

MATHEMATICAL METHODS (CAS)

Teach Yourself Series

Topic 2: Curve Sketching 2 – Semicircles, Circles, Rectangular Hyperbolae, Truncus, Reciprocal Curves and Square Root Curves

Contents

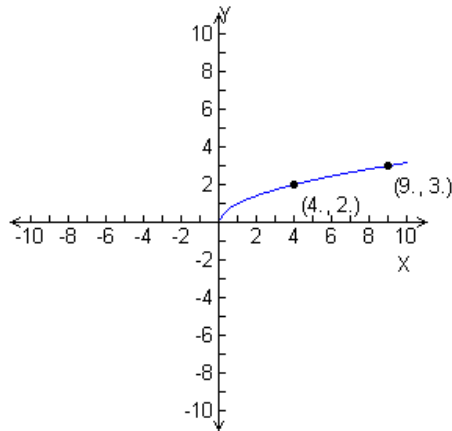
Curve Sketching 2	3
Square root graphs	3
As it appears in Unit 1	3
Truncus graphs	3
As it appears in Unit 1	3
Hyperbolae	4
As it appears in Unit 1	4
Circles	5
As it appears in Unit 1	5
Reciprocal Functions	6
As it appears in Unit 3	6
Review Questions	7
Solutions to Review Questions	12

Curve Sketching 2

Square root graphs

As it appears in Unit 1

The basic square root function is given by $f(x) = \sqrt{x}$.



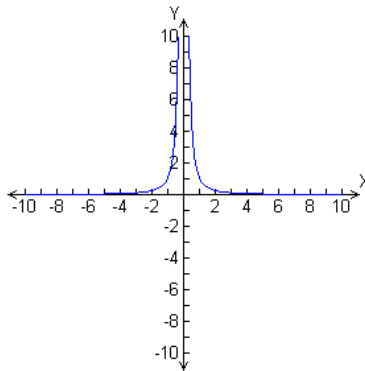
Dom: $\mathbb{R}^+ \cup \{0\}$ Ran: $\mathbb{R}^+ \cup \{0\}$

Truncus graphs

As it appears in Unit 1

These are functions written in the form of $f(x) = \frac{1}{x^2}, (x^{-2})$. The basic truncus looks like:

$$f(x) = \frac{1}{x^2}$$



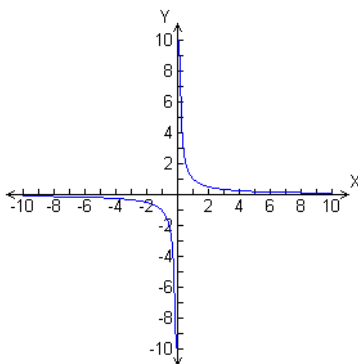
Note that these functions also have vertical and horizontal asymptotes.

Hyperbolae

As it appears in Unit 1

The basic hyperbola has the equation $f(x) = \frac{1}{x}, (x^{-1})$. There are special features on a hyperbola called asymptotes. Refer to the graph below:

$$f(x) = \frac{1}{x}$$



$$\text{Dom: } \mathbb{R} \setminus \{0\}$$

$$\text{Ran: } \mathbb{R} \setminus \{0\}$$

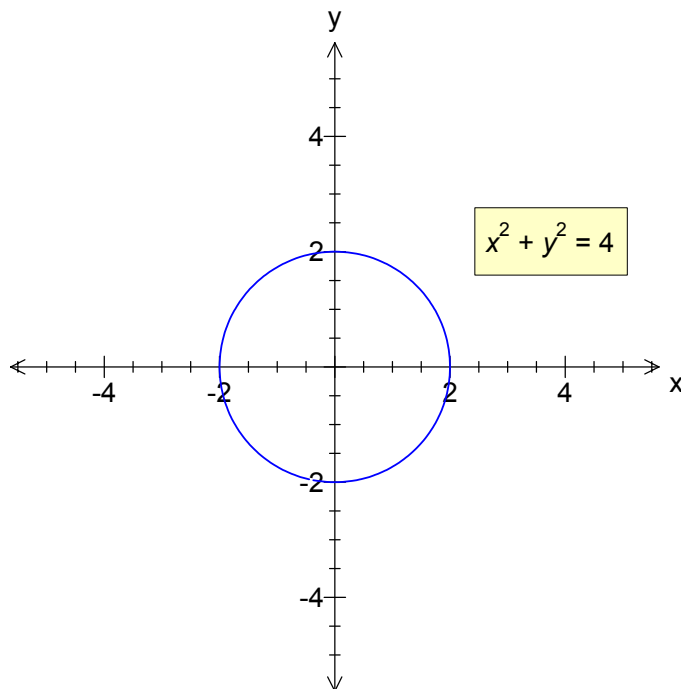
$x = 0$ is an asymptote.

