac{hc}{\lambda} - W$

$\therefore E_{k\ max} \propto \frac{1}{\lambda}$

**8** D. The photoelectric effect demonstrates that $E_{photon} \propto f$. Increasing the intensity will produce more photons per second but will not affect their energy.

**9** A. The energy of a single photon is:

$E = \frac{hc}{\lambda}$

$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{550 \times 10^{-9}}$

$E = 3.62 \times 10^{-19}$ J

There are $2.8 \times 10^{16}$ photons per second so the total power is:

$P = 3.62 \times 10^{-19} \times 2.8 \times 10^{16}$

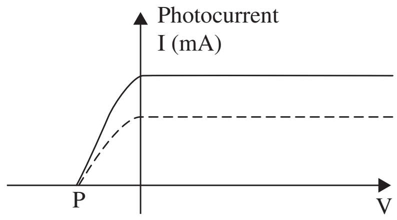
$P = 1.0 \times 10^{-2}$ W

**10** A. A higher work function means a higher cutoff frequency ($x$-intercept). The graph must still have the same gradient (Planck's constant).

### Section B – Short answer questions

**11 a** Stopping voltage (or cut-off voltage or stopping potential). [1 mark]

**b**

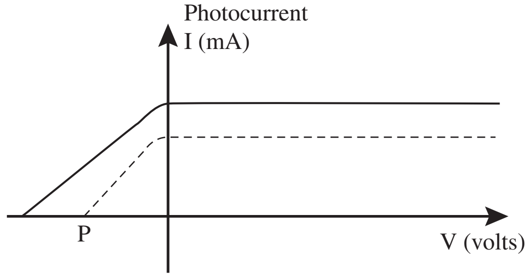



Award 1 mark for the same stopping voltage (increasing intensity of light does not increase nor decrease the stopping voltage) and 1 mark for a greater value of the photocurrent (the number of electrons ejected is proportional to the intensity of light).

***VCAA examination report*** **note:**

Any value of photocurrent greater than the initial value was accepted.

**c**




Award 1 mark for a stopping voltage beyond P (the stopping voltage increases with photon's frequency, and violet light has a higher frequency than green light) and 1 mark for a greater value of the photocurrent (the number of electrons ejected is proportional to the intensity of light).

***VCAA examination report*** **note:**

Any stopping voltage beyond P and any photocurrent greater than the original were accepted.

**d** The work function is found from the $y$-intercept of the graph. The value is 3.4 eV. [1 mark]

***VCAA examination report*** **note:**

Many students chose to calculate the work function using $\phi = hf_0$; however, this assumes the gradient of the graph is $4.14 \times 10^{-15}$ eV s, which may not be the case.

**e** $h$ = gradient

$h = \frac{\text{rise}}{\text{run}}$

$h = \frac{3.4}{8.1 \times 10^{14}}$

$h = 4.2 \times 10^{-15}$ eV s [1 mark]

$= 4.2 \times 10^{-15} \times 1.6 \times 10^{-19}$ to convert eV s to J s

$h = 6.7 \times 10^{-34}$ J s [1 mark]

***VCAA examination report*** **note:**

The most common errors were to calculate the gradient incorrectly as run over rise or to leave the result in eV s. There were a few students who used other, arbitrary points on the graph to calculate the gradient. Students are advised to use existing points.

**f** Answers will vary.

The following are possible examples of limitations of the wave model in explaining the results of the photoelectric effect:

- existence of a threshold frequency
- absence of a time delay

- energy of the photoelectrons is independent of the intensity of the light source

***VCAA examination report* note:**

Students could identify the existence of a threshold frequency, the absence of a time delay or that the energy of the photoelectrons is independent of the intensity of the light source.

**12 a**

| | |
|---|---|
| Intensity | No change [1 mark] |
| Frequency | Doubled [1 mark] |

***VCAA examination report* note:**

The VCAA is aware that increasing the frequency of the light source without increasing its power will lead to a reduction in the rate of photon production, ($E = hf$), and therefore there would be an expected reduction in photocurrent between the two frequencies. However, this question was written to assess the students' understanding of the 1:1 relationship between incident photons and subsequent photoelectrons. It was not expected that students should take the physics of the photon production into consideration for this question.

**b**

| | |
|---|---|
| Intensity | Halved [1 mark] |
| Frequency | No changed [1 mark] |

**c** $E = \dfrac{hc}{\lambda}$

$E = \dfrac{4.14 \times 10^{-15} \times 3.00 \times 10^{8}}{700 \times 10^{-9}}$

$E = 1.77\,\text{eV}$ [1 mark]

As this is less than the work function, there will be no photoelectrons ejected. [1 mark]

**13 a** $E = \dfrac{hc}{\lambda}$

$E = \dfrac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{400 \times 10^{-9}}$

$E = 4.1\,\text{eV}$ [1 mark]

This is below the work function so no photoelectrons will be emitted. [1 mark]

**b** $E_{\text{photon}} = \phi + E_{\text{k max}}$

$5.4\,\text{eV} = 4.9\,\text{eV} + E_{\text{k max}}$

$\therefore E_{\text{k max}} = 0.5\,\text{eV}$ [1 mark]

5.4 eV is greater than the work function so a photoelectron will be emitted with a kinetic energy of 0.5 eV. [1 mark]

**c** The experiment supports the particle model of light. [1 mark]

- The model suggests a threshold frequency and no electrons were emitted by the 400 nm light. [1 mark]
- The model suggests that the energy of the photon is dependent on its frequency and the higher frequency light did produce photoelectrons. [1 mark]

Other findings of other photoelectric effect experiments, such as the absence of a time delay, are not supported by this particular experiment.

**14 a** $E = hf$

$E = 4.14 \times 10^{-15} \times 7.13 \times 10^{14}$ [1 mark]

$E = 2.95\,\text{eV}$ [1 mark]

The work function is 1.95 eV. Therefore, the stopping voltage will be 2.95 − 1.95 = 1.0 V. [1 mark]

**b** $c = f\lambda$

$3.00 \times 10^{8} = 1.04 \times 10^{15} \times \lambda$ [1 mark]

$\therefore \lambda = 2.88 \times 10^{-7}\,\text{m}$

$\lambda = 288\,\text{nm}$ [1 mark]

**c** The particle model predicts a cut-off frequency below which no photoelectrons will be emitted regardless of light intensity. This is supported by April's observations. The wave model predicts that photoelectron energy can be accumulated over time, this is not supported by April's observations. The particle model predicts that the energy of a photon is dependent on its frequency and is delivered in a single instant. This is supported by April's observations.

***VCAA examination report* note:**

Students were required to identify that the particle model predicts a cut-off frequency below which no photoelectrons will be emitted regardless of intensity. The wave model predicts that photoelectron energy can be accumulated over time, while the particle model predicts that the energy of a photon is dependent on its frequency and is delivered in a single instant.

**15 a** $E = \dfrac{hc}{\lambda}$

$E = \dfrac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{500 \times 10^{-9}}$ [1 mark]

$E = 2.5\,\text{eV}$ [1 mark]

***VCAA examination report* note:**

Students are reminded to check which Planck's constant they are using.

**b** From the graph, 1.5 eV

**c** The work function is defined as the difference between the energy of the incident photon and the kinetic energy of the photoelectron.

$W = E_{\text{photon}} - KE_{\text{photoelectron}}$ [1 mark]

$W = 2.5 - 1.5$

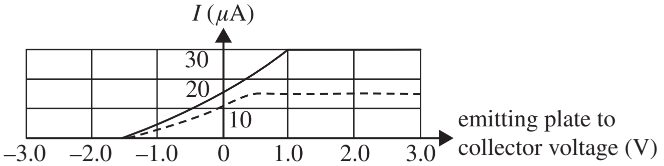
$= 1.0\,\text{eV}$ [1 mark]

**d** At $V = +1.0\,\text{V}$, all of the available photoelectrons are being collected. Since there are no more photoelectrons to be collected [1 mark], increasing the voltage will not result in an increase in photocurrent [1 mark].

**e** The correct response was D.

See the figure at the bottom of the page.*

***15 e**



**f** The increase in photocurrent without increasing the stopping voltage supports the particle model of light rather than the wave model. [1mark]

The particle model predicts that increasing the intensity will increase the number of photons released but not the energy of the photons. [1 mark]

The photocurrent will increase but the stopping voltage will not. [1 mark]

***VCAA examination report*** **note:**

Students were required to identify that the increase in photocurrent without increasing the stopping voltage supports the particle model of light rather than the wave model. The particle model predicts that increasing the intensity will increase the number of photons released but not the energy of the photons. Therefore the photocurrent will increase but the stopping voltage will not.

The wave model predicts that increasing the intensity will increase the kinetic energy of the photoelectrons, which would increase the stopping voltage. This is not what is observed.

# Topic 10 — Matter as particles or waves and the similarities between light and matter

## 10.2 Matter modelled as a type of wave

### Sample problem 1

**a** $\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{10.0 \times 10^{-3} \times 0.100 \times 10^{-3}}$

$= 6.63 \times 10^{-28}$ m

**b** $v = \frac{h}{m\lambda}$

$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.0 \times 10^{-6}}$

$= 7.3 \times 10^{2}$ m s$^{-1}$

The speed of the electron is $7.3 \times 10^{2}$ m s$^{-1}$.

### Practice problem 1

In each case, find the de Broglie wavelength using the relation $\lambda = \frac{h}{m\boldsymbol{v}}$.

Thus, for the proton:

$$\lambda = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 2.0 \times 10^{4}} = 1.99 \times 10^{-11}\text{ m}$$

And for the electron: $\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.0 \times 10^{5}} = 3.6 \times 10^{-9}$ m

The electron has a de Broglie wavelength that is approximately 180 times larger than that of the proton.

## 10.2 Exercise

**1** $\lambda = \frac{h}{\boldsymbol{p}} = \frac{6.63 \times 10^{-34}}{3.5 \times 10^{-4}} = 1.9 \times 10^{-30}$ m

**2** $\boldsymbol{p} = mv = \frac{h}{\lambda} \Rightarrow v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{3.0 \times 5.0 \times 10^{-36}} = 44.2$ m s$^{-1}$.

**3** If $v$ increases, then $\boldsymbol{p}$ increases, as $\boldsymbol{p}$ is proportional to $v(\boldsymbol{p} = mv)$ and thus the de Broglie wavelength will decrease$\left(\boldsymbol{p} = \frac{h}{\lambda}\right)$.

**4** $\lambda = \frac{h}{\boldsymbol{p}} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{0.420 \times 40} = 3.9 \times 10^{-35}$ m

**5** $\boldsymbol{p} = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{1.0 \times 10^{-9}} = 6.63 \times 10^{-25}$ N s

**6** A diffraction experiment where a beam of matter, say electrons, is passed through a narrow opening or around an obstacle. The opening or obstacle would need to be of a similar size or smaller than the de Broglie wavelength associated with the beam of matter.

## 10.2 Exam questions

**1** $p = \frac{h}{\lambda}$

$= \frac{hf}{c}$

$= \frac{6.63 \times 10^{-34} \times 7.0 \times 10^{15}}{3.0 \times 10^{8}}$

$= 1.5 \times 10^{-26}$ kg m s$^{-1}$

**2** C

$\lambda = \frac{h}{mv}$

$\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.0 \times 10^{7}}$

$\lambda = 7.3 \times 10^{-11}$ m

**3** $Vq = \frac{1}{2}mv^2$

$4000 \times 1.6 \times 10^{-19} = 0.5 \times 9.1 \times 10^{-31} \times v^2$

$v^2 = \frac{4000 \times 1.6 \times 10^{-19}}{0.5 \times 9.1 \times 10^{-31}}$

$= 1.4 \times 10^{15}$

$v = 3.7 \times 10^{7}$ m s$^{-1}$ [1 mark]

$\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3.7 \times 10^{7}}$ [1 mark]

$= 2.0 \times 10^{-11}$ m [1 mark]

**4** $\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.0 \times 10^{7}}$ [1 mark]

$= 3.6 \times 10^{-11}$ m [1 mark]

***VCAA examination report*** **note:**

Students are reminded to check which version of Planck's constant they are using.

**5** $\lambda = \frac{h}{mv}$

$v = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.0 \times 10^{-11}}$ [1 mark]

$v = 7.3 \times 10^{7}$ m s$^{-1}$ [1 mark]

***VCAA examination report*** **note:**

The most common error was to use the wrong Planck's constant. This gave an answer of $4.5 \times 10^{26}$ m s$^{-1}$, which is illogical.

## 10.3 The diffraction of light and matter

### Sample problem 2

The de Broglie wavelength of the tennis ball:

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{0.060 \times 30} = 3.7 \times 10^{-34}\,\text{m}$$

The de Broglie wavelength of the electron:

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3.0 \times 10^{6}} = 2.4 \times 10^{-10}\,\text{m}$$

The de Broglie wavelength of the tennis ball is of the order of $10^{-34}$ m and that of the electron is of the order of $10^{-10}$ m. The distances between atoms in a crystal are of the order of $10^{-10}$ m, so diffraction and interference could be observed when these electrons are scattered from a crystal. It is not surprising that diffraction and interference effects are never observed with tennis balls, due to the extremely small wavelength, $10^{-34}$ m, that they have.

### Practice problem 2

The phrase 'to demonstrate diffraction effects' means that the de Broglie wavelength of the neutron is of the same order of magnitude as the width of the slits. Thus, $\lambda$ is of the order $1.0 \times 10^{-6}$ m.

Rearrange the equation $\lambda = \frac{h}{mv}$ to make speed $v$ the subject:

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1.0 \times 10^{-6}} = 0.40\,\text{m s}^{-1}$$

Neutrons with a speed $0.40\,\text{m s}^{-1}$ will have a de Broglie wavelength of order $1.0 \times 10^{-6}$ m. Note: the slower that an object with mass moves, the greater its de Broglie wavelength and vice versa.

### Sample problem 3

$$V = \frac{m_e v^2}{2q_e} = \frac{9.1 \times 10^{-31} \times \left(3.0 \times 10^{6}\right)^2}{2 \times 1.6 \times 10^{-19}} = 26\,\text{V}$$

So, only 26 V is required to accelerate an electron to $3.0 \times 10^{6}\,\text{m s}^{-1}$.

### Practice problem 3

Recall that the kinetic energy of a charged particle of mass $m$ and charge $q$ accelerated from rest to a speed $v$ due to a potential difference $V$ is governed by the relation below. (In this case, $V = 13$ V.)

$$\frac{1}{2}mv^2 = Vq$$

Rearrange the equation to make $v$ the subject:

$$v = \sqrt{\frac{2Vq}{m}} = \sqrt{\frac{2 \times 13 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 2.1 \times 10^{6}\,\text{m s}^{-1}$$

### Sample problem 4

**1** $$\lambda = \frac{h}{\sqrt{2m_e E_k}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 600}} = 5.0 \times 10^{-11}\,\text{m}$$

**2** The de Broglie wavelength of $5.0 \times 10^{-11}$ m is a similar value to the $7.1 \times 10^{-11}$ m wavelength of the X-rays.

### Practice problem 4

**a** The wavelength of the electrons is taken to be the same as that of the X-rays:
$0.053\,\text{nm} = 5.3 \times 10^{-11}$ m
Rearrange the expression $\lambda = \frac{h}{\sqrt{2mqV}}$ to make $V$ the subject:

$$V = \frac{h^2}{2mq\lambda^2} = \frac{\left(6.63 \times 10^{-34}\right)^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times \left(5.3 \times 10^{-11}\right)^2} = 537 = 5.4 \times 10^{2}\,\text{V}$$

**b** For the photon, use the expression $\lambda = \frac{hc}{E}$ with both $h$ and $E$ calculated using common energy units — in this case the electron volt.
Thus, for the photon: $$\lambda = \frac{4.14 \times 10^{-15} \times 3 \times 10^{8}}{100} = 1.2 \times 10^{-8}\,\text{m}$$
For the electron, however, use the expression $\lambda = \frac{h}{\sqrt{2mE_k}}$, being careful to always use the S.I. unit for energy $E_k$.
Thus, for the electron:

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 100}} = 1.2 \times 10^{-10}\,\text{m}$$

Therefore, for the same energy, the photon has a wavelength that is approximately 100 times larger compared to the de Broglie wavelength of the electron.

