

MATHEMATICAL METHODS (CAS)

Teach Yourself Series

Topic 4: Exponential and Log Functions

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Exponential and Log Functions

In this topic you will learn the basic concepts around exponential and logarithmic functions.

Index laws

As it appears in Unit 2 & 3

$$a^x \times a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$a^{-x} = \frac{1}{a^x}$$

$$a^0 = 1$$

$$a^{\frac{x}{y}} = \sqrt[y]{a^x}$$

Some basic tips to remember:

1. Express surds as fractional powers
2. Remove brackets (3rd law)
3. Apply 1st and 2nd laws
4. Use the Reciprocal Law last.
5. Numbers raised to powers expressed in lowest base

Log laws

As it appears in Unit 2 & 3

$$\log_x a + \log_x b = \log(ab)$$

$$\log_x a - \log_x b = \log_x \frac{a}{b}$$

$$\log_x a^n = n \log_x a$$

$$\log_x a = 1$$

$$\log_a x = \frac{\log_{10} x}{\log_{10} a} = \frac{\log_e x}{\log_e a}$$

Some basic rules to remember when using the log laws in simplifying expressions:

- Attempt to reduce the number of logs in the expression to just the one.
- Use 3rd log law before you use the 1st and 2nd.
- Index laws can sometimes help to simplify expressions.
- Look for log statements that can be evaluated.

Equation Solving

Exponential

As it appears in Unit 2 & 3

Type 1: Like base method

Type 2: Use logs – looks like type one

Type 3: Terms added together – substitution method

Type 4: unknown in base – use opposite operations – watch for $\pm$ answers

May have to rearrange equation to make it look like one of the above types - particularly if the equation is written as a fraction.

Log

As it appears in Unit 2 & 3

Type 1: Unknown inside log – use log laws to get multiple logs into singular logs. Opposite to $\log_a$ is to the power of a .

Type 2: Unknown is the base – rewrite in index form and solve.

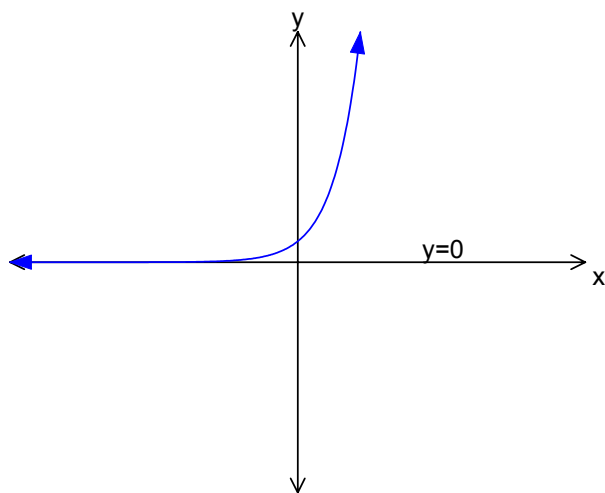
Exponential Graphs

As it appears in Unit 2 & 3

Exponential functions are written in the form $y = a^x$ where a can be any number.

When sketched these have **horizontal** asymptotes. All transformations apply to these graphs.

The basic shape for exponential function:



For functions of the form $f(x) = n.a^{x-h} + k$, $n, h, k \in R$ and $a \in R \setminus \{1\}$:

Maximal Dom is R .

Range is (k, ∞) .

Asymptote: $y = k$.

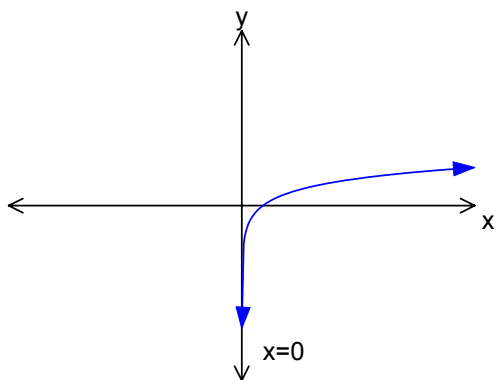
Log Graphs

As it appears in Unit 2 & 3

Log functions are written in the form of $y = \log_a x$. a is the base in which you have to draw the graph in. the calculator will only graph $\log_{10}$ and $\log_e$ or $\ln$. Other bases need to be worked out by you.

