Multiple-choice questions

1 The exact value of $\tan \frac{11\pi}{6}$ is

A $\sqrt{3}$

B $\frac{\sqrt{3}}{2}$

C $-\sqrt{3}$

D $-\frac{\sqrt{3}}{3}$

E $\frac{\sqrt{3}}{3}$

2 Which of the following sequences is geometric?

A 5, 8, 13, 21

B –7, –2, 3, 8

C 24, –12, 6, –3

D $\frac{1}{8}, \frac{1}{6}, \frac{1}{4}, \frac{1}{2}$

E 1, 2, 4, 7

3 A circle defined by equation $x^2 + y^2 - 6x + 8y = 0$ has centre

A (2, 4)

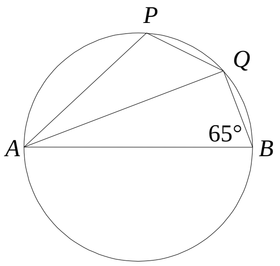
B (–5, 9)

C (4, –3)

D (3, –4)

E (6, –8)

4 P and Q are points on a circle whose diameter is AB. Given that $\angle ABQ = 65°$, the magnitudes of $\angle QAB$ and $\angle APQ$ are



A $\angle QAB = 25°$ and $\angle APQ = 120°$

B $\angle QAB = 25°$ and $\angle APQ = 115°$

C $\angle QAB = 20°$ and $\angle APQ = 120°$

D $\angle QAB = 55°$ and $\angle APQ = 115°$

E $\angle QAB = 25°$ and $\angle APQ = 125°$

5 If $t_1 = 4$ and $t_n = t_{n-1} + 8$, then t_4 is equal to

A 4

B 12

C 20

D 28

E 36

6 From a point on a cliff 500 m above sea level, the angle of depression to a boat is 20°. The distance from the foot of the cliff to the boat, to the nearest metre, is

A 182 m

B 193 m

C 210 m

D 1374 m

E 1834 m

7 The equation of an asymptote of the hyperbola $\dfrac{(y+1)^2}{9} - (x-2)^2 = 1$ is

A $y = -3x - 7$

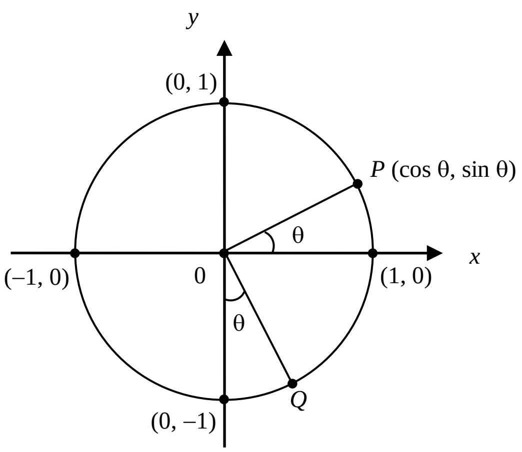
B $y = 3x + 5$

C $y = -3x + 5$

D $y = \dfrac{1}{3}x - 5$

E $y = \dfrac{-1}{3}x - 7$

8 In this diagram, given that $P = (\cos\theta, \sin\theta)$, Q would be the point with coordinates



A $(\sin\theta, \cos\theta)$

B $(\cos\theta, \sin\theta)$

C $(\sin\theta, -\cos\theta)$

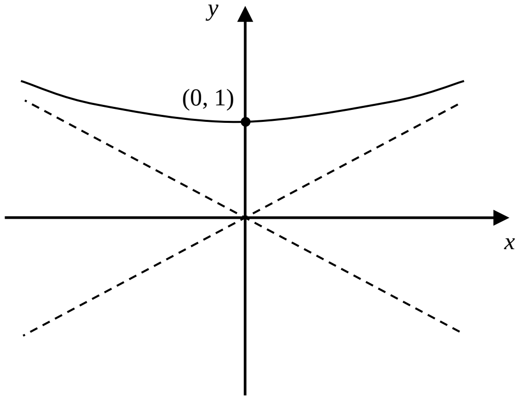
D $(\cos\theta, -\sin\theta)$

E $(-\sin\theta, -\cos\theta)$

9 Which one of the following could be the graph of the curve with equation

$4x^2 - y^2 + 1 = 0$?