## Sample problem 5

**a** $\boldsymbol{p} = \frac{h}{\lambda}$

$= \frac{6.63 \times 10^{-34}}{2.0 \times 10^{-10}}$

$= 3.3 \times 10^{-24}$ N s

The photon and the electron have the same momentum.

**b** Energy of the photon:

$E = \boldsymbol{p}c$

$= 3.3 \times 10^{-24} \times 3.0 \times 10^{8}$

$= 9.9 \times 10^{-16}$ J or 6.2 keV

Energy of the electron:

$$E = \frac{\boldsymbol{p}^2}{2m}$$

$$= \frac{(3.3 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}}$$

$$= 6.0 \times 10^{-18} \text{ J or } 37 \text{ eV}$$

The electron has substantially less kinetic energy than the photon, even though they have the same momentum.

**c** Light and matter with the same wavelength will have the same momentum, and vice versa. However, when photons and electrons have the same momentum, they will not necessarily have the same energy. In this problem, the photon has substantially more energy than the electron.

## Practice problem 5

**a** The momentum of the photon and the electron are governed by the same equation, namely $\boldsymbol{p} = \frac{h}{\lambda}$, and hence both the photon and the electron will have the same momentum because they have the same associated wavelength.

Thus: $p = \frac{6.63 \times 10^{-34}}{1.0 \times 10^{-10}} = 6.6 \times 10^{-24}$ N s

Therefore both the photon and the electron have the same momentum.

**b** To determine the energy of an object from its momentum, the question has to be asked whether the item in the question is a photon or an object with mass, as they have different relationships connecting energy and momentum. The equations are different. For the photon, $E = pc$.

Thus: $E = 6.6 \times 10^{-24} \times 3.0 \times 10^{8}$

$= 1.98 \times 10^{-15}$ J, or 12.4 keV

However, for the electron:

$E = \frac{p^2}{2m}$

Thus: $E = \frac{(6.6 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}}$

$= 2.4 \times 10^{-17}$ J, or approximately 150 eV

The electron again has substantially less kinetic energy than the photon even though they have the same momentum and wavelength.

## 10.3 Exercise

**1 a** $\lambda = \frac{h}{\boldsymbol{p}}$

$= \frac{h}{m\boldsymbol{v}}$

$= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 3.0 \times 10^{7}}$

$= 1.3 \times 10^{-14}$ m

**b** $\lambda = \frac{h}{\boldsymbol{p}}$

$= \frac{h}{\sqrt{2mE}}$

$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 54}}$

$= 1.7 \times 10^{-10}$ m

**c** $\lambda = \frac{h}{\boldsymbol{p}}$

$= \frac{h}{m\boldsymbol{v}}$

$= \frac{6.63 \times 10^{-34}}{0.20 \times 50}$

$= 6.6 \times 10^{-35}$ m

**2 a** $E_k = -\Delta E_p$

$= -Vq_{\text{electron}}$

$= -(5 \text{ kV} \times -1\text{e})$

$= 5$ keV

$= 5 \text{ keV} \times 1.60 \times 10^{-16} \text{ J keV}^{-1}$

$= 8.0 \times 10^{-16}$ J

**b** $E_{\text{photon}} = 8.0 \times 10^{-16}$ J

$= \frac{hc}{\lambda}$

$\lambda = \frac{hc}{E_{\text{photon}}}$

$= \frac{6.63 \times 10^{-34} \text{ J s} \times 3.0 \times 10^{8} \text{ m s}^{-1}}{8.0 \times 10^{-16} \text{ J}}$

$= 2.5 \times 10^{-10}$ m

**3** Light exhibits both wave and particle behaviour, depending on the experiment you are performing at the time. For further discussion of this issue, argue with your friends and teacher!

**4 a** $\boldsymbol{p} = m\boldsymbol{v}$

$= 9.11 \times 10^{-31} \times 2.5 \times 10^{6}$

$= 2.3 \times 10^{-24}$ kg m s$^{-1}$

$\lambda = \frac{h}{\boldsymbol{p}}$

$= \frac{6.63 \times 10^{-34}}{2.3 \times 10^{-24}}$

$= 2.9 \times 10^{-10}$ m

**b** Significant diffraction occurs when $\frac{\lambda}{d} \approx 1$. As the wavelength is a little greater than the atomic spacing, diffraction will be significant.

**5 a** The electrons had a voltage of 3.0 kV or 3000 V of energy. The electrons will have an energy of 3000 eV or $4.8 \times 10^{-16}$ J.

**b** $\boldsymbol{p} = \sqrt{2mE}$

$= \sqrt{2 \times 9.1 \times 10^{-31} \times 4.8 \times 10^{-16}}$

$= 3.0 \times 10^{-23}$ N s

$\lambda = \frac{h}{\boldsymbol{p}}$

$= \frac{6.63 \times 10^{-34}}{3.0 \times 10^{-23}}$

$= 2.2 \times 10^{-11}$ m

**c** $\frac{\lambda}{w} = \frac{2.2 \times 10^{-11}}{5.0 \times 10^{-10}}$
$= 0.044$

Thus the scientist would not expect to observe any diffraction effects since the wavelength of the electrons is too small.

**d** The scientist should make the accelerating voltage smaller. This would reduce the energy of the electrons and hence reduce the momentum, increasing the wavelength.

**e** $2.2 \times 10^{-11}$ m; $3.0 \times 10^{-23}$ N s. To obtain the same diffraction pattern, the wavelength and hence the momentum of the photons must be the same as the electrons used in the previous experiment.

**f** $E = \boldsymbol{p}c$
$= 3.0 \times 10^{-23} \times 3.0 \times 10^{8}$
$= 9.0 \times 10^{-15}$ J
$= 5.6 \times 10^{4}$ eV

**6** $c = f\lambda$
$\lambda = \frac{c}{f}$
$= \frac{3.0 \times 10^{8}}{4.5 \times 10^{14}} = 6.7 \times 10^{-7}$ m
$p = \frac{h}{\lambda}$
$\Rightarrow v = \frac{h}{\lambda m}$
$= \frac{6.63 \times 10^{-34}}{6.7 \times 10^{-7} \times 9.1 \times 10^{-31}}$
$= 1.1 \times 10^{3}$ m s$^{-1}$

**7 a** $\lambda = \frac{h}{\boldsymbol{p}}$
$= \frac{h}{\sqrt{2mE}}$

Since $E$ and $h$ are constant, the larger the value of $m$, the smaller the wavelength. So, the proton will have the shorter wavelength, and the electron will have the larger wavelength.

**b** $\lambda = \frac{h}{\boldsymbol{p}}$
$= \frac{h}{\sqrt{2mE}}$
$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1000 \times 1.6 \times 10^{-19}}}$
$= 3.9 \times 10^{-11}$ m for the electron

using $m = 1.67 \times 10^{-27}$ kg for the proton,
$\lambda = \frac{h}{\boldsymbol{p}}$
$= \frac{h}{\sqrt{2mE}}$
$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1000 \times 1.6 \times 10^{-19}}}$
$= 9.1 \times 10^{-13}$ m for the proton

**8** $\lambda = \frac{h}{\boldsymbol{p}}$
$= \frac{h}{\sqrt{2mE}}$
$= \frac{h}{\sqrt{2mqV}}$
$\Rightarrow V = \frac{h^2}{2mq\lambda^2}$
$= \frac{\left(6.63 \times 10^{-34}\right)^2}{2 \times 9.1 \times 10^{-31} \times 1.60 \times 10^{-19} \times \left(2.0 \times 10^{-10}\right)^2}$
$= 38$ V

**9** For a 10 eV electron:
$\lambda = \frac{h}{\boldsymbol{p}}$
$= \frac{h}{\sqrt{2mE}}$
$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10 \times 1.6 \times 10^{-19}}}$
$= 3.9 \times 10^{-10}$ m

For a 10 eV photon:
$E = \frac{hc}{\lambda}$
$\Rightarrow \lambda = \frac{hc}{E}$
$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{10 \times 1.6 \times 10^{-19}}$
$= 1.2 \times 10^{-7}$ m

Therefore the electron has the smaller wavelength.

## 10.3 Exam questions

**1 a**
$\text{KE} = \frac{1}{2}mv^2$
$10 \times 10^{3} \times 1.6 \times 10^{-19} = 0.5 \times 9.1 \times 10^{-31} \times v^2$ [1 mark]
$\Rightarrow v = 5.93 \times 10^{7}$ ms$^{-1}$ [1 mark]
$\lambda = \frac{h}{mv}$
$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 5.93 \times 10^{7}}$ [1 mark]
$= 1.23 \times 10^{-11}$ m
$= 0.012$ nm [1 mark]

***VCAA examination report* note:**

Some students chose to start with $\lambda = \frac{h}{\sqrt{2mqV}}$.

As is often the case with these multi-step problems, students have trouble working methodically from the start to the end. Of particular concern was the number of students who could not convert from m to nm.

**b** If the speed is increased then the momentum will increase. [1 mark]

Since $\lambda \propto \frac{1}{v}$ [1 mark], if $v$ increases, $\lambda$ will decrease. [1 mark]

***VCAA examination report* note:**

The most common error was to claim that the wavelength would increase as it is directly related to the speed.

**2 a** $E = \frac{hc}{\lambda}$

$400 = \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{\lambda}$ [1 mark]

$\lambda = 3.11 \times 10^{-9}$ m [1 mark]

**b** $\lambda_{\text{x-ray}} = \lambda_{\text{electron}}$

$\lambda_{\text{electron}} = \frac{h}{mv}$ [1 mark]

$3.11 \times 10^{-9} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times v}$ [1 mark]

$v = 2.34 \times 10^{5}$ m s$^{-1}$ [1 mark]

**3 a** $E_k = \frac{1}{2}mv^2$

$= 0.5 \times 9.1 \times 10^{-31} \times \left(1.72 \times 10^{5}\right)^2$

$= 1.35 \times 10^{-20}$ J [1 mark]

$= \frac{1.35 \times 10^{-20}}{1.6 \times 10^{-19}} = 0.08$ eV [1 mark]

***VCAA examination report* note:**

The most common error was to leave the result in joules.

**b** $\lambda_{\text{X-ray}} = \lambda_e$ [1 mark]

$= \frac{h}{p}$

$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.72 \times 10^{5}}$

$= 4.24 \times 10^{-9}$ m [1 mark]

$E_{\text{X-ray}} = \frac{hc}{\lambda}$

$= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{4.24 \times 10^{-9}}$

$= 293$ eV [1 mark]

***VCAA examination report* note:**

Most students were able to identify that the identical patterns indicated that the wavelengths were the same. However, many students were then unable to complete the remaining mathematical process. Students must think about how they plan to show their working before they commence writing.

**4 a** $\lambda = \frac{h}{mv}$

$\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 5.0 \times 10^{5}}$

$\lambda = 1.46 \times 10^{-9}$ m [1 mark]

***VCAA examination report* note:**

There was no common error.

**b** The width of the diffraction pattern can be found from the $\frac{\lambda}{w}$ ratio. The wavelength of the X-rays is found by:

$E = \frac{hc}{\lambda}$

$100 = \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{\lambda}$

$\lambda = 1.24 \times 10^{-8}$ m [1 mark]

This gives a ratio of:

$\frac{\lambda}{w} = \frac{1.24 \times 10^{-8}}{1.24 \times 10^{-6}}$

$\frac{\lambda}{w} = 1.00 \times 10^{-2}$ [1 mark]

The electrons, with a de Broglie wavelength of $1.46 \times 10^{-9}$ m, will also have to have the same ratio of $\frac{\lambda}{w}$.
[1 mark]

$\frac{1.46 \times 10^{-9}}{w} = 1.00 \times 10^{-2}$

$w = 1.46 \times 10^{-7}$ m [1 mark]

***VCAA examination report* note:**

More than half of the students did not attempt the question and 22 per cent attempted the problem but made no significant headway. These multi-step problems continue to pose a significant challenge for students. Only a small percentage of responses demonstrated a clear progression of thinking from beginning to end. The solution above uses signposting to guide the reader. Students are reminded that:

- the number of marks indicates the number of steps they will have to demonstrate
- the purpose of the question is to allow students to demonstrate the depth of their understanding.

**5 a** Moving electrons exhibit wave properties with a wavelength known as the de Broglie wavelength. [1 mark]

Diffraction patterns are dependent on wavelength. [1 mark]

If the de Broglie wavelength of the electron is the same as the wavelength of the X-ray, then the diffraction patterns will be the same. [1 mark]

**b** The momentum of electrons is given by:

$p = \sqrt{(2mE_k)}$

The kinetic energy of the electrons must be in joules.

$E_k = 3.0 \times 10^{3}$ eV

$= (3.0 \times 10^{3}) \times (1.6 \times 10^{-19})$ J

$E_k = 4.8 \times 10^{-16}$ J [1 mark]

$p = \sqrt{2 \times \left(9.1 \times 10^{-31}\right) \times \left(4.8 \times 10^{-16}\right)}$

$p = 2.96 \times 10^{-23}$ kg m s$^{-1}$ [1 mark]

The de Broglie wavelength of the electron can then be calculated:

$\lambda = \frac{h}{p}$

$\lambda = \frac{6.63 \times 10^{-34}}{2.96 \times 10^{-23}}$

$\lambda = 2.24 \times 10^{-11}$ m [1 mark]

The X-ray will have the same wavelength as the electron because they produce same-spaced diffraction patterns.

The frequency of the X-rays can then be calculated.

$v = f \times \lambda$

$f = \frac{v}{\lambda}$

$f = \frac{3 \times 10^{8}}{2.24 \times 10^{-11}}$

$f = 1.34 \times 10^{19}$ Hz [1 mark]

## 10.4 Emission and absorption spectra

### Sample problem 6

**1** For the $n = 4$ to $n = 1$ transition:

$E_{\text{photon}} = E_{\text{initial}} - E_{\text{final}}$

$= (-1.4 \text{ eV}) - (-6.4 \text{ eV})$

$= 5.0$ eV

**2** The remaining five calculations give energies of:

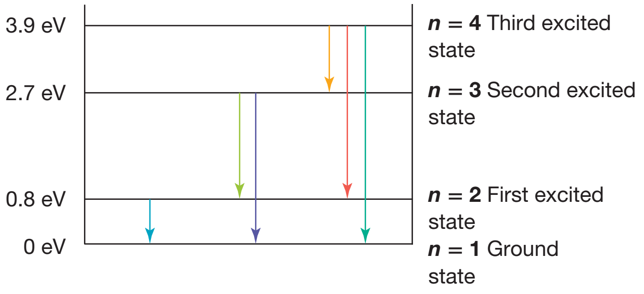
- 3.5 eV (third to first)
- 1.7 eV (third to second)

- 3.3 eV (second to ground state)
- 1.8 eV (second to first)
- 1.5 eV ( first to ground state).

