



MATHEMATICAL METHODS

Problem Solving Task

SAC 2

2016

Student Name: _____

Teacher's Name: _____

Directions

Reading Time 10 Minutes
Writing Time 120 Minutes

- COMPLETE THIS TASK IN THE SPACES PROVIDED.
- **FOR QUESTIONS WORTH MORE THAN 1 MARK THE METHOD OF SOLUTION MUST BE CLEARLY EVIDENT.**
- YOU ARE PERMITTED TO USE A CAS CALCULATOR AND BOUND REFERENCE BOOK.
- UNITS MUST BE INCLUDED WHERE APPROPRIATE.
- GIVE EXACT ANSWERS UNLESS INSTRUCTED OTHERWISE.
- THIS SAC HAS A TOTAL OF 63 MARKS.
- BEFORE YOU COMMENCE WRITING PLEASE SIGN THE STUDENT DECLARATION ON THE NEXT PAGE.

COMPULSORY STUDENT DECLARATION:

I, _____,
acknowledge that I have read the SAC/examination conditions and
understand which items/materials I am permitted to use and have in my
possession.

****If you have any doubts as to what is permitted, raise your hand and
DO NOT sign this declaration****

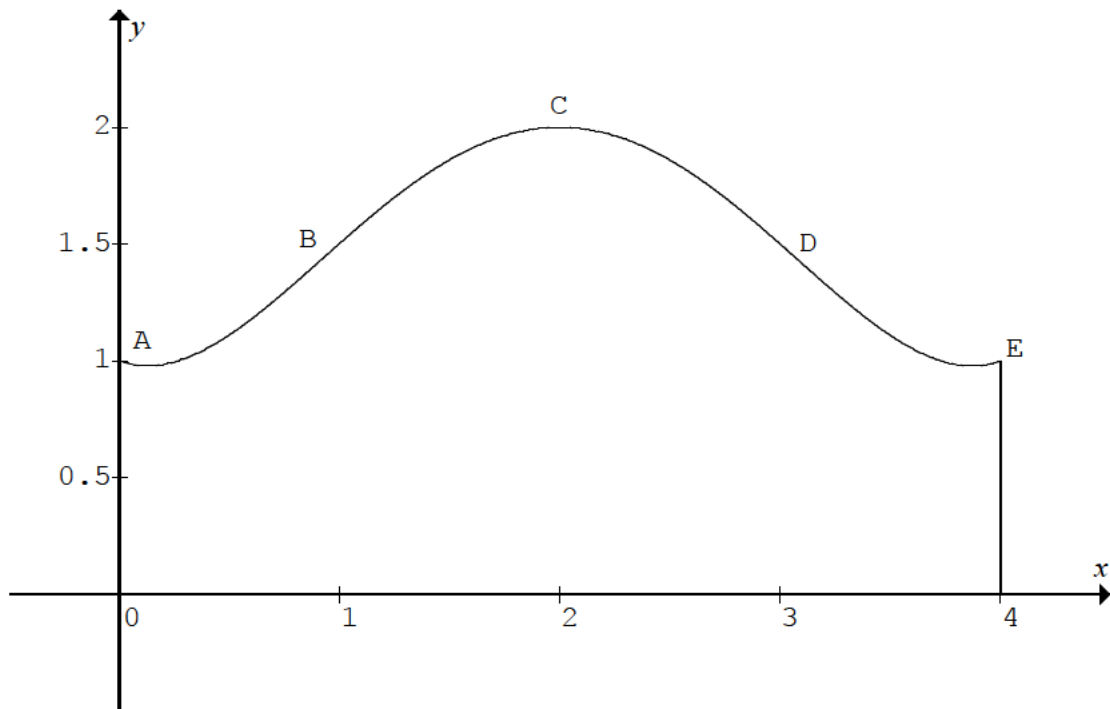
Student's signature: _____

Teacher's name: _____

Question 1

The design of a section of fencing ABCDE is shown below, along with the x and y axes with dimensions in metres. The fence must pass through the points $A(0,1)$, $B\left(1,\frac{3}{2}\right)$, $C(2,2)$

$D\left(3,\frac{3}{2}\right)$ and $E(4,1)$. The point C is the highest point on the fence, while points A and E are the lowest points on the fence.



- a. If the area of the fence is approximated by four equally spaced left rectangles, determine in square metres the area of the fence.