A



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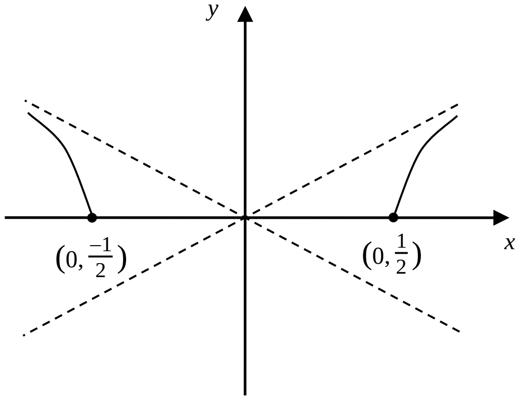

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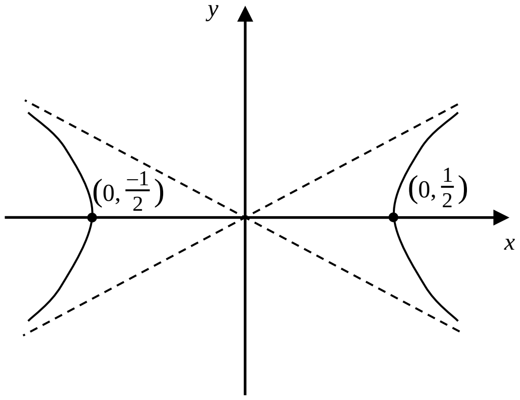

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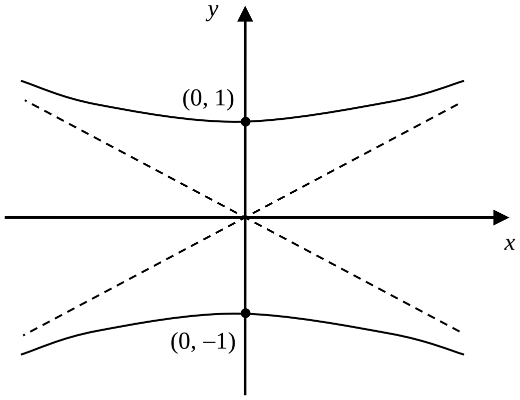
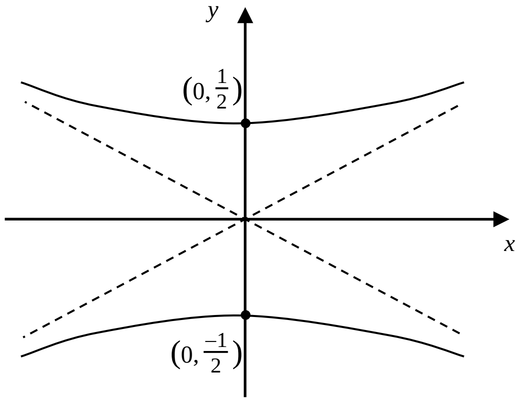


E



10 Which one of the following is not a solution of the equation $2\sin(2x) - \sqrt{3} = 0$?

A $\dfrac{\pi}{6}$

B $\dfrac{\pi}{3}$

C $\dfrac{-5\pi}{3}$

D $\dfrac{13\pi}{6}$

E $\dfrac{-7\pi}{6}$

11 Given that $\cos A = \dfrac{2}{3}$, where A is an acute angle, the exact value of $\cos\left(\dfrac{A}{2}\right)$ is

A $\dfrac{1}{12}$

B $\dfrac{\sqrt{30}}{6}$

C $\dfrac{\sqrt{15}}{6}$

D $\dfrac{3}{2}$

E $\dfrac{2}{\sqrt{6}}$

12 The equations of the asymptotes of $y = 4\tan^{-1}x + \pi$ are

A $y = -\pi$ and $y = 3\pi$

B $y = 0$ and $y = 2\pi$

C $y = \dfrac{\pi}{2}$ and $y = \dfrac{3\pi}{2}$

D $y = -2 + \pi$ and $y = 2 + \pi$

E $y = -\pi + 2$ and $y = 3\pi + 2$

13 The range of the function $f\colon \left(0, \frac{\pi}{2}\right) \to \mathbb{R}$, where $f(x) = 2\operatorname{cosec}(2x) + 1$, is