When these are sketched, they have **vertical** asymptotes. All transformations apply to these graphs.

The basic shape for log functions are:



For graphs of the form $f(x) = n \log_a(x - h) + k$, where $n, h, k \in R$ and $a \in R^+ \setminus \{1\}$:

Maximal Domain (h, ∞)

Range R

Vertical Asymptote: $x = h$. This can be found by equating the bracket to zero and solving.

Be careful of reflections across the y axis.

Calculator Skills

Define functions and evaluate them

Solve functions

Graph functions

Review Questions

1. Simplify $\frac{4(xy^2)^3 \times (2x^3y)^2}{xy^2(2x)^4}$.

2. Simplify $\left(\frac{64}{27}\right)^{-\frac{2}{3}} \div \left(\frac{9}{16}\right)^{\frac{3}{2}}$.

3. Simplify $\frac{x}{1+x^{-1}} + \frac{x}{x^{-1}-1}$.

a. Solve for x where: $2\log_5(x-3) = 4$.

b. Simplify $\frac{\log_4 x^5}{\log_4 \sqrt[3]{x}}$.

4. Solve for x where $2^{2x} + 4 \times 2^x - 32 = 0$.

5. A solution for x in $4^{-x} = 9$ exact to three decimal places is.

6. Solve for x where: $\log_2 x + 2\log_2 10 - 3\log_2 5 = 3$.

7. Solve for x where $\log_x 96 - \log_x 12 = 6$.

8. Solve for x where: $(\log_3 x)^2 - 4\log_3 x + 3 = 0$.

9. Simplify $5\log_3 243 - 6\log_2 \left(\frac{1}{32}\right)$.

10. If $R = 5\log_3 \left(\frac{a}{T}\right) + B$, calculate a value for R , when $a = 72, T = 8$ and $B = -4$.

11. Solve for x where: $256^{-x} = \frac{4^x}{16^{3-x}}$.

12. Solve for x where $e^{5-x} = \frac{2}{3e^{4x}}$.

13. Solve for x where: $\log_e(1-x) = 3$.

14. Solve for x where $\log_e(x-3) + \log_e(x-2) = \log_e 12$.

15. Solve for y where: $4 - \log_e x = 2 \log_e y$.

16. The population of bacteria is given by: $P(t) = 500e^{0.3t}$, where t is the time measured in weeks. The number of bacteria in the pond after 10 weeks would be closest to:

17. The height of a certain species of tree can be modelled by the function: $H = \frac{12}{8 + 100e^{-0.3t}}$, where H is the height of the tree in metres and t is the age of the tree in years since it was planted. The height of the tree when planted is approximately:

18. Sketch the graphs of the following stating domain and range:

a. $f(x) = 2e^{3x} - 6$.

b. $f(x) = -3e^x + 4$.

19. Sketch the following graphs stating domain and range:

a. $f(x) = \log_e(2 - x)$.

b. $f(x) = -3\log_e(2x + 3) + 4$.

20. The size of a population of rabbits is determined by the rule $P = 6400 \times 3^{0.2t} - 400$ where P is the size of the population t years after January 2006.

a. Find the size of the population when $t = 0$ and $t = 15$.

b. Find the value of t when the population exceeds 1,000,000.

c. Sketch the graph of P vs. t .

Solutions to Review Questions

1.

$$x^4y^6$$

Remove brackets.

Then use the addition and subtraction laws.

2.

$$\frac{4}{3}$$

Express numbers in bases in lowest base (2 and 3).

Remove brackets and simplify.

3.

$$\frac{x^2}{(x+1)(x-1)}$$

Express denominators over lowest common denominator.

Then add fractions.

4.

$$x = 28$$

Divide through by 2.