2 mark

b. One design for the fence is a function of the form $g(x) = p + q \cos(nx)$ for $0 \leq x \leq 4$.

i. Find the values of p , q and n .

3 marks

ii. Using this design, write down a definite integral which gives the average height of the fence.

1 mark

iii. Find the average height of the fence in metres

1 mark

Question 2

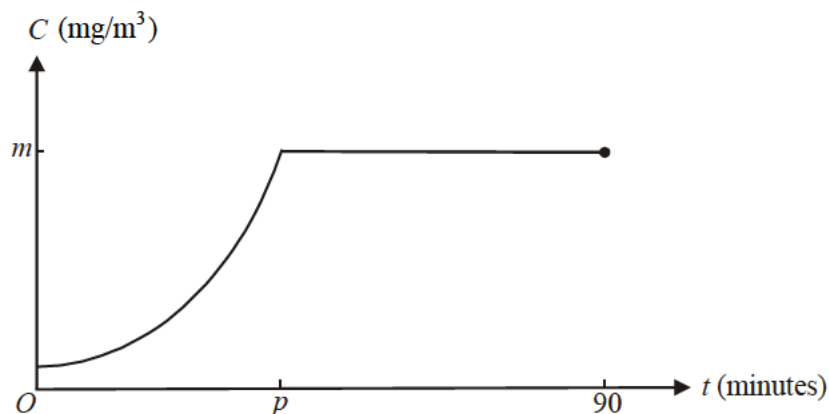
Rugged Rob is a spy. He is trapped in a space where poisonous gas is leaking.

The concentration C , in mg/m^3 , of the gas t minutes after Rugged Rob became trapped is given by the continuous function

$$C(t) = \begin{cases} \frac{500}{100-t}, & 0 \leq t \leq p \\ m, & p < t \leq 90 \end{cases}$$

where m and p are constants.

A graph of the function is shown below.



- a. What is the initial concentration of the gas in mg/m^3 ?

1 mark

- b. Find an expression for m in terms of p .

1 mark

- c. Find the minimum and maximum values of m .

2 marks

- d. Find the function $C'(t)$ for $0 < t < p$.

1 mark

- e. If the rate at which the concentration of the gas is increasing was 1 mg/m^3 per minute, find the value of t . Express your answer in minutes correct to 2 decimal places.

2 marks

- f. If $p = 10$, find the average concentration of the gas between $t = 0$ and $t = p$, correct to 2 decimal places.

2 marks

If the concentration of the gas reaches 6 mg/m^3 then a human cannot survive.

- g. Given that Rugged Rob is trapped for 90 minutes in this space, find the possible values of p in order for him to survive.

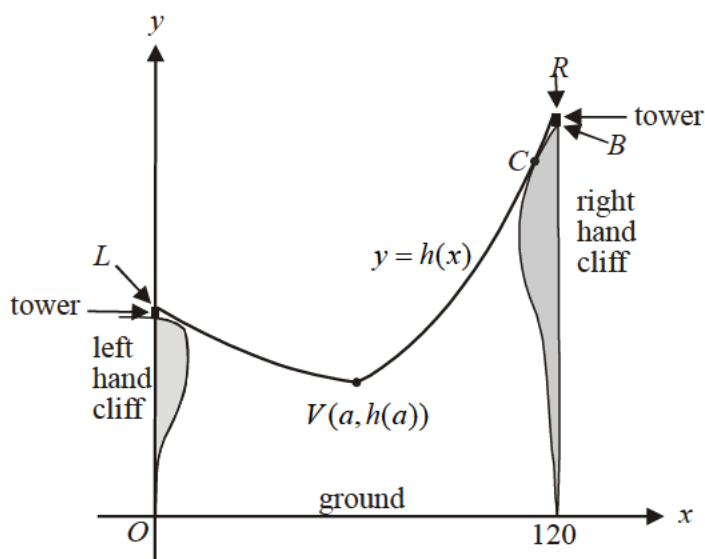
2 marks

Question 3

Flying Frances is a spy. Whilst on a chairlift that spans a valley between two cliffs, her enemies destroyed the control tower causing the chairlift cable to slacken. As a result, the position of the cable above the ground can be described by the **continuous** function

$$h(x) = \begin{cases} \frac{1}{100}(x-50)^2 + 40, & \text{for } x \in [0, a] \\ \frac{1}{100}(x-30)^2 + 40, & \text{for } x \in (a, 120] \end{cases}$$

where x represents the horizontal distance in metres of the cable from the base of the left hand cliff and y represents the height in metres of the cable above the ground.