A $(-1, 3)$

B $(-2, 2)$

C $\mathbb{R}\backslash(-1, 3)$

D $(-\infty, 1) \cup (3, \infty)$

E $[3, \infty)$

14 The maximum and minimum values of $\dfrac{1}{3\sec x + 3}$ for $x \in \left[0, \frac{\pi}{3}\right]$ are

A maximum value = $\frac{1}{3}$ and minimum value = $\frac{1}{6}$

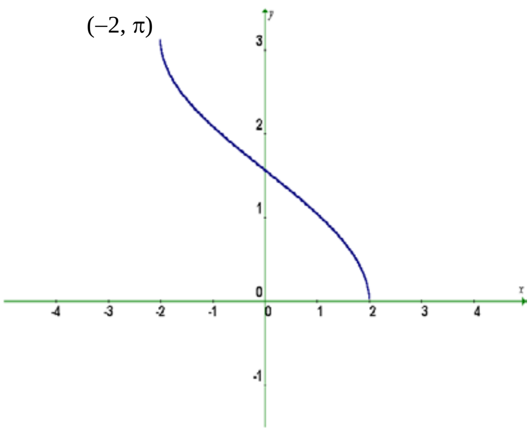
B maximum value = $\frac{1}{6}$ and minimum value = $\frac{1}{9}$

C maximum value = 1 and minimum value = 0

D maximum value = $\frac{\sqrt{3}}{3}$ and minimum value = $\frac{1}{3}$

E maximum value = $\frac{\sqrt{3}}{2}$ and minimum value = $\frac{1}{3}$

15 The rule of the graph shown is



A $y = \cos^{-1}(2x)$

B $y = \cos^{-1}(x)$

C $y = \cos^{-1}\left(x - \frac{1}{2}\right)$

D $y = \cos^{-1}\left(x + \frac{1}{2}\right)$

E $y = \cos^{-1}\left(\frac{x}{2}\right)$

16 The maximal domain of the function with rule $f(x) = 2\sin^{-1}(1 - 5x)$ is

A $\left[0, \frac{2}{5}\right]$

B $[-0.2, 0{,}2]$

C $\left[0, \frac{1}{5}\right]$

D $\left[\frac{-1}{5}, \frac{2}{5}\right]$

E $[-2, 2]$

17 The range of $y = 2\cos^{-1}(x + 1)$ is

A $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

B $[0, \pi]$

C $[-\pi, \pi]$

D $[0, 2\pi]$

E $[-2, 0]$

18 The gradient of the normal to the ellipse with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at the point $\left(1, \frac{4\sqrt{2}}{3}\right)$ is

A $\frac{-2}{3}$

B $\frac{3}{2}$

C $\frac{-3}{4\sqrt{2}}$

D $-\sqrt{3}$

E $3\sqrt{2}$

19 Using an appropriate substitution, $\int_2^3 \frac{1}{x^2} e^{\frac{6}{x}}\, dx$ is equal to

A $\frac{-1}{6}\int_2^3 e^u\, du$

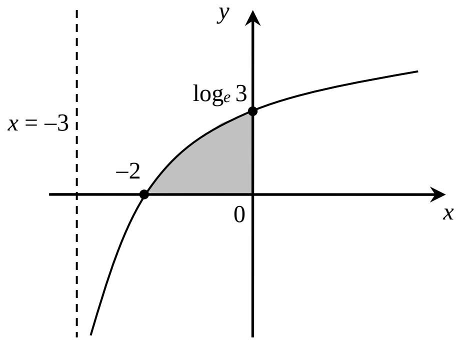
B $\frac{1}{6}\int_3^2 e^u\, du$

C $\frac{1}{6}\int_2^3 \frac{1}{u^2} e^u\, du$

D $\int_2^3 e^u\, du$

E $\frac{1}{6}\int_2^3 e^u\, du$

20 A section of the graph of $y = \log_e(x + 3)$ is shown below.



The area of the shaded region is given by

A $\int_0^{\log_e 3} \log_e(x+3)\, dx$

B $\int_0^{\log_e 3} e^y\, dy - 3$

C $\int_{-2}^{0} e^y - 3\, dy$

D $\int_{0}^{2} \log_e x\, dx$

E $\int_{0}^{\log_e 3} 3 - e^x\, dx$

21 A chemical dissolves in water at a rate equal to 10% of the amount of undissolved chemical per hour. At time t hours, the amount of undissolved chemical is x grams. Initially the amount of undissolved chemical is 6 grams. Which one of the following differential equations applies to this situation?