**3** Arranged in ascending order, the six photon energies are 1.5 eV, 1.7 eV, 1.8 eV, 3.3 eV, 3.5 eV and 5.0 eV.

## Practice problem 6

**a**




**b** $\Delta E = 3.9 - 0 = 3.9$ eV, therefore

$$f = \frac{\Delta E}{h}$$
$$= \frac{3.9\text{ eV}}{4.14 \times 10^{-15}\text{ eV s}}$$
$$= 9.4 \times 10^{14}\text{ Hz}$$

**c** $\Delta E = 2.7 - 0.80 = 1.9$ eV, therefore

$$f = \frac{\Delta E}{h}$$
$$= \frac{1.9\text{ eV}}{4.14 \times 10^{-15}\text{ eV s}}$$
$$= 4.6 \times 10^{14}\text{ Hz}$$

## Sample problem 7

For $n = 4$ to $n = 1$ transition:

$$E_{\text{photon}} = E_{\text{initial}} - E_{\text{final}}$$
$$= (-0.85\text{ eV}) - (-13.61\text{ eV})$$
$$= 12.76\text{ eV}$$
$$= 12.76\text{ eV} \times 1.6 \times 10^{-19}\text{ J eV}^{-1}$$
$$= 2.0 \times 10^{-18}\text{ J}$$

$$\lambda_{\text{photon}} = \frac{hc}{E_{\text{photon}}}$$
$$= \frac{6.63 \times 10^{-34}\text{ J s} \times 3.0 \times 10^{8}\text{ m s}^{-1}}{2.0 \times 10^{-18}}$$
$$= 9.7 \times 10^{-8}\text{ m}$$

This is ultraviolet radiation.

## Practice problem 7

The lowest frequency is related to the smallest energy change:

$$E_{\text{photon}} = E_{\text{initial}} - E_{\text{final}}$$
$$= (-0.85\text{ eV}) - (-1.51\text{ eV})$$
$$= 0.66\text{ eV}$$

$$f = \frac{\Delta E}{h}$$
$$= \frac{0.66\text{ eV}}{4.14 \times 10^{-15}\text{ eV s}}$$
$$= 1.59 \times 10^{14}\text{ Hz}$$

$$\lambda = \frac{c}{f}$$
$$= \frac{3.0 \times 10^{8}}{1.59 \times 10^{14}}$$
$$= 1.9 \times 10^{-6}\text{ m}$$

## 10.4 Exercise

**1** A fire glows with a continuous range of wavelengths, and, in a red fire, the red wavelengths have the greatest intensity. Neon in a discharge tube glows red because the electrons in the neon atoms are excited to specific energies. When the electrons return to the ground state, they produce light of a few fixed wavelengths, mainly in the red part of the spectrum.

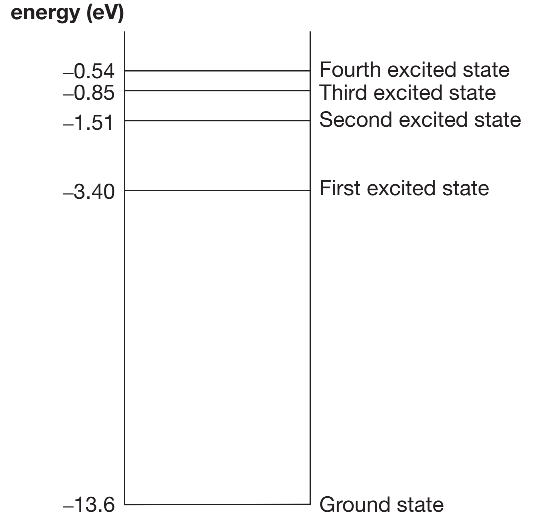
**2** Emission lines are produced when electrons return from an excited state to a lower energy state. The energy is released in the form of photons of particular frequencies. Absorption lines are produced when light from a continuous spectrum passes through a gas. This light excites some of the electrons in the atoms making up the gas, so photons with the energies allowed by the atoms will be removed from the continuous spectrum. As the energy required to raise an electron to a more excited state is equal to the energy released when the electrons drop back to the lower state, the emission lines and absorption lines for a particular element will be the same.

**3** Possible answers include refracting the light through a prism. Spectral yellow will remain yellow, whereas a mixture of green and red light will separate into two beams.

**4** The ground state is 10.4 eV below ionisation.
ground state: $-10.4$ eV;
The first excited state is 4.9 eV above the ground state,
so $-10.4 + 4.9 = -5.5$ eV;
The second excited state is 6.7 eV above the ground state,
so $-10.4 + 6.7 = -3.7$ eV;
The third excited state is 8.8 eV above the ground state,
so $-10.4 + 8.8 = -1.6$ eV.

**5** **a** The term 'ground state' defines the lowest energy state of an electron.

**b**




**c** The Balmer series is a collection of transitions from the 3rd, 4th, 5th and 6th excited states to the 2nd excited state in a hydrogen atom. They are grouped because each transition produces a photon in the visible part of the electromagnetic spectrum.

**6** Red light

$E = 3.14 \times 10^{-19}$ J

$= \dfrac{3.14 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.96$ eV

$p = \dfrac{E}{c}$

$= \dfrac{3.14 \times 10^{-19}}{3.0 \times 10^{8}}$

$= 1.05 \times 10^{-27}$ N s

$f = \dfrac{E}{h}$

$= \dfrac{3.14 \times 10^{-19}}{6.63 \times 10^{-34}}$

$= 4.74 \times 10^{14}$ Hz

$\lambda = \dfrac{c}{f}$

$= \dfrac{3.0 \times 10^{8}}{4.74 \times 10^{14}}$

$= 6.33 \times 10^{-7}$ m

$= 633$ nm

Electron:

$E = 1.96$ eV

$= 1.96 \times 1.6 \times 10^{-19}$

$= 3.14 \times 10^{-19}$ J

$p = \sqrt{2mE}$

$= \sqrt{2 \times 9.1 \times 10^{-31} \times 3.14 \times 10^{-19}}$

$= 7.56 \times 10^{-25}$ N s

$\lambda = \dfrac{h}{p}$

$= \dfrac{6.63 \times 10^{-34}}{7.56 \times 10^{-25}}$

$= 8.77 \times 10^{-10}$ m

$= 0.877$ nm

Blue light:

$f = \dfrac{c}{\lambda}$

$= \dfrac{3.0 \times 10^{8}}{4.05 \times 10^{-7}}$

$= 7.41 \times 10^{14}$ Hz

$E = hf$

$= 6.63 \times 10^{-34} \times 7.41 \times 10^{14}$

$= 4.91 \times 10^{-19}$ J

$= \dfrac{4.91 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.07$ eV

$p = \dfrac{E}{c}$

$= \dfrac{4.91 \times 10^{-19}}{3.0 \times 10^{8}}$

$= 1.64 \times 10^{-27}$ N s

Electron:

$p = \dfrac{h}{\lambda}$

$= \dfrac{6.63 \times 10^{-34}}{4.05 \times 10^{-7}}$

$= 1.64 \times 10^{-27}$ N s

$E = \dfrac{p^2}{2m}$

$= \dfrac{\left(1.64 \times 10^{-27}\right)^2}{2 \times 9.1 \times 10^{-31}}$

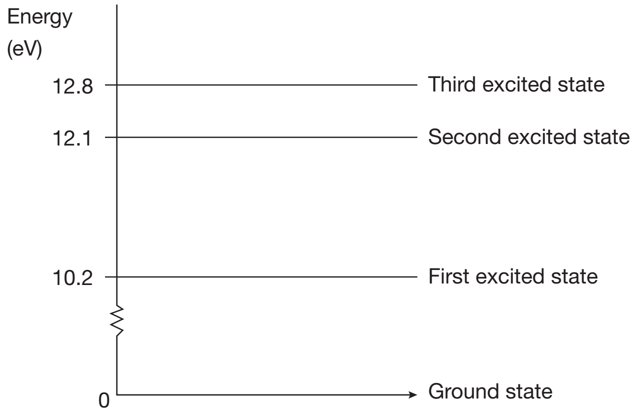
$= 1.48 \times 10^{-24}$ J

$= \dfrac{1.48 \times 10^{-24}}{1.60 \times 10^{-19}}$

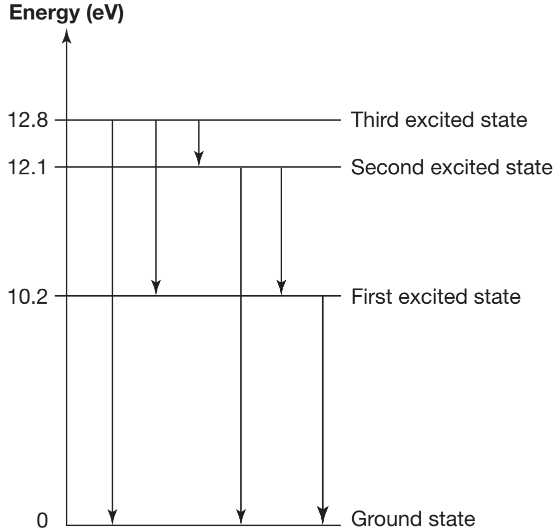
$= 9.24 \times 10^{-6}$ eV

| | $\lambda$(nm) | $f$(Hz) | $E$(J) | $E$(eV) | $p$(N s) |
|---|---|---|---|---|---|
| Red light | 633 | $4.74 \times 10^{14}$ | $3.14 \times 10^{-19}$ | 1.96 | $1.05 \times 10^{-27}$ |
| Electron | 0.877 | — | $3.14 \times 10^{-19}$ | 1.96 | $7.56 \times 10^{-25}$ |
| Blue light | 405 | $7.41 \times 10^{14}$ | $4.91 \times 10^{-19}$ | 3.07 | $1.64 \times 10^{-27}$ |
| Electron | 405 | — | $1.48 \times 10^{-24}$ | $9.24 \times 10^{-6}$ | $1.64 \times 10^{-27}$ |

**7 a**




**b**




**c** $E_1 = 12.8 - 12.1 = 0.7$ eV

$E_2 = 12.8 - 10.2 = 2.6$ eV

$E_3 = 12.8$ eV

$E_4 = 12.1 - 10.2 = 1.9$ eV

$E_5 = 12.1$ eV

$E_6 = 10.2$ eV

**d** The least energetic photon has an energy of 0.7 eV:

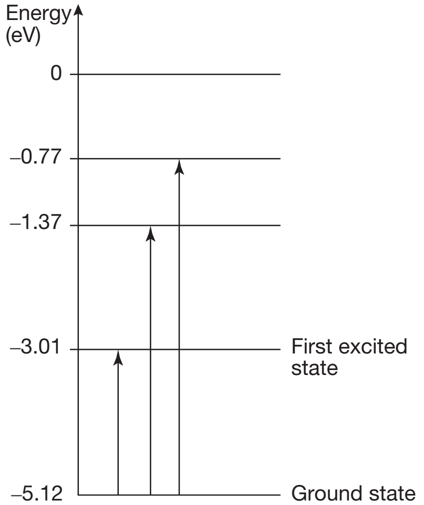
$$\lambda = \frac{hc}{E}$$
$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.60 \times 10^{-19} \times 0.7}$$
$$= 1.78 \times 10^{-6}\,\text{m}\ (1.8 \times 10^{-6}\,\text{m})$$

The greatest energetic photon has an energy of 12.8 eV:

$$\lambda = \frac{hc}{E}$$
$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 12.8}$$
$$= 9.7 \times 10^{-8}\,\text{m}$$

**8** The dark lines in an absorption spectrum correspond to upward transitions from the ground state to excited states of atoms of a gaseous sample. The particles absorb those specific frequencies of light, and thus a continuous spectrum has these colours missing.

**9 a, b**




**c** $E_{\text{photon}} = -\Delta E_{\text{atom}}$

$= E_{\text{atom initial}} - E_{\text{atom final}}$

First excited state:

$E_{\text{photon}} = -\Delta E_{\text{atom}}$

$= -3.01\,\text{eV} - (-5.12\,\text{eV})$

$= 2.11\,\text{eV} = 3.38 \times 10^{-19}\,\text{J}$

$$\Rightarrow \lambda = \frac{hc}{E_{\text{photon}}}$$
$$= \frac{6.63 \times 10^{-34}\,\text{J s} \times 3.0 \times 10^{8}\,\text{m s}^{-1}}{3.38 \times 10^{-19}\,\text{J}}$$
$$= 5.9 \times 10^{-7}\,\text{m}$$

Second excited state:

$E_{\text{photon}} = -\Delta E_{\text{atom}}$

$= -1.37\,\text{eV} - (-5.12\,\text{eV})$

$= 3.75\,\text{eV} = 6.0 \times 10^{-19}\,\text{J}$

$$\Rightarrow \lambda = \frac{hc}{E_{\text{photon}}}$$
$$= \frac{6.3 \times 10^{-34}\,\text{J s} \times 3.0 \times 10^{8}\,\text{m s}^{-1}}{6.0 \times 10^{-19}\,\text{J}}$$
$$= 3.2 \times 10^{-7}\,\text{m}$$

Third excited state:

$E_{\text{photon}} = -\Delta E_{\text{atom}}$

$= -0.77\,\text{eV} - (-5.12\,\text{eV})$

$= 4.35\,\text{eV} = 6.96 \times 10^{-19}\,\text{J}$

$$\Rightarrow \lambda = \frac{hc}{E_{\text{photon}}}$$
$$= \frac{6.63 \times 10^{-34}\,\text{J s} \times 3.0 \times 10^{8}\,\text{m s}^{-1}}{6.96 \times 10^{-19}\,\text{J}}$$
$$= 2.9 \times 10^{-7}\,\text{m}$$

The energy change from the first excited state is responsible for the yellow glow. Yellow light is between red and blue light in the spectrum — between 405 nm and 633 nm. The first excited state is the only one that produces a wavelength in this range.

## 10.4 Exam questions

**1 a** $E = \dfrac{hc}{\lambda}$

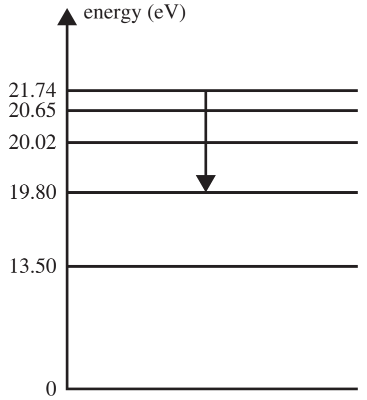
$E = \dfrac{4.14 \times 10^{-15} \times 3.00 \times 10^{8}}{640 \times 10^{-9}}$ [1 mark]

$E = 1.94\,\text{eV}$

***VCAA examination report* note:**

There were a number of students who identified that 21.74 − 19.80 = 1.94, however, this only identifies the levels for the transition. It does not relate this transition to the wavelength.

**b**




Award 1 mark for correctly drawing an arrow between the level with energy 21.74 eV and the level with energy 19.80 eV.

**2** Photons with energies matching the transition energies between shells [1 mark] will be absorbed as the electrons are excited to higher energy states [1 mark]. These photons are missing from the spectrum [1 mark].

**3** $E_{\text{max}} = 3.61\,\text{eV}$

$E = hf$

$3.61 = 4.14 \times 10^{-15} \times f$ [1 mark]

$f = 8.72 \times 10^{14}\,\text{Hz}$ [1 mark]

**4 a** $E = \dfrac{hc}{\lambda}$

$E = \dfrac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{5.65 \times 10^{-7}}$

$E = 2.2\,\text{eV}$ [1 mark]

This corresponds to a transition from 8.9 eV to 6.7 eV. [1 mark]

***VCAA examination report* note:**

Students could either state the transition as shown or draw an arrow on Figure 16. The most common error was to correctly calculate 2.2 eV but then show it on the diagram as an upwards transition.