$y = 0$ is an asymptote.

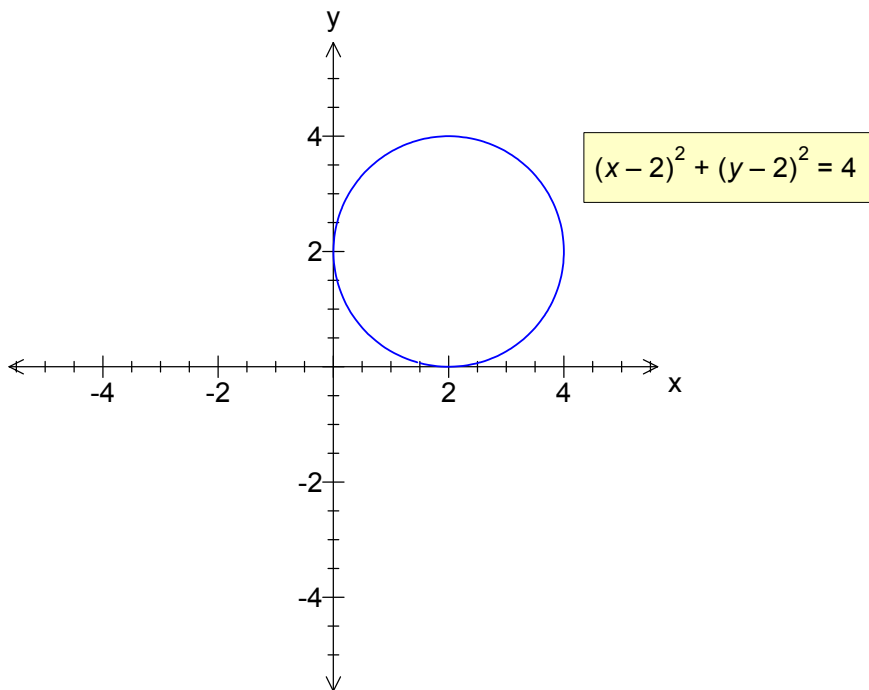
Circles

As it appears in Unit 1

The basic equation is $x^2 + y^2 = r^2$. This is a circle centered at the origin with a radius of r .



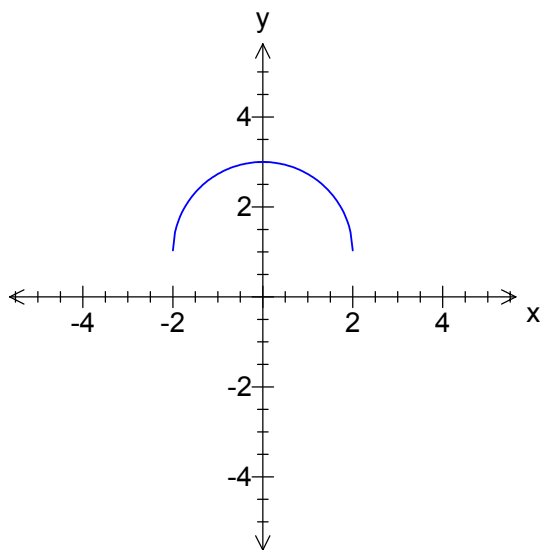
This equation can be translated in the x and y direction: $(x - h)^2 + (y - k)^2 = r^2$. This is a circle centered at (h,k) with a radius of r .