A $\frac{dx}{dt} = \frac{-x}{10}$

B $\frac{dx}{dt} = 6 - \frac{x}{10}$

C $\frac{dx}{dt} = \frac{x}{10}$

D $\frac{dx}{dt} = \frac{x-6}{10}$

E $\frac{dx}{dt} = \frac{6-x}{10}$

22 A particle moves along a straight line such that its acceleration at time t is given by $a = 6t - 4$ m/s^2. Initially the particle is at the point O ($x = 0$) and has a velocity of -2 m/s^2. The position of the particle from O at time t is

A $t^3 - 2t^2 - 2t$

B $t^3 - 2t^2$

C 6

D $t^3 - 2t^2 + 2$

E 0

23 A particle moves in a straight line with acceleration $2x$ where x is its displacement from a fixed point on the line. Also $v = 0$ when $x = 0$. The velocity when its displacement is x is given by

A x

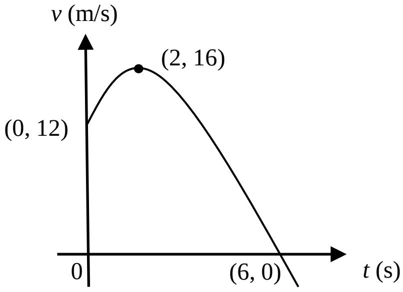
B x^2

C $-x\sqrt{2}$

D $x\sqrt{2}$

E $\sqrt{2x}$

24 For the velocity–time graph shown below, the relationship between velocity v and time t is of the form $v = k - (t - b)^2$, $t \geq 0$. The acceleration of the particle (in m/s^2) when t = 2 seconds is



A –4

B –2

C 0

D 2

E 4

25 A stone is projected vertically upwards from the top of a 10 metre tall platform with an initial velocity of 20 m/s. Its velocity (in m/s) when it hits the ground at the base of the building is closest to

A –24.4

B –20.0

C –14.3

D 14.3

E 24.4

26 The velocity v of a particle with displacement x is given by $v = x^2$. Its acceleration a is given by

A $a = 2x$

B $a = \sqrt{v}$

C $a = \dfrac{v}{x}$

D $a = 2\dfrac{v}{x}$

E $a = 2x^3$

27 The displacement x metres from the origin at time t seconds of a particle travelling in a straight line is given by $x = e^{-\frac{t}{2}} \sin(2t)$. The time when the particle will first be instantaneously at rest is

A $\dfrac{1}{2}\tan^{-1}(4)$

B $\dfrac{1}{2}\tan^{-1}\left(\dfrac{1}{4}\right)$

C $\dfrac{\pi}{4}$

D $\dfrac{\pi}{8}$

E 0

28 A particle moves in a straight line with acceleration of $12t - 5$ m/s^2 at time t seconds. The particle has an initial velocity of 1 m/s. The velocity of the particle (in m/s) at $t = 1$ is

A 1

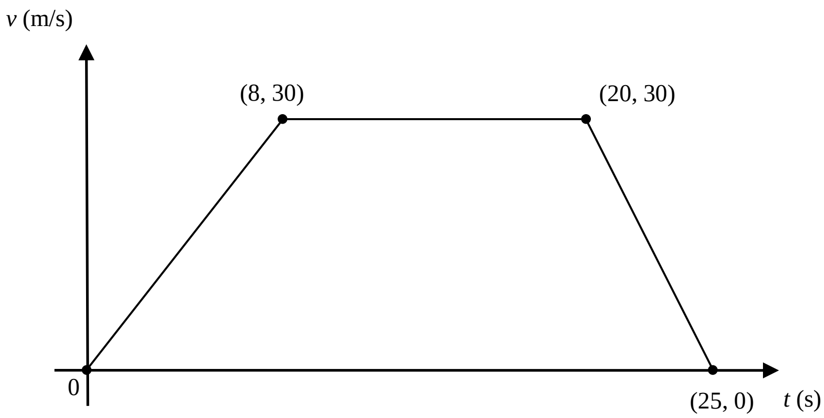
B –5

C 7

D 2

E 3

29 A vehicle's motion is represented by the velocity–time graph shown.



The distance in metres travelled by the vehicle over the 25 seconds is

A 30

B 750

C 500

D 555

E 600

30 A body initially travelling at 20 m/s is subject to a constant deceleration of 4 m/s^2. The time it takes to come to rest (t seconds) and the distance travelled before it comes to rest (s metres) are