**b** There were 10 transitions. [1 mark]

The transition from 9.8 eV to 4.9 eV has the same energy as the transition from 4.9 eV to 0 eV and therefore there will be nine spectral lines. [1 mark]

***VCAA examination report* note:**

Students could either state that there would be four transitions from the 9.8 eV level, three from the 8.9 eV level, two from the 6.7 eV level and one from the 4.9 eV level or they could draw arrows on the diagram.

The most common error was to miscalculate the number of transitions.

**5 a** 668 nm [1 mark]

The lowest energy has the longest wavelength.

**b** $f = \dfrac{c}{\lambda}$

$= \dfrac{3.0 \times 10^8}{588 \times 10^{-9}}$ [1 mark]

$= 5.1 \times 10^{14}$ Hz [1 mark]

**c** Only certain energies are visible because the electrons exist only in certain discrete energy levels [1 mark]. As the electrons transition between these energy levels they can emit only discrete amounts of energy [1 mark].

***VCAA examination report* note:**

Many responses answered the question in one of two ways: either they restated the question as a statement (e.g. 'Only discrete energies are seen because atoms can only absorb and emit discrete energies'), or they provided nonsensical responses (e.g. 'Electrons in disallowed orbits result in destructive interference which is why no emission occurs').

There were also a number of responses that either focused entirely on the quantisation of electron shells or the process of transition without linking the two concepts. This suggested some students were copying stock responses from their A3 sheets.

## 10.5 Electrons, atoms and standing waves

### 10.5 Exercise

**1 a** The classical model of a hydrogen atom has an electron orbiting around the proton, which lies at the centre of the atom. This model is often referred to as a planetary model.

**b** An electron orbiting a proton is accelerating and, according to Maxwell, should emit electromagnetic radiation continuously as it orbits. This loss of energy for the electron would cause it to spiral into the nucleus in a very short time. This model does not predict that atoms are stable.

**2** In the Bohr model of a hydrogen atom, the electron is modelled as a type of standing wave around the proton. This stationary state is simply asserted not to emit light.

*Note:* As an interesting aside, the velocity of a stationary wave is zero and so in this state, the acceleration, if we were to mix models, would be zero. In this stationary state no light would be emitted, which is more in keeping with Maxwell's electromagnetic model for light. It must be pointed out that there is no compulsion for two differing models to be compatible.

**3** In the Bohr model for an atom, light in the form of photons is emitted only when an atom undergoes a transition from one excited state to a lower state. The light emitted is a specific energy (hence frequency, hence colour) accounting for the discrete nature of emission spectra. In a similar fashion, light of specific energies is absorbed when the energies match energy differences in stationary states. Usually, the initial state is the ground state and the final state is an excited state.

### 10.5 Exam questions

**1** Electrons exhibit a wave property. [1 mark]

Only orbits with circumferences that are a whole multiple of this wavelength will permit a standing wave to form. [1 mark]

**2** Answers will vary.

Electrons possess wave properties and their wavelength is known as their de Broglie wavelength, they produce a diffraction pattern. [1 mark]

Mary is correct and Roger is incorrect. [1 mark]

- Electrons passed through a crystal will produce a diffraction pattern, just as if X-rays were passed through the crystal. [1 mark]
- Electrons passed through a single slit will produce a diffraction pattern. [1 mark]

***VCAA examination report* note:**

Students were required to identify the following:

- Mary is correct and Roger is incorrect.
- Electrons do possess wave properties and their wavelength is known as their de Broglie wavelength.

Students were then required to refer to two of the following:

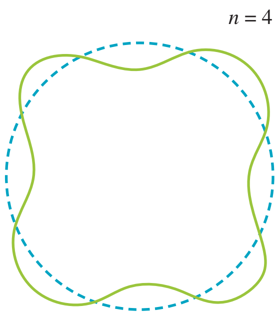
- If electrons are passed through a crystal they will produce a diffraction pattern, just as if X-rays were passed through the crystal.
- If electrons are passed through a single slit they will also produce a diffraction pattern.
- If electrons are passed through two closely spaced slits they will produce an interference pattern.

Most students were unable to demonstrate any understanding of the question or the subject of electron diffraction. The most common error was the assertion that 'Mary is correct for electrons and Roger is correct for light'. Any suggestion that both were correct was seen as a contradiction, which meant that full marks could not be awarded. The question did not relate to whether light diffracts or not and most students were unable to understand that Roger's assertion that only light diffracts is incorrect. Also of concern was the reference to Young's double-slit experiment with electrons. Young did not perform his experiment with electrons, and students who asserted that Young observed electron interference were incorrect. Once again there seemed to be a reliance on prepared responses copied from the student's sheet of notes. Responses such as this did not receive any marks as they did not refer to the question. Students are reminded that copying responses from their sheet(s) of notes will not result in marks being awarded as prepared responses will not satisfactorily address the question.

**3** Electrons exhibit a wave behaviour. [1 mark]

Electrons form standing waves in orbits where the circumference is a whole multiple of the electron wavelength. [1 mark]

This means that only certain discrete energy states can exist. [1 mark]




Award 1 mark for correct diagram.

**4 a** Answers will vary.

Electrons have a wavelike nature, with wavelength $\lambda$ as defined by Louis de Broglie. [1 mark]

Electrons can be modelled as travelling along one of the allowed orbits around the nucleus, together with their associated wave. [1 mark]

The circumference of each allowed orbit contains a whole number of wavelengths of the electron-wave: $n\lambda = 2\pi r$. [1 mark]

***VCAA examination report* note:**

Students were required to identify the wave nature of electrons. They were then required to explain the standing wave theory, where $n\lambda = 2\pi r$.

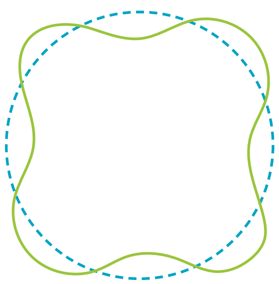
Finally, they were required to identify that the different allowed orbits had different whole values of $n$. While most students knew that 'standing waves' was a key term, they could not apply it in a coherent way. Of further concern was the number of students who seemed to believe that the electrons wobble or follow a sinusoidal path around the nucleus. It should be understood that the Bohr model of the atom is a simplistic model and the electrons do not orbit at all. Even if they did, they would not follow a sinusoidal path. Students should familiarise themselves with de Broglie's work.

**b** Answers will vary.

Award 2 marks for correct diagram.

***VCAA examination report* note:**

The following is an example of a high-scoring response.



Students who tried to draw a strip of paper joined end to end with a sinusoidal wave pattern on it generally had difficulty identifying the standing wave nature of the wave function in the orbit.

**5** In the first model, the electron modelled as a particle is in a circular orbit and thus accelerating. Classical theory predicts that the accelerating electron would continuously emit light and hence spiral into the nucleus. Atoms are inherently unstable using this model and not consistent with observation. The second model treats the electron as a type of standing wave — a resonance. The standing wave is considered stable and does not emit a photon. Only transitions from a higher energy state to a lower energy state are associated with the emission of a single photon. This second model is consistent with observations of emission spectra.

## 10.6 Review

## 10.6 Review questions

**1 a** $\lambda_\gamma = \dfrac{c}{f}$

$= \dfrac{3.0 \times 10^8}{5.2 \times 10^{14}}$

$= 5.8 \times 10^{-7}$ m

$\lambda_e = \dfrac{h}{mv}$

$= \dfrac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.8 \times 10^6}$

$= 2.6 \times 10^{-10}$ m

The photon has the longer wavelength.

**b** $p_\gamma = \dfrac{h}{\lambda_\gamma}$

$= \dfrac{6.63 \times 10^{-34}}{5.8 \times 10^{-7}}$

$= 1.2 \times 10^{-27}$ N s

$p_e = \dfrac{h}{\lambda_e}$

$= \dfrac{6.63 \times 10^{-34}}{2.6 \times 10^{-7}}$

$= 2.6 \times 10^{-24}$ N s

The electron has the larger momentum.

**c** $E_\gamma = hf$

$= 6.63 \times 10^{-34} \times 5.2 \times 10^{14}$

$= 3.5 \times 10^{-19}$ J

$E_e = \dfrac{p^2}{2m}$

$= \dfrac{(2.6 \times 10^{24})^2}{2 \times 9.1 \times 10^{-31}}$

$= 3.6 \times 10^{-18}$ J

The electron has the larger energy.

**2 a** $p_\gamma = \dfrac{E_\gamma}{c}$

$= \dfrac{4.8 \times 10^{-19}}{3.0 \times 10^8}$

$= 1.6 \times 10^{-27}$ N s

$p_e = \sqrt{2m_e E_e}$

$= \sqrt{2 \times 9.1 \times 10^{-31} \times 4.8 \times 10^{-19}}$

$= 9.3 \times 10^{-25}$ N s

The electron has the larger momentum.

**b** $\lambda_\gamma = \dfrac{h}{p_\gamma}$

$= \dfrac{6.63 \times 10^{-34}}{1.6 \times 10^{-27}}$

$= 4.1 \times 10^{-7}$ m

$\lambda_e = \dfrac{h}{p_e}$

$= \dfrac{6.63 \times 10^{-34}}{9.3 \times 10^{-25}}$

$= 7.1 \times 10^{-10}$ m

The photon has the longer wavelength.

**c** As the energy of the photon and electron increases, the momentum of both also increases, but the momentum of the photon increases at a faster rate than the momentum of the electron since $p_\gamma \propto E_\gamma$ but $p_e \propto \sqrt{E_e}$. The wavelengths of both the photon and the electron will decrease as their momentum increases, as wavelength is inversely proportional to momentum.

**3** The observation that a beam of electrons directed at a thin crystal will diffract is strong evidence for the wavelike behaviour of individual electrons in the beam. Objects modelled as particles do not display diffraction.

**4** In a typical emission spectrum, a series of discrete coloured lines is observed. These lines occur due to atomic transitions from one energy level to a lower energy level within each excited atom. In an absorption spectrum, a continuous spectrum consisting of all colours with black lines is observed. The missing colours (where the black lines are positioned) occur when atoms in their ground state are excited into a higher energy state by the absorption of photons of specific energy consistent with the dark lines in an otherwise continuous spectrum.

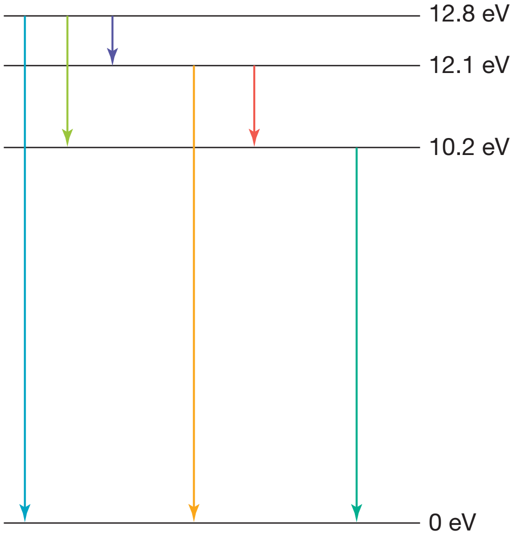
**5** use $\lambda_e = \dfrac{h}{\sqrt{2m_e V q_e}}$ rearranged to make $V$ the subject. Thus:

$V = \dfrac{h^2}{2m_e q_e \lambda_e^2}$

$= \dfrac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times (1.0 \times 10^{-11})^2}$

$= 1.5 \times 10^4$ V

**6** **a**




**b** There are six different energy photons. In order from left to right in the diagram in part **a**, they have energy 12.8 eV, 2.6 eV, 0.70 eV, 12.1 eV, 1.90 eV, and 10.2 eV.

**c** The photon with the least energy is 0.70 eV. Its wavelength is

$\lambda_\gamma = \dfrac{hc}{E_\gamma}$

$= \dfrac{4.14 \times 10^{-15} \times 3.0 \times 10^8}{0.70}$

$= 1.8 \times 10^{-6}$ m

**7** **a** **i** The energy of the emitted photon equals the loss of energy of the atom.

Thus:

$E_\gamma = (-3.7) - (-5.5) = 1.8$ eV

**ii** $f = \dfrac{E_\gamma}{h}$

$= \dfrac{1.8}{4.14 \times 10^{-15}} = 4.3 \times 10^{14}$ Hz

$\lambda_\gamma = \dfrac{c}{f}$

$= \dfrac{3.0 \times 10^8}{4.348 \times 10^{14}} = 6.9 \times 10^{-7}$ m

**b** The photon absorbed has the following energy:

$E_\gamma = -1.6 - (-10.4)$

$= 10.4 - 1.6 = 8.8$ eV

Thus the wavelength of this photon is:

$\lambda_\gamma = \dfrac{hc}{E_\gamma}$

$= \dfrac{4.14 \times 10^{-15} \times 3.0 \times 10^8}{8.8}$

$= 1.4 \times 10^{-7}$ m

**8** Emission spectra consist of coloured emission lines resulting from photons emitted from atoms when transitions occur between excited states to lower energy states, one of which would be the ground state. Absorption spectra on the other hand consist of absorption lines resulting from atoms being excited from the ground state to higher energy states. There are fewer absorption transitions than there are emission transitions possible.

**9** Taylor's experiment tells us that each particle of light, a photon, behaves like a wave when passing through the apparatus and also like a particle when it strikes the photographic plate or screen used.

## 10.6 Exam questions

### Section A — Multiple choice questions

**1** B. The emission energies are 0.7 eV, 1.9 eV, 10.2 eV.

**2** C. Matter particles have de Broglie wavelengths related to their momentum by $\lambda = \dfrac{h}{p}$.

**3** B. Absorption means that the arrow should point up to indicate the electron gaining energy.

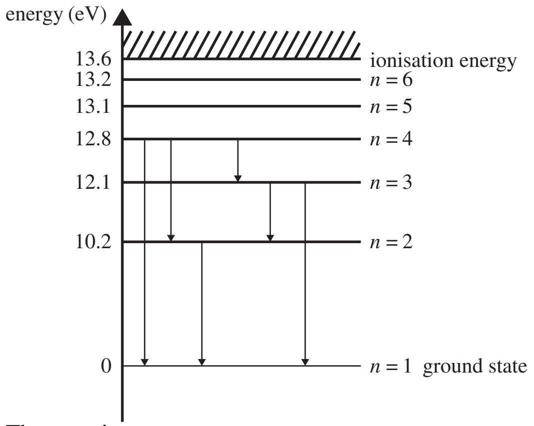
**4** B

$\lambda = \dfrac{h}{mv}$

$\lambda = \dfrac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 4.2 \times 10^7}$

$\lambda = 1.7 \times 10^{-11}$ m

**5** C




The energies are:
12.8 eV, 12.1 eV, 10.2 eV, 2.6 eV, 1.9 eV, and 0.7 eV.

**6** B. The de Broglie wavelength ($\lambda$) of an electron can be calculated using the equation

$$\lambda = \frac{h}{mv}$$

Therefore, as $v$ increases, $\lambda$ decreases.
The width of a diffraction pattern is proportional to the de Broglie ($\lambda$) of the electrons.
As $v$ increases, $\lambda$ decreases. Therefore, the fringe spacing will decrease.

**7** C is the greatest emitted energy difference. Note that A is an absorption.

**8** B. There are three wavelengths in each pattern so $n = 3$.

**9** D. Quantised energy levels within atoms is explained by electrons behaving as waves, which have standing waves of particular wavelengths corresponding to the quantised energy levels.

**10** D. To allow a standing wave to be formed by constructive interference, the circumference of the orbit must be an integral multiple of the de Broglie wavelength of the electron.