Raise both sides to the power of 5.

5.

$$15$$

Express the cube root as a power of $\frac{1}{3}$.

Bring both powers to the front using index law.

Cancel logs

6.

$$x = 2$$

Let $2^x = A$ to set up quadratic equation $A^2 - 4A - 32 = 0$

Solve in terms of A

Then substitute 2^x back instead of having A and solve.

7.

$$x = -\frac{\log 9}{\log 4} = -\frac{\log 3}{\log 2} = -1.585$$

Use index law to bring the power of x out of the log. Then solve.

8.

$$x = 10$$

Put the 2 and the 3 inside the log using log law.

Simplify the multiple logs into single log expression

9.

$$x = \sqrt{2}$$

Use division log law. Rewrite into index form then solve.

10.

$$x = 27 \text{ or } x = 3$$

Let $\log_3 x = A$ to set up quadratic equation $A^2 - 4A + 3 = 0$

Solve in terms of A

Then substitute $\log_3 x$ back instead of having A and solve.

11.

$$55$$

Evaluate logs.

12.

$$R = 6$$

Substitute values into expression and simplify.

13.

$$x = \frac{6}{7}$$

Express each term as a base of 2.

Express right hand side in terms of one base.

Equate the powers.

14.

$$x = \frac{1}{3} \log_e \left(\frac{2}{3e^5} \right)$$

Multiply both sides by e^{4x} .

Add the powers of e .

Divide through by e^5

Take $\log_e$ of both sides of the equation.

15.

$$x = 1 - e^3$$

Raise both sides as a power of e .

16.

$$x = 6$$

Use addition log law.

Equate expression inside log.

Solve quadratic equation by factorising.

Can't have $x = -1$ as you can't log a negative number when you test the solution.

17.

$$y = e^2 / \sqrt{x}$$

Divide through by 2.

Raise both sides as a power of e .

18.

10043

Evaluate $P(10)$.

19.

11.1 cm

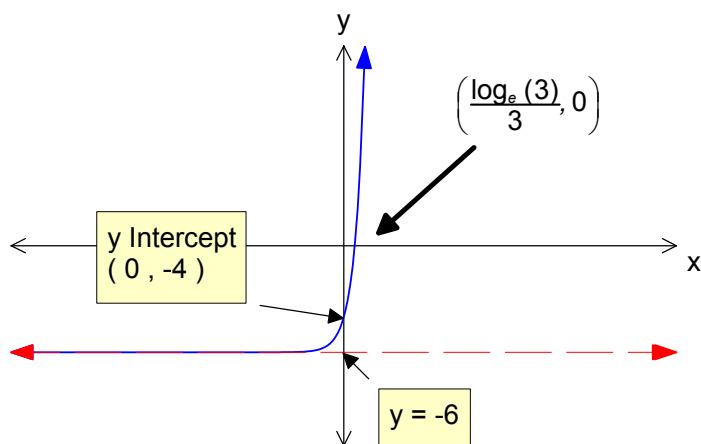
Evaluate $H(0)$

20.

a.

Dom: $\mathbb{R}$

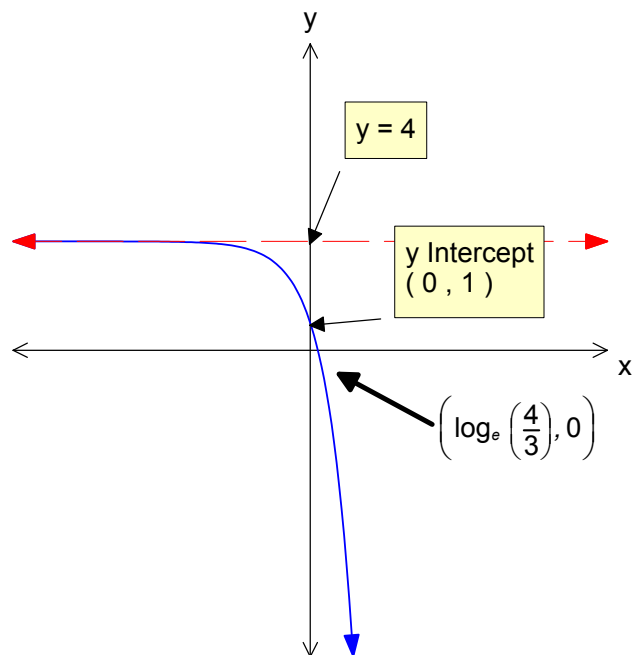
Ran: $(-6, \infty)$



b.