A $t = 5$, $s = 50$

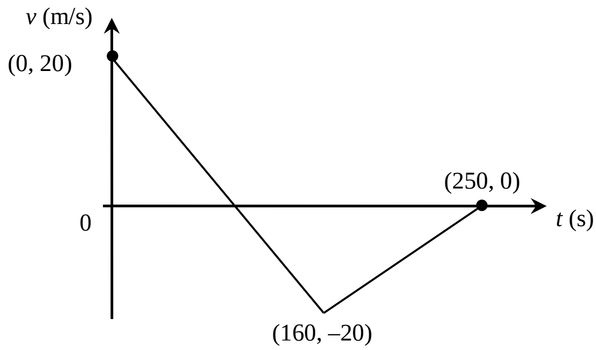
B $t = 5$, $s = 45$

C $t = 4$, $s = 20$

D $t = 5$, $s = 40$

E $t = 4$, $s = 35$

31 The velocity–time graph shown describes the motion of a particle.



The distance travelled (in km) during the first 250 seconds is

A 0.8

B 0.9

C 1.6

D 2.5

E 5.0

32 The velocity, v m/s, of a particle at time t seconds is given by the equation $v = t(4t - 1)$, $t \geq 0$. The acceleration at time $t = 2$, in m/s^2, is

A 10

B 12

C 14

D 15

E 16

33 A particle moves in a straight line so that its position x at time t is given by $x = 8t - t^2$. The velocity of the particle at time $t = 4$ is

A 16

B –16

C 0

D 8

E –8

34 The velocity of an object at time t is given by $v = 100(1 - e^{-5t})$ m/s. If the object starts from the origin, its displacement at time t ($t \geq 0$) is given by

A $100(t + e^{-5t} - 1)$

B $20(5t + e^{-5t} - 1)$

C $20(5t + e^{-5t})$

D $20(t - 5e^{-5t} + 5)$

E $500e^{-5t}$

35 The velocity v m/s at time t seconds of an object is given by $v = 3t^2 - 24t$, $t \geq 0$. The object changes direction after

A 3 seconds

B 4 seconds

C 5 seconds

D 7 seconds

E 8 seconds

36 If the velocity v m/s of an object whose displacement is x m from the origin is given by $v = 10e^{-0.01x}$, its acceleration a is given by

A $a = -1000e^{-0.01x}$

B $a = -10e^{-0.01x}$

C $a = -2e^{-0.02x}$

D $a = -e^{-0.02x}$

E $a = -0.1e^{-0.01x}$

37 A particle starts from rest at $t = 0$ and moves in a straight line so that its acceleration, a, at time t is given by $a = 5\,e^{-0.1t}$. The velocity of the particle at $t = 1$, correct to two significant figures, is

A -50

B -4.8

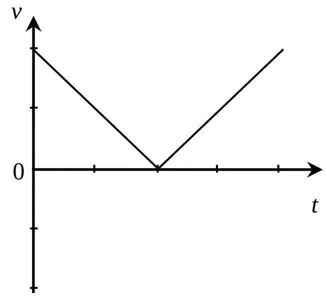
C -0.50

D 4.8

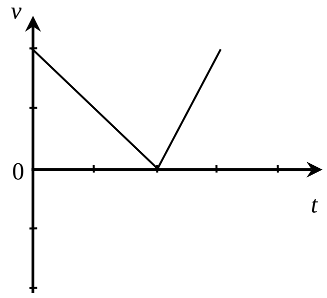
E 50

38 A ball is dropped from rest onto a concrete floor and bounces vertically to half its drop height. Which one of the following velocity–time graphs could represent the motion of the ball?

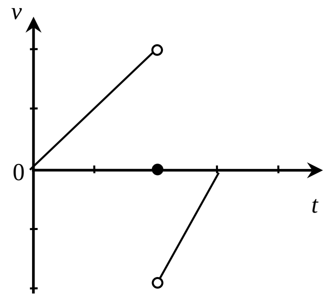
A



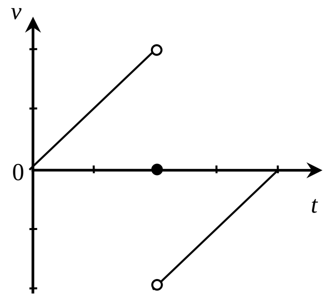
B



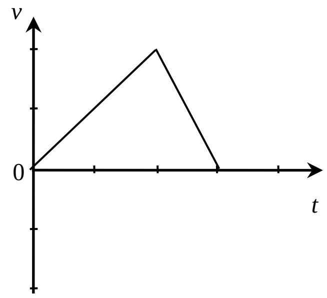
C



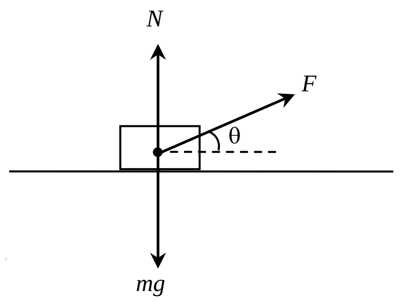
D



E



39 A body of mass m kg is being pulled along a smooth horizontal table by means of a string inclined at angle θ to the horizontal. The diagram below indicates the forces acting on the body.