This equation can be transposed to make y the subject:

$$y = \pm\sqrt{r^2 - (x - h)^2} + k .$$

This gives us semi circles:



Reciprocal Functions

As it appears in Unit 3

These are graphs that are written in the form of $y = \frac{1}{f(x)}$. To sketch these use the basic process of:

1. Sketch $y = f(x)$ first.
2. Mark in vertical asymptotes. These are where $f(x) = 0$.

3. Now sketch $y = \frac{1}{f(x)}$ on all side of asymptotes. You need to take note of the reciprocal behaviour of the function.

Calculator skills

Graphing functions – setting up an appropriate window

Defining functions and evaluating them.

Factor

Prop Frac – to convert a hyperbola into a form that can be graphed.

Review Questions

1. State the maximal domain and the corresponding range of each of these functions:

a. $f(x) = \sqrt{x} + 4$.

b. $g(x) = \frac{1}{x-2}$.

c. $h(x) = \frac{1}{(x+2)^2} + 3$.

2. Sketch the graphs of each of the following. Label all important features. State the maximal domain and range of each of the functions.

a. $f(x) = \sqrt{x-4}$.

b. $f(x) =$

c. $f(x) = \frac{4}{(x-3)^2} - 4$.

d. $f(x) = 3 - \frac{2}{x-4}$.

e. $f(x) = \frac{5}{x+6}f$.

f. $f(x) = 6 + \sqrt{4x-1}$.

3. Sketch the following graphs stating the domain and range:

a. $x^2 + y^2 = 9$.

b. $x^2 + (y + 5)^2 = 25$.

c. $(x - 7)^2 + (y + 7)^2 = 49$.

d. $y = \sqrt{9 - x^2} - 3$.

e. $y = \sqrt{(25 - (x - 5)^2)}$.

4. If $f(x) = x^2 + 4$, Sketch $\frac{1}{f(x)}$.

5. If $f(x) = x^2 + 6x + 8$, Sketch $\frac{1}{f(x)}$.

Solutions to Review Questions

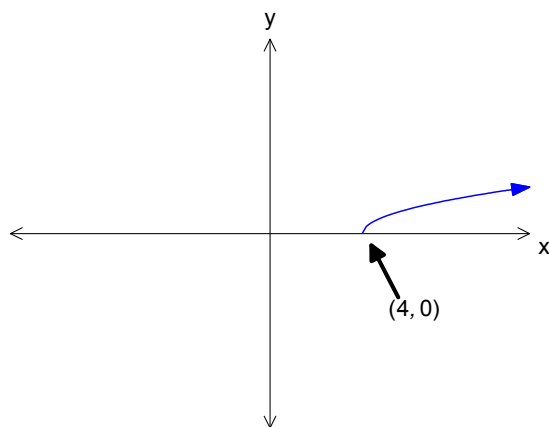
1.

- a. $\mathbb{R}^+$. Range is $[4, \infty)$
- b. $\mathbb{R} \setminus \{2\}$ Range is $\mathbb{R} \setminus \{0\}$
- c. $\mathbb{R} \setminus \{-2\}$ Range is $(3, \infty)$

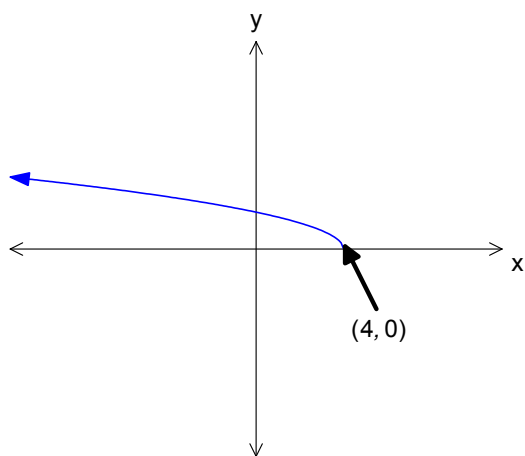
Sketch a graph of each to work out domain.

