

Student NAME: _____

XAVIER COLLEGE
MATHEMATICAL METHODS UNIT 4
School Assessed Coursework 3
SAC 3: Problem-Solving Task 2
(Probability and Statistics)



Reading Time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
6	6	50

SAC 3 Problem-Solving Task 2 consists of 6 extended-response questions.

- Students are permitted to bring into the assessment room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aides for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are **NOT** permitted to bring into the assessment room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book, sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the assessment room.

Mathematical Methods formulae

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{p} = \frac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$

COMPULSORY STUDENT DECLARATION:

I, _____, acknowledge that I have read the SAC/examination conditions and understand which items/materials I am permitted to use and have in my possession.

****If you have any doubts as to what is permitted, raise your hand and DO NOT sign this declaration****

Student's signature: _____

Teacher's name: _____

Teacher's name: _____

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given *unless otherwise specified*.
In questions where more than **one mark** is available, appropriate working **must** be shown.

Question 1

The probability distribution of a discrete random variable, W , is given by the following table:

w	0	1	2	3	4	5
$Pr(W = w)$	0.2	$0.3k^2$	$0.4k$	0.3	$0.1k$	$0.7k^2$

- a) Show that k must be equal to $\frac{1}{2}$.

2 marks

- b) Find:

- i. $E(W)$ *correct to 2 decimal places*

1 mark

- ii. $\text{Var}(3W + 1)$ *correct to 4 decimal places*

2 marks

Total 5 marks

Question 2

A new virus kills 65% of fish with which it comes into contact. A tank has a large number of fish when the virus is introduced. It is expected that 129 of the fish will survive.

a) Find:

- i.** the number of fish in the tank. *Give answer to the nearest whole number and use this value for all parts that follow.*

1 mark

- ii.** the variance, *correct to 4 decimal places*, in the number of fish that will survive.

1 mark

- iii.** the interval for the number of fish to survive, for which we can be 95% certain.

2 marks

b) Find the probability, *correct to 4 decimal places*, that:

- i.** exactly 250 of the fish in the tank die.

2 marks

- ii.** exactly no more than 200 fish die given that at least 150 fish survive.

3 marks

Total 9 marks

Question 3

The weight, in kg, of a particular breed of cat is given by the probability distribution

$$f(w) = \begin{cases} \frac{2}{27}w(w-1) & \text{for } 1 \leq w \leq 4 \\ 0 & \text{everywhere else} \end{cases}$$

a) Find:

i. the maximum weight of a cat

1 mark

ii. the average weight of a cat. *Give answer in exact form.*

2 marks

iii. the standard deviation of the weight of a cat, *correct to the nearest gram.*

3 marks

b) Find the median weight of a cat, *correct to two decimal places.*

2 marks

c) Calculate the probability, *correct to 4 decimal places*, that a cat:

i. weighs more than 3 kg

1 mark

ii. weighs between 3.5 kg and 4 kg

1 mark

Total 10 marks

Question 4

The time taken to eat out at a particular restaurant, T , is normally distributed with a mean of 2.25 hours and a variance of 45 minutes.

- a) What time range will 95% of customers dine for? *Give answer expressed in nearest whole number of minutes.*

2 marks

- b) Find the probability, *correct to 4 decimal places*, that a couple will dine out for greater than 2 hours but less than 4 hours

1 mark

- c) 70% of diners will dine longer than a certain time. Find this time. *Give answer in nearest whole number of hours and minutes.*

1 mark

- d) If $\Pr(T < 1) = \Pr(Z > k)$, find the value of k , *correct to 2 decimal places*.

2 marks

- e) A couple who are known to never dine for less than the average time book a table at 6 pm. At what time should this table next be made available if the proprietor of the restaurant wants to be 90% certain that it will be free?

4 marks

Total 10 marks

Question 5

At a seminar for Mathematical Methods, teachers are quizzed on the other mathematics subjects that they teach. It is found that 45% teach Further Mathematics, 5% teach both Specialist and Further, and 25% teach no other mathematics aside from Mathematical Methods.

a) What is the probability that a teacher:

i. teaches Further but not Specialist?

1 mark

ii. does not teach Specialist?

1 mark

iii. teaches Specialist knowing that they do not teach Further, *correct to 2 decimal places?*

1 mark

At the conference, there are two options for lunch: vegetarian and non-vegetarian. It is estimated that teachers who teach only Further are 20% likely to choose vegetarian, those who teach only Specialist are 45% likely to choose vegetarian and those who don't teach either or teach both are 40% likely to choose vegetarian. *Use a Venn diagram, tree diagram or Karnaugh map.*

- b)** What is the probability that the next person in line will choose the vegetarian option?
Give answer correct to 3 decimal places.

2 marks

- c)** Given that a person chose a non-vegetarian option for lunch, what is the probability that they teach only Specialist? *Give your answer correct to four decimal places.*

1 marks

Total 6 marks

Question 6

A new chocolate is being trialled by a burgeoning company, and in a blind taste testing it was found that 78% of people preferred it over big-name brands. Further analysis of the sample produced a standard deviation of 4.5%.

- a) Find the size of the sample, *correct to the nearest whole number*.

2 marks

- b) Give a 95% confidence interval, *correct to 3 decimal places*, for the proportion of people who prefer the new chocolate.

1 mark

A second sample of 1000 people is taken, for which the standard deviation is found to be 1.5%.

- c) What is the population proportion, *correct to 2 decimal places*?

2 marks

- d) Assuming that $\hat{p} = 0.66$, if the lower limit of a confidence interval is 0.6213, find the percentage confidence interval

2 marks

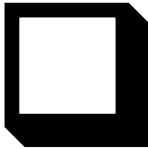
- e) By what number of people would the company need to increase the size of the sample in order to achieve a margin of error of 2% for the confidence interval and proportion found previously?

3 marks

Total 10 marks

END OF TASK.

Additional Working Space (*if needed*)

Assessment Grade for Task


This grade is subject to statistical moderation at the Victorian Curriculum and Assessment Authority (VCAA) and is likely to change.
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