Which one of the following statements is true?

A $N - mg = 0$

B $N + F\sin\theta - mg = 0$

C $N - F\sin\theta - mg = 0$

D $N + F\cos\theta - mg = 0$

E $N - F\cos\theta - mg = 0$

40 A particle of mass 5 kg is acted on by two forces, $24\boldsymbol{i} + 7\boldsymbol{j}$ and $7\boldsymbol{i} - 24\boldsymbol{j}$, measured in Newtons. The acceleration of the particle is

A 10 m/s^2

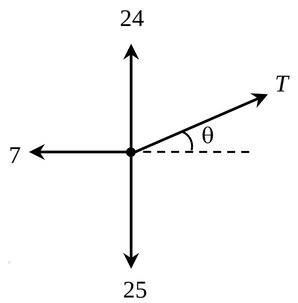
B $10(31\boldsymbol{i} - 17\boldsymbol{j})$ m/s^2

C $31\boldsymbol{i} - 17\boldsymbol{j}$ m/s^2

D 50 m/s^2

E $\frac{1}{5}(31\boldsymbol{i} - 17\boldsymbol{j})$ m/s^2

41 The diagram shows a system of forces, measured in Newtons, acting on a particle.



Given that the particle is at rest, T is equal to

A $7\cos\theta$

B 50

C $\dfrac{7}{\sin\theta}$

D $5\sqrt{2}$

E $2\sqrt{5}$

42 A particle is brought to a halt from 30 m/s over a horizontal distance of 60 m. If the resultant force acting on the particle is constant, then the magnitude of the acceleration of the particle is

A 1 m/s^2

B $\dfrac{1}{2}$ m/s^2

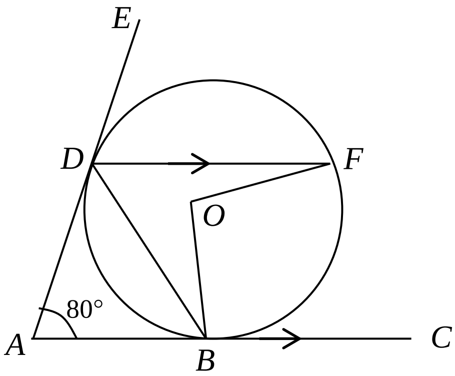
C 2 m/s^2

D 7 m/s^2

E $\dfrac{15}{2}$ m/s^2

Short-answer questions (technology-free)

1 The diagram shows a circle with centre O, and tangents to the circle at D and B from a point A. F is a point on the circle such that DF is parallel to AC. The magnitude of angle CAE is 80°.



Find the magnitude of

a angle ADB

b angle FDB

c obtuse angle FOB

d angle FBC.

2 The first term of an arithmetic sequence is –2 and its last term is 85. The sum of all its terms is 1245. Find:

a the number of terms in the sequence

b the common difference.

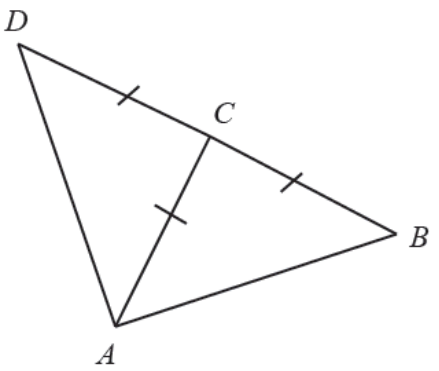
3 For the sequence defined by $t_n = 2t_{n-1} + 6$ with $t_0 = 1$, find the sum $t_0 + t_1 + t_2$.

4 Solve the equation $2\sin\left(2\left(x - \frac{\pi}{6}\right)\right) = -1$ for $x \in [0, 3\pi]$.

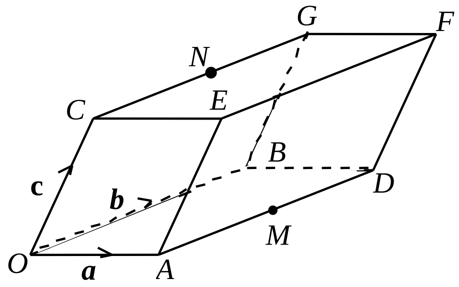
5 P is the point (2, 3, 4), Q is the point (–4, 6, 1) and R is the point (–2, 5, 2). Show that P, Q and R are collinear (i.e. show that P, Q and R lie on a straight line).

