

2018 SAC PREP 1

Question 1

Brett and Dora each subscribe to a particular website.

The time X , in seconds, it takes Brett to log-in to this website has a probability density function given by

$$f(x) = \begin{cases} 0 & x < 0 \\ 0.3125 & 0 \leq x \leq 3 \\ \frac{5}{16}e^{-5(x-3)} & x > 3 \end{cases}$$

The time Y , in seconds, it takes Dora to log-in to the same website is normally distributed with a mean of 2 and a standard deviation of 0.6.

- a. Find the probability of it taking less than 1 second for Dora to log in. Express your answer, correct to four decimal places.

1 mark

- b. Sixty percent of the time, Dora can log-in to the website in under p seconds. Find the value of p correct to 4 decimal places. Include in your answer a suitable diagram.

2 marks

- c. Sketch the continuous probability for Brett's login times on a suitably labelled axis.

- d. Find the probability that it takes Brett between 2 and 4 seconds to log in. Express your answer, correct to four decimal places.

3 marks

- e. Find the variance of
- Y
 - X , express your answer, correct to four decimal places.

1 + 3 = 4 marks

- f. Six of Dora's log-in times are randomly selected to check the efficiency of her computer. What is the probability, correct to 4 decimal places that:
- exactly one of these log-in times took less than 2 seconds?
 - at least one of these log-in times took less than 2 seconds?

2 + 1 = 3 marks

Brett and Dora's subscriptions to the website expire after a certain, large number of log-ins have been made. Both their subscriptions have just expired. The log-in times for both Brett and Dora during their subscriptions were analysed and a randomly selected log-in time was found to be less than 1 second.

- g.
- Use a tree diagram to represent this situation. Consider who made the login and the length of time it took to connect.
 - Hence, find the probability correct to three decimal places that this log-in was one that Dora made given that it was found to have taken less than 1 second.

1 + 3 = 4 marks

Total 16 marks

Question 2

At a seminar for Mathematical Methods, teachers are quizzed on the other mathematics subjects that they teach. It is found that 60% teach Further Mathematics, 10% teach both Specialist and Further and 22% teach no other mathematics aside from Mathematical Methods.

- a. Use a suitable probability technique to display this information.
- b. Hence find the probability that a teacher:
- i. teaches Further Mathematics but not Specialist Mathematics.
 - ii. does not teach Specialist Mathematics.
 - iii. teaches Specialist Mathematics if it is known they do not teach Further Mathematics.

2 + 1 + 1 + 2 = 6 marks

- c. Explain whether the events 'teaches Further Mathematics' and 'teaches Specialist Mathematics' are independent.

2 marks

- d. Explain whether the events 'teaches Further Mathematics' and 'teaches Specialist Mathematics' are mutually exclusive.

2 marks

At a three-day conference, there are two options for lunch: vegetarian and non-vegetarian. John, a mathematics teacher finds that if orders a vegetarian lunch one day, the probability that he orders the non-vegetarian option the following day is 0.75. However, if he orders non-vegetarian one day, the probability that he orders vegetarian the following day is 0.55. John chose the vegetarian option on the first day (Monday) of the conference.

- e. Construct a tree diagram to represent John's choices for the three days of the conference.

3 marks

- f. Find the probability that Jamie will choose the non-vegetarian option at least once over the next two days. Give answer correct to four decimal places.

2 marks

- g. Find the probability that John chooses the vegetarian option on the last day of the conference given that he chose the non-vegetarian option at least once over the next two days.

2 marks

John has found over time that at a three-day conference the number of days he chooses a vegetarian option is a discrete random variable, X . The discrete random variable has probability distribution

x	0	1	2	3
$\Pr(X=x)$	$2p$	$2p^2$	$p^2 + p$	$2p^2 + p$

- h. Find the value of p .

2 marks

- i. Find the expected number of vegetarian meals that John orders at the three-day conference.

2 marks

- j. Find the probability that John will have at least 2 vegetarian meals at the conference.

2 marks

- k.** Determine a 95% confidence interval for the number of vegetarian lunches ordered at a three-day conference.

3 marks

John plans to attend 3 conferences in 2017.

- l.** Find the probability that he will order the same number of vegetarian lunches at the three conferences.

2 marks

- m.** Find the probability that he will order a total of 2 vegetarian lunches over the three conferences.

2 marks