Dom: $\mathbb{R}$

Ran: $(-\infty, 4)$

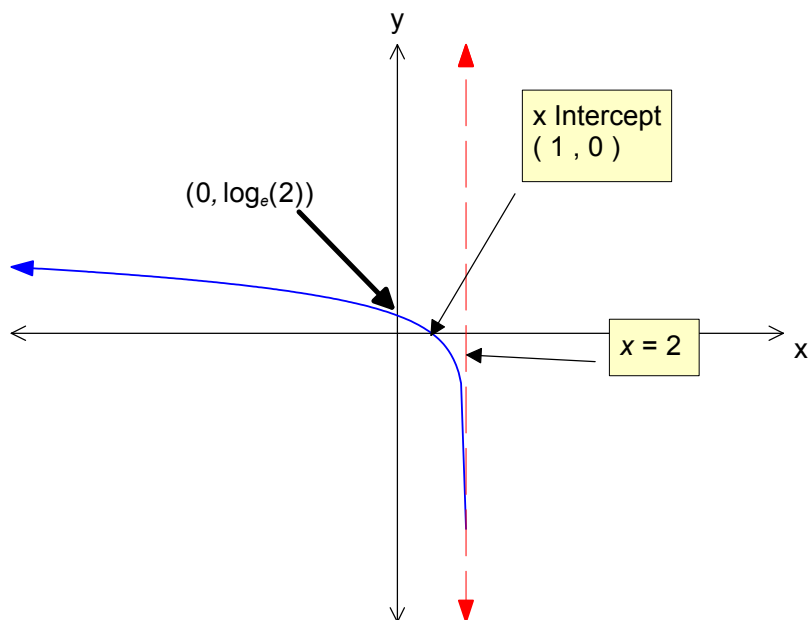


21.

a.

Dom: $(-\infty, 2)$

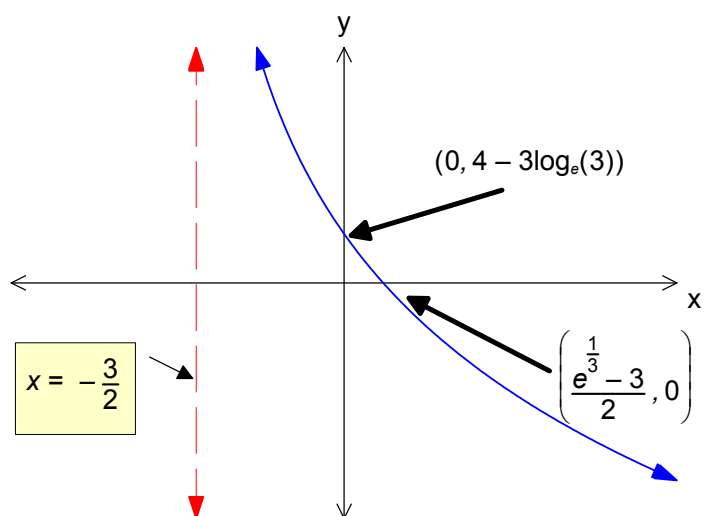
Ran: $\mathbb{R}$



b.

Dom: $\left(-\frac{3}{2}, \infty\right)$

Ran: $\mathbb{R}$



22.

a.

$$P(0)=6000$$

$$P(15)=172400$$

b.

$$t \approx 23 \text{ years}$$

$$\text{Solve } 1000000 = p(t)$$

Could use calculator.

c.