## Section B — Short answer questions

**11 a** $E = \frac{hc}{\lambda}$

$$E = \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{2.0 \times 10^{-9}} \quad \text{[2 marks]}$$

$$E = 6.2 \times 10^{2}\ \text{eV} \quad \text{[1 mark]}$$

**b** Diffraction depends on the ratio: $\frac{\lambda}{w}$. [1 mark]

In this case the ratio is $\frac{2.0 \times 10^{-9}}{5 \times 10^{-8}} = 4 \times 10^{-2}$ [1 mark] so little, if any, diffraction will be observed. [1 mark]

**12 a** The electron and the X-ray must have the same wavelength if they produce the same diffraction pattern.

$$E = \frac{hc}{\lambda}$$

$$8000 = \frac{4.14 \times 10^{-15} \times 3.00 \times 10^{8}}{\lambda} \quad \text{[1 mark]}$$

$$\lambda = 1.55 \times 10^{-10}\ \text{m}$$

$$\lambda = 0.155\ \text{nm} \quad \text{[1 mark]}$$

***VCAA examination report* note:**
Many students struggled to begin responding to this question. Of those who did know how to complete the problem, many could not convert to nanometres. Converting between units is an expected skill.

**b**

$$\lambda = \frac{h}{mv} \quad \text{[1 mark]}$$

$$1.55 \times 10^{-10} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times v} \quad \text{[1 mark]}$$

$$v = 4.7 \times 10^{6}\ \text{m s}^{-1} \quad \text{[1 mark]}$$

$$E = \frac{1}{2}mv^2$$

$$E = 0.5 \times 9.1 \times 10^{-31} \times \left(4.7 \times 10^{6}\right)^2 \quad \text{[1 mark]}$$

$$E = 1.0 \times 10^{-17}\ \text{J} \quad \text{[1 mark]}$$

***VCAA examination report* note:**
The majority of students struggled to begin responding. There was a range of incorrect formulas used as a starting point with no progress beyond.

**13 a** D. The 625–750 nm range corresponds to red. [1 mark]

**b** Answers will vary. Students were required to identify the following ideas/concepts:

- that electrons orbit nuclei in shells with discrete energy levels [1 mark]
- electrons can transition between these shells by absorbing or emitting discrete amounts of energy equal to the difference between the two shells [1 mark]
- since transitioning electrons can only release discrete amounts of energy, only discrete spectral lines will be observed [1 mark]

***VCAA examination report* note:**
Students who did not score any marks generally spoke of absorption spectra or gave answers that restated the stem such as '…the hydrogen atom emits discrete spectral lines because all of the photons have the same wavelength…'. Some students responded with information about electron orbits. While this was the basis for the argument they then went on to give detail about the orbits themselves, often referencing the relationship between the circumference and the de Broglie wavelength of the electron, but not describing the origin of the spectral lines. Some students went on to discuss electron shell transitions but did not relate this back to the spectral lines. Students are reminded that they are required to respond to the question asked and should not attempt to present prepared responses as these will rarely fit the question.

**14 a** Answers will vary.
For the X-rays to have the same diffraction pattern as the electrons they must have the same wavelength. [1 mark]
For the X-rays and electrons to have the same wavelength they must have the same momentum. [1 mark]
The energies will thus be different.

***VCAA examination report* note:**
Students were required to state that for the X-rays to have the same diffraction pattern as the electrons they must have the same wavelength. For the X-rays and electrons to have the same wavelength they must have the same momentum. In this case the energies will be different.

**b**

$$E = \frac{1}{2}mv^2$$

$$5000 \times 1.6 \times 10^{-19} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{5000 \times 1.6 \times 10^{-19}}{0.5 \times 9.1 \times 10^{-31}}}$$

$$v = 4.2 \times 10^7 \text{ m s}^{-1}$$ [1 mark]

$$\lambda_{\text{elec}} = \frac{h}{mv}$$

$$\lambda_{\text{elec}} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 4.2 \times 10^7}$$

$$\lambda_{\text{elec}} = 1.74 \times 10^{-11} \text{ m}$$ [1 mark]

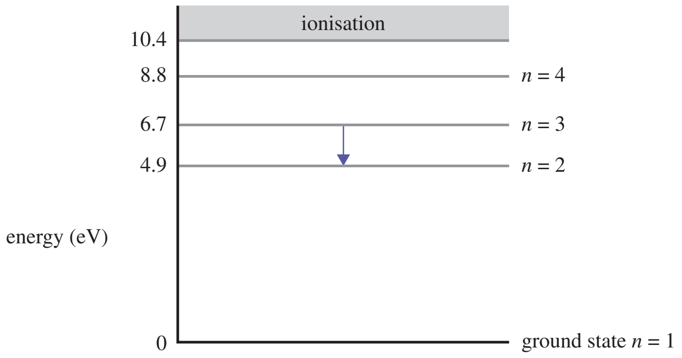
The X-rays must have the same wavelength as the electron to produce the same diffraction pattern. [1 mark]

$$E = \frac{hc}{\lambda}$$

$$E = \frac{4.14 \times 10^{-15} \times 3.0 \times 10^8}{1.74 \times 10^{-11}}$$

$$E = 7.1 \times 10^4 \text{ eV}$$ [1 mark]

**15 a**




Award 1 mark for correctly drawing the arrow from $n = 3$ to $n = 2$ (1.6 eV).

***VCAA examination report* note:**

The diagram may have suggested to students that there is a discrete energy level at 10.4 eV with ionisation occurring beyond this point. While the intent of the question was that ionisation would occur at, or even slightly before, 10.4 eV and hence no electron transition down from this level would be possible, the assessment process favoured the students who may not have interpreted the diagram in that way.

**b** $\Delta E_{\text{eV}} = 6.7$ eV

$\Delta E_{\text{J}} = 6.7 \times 1.6 \times 10^{-19}$ [1 mark]

$\Delta E_{\text{J}} = 1.07 \times 10^{-18}$ J [1 mark]

**c** Nothing will happen. [1 mark]

2.1 eV is not enough energy to transition to any higher state. [1 mark]

# Topic 11 — Einstein's special theory of relativity and the relationship between energy and mass

## 11.2 Einstein's special theory of relativity

### Sample problem 1

In the first scenario, the first car is travelling at 80 km $h^{-1}$ relative to the second car.

In the second scenario, the first car is travelling at $v_A - v_B = 100 - 20 = 80$ km $h^{-1}$ relative to the second car.

In the third scenario, the first car is travelling at $v_A - v_B = 100 - (-20) = 120$ km $h^{-1}$ relative to the second car.

Although in each case the velocities relative to the road are different, the relative velocities of the cars in scenarios 1 and 2 are the same and will cause similar effects upon colliding. When examining scenarios 2 and 3, the cars are travelling at the same speeds but in opposite directions, and therefore the relative velocities are different and there will be different effects upon the cars colliding.

### Practice problem 1

b. velocity

Acceleration, time, and mass are all invariants in Galilean relativity, but velocity is relative to the observer.

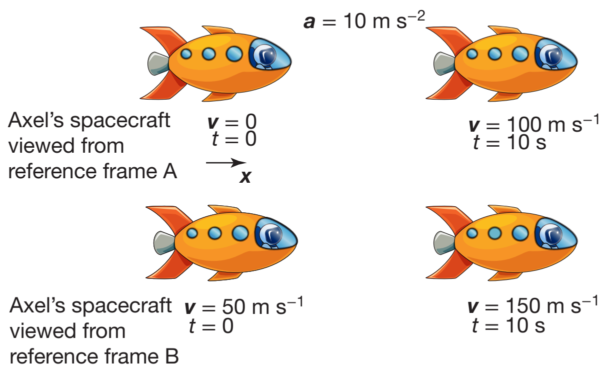
### Sample problem 2

According to the measurements made in A, the rocket accelerated for 10 s at:

$$\begin{aligned} \boldsymbol{a} &= \frac{\Delta \boldsymbol{v}}{t} \\ &= \frac{100-0}{10} \\ &= \frac{100}{10} \\ &= 10 \text{ m s}^{-2} \end{aligned}$$

Axel would feel a force in the forward direction.

Now choose a different reference frame. Effie is in reference frame B in another spacecraft, moving at 50 m $s^{-1}$ relative to A.




Effie measures the velocity of Axel's spacecraft to change from 50 m $s^{-1}$ to (50 + 100) m $s^{-1}$ in 10 s. From B:

$$\begin{aligned} \boldsymbol{a} &= \frac{\Delta \boldsymbol{v}}{t} \\ &= \frac{150-50}{10} \\ &= \frac{100}{10} \\ &= 10 \text{ m s}^{-2} \end{aligned}$$

The acceleration is the same whether it is measured from frame A or frame B. It will still be 10 m $s^{-2}$ regardless of the speed of the reference frame.

### Practice problem 2

**a** The measured speed depends entirely on the relative motion of the person measuring the speed. Therefore, it depends on the observer's frame of reference.

**b** Newton's three laws of motion do not require the entering of a value of velocity at any point. However, the second law (net force = $m\boldsymbol{a}$) includes a value for acceleration. If the value for acceleration is different, then the net force would be different. By the principle of relativity, physical laws need to give the same answer in all inertial reference frames.

### Sample problem 3

Galileo proposed that all inertial frames of reference are equally valid. Maxwell's concept of electromagnetic waves suggested the presence of an absolute frame of reference — the luminiferous aether. That is, observers moving relative to the aether should experience light at different velocities, and therefore not all inertial frames of reference are equally valid, as stated by Galileo.

### Practice problem 3

The light is moving at $c$ relative to the aether, and the spacecraft is moving at $0.5c$ in the other direction; therefore, the light is moving away from the spacecraft at $1.5c$, assuming that light moves at $c$ relative to the aether and that the Galilean understanding of relativity is correct.

### Sample problem 4

Consider Einstein's first postulate, in which the laws of physics are the same in all inertial frames of reference.

Firstly, the principle of relativity is applied to all laws of physics, not just the mechanics of Galileo and Newton.

Consider Einstein's second postulate, in which the speed of light has a constant value for all observers regardless of their motion.

Secondly, the speed of light is constant for all observers. Before Einstein, the speed of light was assumed to be relative to its medium, the luminiferous aether.

## Practice problem 4

Einstein found that the speed of light was invariant and that time, distance, simultaneity, mass and kinetic energy were all dependent on the motion or the reference frame of the observer, and hence relative.

## Sample problem 5

$\boldsymbol{u} = 0\,\text{m s}^{-1}$, $\boldsymbol{v} = 0.1c = 3.0 \times 10^7\,\text{m s}^{-1}$, $\boldsymbol{a} = 9.8\,\text{m s}^{-2}$, $t = ?$

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}}$$

$$= \frac{3.0 \times 10^7 - 0}{9.8}$$

$$= 3.06 \times 10^6 \text{ seconds}$$

$$= 35.4 \text{ days}$$

It would take more than 35 days to achieve a speed of $0.1c$! (This is the fastest speed for which use of Newtonian kinematics still gives a reasonable approximation.)

## Practice problem 5

Even small increases in the acceleration would make the astronauts feel very heavy. Humans cannot withstand large accelerations for long periods of time. If the acceleration was large enough, they would be squashed flat, but, long before that, they would be rendered unconscious by the lack of blood flow through the brain.

## Sample problem 6

Figure 11.16a in the text shows the situation for planet A. The light radiates in all directions at the same rate, and the diagram shows where the light in one direction and the opposite direction would be after one year.

Figure 11.16b in the text shows what is happening on planet B according to observers on planet A. The light moving out behind the moving planet reaches the one-light-year distance sooner than the light moving out from the front! But it is known that planet B is at the centre of this light circle. The way to achieve this is to move away from absolute space and time and understand that these are relative to the observer. When this is done, it is seen as possible for planet B to be at the centre of the light circle. However, this requires that A and B disagree about when two events occur. According to planet A, the different sides of the light circle reach the light-year radius at different times, but from planet B this must occur simultaneously.

## Practice problem 6

**a** In classical physics, simultaneity is invariant: all observers can agree when events happen.

**b** In special relativity, simultaneity is relative: events that are simultaneous in one reference frame are not simultaneous across other reference frames that are in relative motion.

## 11.2 Exercise

**1** The value of the velocity depends on the reference frame in which it is measured.

**2** A frame of reference is a set of length and time coordinates that an observer uses to measure an event.

**3** $\boldsymbol{v} = 50 - (-50) = 100\,\text{km h}^{-1}$

**4** **a** The Earth's direction changes by only about 1° each day so, from an observer's point of view, it is travelling in a straight line with constant speed. The acceleration due to the revolution of the Earth around the Sun is about $6 \times 10^{-2}\,\text{m s}^{-2}$, which is tiny. According to the principle of relativity, if the Earth is not accelerating, the movement of the Earth cannot be felt no matter what its speed is.

**b** Daily rotation about its axis and precession of the equinoxes (the motion of the Earth's axis), which has a period of about 22 000 years

**c** The accelerations are so tiny that they are difficult to detect. Therefore, for most purposes, Earth can be considered an inertial reference frame.

**5** hypothesised speed of light

$$\text{emitted from the Earth} = c + \boldsymbol{v}$$
$$= 2.9979 \times 10^8 + 0.0010 \times 10^8$$
$$= 2.9989 \times 10^8\,\text{m s}^{-1}$$

**6** **a** Newton's laws are an excellent approximation at speeds lower than light speed. For most of human history, there was no way of observing extremely high speeds, meaning that the limitations of Newton's laws were unidentifiable.

**b** Newton's laws are still useful at the lower speeds that are seen in everyday life, providing accurate results at those speeds. They are also much easier to use and learn than Einstein's laws.

**7** It would seem impossible. An object travelling at a speed of $0.99c$ towards a stationary observer emits light in all directions. The light travels at speed $c$. According to Newtonian physics, the observer would measure the speed of the light from the approaching object to be $1.99c$. However, the observer actually measures the speed of light to be $c$.

**8** The speed of light is $c$, regardless of the frame of reference. Therefore, the speed of light when it hits the second star is $3.0 \times 10^8\,\text{m s}^{-1}$.

## 11.2 Exam questions

**1** D. Inertial frames of reference are non-accelerating. Therefore, no observer within the frame of reference will be able to detect any acceleration of the frame of reference.

**2** Einstein's second postulate is 'The speed of light has a constant value for all observers regardless of their motion or the motion of the source.' [1 mark]
Classical physics states that if there is relative motion between the source of light and the observer then the measurement of the speed of light would vary. [1 mark]
Specifically, if the source and the observer were approaching each other, the speed of light would appear faster than $3 \times 10^8\,\text{m s}^{-1}$. If the source and observer were retreating from each other, the speed of light would appear slower than $3 \times 10^8\,\text{m s}^{-1}$. [1 mark]

3 No, the velocity might not be constant (as constant speed is not the same as constant velocity). [1 mark]
The ship in question could be travelling in a circular path, or it could be in orbit and still be travelling at a constant speed. Therefore, the spaceship may not be in an inertial frame of reference. [1 mark]
***VCAA examination report* note:**
Many students made statements such as 'Jani is in an inertial frame but the spaceship is not'.

4 B. The laws of physics are the same in all inertial reference frames.
However, an accelerating train is not an inertial frame, so the results of motion experiments will be different in B.

5 B. From Anna's perspective, the space lab is moving towards her while Barry is moving away. So she sees the signal travelling a smaller distance to the space lab and arriving first.

## 11.3 Time dilation

### Sample problem 7

The proper time $t_0$ is the time interval between the two events: when the clock shows 12 pm and when the clock shows 12.05 pm. The difference is 5 minutes.

Calculate the Lorentz factor using $v = 0.5c$:

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\\ &= \frac{1}{\sqrt{1-\frac{(0.5c)^2}{c^2}}}\\ &= \frac{1}{\sqrt{1-\frac{0.25c^2}{c^2}}}\\ &= \frac{1}{\sqrt{1-0.25}}\\ &= 1.155\end{aligned}$$

Calculate the time taken for the hands to change from 12 pm to 12.05 pm in James's frame of reference, using $t = t_0\gamma$:

$$\begin{aligned}t &= t_0\gamma\\ &= 5\text{ minutes} \times 1.155\\ &= 5.774\text{ minutes}\end{aligned}$$

James notices that the moving clock takes 5.774 minutes for its hands to move from 12 pm to 12.05 pm.