2.

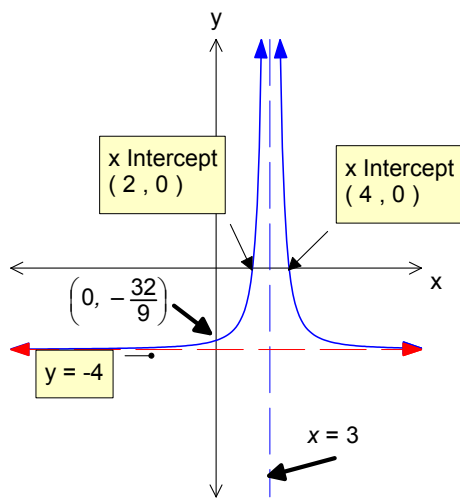
- a. Dom: $[4, \infty)$
Ran: $\mathbb{R}^+ \cup \{0\}$



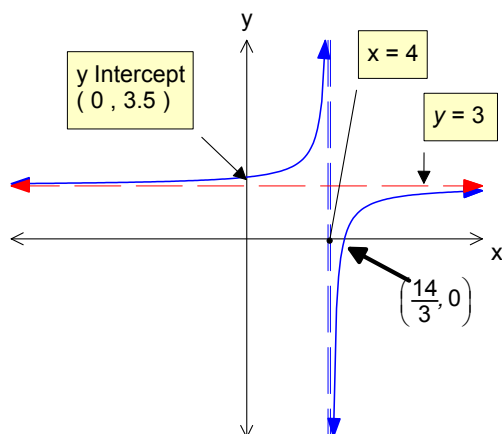
- b. Dom: $(-\infty, 4]$
 Ran: $\mathbb{R}^+ \cup \{0\}$



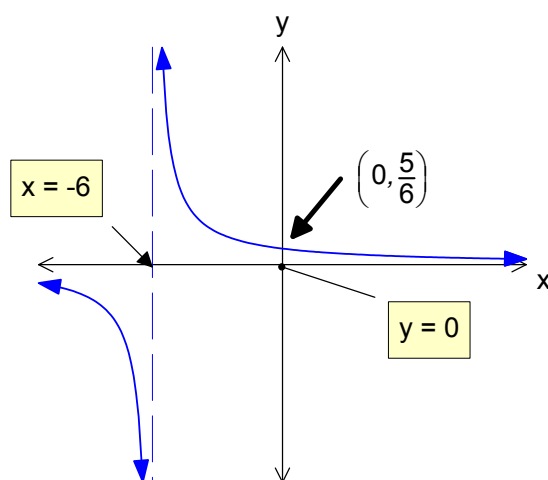
- c. Dom: $\mathbb{R} \setminus \{3\}$
 Ran: $(-4, \infty)$



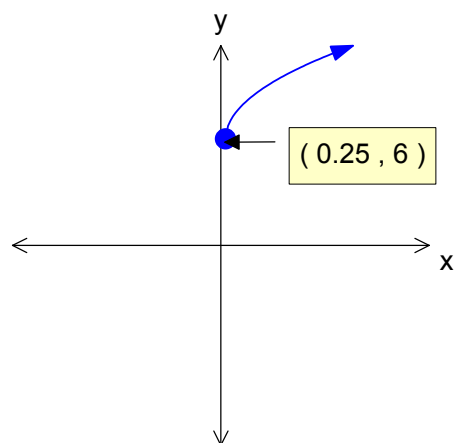
- d. Dom: $\mathbb{R} \setminus \{4\}$
 Ran: $\mathbb{R} \setminus \{3\}$