The graph of h is shown above.

The cable is attached to the top of 5m high towers on the left and right hand cliffs at points L and R respectively. The base of the tower on the right hand cliff is indicated by point B .

The cable touches the right hand cliff at point C and starts to fray. Frances is left stationary at the point $V(a, h(a))$.

- a. Show that $a = 40$.

(2 marks)

- b.** How high is Frances above the ground?

(1 mark)

The gradient of the cable at point C is 1.48.

- c.** Find the coordinates of point C giving your answer correct to two decimal places.

(3 marks)

Frances has equipment with her that enables her to lower herself to the ground from her stationary position. Let v be her height, in metres, above the ground t seconds after she starts her descent. The rate at which she descends is given by

$$\frac{dv}{dt} = \frac{-23}{21} e^{\frac{-t}{42}}.$$

- d. i.** Find an expression for v in terms of t .

(3 marks)

- ii.** Hence find the time that it takes for Frances to reach the ground. Express your answer correct to 3 decimal places.

(1 mark)

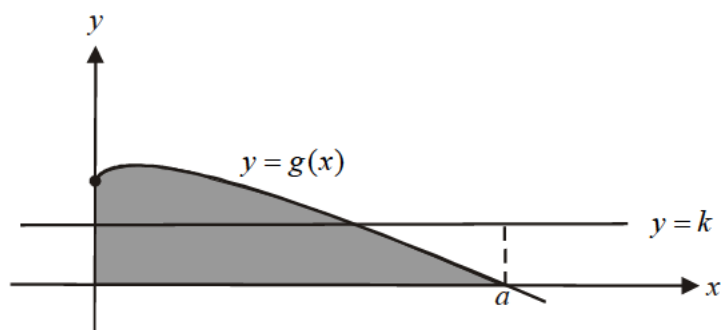
At the same instant that Frances starts her descent, an enemy spy who is located at point B , starts to abseil down to point C . He drops 0.24 metres vertically each second. Once at C he takes n seconds to cut the cable.

- e. i. Find an expression for the total time T , in terms of n , that it takes for him to abseil down from point B and cut the cable. (3 marks)

- ii. Assuming Frances will be safe once she reaches the ground, find the values of n for which Frances will be safe. (2 mark)

Question 4

Let $g : [0, \infty) \rightarrow \mathbb{R}$, $g(x) = \sqrt{x} + 2 - x$. The graphs of $y = g(x)$ and the line with equation $y = k$, where k is a positive constant, are shown below.



The graph of g has an x-intercept at the point $(a, 0)$ where $a > 1$. The area enclosed by the graph of $y = g(x)$ and the x and y axes is shaded.

- a. Show that $a = 4$.

(3 marks)

- b. Find the values of k for which there is one point of intersection with the line $y = k$ and the graph of $y = g(x)$.

(3 marks)

- c. If the area enclosed by the lines with equations $y = k$ and $x = a$ and the x and y axes, is equal to the area of the shaded region, then find the value of k . (2 marks)

- d. Find the area of the shaded region. (1 mark)

The graphs of $y = g(x)$ and $y = k$, intersect at a point where $x = 0.25(\sqrt{9 - 4k} + 1)^2$.

- e. Find the value of k for which the area of the shaded region above the line $y = k$ is equal to the area of the shaded region below the line $y = k$. Express your answer correct to 2 decimal places. (3 marks)

Question 5

Given the cubic function $f: R \rightarrow R$, $f(x) = x^3 + bx^2 + cx + 6$ where b and c are real constants.

- i. Find $f'(x)$ in terms of b and c . (1 mark)

- ii. Find in terms of b and c , the equation of the tangent $t(x)$ to the cubic at the point P where $x = 2$. Give the answer in simplest form. (3 marks)

- iii. This tangent $t(x)$ intersects the cubic again at the point $Q(-1, 6)$.
Write down simultaneous equations involving b and c , and hence show that $b = -3$
and $c = -4$. (3 marks)

- iv. Write down a definite integral A_1 not involving b and c , which gives the area between the tangent $t(x)$ and the cubic function.