6 A particle is subject to two forces, one of 8 units acting due east, the other 3 units acting on a bearing of N30°E. Describe the resultant force (vector sum) of the two forces, using vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the east and north direction respectively.

7 In triangle *ABD*, $AC = CD = CB$ .Let $\overrightarrow{AB} = \boldsymbol{u}$, and $\overrightarrow{BC} = \boldsymbol{v}$. Prove that *DAB* is a right angle.



8 *OADBCEFG* is a parallelepiped with $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OC} = \boldsymbol{c}$. *M* and *N* are the midpoints of *AD* and *CG* respectively.



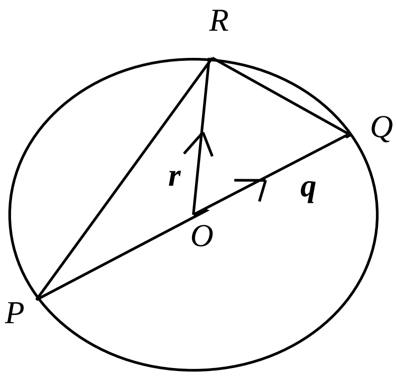
a Express the position vector of *F* (i.e. $\overrightarrow{OF}$) in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ and hence find an expression in $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ for the position vector of the midpoint of *OF*.

b By similarly finding expressions for the position vectors of the midpoints of *AG* and *MN*, show that *OF*, *AG* and *MN* are concurrent at their point of bisection.

9 Let $\boldsymbol{u} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{v} = \sqrt{2}\boldsymbol{i} + 2a\boldsymbol{j} - \sqrt{2}\boldsymbol{k}$, where $a \in R$. The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is 60°. Find the exact value of *a*.

10 Find m and n if $(m + n)\boldsymbol{i} + (2m + n)\boldsymbol{j} + \boldsymbol{k} = \boldsymbol{a}$, where $\boldsymbol{a} = 3\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k}$.

11 R is a point on the circumference of a circle (or a sphere) with centre O and diameter PQ. Let $\boldsymbol{r}$ and $\boldsymbol{q}$ be the position vectors of R and Q respectively.



a Express $\overrightarrow{RQ}$ and $\overrightarrow{RP}$ in terms of $\boldsymbol{r}$ and $\boldsymbol{q}$.

b Hence show that $\angle PRQ$ is a right angle.

12 Describe, through a Cartesian equation, the set of points with position vector $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$ satisfying $|\boldsymbol{r} - 4\boldsymbol{i}| + |\boldsymbol{r} + 4\boldsymbol{i}| = 10$.

13 Evaluate

a $\cos^{-1}\left(\sin\left(\frac{-\pi}{3}\right)\right)$

b $\sin^{-1}\left(\cos\left(\frac{\pi}{4}\right)\right)$.

14 State the implied domain and range of $y = \sin^{-1}(3 - x)$.

15 **a** State the maximal domain of the function with rule $f(x) = 3\sin^{-1}(2x)$.

b Sketch the graph of $f(x) = 3\sin^{-1}(2x)$.

c State the range of f.

16 Sketch the graph of $y = \sin^{-1}(x - 1)$.

17 Use the double angle formula for tan $(2x)$ to find the exact value of tan $\left(\frac{-\pi}{8}\right)$.

18 For $z = 1 + i$, find:

a Arg (z)

b Arg $(-z)$

c Arg $\left(\frac{1}{z}\right)$

19 Show that if $\frac{-\pi}{2} < \text{Arg}\,(z_1) < \frac{\pi}{2}$ and $\frac{-\pi}{2} < \text{Arg}\,(z_2) < \frac{\pi}{2}$ then

a Arg (z_1z_2) = Arg (z_1) + Arg (z_2)

b $\text{Arg}\left(\frac{z_1}{z_2}\right)$ = Arg (z_1) – Arg (z_2)

20 Show that if $\frac{-\pi}{2} < \text{Arg}\,(z) < 0$ then

$\text{Arg}\left(\frac{1}{z}-1\right) = Arg\left(\frac{1-z}{z}\right)$ = Arg (1 – z) – Arg (z).