- e. Dom: $\mathbb{R} \setminus \{-6\}$
 Ran: $\mathbb{R} \setminus \{0\}$



f. Dom: $\left[\frac{1}{4}, \infty\right)$
 Ran: $[6, \infty)$

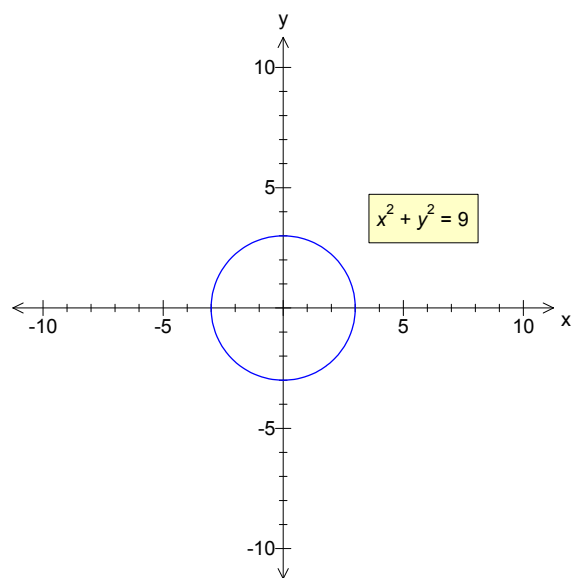


3.

a.

Domain: $[-3, 3]$

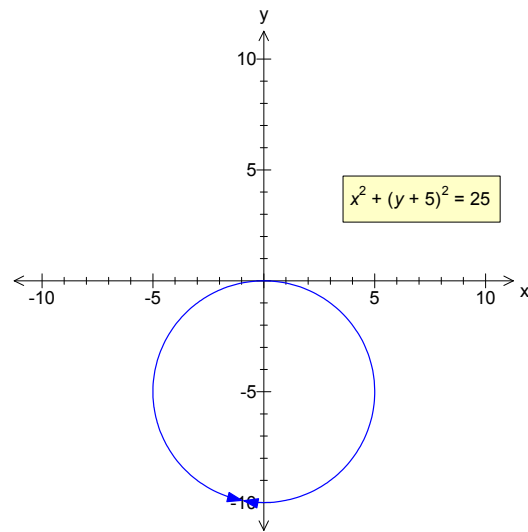
Range: $[-3, 3]$



b.

Domain: $[-5, 5]$

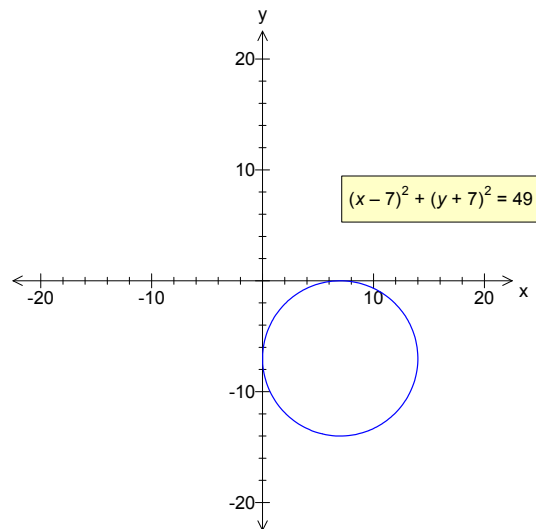
Range: $[-10, 0]$



c.

Domain: $[0, 14]$

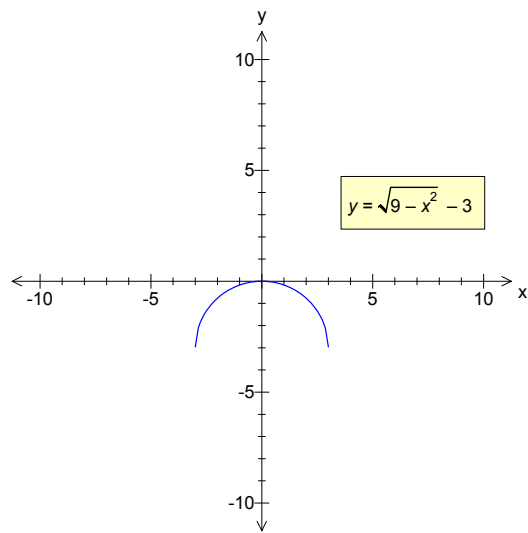
Range: $[-14, 0]$



d.

