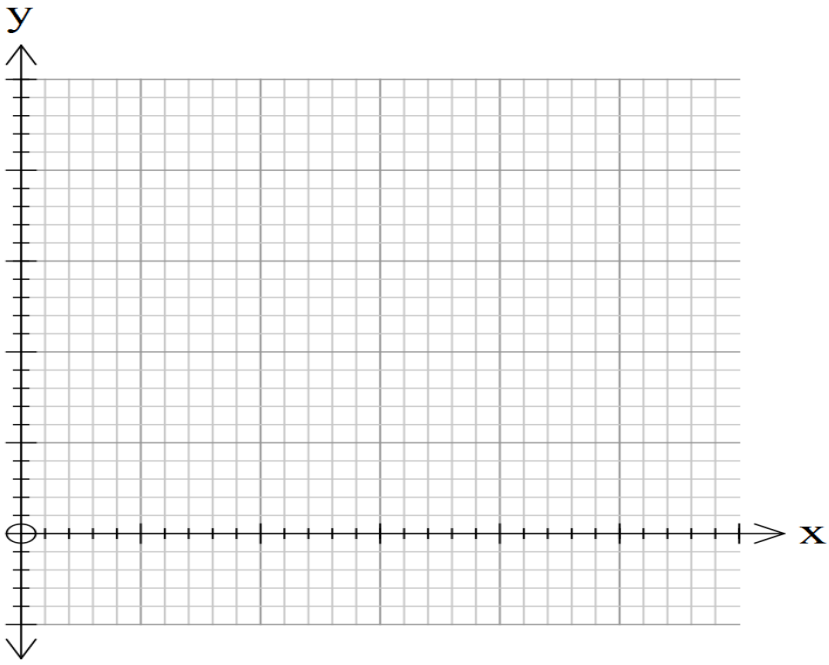
2018 SAC PREP 3

John investigates a path of the form $f(x) = -5px^2(2x-3)^3, x \in \left[0, \frac{3}{2}\right]$.

a) Sketch a graph of the robot's path when $p = 2$.



b) Find the average rate of change between $x = 0$ *and* $x = 1$.

c) Sketch a graph of the derivative function $f'(x)$ when $p = 2$, for a suitable domain. Turning points can be shown to three decimal places. State the domain.

d) Find the coordinates, correct to 3 decimal places, of the point on the robot's path where the gradient is the same as the average rate of change found in **part b)**.

e) Find the angle θ from the positive direction of the x-axis to the tangent to the graph of f at $x = 1$, measured in an anti-clockwise direction.

John wants the robot's path to be smooth and has decided that he wants the angle θ from the positive direction of the x-axis to the tangent to the graph of f to always be less than $75^{o.}$

f) Use a suitable method to determine if when $p = 2$ this requirement is met.

f) If the condition is not met, determine a suitable value of p that would achieve this requirement.

During some competitions more than one robot can be active in the demonstration area. One of the other robots brushes past John's robot (tangent) when $x = 1$. Return to the curve when $p = 2$

g) Determine the equation of the tangent at this point.

h) An out of control robot, travelling in a straight line hits John's robot. Their paths are at right angles to each other. If they collide when $x = 1$, determine the equation of the other robot's path.

i) Determine the equation of the tangent when $x = a$. Check your result on your CAS.