### Practice problem 7

James is measuring Mabry's time with a clock that is moving relative to her, so Mabry's clock tells the proper time.

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\\ &= 1.155 \text{ when } v = 0.5c\end{aligned}$$

$$t = t_0\gamma$$

So, $t_0 = \dfrac{t}{\gamma}$

$$\begin{aligned}&= \frac{5}{1.155}\\ &= 4.329\text{ minutes}\end{aligned}$$

### Sample problem 8

Mabry notices that 5.774 minutes pass when James's watch shows 5 minutes passing.

### Practice problem 8

Aixi times the wrestling to last for 3 minutes, but she is in a reference frame moving at $0.8c$ past them.

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\\ &= 1.667 \text{ when } v = 0.8c\end{aligned}$$

The time dilation equation is:

$$t = t_0\gamma$$

The time being found is $t_0$ because the time that is known is in Aixi's reference frame, which is moving relative to Xiaobo's wrestling (the event of interest) at $0.8c$.

$$\begin{aligned}t_0 &= \frac{t}{\gamma}\\ &= \frac{3}{1.667}\\ &= 1.8\text{ minutes, or 1 minute and 48 seconds.}\end{aligned}$$

Aixi would see them wrestling in slow motion because the event is dilated to 3 minutes in her reference frame. Xiaobo times Aixi's song as lasting $3 \times 1.667 = 5$ minutes. It is only Aixi who thinks that her song and his wrestling started and ended at the same time; from Xiaobo's perspective, these events are not simultaneous.

### Sample problem 9

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\\ &= \frac{1}{\sqrt{1-\frac{20^2}{\left(3\times 10^8\right)^2}}}\\ &= 1.000\,000\,000\,000\,0022\end{aligned}$$

$$\begin{aligned}t &= t_0\gamma\\ &= 1.000\,000\,000\,000\,0022t_0\end{aligned}$$

The difference between the rates of time in the two perspectives is so small that it is difficult to calculate, much less notice it.

### Practice problem 9

$\dfrac{t}{t_0} = 2$, so $\gamma = 2$

$$\begin{aligned}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} &= 2\\ \sqrt{1-\frac{v^2}{c^2}} &= \frac{1}{2}\\ 1-\frac{v^2}{c^2} &= \frac{1}{4}\\ \frac{3}{4} &= \frac{v^2}{c^2}\end{aligned}$$

$$v = \sqrt{\frac{3}{4}}c$$
$$= 2.6 \times 10^8 \text{ m s}^{-1}$$

OR

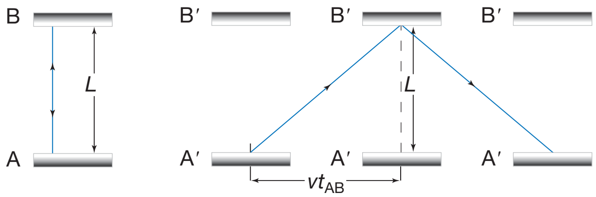
$$v = \sqrt{\frac{3}{4}}c$$
$$= 0.866c$$

## 11.3 Exercise

1 Time dilation is the slowing of time for objects in motion relative to the observer. This is only significant at very great speeds. However, it can be detected with very sensitive instruments when comparing times on board aeroplanes with the time on the ground.

2 The clock in motion relative to you and it will therefore run slow from your frame of reference. However, it is equally valid to state that from the clock in motion's frame of reference, your clock runs slow.

3 As can be seen in the two figures, the distance the light travels in the moving clock is further than in the stationary clock. As the speed of light is constant, the time taken for the light to complete one cycle in the moving clock is longer. Therefore it 'ticks' more slowly.




4 Proper time is $t_0$, which is the time as measured in the reference frame of the event. Conversely, $t$ is the time as measured in a different inertial reference frame.

5 **a** $\gamma = \dfrac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$

$$= \frac{1}{\sqrt{\left(1 - \left(\frac{0.3c}{c}\right)^2\right)}}$$
$$= 1.048\,28$$

**b** $T = \dfrac{1}{f}$

$$= \frac{1}{100} = 0.01 \text{ min}$$
$$t = t_0\gamma$$
$$= 0.01 \times 1.048\,28$$
$$= 0.010\,482\,8 \text{ minutes}$$
$$f' = \frac{1}{t}$$
$$= 95.4 \text{ beats per minute. (It beats more slowly.)}$$

## 11.3 Exam questions

1 A

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$= \frac{1}{\sqrt{1 - \frac{\left(2.50 \times 10^8\right)^2}{\left(3.0 \times 10^8\right)^2}}}$$
$$= 1.81$$

2 **a** The distance of 8.61 light-years is the proper length in Earth's frame of reference. The proper time in Earth's frame of reference is:

$$t = \frac{d}{v}$$
$$= \frac{8.61}{0.8}$$
$$= 10.76 \text{ years} \quad [1 \text{ mark}]$$

This time is the dilated time in the astronaut's frame of reference. The proper time as measured by the astronaut is:

$$t = t_0\gamma$$
$$10.76 = t_0 \times 1.67$$
$$t_0 = 6.44 \text{ years} \quad [1 \text{ mark}]$$

***VCAA examination report* note:**

Many students understood a light-year as a measure of time rather than distance. The range of workings indicates a varied level of understanding of proper time and dilated time.

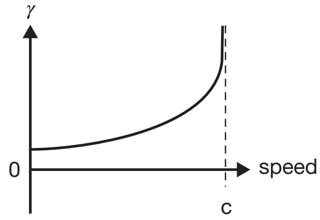
**b** The time measured by the astronaut will be proper time [1 mark] because the clock is stationary in the astronaut's frame of reference [1 mark].

***VCAA examination report* note:**

While most students were able to identify the time as proper time, many struggled with the explanation. Some incorrect reasonings included statements that 'anything the astronaut observes is "proper" since it is the astronaut doing the observing', 'the astronaut measures proper time because he is stationary inside the spaceship' and 'the shorter time is always proper time'.

Some students confused a frame of reference as a physical creation, such as a spaceship, rather than something applied to an observer.

3 D. At $v << c$ the Lorentz factor is 1 and it increases exponentially as $v$ approaches $c$.




4 $t_{\text{scientist}} = \dfrac{d}{v} = \dfrac{9.14 \times 10^{-5}}{0.99875 \times 3 \times 10^8}$ [1 mark]

$$= 3.05 \times 10^{-13} \text{ s} \; [1 \text{ mark}]$$

***VCAA examination report* note:**

The most common error was to calculate the correct time then apply a time dilation to the result. This suggested that students did not understand how to interpret frames of reference.

5 C. Anna and Barry are both observing their own clocks, in their own frames of reference. They will both observe their proper times.

## 11.4 Length contraction

### Sample problem 10

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$= \frac{1}{\sqrt{1-\frac{(0.5c)^2}{c^2}}}$$
$$= \frac{1}{\sqrt{1-\frac{0.25c^2}{c^2}}}$$
$$= \frac{1}{\sqrt{1-0.25}}$$
$$= 1.155$$
$$L = \frac{L_0}{\gamma}$$
$$\frac{L}{L_0} = \frac{1}{\gamma}$$
$$= \frac{1}{1.155}$$
$$= 0.866$$

The spacecraft appears to be only 0.866 or 86.6% of its proper length. This is a contraction of 13.4%.

### Practice problem 10

**a** Rebecca is in the rest frame of the distance from Melbourne to Sydney whereas Madeline is moving at high speed in the direction of Sydney. Madeline sees the distance contracted, and therefore her recorded distance will be shorter than the distance recorded by Rebecca.

**b** In this situation, the proper length of the journey is that recorded by Rebecca. The proper length is the length measured from the frame of reference at which objects are at rest, which is the situation for Rebecca, as she stays in Melbourne.

## 11.4 Exercise

**1** Depth. Only the length in the direction of the motion is contracted.

**2 a** Its length would be unchanged from the perspective of the alien on board.

**b** $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$$= \frac{1}{\sqrt{1-\frac{(0.8c)^2}{c^2}}}$$
$$= \frac{1}{\sqrt{1-0.64}}$$
$$= 1.667$$
$$L = \frac{L_0}{\gamma}$$
$$= \frac{L_0}{1.667}$$
$$= 0.6L_0$$

The length of the spaceship from the perspective of the Sun would be 60% of its proper length.

**c** The speed that light from the Sun reaches it is $c$.

**3 a** $\gamma = \frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$

$$= 7.0888$$
$$t_0 = \frac{t}{\gamma}$$
$$= \frac{20}{7.0888}$$
$$= 2.82\text{ s}$$

**b** $L = \frac{L_0}{\gamma}$

$$= \frac{5}{7.0888}$$
$$= 0.7053\text{ m}$$

**4** $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$$= \frac{1}{\sqrt{1-\frac{(0.9c)^2}{c^2}}}$$
$$= \frac{1}{\sqrt{1-0.81}}$$
$$= 2.294$$
$$L = \frac{L_0}{\gamma}$$
$$= \frac{100}{2.294}$$
$$= 43.59\text{ m}$$

The length of the spaceship from the perspective of the astronaut is 43.59 m.

**5** Use $L_0 = \gamma L$ and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

where $L = 8.0$ m and $L_0 = 12.0$ m

$$\gamma = \frac{L_0}{L} = \frac{3}{2}$$

and $\frac{1}{\gamma^2} = \left(\frac{2}{3}\right)^2 = 1 - \frac{v^2}{c^2}$

Thus $\frac{v^2}{c^2} = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$

and $v = \frac{\sqrt{5}}{3}c \simeq 0.75c$

## 11.4 Exam questions

**1 a** The technician is observing length contraction [1 mark], which only occurs in the axis/direction of motion. [1 mark]

***VCAA examination report*** **note:**

The most common error was to refer to length dilation, suggesting that this area of study is confusing to a number of students.

**b** The first step was to calculate gamma from the relative velocity provided.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$\gamma = \frac{1}{\sqrt{1-0.7^2}}$$

$\gamma = 1.4$ [1 mark]

If the spaceship is moving in the technician's frame of reference, then they will observe contracted length. The proper length, $L_0$, is found:

$L = \frac{L_0}{\gamma}$

$135 = \frac{L_0}{1.4}$

$L_0 = 189$ m [1 mark]

***VCAA examination report* note:**

The most common errors were to incorrectly calculate gamma or confuse proper and contracted length. While not a common error, students should know that gamma can never be less than one.

**2** C

$L = \frac{L_0}{\gamma}$

$L_0 = 150 \times 3$

$L_0 = 450\,\text{m}$

**3** $L = \frac{1}{\gamma}L_0 = \sqrt{\left(1 - \frac{v^2}{c^2}\right)} \times L_0$ but $L = \frac{1}{3}L_0$

$\Rightarrow \sqrt{\left(1 - \frac{v^2}{c^2}\right)} = \frac{1}{3}$ [1 mark]

$\Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{9}$

$\Rightarrow \frac{v^2}{c^2} = \frac{8}{9}$ $\quad v = \sqrt{\frac{8}{9}}c = 0.94c$ [1 mark]

**4** The distance travelled by the particle, in the scientists' frame of reference, is called its 'proper length', $L_0$.

This distance, as measured by the moving particle, will be contracted (shortened).

$L_{\text{particle}} = \frac{L_0}{\gamma} = \frac{9.14 \times 10^{-5}}{20}$ [1 mark]

$= 4.57 \times 10^{-6}\text{m}$ [1 mark]

***VCAA examination report* note:**

The most common error was to correctly identify the form of the equation but then confuse proper length with contracted length, resulting in length dilation rather than contraction. Some students did not recognise that the calculated contracted length was longer than the proper length. This suggested that students had very little understanding of this phenomenon.

**5** A. The separation distance will be contracted from the moving particle's perspective.

$L = \frac{L_0}{\gamma}$

$= \frac{2.0}{2.4}$

$= 0.83$

## 11.5 Relativity is real

### Sample problem 11

**a** The proper time for the half-life of muons is 1.56 microseconds.

**b** $t = t_0\gamma$

$\gamma = \frac{t}{t_0}$

$= \frac{6.5}{0.7}$

$= 9.29$

**c** $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$v = c\sqrt{1 - \frac{1}{\gamma^2}}$

$= c\sqrt{1 - \frac{1}{9.29^2}}$

$= 0.994c$

The muons were travelling at 99.4% of the speed of light.

**d** $t = \gamma t_0$

$= 9.29 \times 1.56\,\mu\text{s}$

$= 14.5\,\mu\text{s}$

The half-life of muons as viewed from Earth is 14.5 microseconds compared to the 1.56 microseconds as experienced by the muons.

### Practice problem 11

**a** $t = 6.5 \times 10^{-6}$ s, $v = 0.994c$

$d = vt$

$= 0.994 \times 3.0 \times 10^8 \times 6.5 \times 10^{-6}$

$= 1938\,\text{m}$

**b** $d = vt$

$= 0.994 \times 3.0 \times 10^8 \times 0.7 \times 10^{-6}$

$= 209\,\text{m}$

### 11.5 Exercise

**1** Classical physics does not consider Einstein's second postulate. As a result of the muons travelling at very near light speed, their half-lives according to scientists on Earth are much longer than in their rest frame due to time dilation. This means that more of the muons have sufficient time to reach the Earth's surface before decaying.

**2** $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.98^2}} = 5.0252$

$L_0 = 30\,\text{km}$

$L = \frac{L_0}{\gamma} = \frac{30}{5.0252} = 5.97\,\text{km}$

**3** $t = t_0\gamma$

$= 1.56 \times 5.0252$

$= 7.839$ microseconds

**4** In the early days of special relativity, few experiments were available to test whether the phenomena predicted by the theory matched up with measurements. Muons, however, were detectable, were travelling close to the speed of light and had features of their journey that were measurable, in particular the distance travelled and their half-life. The number of muons reaching the ground agreed with the predictions of special relativity but not classical physics.

**5** Use $L = \frac{L_0}{\gamma}$ to calculate $l$ where $L_0 = 1200$ m.

$\gamma = \frac{1}{\sqrt{-1\left(\frac{0.99c}{c}\right)^2}} = 7.089$

Hence $L = \frac{1200}{7.089} = 1.7 \times 10^2$ m (to 2 significant figures)

**6 a** The 140.0-m length in the laboratory frame is reduced to a 20-m length, by length contraction, in the rest frame of the electrons.

So $\gamma = \frac{l_0}{l} = \frac{140.0}{20.0} = 7.00$.

The speed of the electrons is thus given by the expression

$$\gamma = 7.00 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.$$

By rearranging the expression:

$$1-\frac{v^2}{c^2} = \frac{1}{49.0}$$

$$\Rightarrow v = \frac{\sqrt{48}}{7.00}c$$

$$v = 2.97\times10^8\ \text{m s}^{-1}.$$

The speed of the electrons is $2.97\times10^8\ \text{m s}^{-1}$.

**b** The electrons are moving relative to the laboratory with speed $2.97\times10^8\ \text{m s}^{-1}$. They will travel a laboratory rest frame distance of 140.0 m in

$$t = \frac{d}{v} = \frac{140.0}{2.97\times10^8} = 4.71\times10^{-7}\ \text{s}$$

It takes the electrons $4.71\times10^{-7}$ s.