Domain: $[-3,3]$

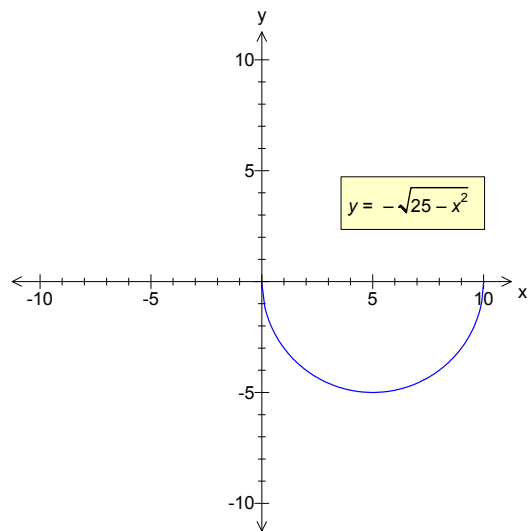
Range: $[-3,0]$



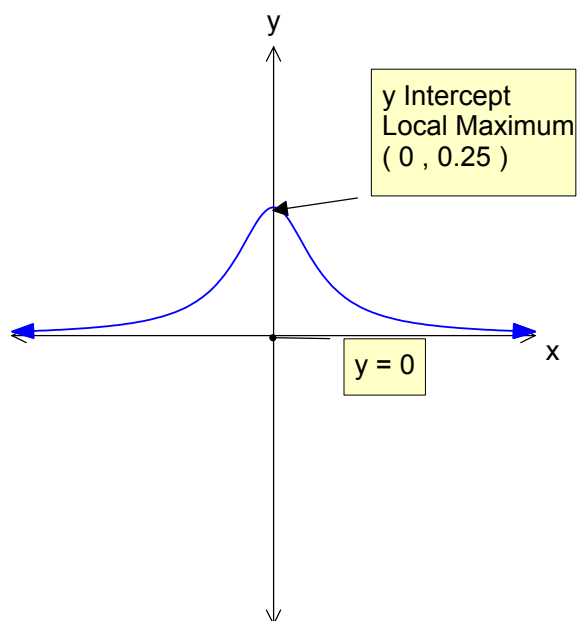
e.

Domain: $[0,10]$

Range: $[-5,0]$

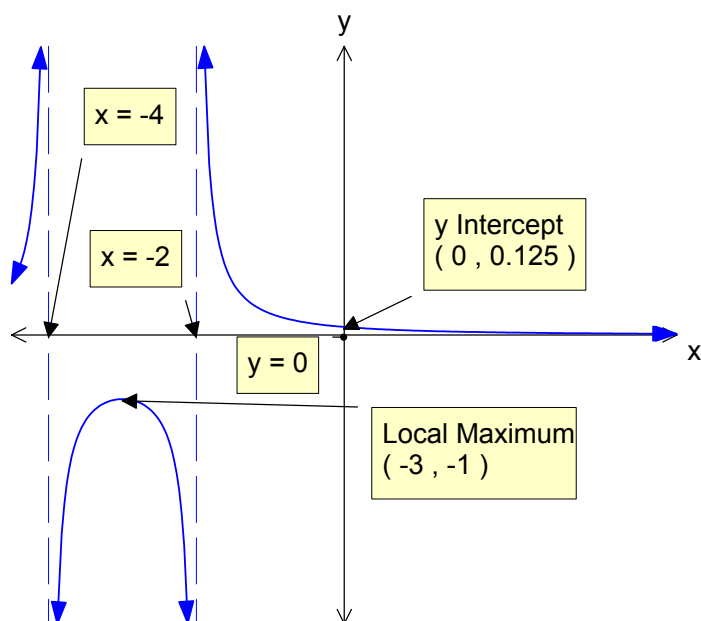


4. If $f(x) = x^2 + 4$, Sketch $\frac{1}{f(x)}$



Sketch $y = x^2 + 4$ first.
Then reciprocate the y values.

5.



Sketch $x^2 + 6x + 8$ first, then reciprocate y values.
 x intercepts on first graph become vertical asymptotes.