(2 marks)

- v. Find the equation of the tangent $t_1(x)$ to the cubic at the point $Q(-1, 6)$. (2 marks)

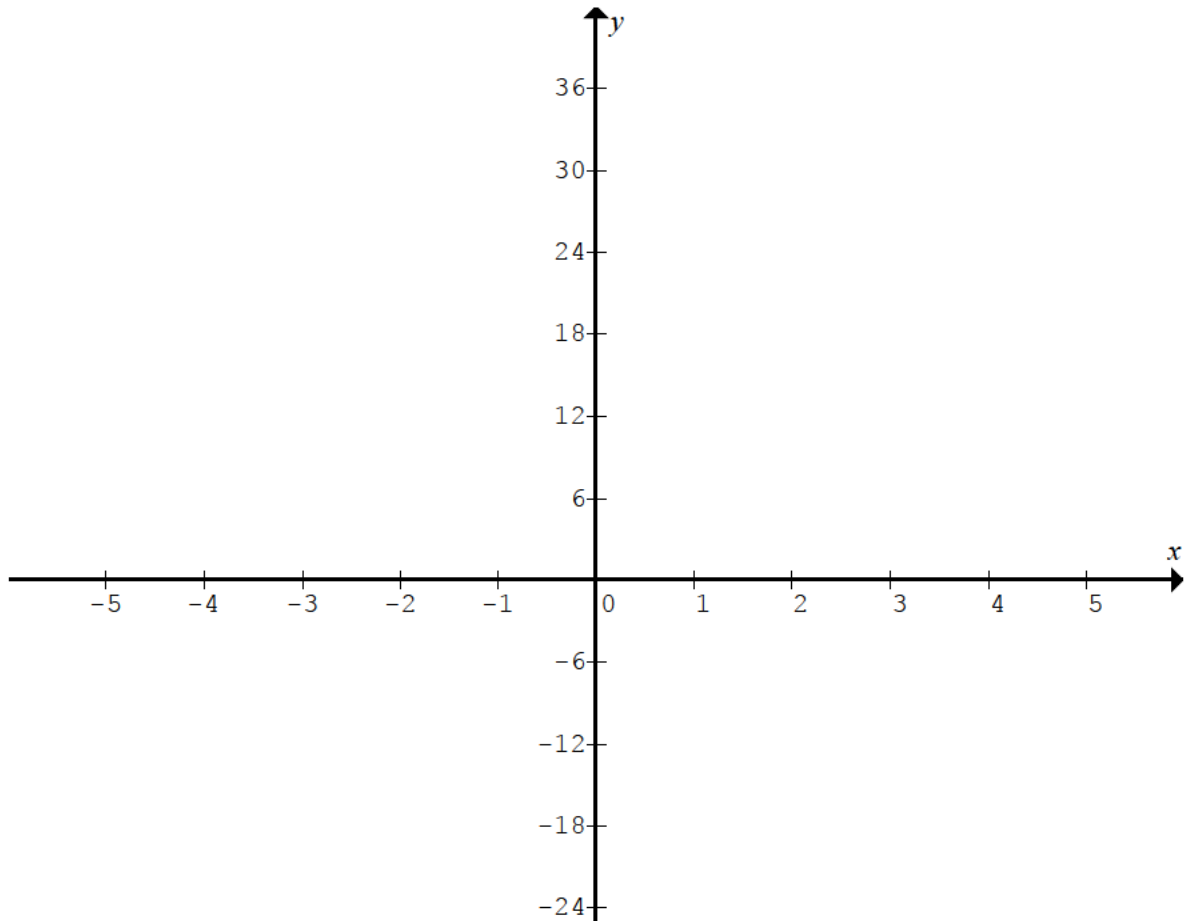
- vi. At point R, this tangent $t_1(x)$ intersects the cubic function again. Find the coordinates of the point R. (1 mark)

- vii. Write down a definite integral A_2 not involving b and c , which gives the area between the tangent $t_1(x)$ and the cubic function.

(1 mark)

- viii. Find the value of $\frac{A_2}{A_1}$ (1 mark)

- ix. On the axes below, sketch the graphs of $y = f(x)$ and the tangents at P and Q and shade the area A_2 . Give the coordinates of the axes intercepts and intersection points. (4 marks)



End of SAC