## 11.5 Exam questions

**1** C

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{(2.99\times10^8)^2}{(3.00\times10^8)^2}}}$$

$\gamma = 12.3$

**2** Answers will vary.

This is due to time dilation. [1 mark]

The half-life of the muon as measured in Earth's frame of reference is longer than the half-life measured in the muon's frame of reference. This explains why muons travel further before they decay. [1 mark]

OR

This is due to length contraction. [1 mark]

The distance to the surface as measured in the muon's frame of reference is shorter than the distance measured in Earth's frame of reference. This explains why muons can reach the surface before they decay. [1 mark]

***VCAA examination report* note:** This question was not done well. Students had difficulty identifying the two frames of reference and how observations from one to the other are affected. Teachers and students need to spend more time reviewing these situatiins.

**3 a** $t = t_0\gamma$

$t = 2.3\times3.2$ [1 mark]

$t = 7.4\ \mu\text{s}$ [1 mark]

**b** The length would appear shorter due to length contraction. [1 mark]

$$L = \frac{L_0}{\gamma}$$

$$L = \frac{1.5}{3.2}$$

$L = 0.47$ km [1 mark]

**4** C

$d = \boldsymbol{v}t_0$

$d = 0.994\times c\times0.32\times10^{-6}$

$= 0.994\times3.0\times10^8\times0.32\times10^{-6}$

$= 95.424$

$= 95$ m

**5** In 7 µs light travels $d = 3.0\times10^8\times7\times10^{-6} = 2\times10^3$ m, whereas in 1 ns light travels $d = 3.0\times10^8\times1\times10^{-9} = 0.3$ m. [1 mark]

The objective of a GPS is to provide an accurate position on Earth; however, if the clock in the GPS lags by 7 µs each day compared to a clock on Earth, then the GPS would provide you with a position accurate to 2 km [1 mark]. This is not useful compared to the 0.3 m accuracy that would result from the clock in the satellite being accurate to the nanosecond. [1 mark]

## 11.6 Einstein's relationship between mass and energy

### Sample problem 12

As $\boldsymbol{v}\to c$,

$$\gamma \to \frac{1}{\sqrt{1-\frac{c^2}{c^2}}} \to \infty$$

So as $\boldsymbol{v}\to c$, $\gamma$ becomes infinitely large

$m = m_0\gamma$, so

as $\boldsymbol{v}\to c, \gamma\to\infty, m\to\infty$.

An object travelling at $c$ would have infinite mass.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

If $\boldsymbol{v} > c$, then $\frac{\boldsymbol{v}^2}{c^2} > 1$ and $1-\frac{\boldsymbol{v}^2}{c^2} < 0$.

Speeds larger than $c$ would produce a negative under the square root sign, so these speeds are not possible.

### Practice problem 12

$$\gamma = \frac{1}{\sqrt{1-\frac{30\,000^2}{3.0\times10^8}}} = 1.000\,000\,005$$

$m = \gamma m_0 = 1.000\,000\,005\times6\times10^{24}$

$= 6.000\,000\,03\times10^{24}$ kg

The increase in mass is
$6.000\,000\,03\times10^{24} - 6.0\times10^{24}\ \text{kg} = 3\times10^{16}$ kg.

Even though gamma is relatively close to one, as the mass of the Earth is so large, the mass increase is also large.

## Sample problem 13

$$\Delta E = 11 \text{ GeV}$$
$$= 11 \times 10^9 \times 1.6 \times 10^{-19} \text{ J}$$
$$= 1.76 \times 10^{-9} \text{ J}$$

$$\Delta E = \Delta mc^2$$

$$\Delta m = \frac{\Delta E}{c^2}$$
$$= \frac{1.76 \times 10^{-9} \text{ J}}{(3 \times 10^8 \text{ m s}^{-1})^2}$$
$$= 1.96 \times 10^{-26} \text{ kg}$$

Note that the rest mass of a proton is $1.67 \times 10^{-27}$ kg, so the accelerated proton behaves as though its mass is nearly 12 times its rest mass.

## Practice problem 13

$$\Delta m = \frac{\Delta E}{c^2}$$
$$= \frac{3.45 \times 10^{-10}}{(3 \times 10^8)^2}$$
$$= 3.83 \times 10^{-27} \text{ kg}$$

The proton's mass is approximately two times larger when accelerated by $3.45 \times 10^{-10}$ J of energy.

## Sample problem 14

$$E_\text{k} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_\text{k}}{m}}$$

$$= \sqrt{\frac{2 \times 11 \times 10^9 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}}$$

$$= 1.45 \times 10^9 \text{ m s}^{-1}$$

This speed is not possible as the maximum speed attainable is $3.0 \times 10^8$ m s$^{-1}$.

## Practice problem 14

$$E = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E}{m}}$$

$$= \sqrt{\frac{2 \times 3.45 \times 10^{-10}}{1.67 \times 10^{-27}}}$$
$$= 6.43 \times 10^8 \text{ m s}^{-1}$$

Since the maximum speed possible is $3.0 \times 10^8$ m s$^{-1}$, the calculated speed is not possible, showing that Newton's laws are not useful when dealing with objects travelling at high speeds.

## Sample problem 15

$$E_\text{k} = (\gamma - 1)m_0c^2$$
$$= \left(\frac{1}{\sqrt{1 - 0.5^2}} - 1\right) \times 10\,000 \times (3 \times 10^8)^2$$
$$= 1.39 \times 10^{20} \text{ J}$$

$$E_\text{k} = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 10\,000 \times (0.5 \times 3 \times 10^8)^2$$
$$= 1.13 \times 10^{20} \text{ J}$$

The kinetic energy is $\frac{1.39}{1.13} = 1.23$ times the value predicted by classical physics.

## Practice problem 15

$$E_\text{k} = (\gamma - 1)\, m_0c^2$$
$$3 \times 10^9 \times 1.6 \times 10^{-19} = (\gamma - 1) \times 9.1 \times 10^{-31} \times \left(3.0 \times 10^8\right)^2$$
$$4.8 \times 10^{-10} = (\gamma - 1) \times 8.19 \times 10^{-14}$$
$$\gamma = \frac{1.6 \times 10^{-9}}{8.19 \times 10^{-14}} + 1 = 5.86 \times 10^3$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 5.86 \times 10^{13}$$
$$\sqrt{1 - \frac{v^2}{c^2}} = 1.71 \times 10^{-4}$$
$$1 - \frac{v^2}{c^2} = 2.91 \times 10^{-8}$$
$$0.999\,999\,970\,9 = \frac{v^2}{c^2}$$
$$v = 0.999\,999\,985\,4c$$

This is very close to the speed of light.

## Sample problem 16

Deuterium (or hydrogen-2) has a mass of $3.3435 \times 10^{-27}$ kg. A proton has a mass of $1.6726 \times 10^{-27}$ kg and a neutron has a mass of $1.6759 \times 10^{-27}$ kg. This leads to a combined mass of $3.3475 \times 10^{-27}$ kg, showing a mass difference of $0.004 \times 10^{-27}$ kg or $4.0 \times 10^{-30}$ kg.

Deuterium has a rest-mass and also binding energy that account for this mass deficit. Using $\Delta E = mc^2$, the energy difference from this mass deficit would be $3.6 \times 10^{-13}$ J.

It is clear that the mass of the nucleus is different to the mass of the individual particles, but when the binding energy of the hydrogen-2 nucleus is included, the mass–energy of both is found to be the same. The separate particles have their mass and zero potential energy. The particles bound in the nucleus have a reduced mass and the binding energy of the nucleus. (The binding energy is the energy required to separate the particles. It is released as a combination of increased kinetic energy of the particles and gamma rays.)

### Practice problem 16

$\Delta E = 30 \times 10^6 \times 1.6 \times 10^{-19}$
$= 4.8 \times 10^{-12}$ J

$\Delta m = \dfrac{\Delta E}{c^2}$
$= \dfrac{4.8 \times 10^{-12}}{(3 \times 10^8)^2}$
$= 5.33 \times 10^{-29}$ kg

### Sample problem 17

$E = mc^2$
$= 4.4 \times 10^9 \times (3.0 \times 10^8)^2$ J
$= 4.0 \times 10^{26}$ J

$P = \dfrac{E}{t}$
$= \dfrac{4.0 \times 10^{26}\text{ J}}{1\text{ s}}$
$= 4.0 \times 10^{26}$ W

The mass loss of $4.4 \times 10^9$ kg s$^{-1}$ equates to a power output of $4.0 \times 10^{26}$ W.

### Practice problem 17

$t = \dfrac{2.0 \times 10^{30}}{4.4 \times 10^9}$
$= 4.55 \times 10^{20}$ s
$= \dfrac{4.55 \times 10^{20}}{86\,400}$

$= 5.26 \times 10^{15}$ days
$= \dfrac{5.26 \times 10^{15}}{365}$
$= 1.44 \times 10^{13}$ years

### Sample problem 18

$\Delta E = \Delta mc^2$
$= 2 \times 9.1 \times 10^{-31} \times \left(3 \times 10^8\right)^2$
$= 1.64 \times 10^{-13}$J

Each photon has an energy $\dfrac{1.64 \times 10^{-13}\text{ J}}{2}$.

Thus, the energy is $8.2 \times 10^{-14}$ J.

### Practice problem 18

$\Delta E = \Delta mc^2$
$= 2 \times 1.6 \times 10^{-27} \times \left(3 \times 10^8\right)^2$
$= 2.9 \times 10^{-10}$J

## 11.6 Exercise

**1** The mass of an object increases with its motion, according to the factor $\gamma$. Therefore an object's mass becomes infinite at the speed of light. Also, the length of the object would be zero at the speed of light, and time would cease. Mathematically, speeds beyond the speed of light would require taking the square root of a negative number in the equation for $\gamma$.

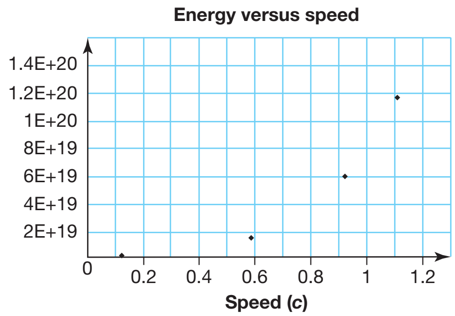
**2 a i** $= \left(\dfrac{1}{\sqrt{1-0.1^2}} - 1\right) \times 1000 \times (3 \times 10^8)^2$
$= 4.5 \times 10^{17}$ J

**ii** $= \left(\dfrac{1}{\sqrt{1-0.5^2}} - 1\right) \times 1000 \times (3 \times 10^8)^2$
$= 1.4 \times 10^{19}$ J

**iii** $= \left(\dfrac{1}{\sqrt{1-0.8^2}} - 1\right) \times 1000 \times (3 \times 10^8)^2$
$= 6.0 \times 10^{19}$ J

**iv** $= \left(\dfrac{1}{\sqrt{1-0.9^2}} - 1\right) \times 1000 \times (3 \times 10^8)^2$
$= 1.2 \times 10^{20}$ J

**b**




**3** Energy and mass are equivalent. We cannot talk about an increase in energy without an increase in mass, and vice versa.

**4** $E = mc^2$
$= 5.98 \times 10^{24} \times \left(3.0 \times 10^8\right)^2$
$= 5.4 \times 10^{41}$ J

**5** $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
$= \dfrac{1}{\sqrt{1 - \frac{\left(3 \times 10^4\right)^2}{c^2}}}$
$= \dfrac{1}{\sqrt{1 - 0.0001^2}}$
$= 1.000\,000\,005$

$m = m_0\gamma$
$= 5.98 \times 10^{24} \times 1.000\,000\,005$
$= 5.980\,000\,029\,9 \times 10^{24}$ kg

$m - m_0 = 5.980\,000\,029\,9 \times 10^{24}$ kg $- 5.98 \times 10^{24}$ kg
$= 2.99 \times 10^{16}$ kg

The difference between the rest mass and the mass from the point of view of the stationary observer is $2.99 \times 10^{16}$ kg.

**6** Inside the Sun, fusion is converting mass into energy that is eventually radiated as heat and light. This results in the mass of the Sun decreasing over time.

## 11.6 Exam questions

**1** C
$E = mc^2$
$= 6.6 \times 10^9 \times \left(3.0 \times 10^8\right)^2$
$= 6.0 \times 10^{26}$ J

Since this conversion occurs every second, the output is $6.0 \times 10^{26}$ W.

**2** B. At $v << c$ the relativistic $E_k$ is the same as the classical $E_k$, but as $v$ approaches $c$, relativistic $E_k$ increases at a greater rate than classical $E_k$.

**3** $\gamma = \frac{t}{t_0} = 8$ [1 mark]

$E = (\gamma - 1)\,mc^2$

$E = 7 \times 1000 \times \left(3 \times 10^8\right)^2$ [1 mark]

$E = 6.3 \times 10^{21}$ J [1 mark]

***VCAA examination report* note:**

The most common errors were to calculate gamma as $\frac{1}{8}$ rather than 8 or to forget to square $c$ in the substitution.

**4** $E = mc^2$

$E = 2.5 \times 10^{-28} \times \left(3.0 \times 10^8\right)^2$ [1 mark]

$E = 2.3 \times 10^{-11}$ J [1 mark]

**5** B. When a mass ($m$) is converted to energy ($E$):

$m = \frac{E}{c^2} = \frac{3.8 \times 10^{26}}{\left(3 \times 10^8\right)^2} = 4.2 \times 10^9$ kg

## 11.7 Review

### 11.7 Review questions

**1** Inertial reference frames are those that are not subject to acceleration. Non-inertial frames are accelerating.

**2** There are no indications of movement with constant velocity. While accelerating, however, a force is required and you feel this as a push from your seat.

**3** Head-on collisions are particularly dangerous because the velocity of one car relative to the other can be much greater than the velocity of each car individually. For example, two cars, both travelling at 50 km h$^{-1}$ in opposite directions, collide head-on. The velocity of one car relative to the other is 100 km h$^{-1}$: $\left(v = 50 - (-50) = 100\,\text{km h}^{-1}\right)$.

**4** **a** The laws of physics are the same in all inertial reference frames, and light speed is constant in a vacuum, for all observers.

**b** In previous physics, the laws of electromagnetism were not the same in all inertial reference frames — light speed depended on the motion of the source and the receiver, so was not the same for all observers.

**5** False. By the principle of relativity, the situation must be symmetrical so that any two inertial reference frames are indistinguishable. Both observers see the other frame as moving and, therefore, both see the other's time running slow.

**6** **a** The proper time is the time that passes in the frame of reference in question. In this case, the proper time is 5 minutes.

**b** $t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$

$v = c\sqrt{\left(1 - \left(\frac{t_0}{t}\right)^2\right)}$

$= c\sqrt{\left(1 - \left(\frac{5}{8}\right)^2\right)}$

$= 0.78c$

**7** Accelerating even relatively small masses to near light speed involves enormous amounts of energy. This is not currently feasible. Additionally, humans are not able to sustain large accelerations for extended periods of time. Reaching near light speeds with safe accelerations would take years.

**8** $E = mc^2$

$= 0.25 \times \left(3 \times 10^8\right)^2$

$= 2.25 \times 10^{16}$ J

**9** All the chemical products of combustion must add up to less than the mass of the initial coal because energy has been released.

**10** $E = mc^2$

$m = \frac{E}{c^2}$

$= \frac{0.42 \times 10^6 \times 1.6 \times 10^{-19}}{\left(3 \times 10^8\right)^2}$

$= 7.47 \times 10^{-31}$ kg

## 11.7 Exam questions

### Section A — Multiple choice questions

**1** A. A bus travelling at a constant speed is an example of an inertial frame of reference. All the others are accelerating in some way.

**2** A. The 4.25 light-year distance is the proper length measured from Earth in which neither the Earth nor the star is moving. As this length is moving in Ning's frame of reference, it will appear contracted or less than 4.25 light-years.

**3** C

$\Delta m = 3 \times \left(6.645 \times 10^{-27}\right) - 1.992 \times 10^{-26}$

$\Delta m = 5 \times 10^{-30}$ kg

$E = mc^2$

$E = 5 \times 10^{-30} \times \left(3.0 \times 10^8\right)^2$

$E = 4.5 \times 10^{-12}$ J

**4** D. The description of the clock as stationary indicated that it too is travelling at $0.943c$ ($\gamma = 3.00$).

The description of the time interval of 75.0 s as a proper time indicates that the clock is operating as expected in its frame of reference. However, the clock is moving in Joanna's frame of reference so she will observe dilated time on the clock.

$t = t_0 \times \gamma$

$t = 75.0 \times 3.00$

$t = 225$ s

**5** A

$E = mc^2$

$\Delta m = \frac{3.6 \times 10^{-13}}{\left(3 \times 10^8\right)^2}$

$\Delta m = 4 \times 10^{-30}$ kg

The final mass will be $M_i - 4 \times 10^{-30}$ kg.

**6** D

$t = \frac{d}{v}$

$t = \frac{3 \times 10^{11}}{3 \times 10^8}$

$t = 1000$ s

**7** A

$\gamma = \frac{1}{\sqrt{1-0.2^2}}$

$\gamma = 1.02$

**8** D

$t = t_0\gamma$

$\gamma = \frac{16}{2.2}$

$\gamma = 7.3$

**9** C

$E = (\gamma - 1)mc^2$

$\Rightarrow (\gamma - 1) = 10$

$\therefore \gamma = 11$

**10** A

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$\gamma = \frac{1}{\sqrt{1-0.999^2}}$

$\gamma = 22.4$

## Section B — Short answer questions

**11** The transformation is a mass–energy transformation due to nuclear fusion. [1 mark] As the energy is radiated away, the mass of the star will decrease. [1 mark]

***VCAA examination report* note:** This question was not answered well. Almost 40% of students either did not respond to the question or gave a response that demonstrated no understanding of stellar processes. There was a group of students who explained the solar fusion process in detail but did not address the change in mass of the star and, conversely, a group who knew the mass would decrease but could not provide a coherent reason. The fission process was identified a number of times.

**12** **a** $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$\gamma = \frac{1}{\sqrt{1-0.954^2}}$ [1 mark]

$\gamma = 5.61$ [1 mark]

**b** $t = \frac{d}{v}$

$t = \frac{4.37}{0.984}$ [1 mark]

$t = 4.44$ years [1 mark]

**13** $t_{\text{Earth}} = \frac{10.5}{0.85}$

$= 12.35$ years [1 mark]

This is the dilated time as observed from the spaceship's frame of reference.

$t = t_0\gamma$

$t_{\text{Earth}} = t_{\text{spaceship}} \times \gamma$

$12.35 = t_{\text{spaceship}} \times 1.90$ [1 mark]

$t_{\text{spaceship}} = 6.5$ years [1 mark]

**14**

| | |
|---|---|
| length of side along *x*-axis | $4.5 \times 10^2$ m |
| length of side along *y*-axis | $3.2 \times 10^3$ m |
| length of side along *z*-axis | $3.2 \times 10^3$ m |

Award 1 mark for the 3 correct lengths.

$L = \frac{L_0}{\gamma}$

$L = \frac{3.2 \times 10^3}{7.09}$

$L = 4.5 \times 10^2$ m [1 mark]

Length contraction only occurs in the direction of travel, the *y* and *z* dimensions would remain unchanged. [1 mark]

***VCAA examination report* note:**

Students were required to identify that length contraction only occurs in the direction of travel and, therefore, the *y* and *z* dimensions would remain unchanged.

**15** $L = \frac{L_0}{\gamma}$

$L = \frac{1.0 \times 10^{18}}{7.1}$

$l = 1.41 \times 10^{17}$ m [1 mark]

$t = \frac{d}{v}$

$t = \frac{1.41 \times 10^{17}}{0.99 \times 3.0 \times 10^8}$ [1 mark]

$t = 15$ years [1 mark]

Alternatively:

$t = \frac{d}{v}$

$t = \frac{1.41 \times 10^{17}}{0.99 \times 3.0 \times 10^8}$

$t = 107$ years [1 mark]

$t = t_0\gamma$

$107 = t_0\gamma$ [1 mark]

$t_0 = 15$ years [1 mark]

# Unit 4 — Area of Study 1 review

## Practice examination

### Section A — Multiple choice questions

**1** D

The photons emitted during the transitions between these energy levels are expected to emit photons with a frequency given by

$$f = \frac{\Delta E}{h} = \frac{2.78}{4.14 \times 10^{-15}} = 6.7 \times 10^{14}\text{ Hz}$$

**2** C

According to the second postulate of Einstein's Special Theory of Relativity, the speed of light is constant in all inertial frames of reference, and is equal to $c$ in a vacuum.

**3** B

The de Broglie wavelength is given by $\lambda = \frac{h}{p} = \frac{h}{mv}$.

Rearranging the equation we obtain

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{1.20 \times 10^{-24} \times 2.5 \times 10^{-12}} = 221\text{ ms}^{-1}$$

**4** B

The wave in diagram (b) has a lower frequency and a lower amplitude. The lower amplitude means that the light is dimmer. Blue light has a higher frequency than green light, whereas red light has a lower frequency. Hence, the wave in diagram (b) is dimmer and red.

**5** A

The amplitude of the wave is a measure of the intensity of the energy propagated by the wave, and so it determines the brightness of the light. Note that this question relates to the wave model of light. The particle model of light suggests that the brightness of light depends on the number of light photons.

**6** D

2.2 μs is the proper time of the muons.

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{1 - 0.995^2} = 10.01$$

$$t = t_0\gamma$$

$$t \approx 2.2 \times 10.01 = 22\text{ s}$$

**7** C

The standing wave depicted is the 4th harmonic as it has 4 antinodes (maxima). Hence the frequency is $4 \times f_0 = 4 \times 220 = 880\text{ Hz}$.

**8** A

The wavelengths of X-rays are in the range of $5 \times 10^{-1}$ m, whereas DNA molecules are in the range of $3 \times 10^{-9}$ m. Therefore, the ratio $\frac{\lambda}{w}$ is much less than 1; with little diffraction occurring, it is possible to distinguish the edge of objects on the scale of DNA.

**9** C

If a higher frequency light source is used, it is expected that the photocurrent will increase due to the higher rate of photoelectrons produced (rather than increasing the kinetic energy). All the other statements are consistent with the wave model of light. For example, wave energy is dependent on the intensity of the light source; thus, a higher intensity light source would increase the kinetic energy of the photoelectrons.

**10** D

The gradient of the $E_k$ vs $f_{photon}$ graph is unchanged, being Planck's constant, $h$. A higher work function should result in a higher threshold frequency, hence option D.

**11** A

The particle model of light suggests that low-frequency light, which has a lower photon energy, is unable to produce a photocurrent.

**12** A

The photon energy of the light is

$$E = hf = 4.14 \times 10^{-15} \times 7.30 \times 10^{14} = 3.02\text{ eV}$$

The kinetic energy of the photoelectron is

$$E_k = E_{photon} - \phi = 3.02 - 1.23 = 1.79\text{ eV}$$

**13** D

The work function of a metal is given by

$$\phi = hf_0 = 6.63 \times 10^{-34} \times 5.56 \times 10^{14} = 3.7 \times 10^{-19}\text{ J}$$

**14** C

Matching diffraction patterns indicates that the wavelength of the X-rays matches the de Broglie wavelength of the electrons. Since the wavelength is related to the momentum by $p = \frac{h}{\lambda}$, the momentum of the X-rays must also match the momentum of the electrons.

**15** A

The de Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{0.046 \times 70} = 2.1 \times 10^{-34}\text{ m}$$

**16** B

The Michelson–Morley experiment measured the speed of light as being the same in different directions, meaning that there is not a medium (the luminiferous aether) through which light propagates. The conclusion from this experiment is that the speed of light is constant, independent of the motion of the source of light or of the observer.

**17** C

$E_k = (\gamma - 1)mc^2$

$$\gamma = \frac{E_k}{mc^2} + 1$$

$$= \frac{8.2 \times 10^{-14}}{9.1 \times 10^{-31} \times (3 \times 10^8)^2} + 1$$

$= 2.0$

**18** D

The Lorentz factor is given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - \frac{0.7^2}{1^2}}}$$

$= 1.4$

According to observers on Earth, the trip takes:

$$t = \frac{d}{v}$$

$$= \frac{3.9 \times 10^{18}}{0.7 \times 3 \times 10^8}$$

$= 1.9 \times 10^{10}$ s

The distance between Earth and the star, as measured by the occupants of the spaceship, is length-contracted:

$$L = \frac{L_0}{\gamma}$$

$$= \frac{3.9 \times 10^{18}}{1.4}$$

$= 2.786 \times 10^{18}$ m

According to the astronauts on the spaceship, the trip takes:

$$t = \frac{d}{v}$$

$$= \frac{2.786 \times 10^{18}}{0.7 \times 3 \times 10^8}$$

$= 1.3 \times 10^{10}$ s

**19** B

The Lorentz factor for $v = 0.87\,c$ is

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{0.87^2}{1^2}}} = 2.03$$

Measured from the neutrino's frame of reference, Earth's diameter would have undergone length contraction, thus its length according to the neutrino's frame of reference is

$$L = \frac{L_0}{\gamma} = \frac{1.3 \times 10^7}{2.03} = 6.4 \times 10^6 \text{ m.}$$

**20** B

Electromagnetic waves can be produced by electron energy-level transitions, accelerating electric charges, as well as by various other means.

## Section B – Short answer questions

**21 a** The wavelength of the photon is given by

$$\lambda = \frac{c}{f}$$

$$= \frac{3.0 \times 10^8}{4.7 \times 10^{14}}$$

$= 6.4 \times 10^{-7}$ m [1 mark]

**b** The energy of a photon is given by

$E = hf$

$= 4.14 \times 10^{-15} \times 4.7 \times 10^{14}$

$= 1.9$ eV [1 mark]

**c** The momentum of a photon is given by

$$p = \frac{h}{\lambda}$$

$$= \frac{6.63 \times 10^{-34}}{6.4 \times 10^{-7}}$$

$= 1.0 \times 10^{-27}$ N s [1 mark]

**22 a** Constructive and destructive interference of coherent waves in terms of path differences: $n\lambda$ and $\left(n + \frac{1}{2}\right)\lambda$ where $n = 0, 1, 2,$ . At a dark band, destructive interference is taking place [1 mark].

The third dark band is therefore the result of a path difference of:

$$\left(2 + \frac{1}{2}\right)\lambda = 2.5\lambda$$

$= 2.5 \times 530.0$

$= 1325$ nm [1 mark]

**b** Constructive and destructive interference of coherent waves in terms of path differences: $n\lambda$ and $\left(n + \frac{1}{2}\right)\lambda$ where $n = 0, 1, 2, \cdot$. At a bright band, constructive interference is taking place [1 mark].

The second bright band is therefore the result of a path difference $2\lambda$. The path difference is $2 \times 530 = 1060$ nm [1 mark].

**23 a** The work function of a metal is given by

$\Phi = hf_0$

$= 6.63 \times 10^{-34} \times 4.3 \times 10^{14}$

$= 2.9 \times 10^{-19}$ J [1 mark]

**b** The maximum kinetic energy of the photoelectrons is given by

$E_k = E_{photon} - \Phi$

$= hf - \Phi$ [1 mark]

$= 6.63 \times 10^{-34} \times 5.7 \times 10^{14} - 2.9 \times 10^{-19}$

$= 8.8 \times 10^{-20}$ J [1 mark]

**c** The stopping voltage is the electron volt equivalent of the maximum kinetic energy,

$$V = \frac{E_k}{e}$$

$$= \frac{8.8 \times 10^{-20}}{1.6 \times 10^{-19}}$$ [1 mark]

$= 5.5 \times 10^{-1}$ V [1 mark]

**d** According to the particle model of light, intensity is related to the number of photons [1 mark]. Hence, increasing the number of photons will increase the magnitude of the photocurrent as more electrons are liberated from the metal [1 mark]; however, the stopping voltage will remain the same as the energy of each light photon is the same [1 mark].

**24** The wavelength of the light source,

$$\lambda = \frac{w\Delta x}{L} \quad \text{[1 mark]}$$

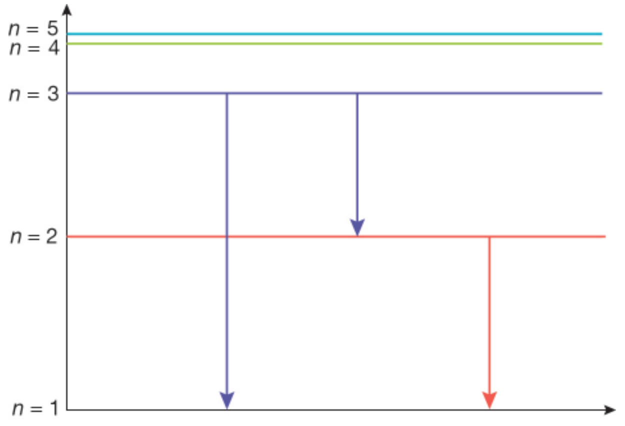
$$= \frac{0.15\times10^{-3}\times5.8\times10^{-3}}{1.65}$$

$$= 5.3\times10^{-7}\text{ m} \quad \text{[1 mark]}$$

**25** **a**
- Diffraction is a property of waves. [1 mark]
- Electrons can undergo diffraction when passed through atomic crystals, and electrons are matter. [1 mark] Thus electron diffraction is evidence for the wave-like nature of matter.

**b**
- The photoelectric effect supports the particle theory of light because the energy required to release photoelectrons from a metal is dependent upon the frequency of light, and not the intensity of light. [1 mark]
- The photoelectric effect proves that energy is quantised. This contradicts the wave model for light with energy arriving continuously. [1 mark]

**26** **a**




Award (1 mark) for all 3 correctly drawn arrows.

**b** The difference in the energy levels is

$$\Delta E = 23.8 - 20.6$$

$$= 3.2\text{ eV} \quad \text{[1 mark]}$$

The frequency of the photon emitted is

$$f = \frac{\Delta E}{h}$$

$$= \frac{3.2}{4.14\times10^{-15}}$$

$$= 7.7\times10^{14}\text{ Hz} \quad \text{[1 mark]}$$

**27** **a** The wavelength of the X-rays used is given by

$$\lambda = \frac{hc}{E}$$

$$= \frac{4.14\times10^{-15}\times3.00\times10^{8}}{2.45\times10^{4}}$$

$$= 5.07\times10^{-11}\text{ m} \quad \text{[1 mark]}$$

**b** As the diffraction patterns are similar, the de Broglie wavelength of the electrons is the same as that for the X-rays, $5.07\times10^{-11}$ m. [1 mark]

**c** The kinetic energy of the electrons may be calculated from the de Broglie wavelength,

$$E_k = \frac{h^2}{2m\lambda^2} \quad \text{[1 mark]}$$

$$= \frac{(6.63\times10^{-34})^2}{2\times9.1\times10^{-31}\times(5.07\times10^{-11})^2}$$

$$= 9.4\times10^{-17}\text{ J} \quad \text{[1 mark]}$$

**d** The accelerating voltage is the electron volt equivalent of the kinetic energy, that is

$$V = \frac{E_k}{e} \quad \text{[1 mark]}$$

$$= \frac{9.4\times10^{-17}}{1.6\times10^{-19}}$$

$$= 5.9\times10^{2}\text{ V} \quad \text{[1 mark]}$$