21 Shade the region of the complex plane defined by $\left\{z : \text{Arg}(z) \le \frac{2\pi}{3}\right\}$.

22 Find the cube roots of $-3\sqrt{2} + 3i$ in Cartesian form, correct to three significant figures.

23 Solve $z^3 = 8i$, giving your answer(s) in exact polar form.

24 Shade the region of the complex plane defined by $\left\{z : \text{Arg}(z) > \frac{-\pi}{4}\right\}$.

25 For $f(x) = \sin^{-1}(2x - 1)$, find the values of x for which $f'(x) = 3$.

26 Consider the relation $3y - xy^2 = 2$.

a Find an expression for $\frac{dy}{dx}$ in terms of x and y.

b Find the exact value of $\dfrac{dy}{dx}$ when $y = 1$.

27 Evaluate each of the following definite integrals.

a $\displaystyle\int_{\frac{1}{2}}^{1} x\sqrt{2x-1}\ dx$

b $\displaystyle\int_{0}^{\frac{\pi}{2}} 4\sin^2 t\ dt$

c $\displaystyle\int_{0}^{1} \frac{x}{x^2+4}\ dx$

d $\displaystyle\int_{0}^{\frac{\pi}{2}} \sin(6x)\sin(3x)\ dx$

e $\displaystyle\int_{-1}^{1} \frac{2x+1}{x^2+1}\ dx$

f $\displaystyle\int_{2}^{4} \frac{2x}{\sqrt{x^2-1}}\ dx$

28 **a** Show that the curve with equation $y = \dfrac{6}{x^2+6x+12}$ has no vertical asymptotes.

b Find $\displaystyle\int \frac{6}{x^2+6x+12}\ dx$.

c Hence find the area enclosed by the curve with equation $y = \dfrac{6}{x^2+6x+12}$, the x-axis and the lines $x = 0$ and $x = a$, where a is the x coordinate of the turning point of the curve.

29 The normal to the curve with equation $y = e^x$ at the point B with coordinates $(1, e)$ meets the x-axis at the point C. The finite region bounded by the curve, the line BC, the y-axis and the x-axis is rotated around the x-axis to form a solid of revolution. Find the volume of this solid.

30 The rate of decay of a certain radioactive substance is modelled by the differential equation $\frac{dR}{dt} = (-4.35 \times 10^{-4})R$ where R units is the amount of the substance at time t years.

a If R_0 represents the initial amount of the substance present, find the time it takes, to the nearest year, for this to reduce to $\frac{1}{2}R_0$.

b What percentage of the substance remains after 20 years, correct to one decimal place?

31 At all points on a certain curve $\frac{d^2y}{dx^2} = 6x + 6$. At the point $(-2, 0)$ the tangent is parallel to the x-axis. Find the equation of the curve.

32 When the outlet at the bottom of a tank of water is opened, water flows out at a rate given by $\frac{dv}{dt} = k\sqrt{h}$ where v cm^3 is the volume of water in the tank and the depth is h cm at time t seconds. The tank initially contains water to a depth of 6 metres and is in the shape of a cylinder sitting with the circular base horizontal. After 10 minutes the depth has dropped by one metre.

a Express t in terms of h.

b Hence find how long it will take, to the nearest second, for the water level to drop a further metre.

33 If $y = x\tan^{-1}\left(\frac{c}{x}\right)$ is a solution of the differential equation $\frac{dy}{dx} - \frac{y}{x} = \frac{2x}{4x^2+1}$, find the value of the constant c.

34 A tank initially contains 500 litres of salt solution of concentration 0.02 kg/L. A solution of the same salt, but of concentration 0.05 kg/L, flows into the tank at the rate of 5 L/min. The mixture in the tank is kept uniform by stirring and the mixture flows out at the rate of 3 L/min. Let Q kg be the mass of salt in the tank after t minutes. Set up the differential equation for $\frac{dQ}{dt}$ in terms of t and specify the initial conditions.

35 If $y = xe^{3x}$ is a solution of the differential equation $\dfrac{d^2y}{dx^2} + m\dfrac{dy}{dx} + ny = 0$, where $m, n \in \mathbb{R}$, find the values of m and n.

36 Solve the differential equation $\dfrac{dy}{dx} = \dfrac{2}{(1-x)^2}$ given that $y = 0$ when $x = 0$.

37 A vessel full of water has the shape of an inverted right circular cone of base radius 2 m and height 5 m. Water flows from the apex of the cone at the constant rate of 0.2 m^3/min. Given that the volume of water in the cone is v m^3 at time t minutes, express:

a v in terms of h

b $\dfrac{dh}{dt}$ in terms of h.

38 A particle moves in a line so that its position x metres at time t seconds, relative to a fixed point O, is given by $x = 3\sin\left(\dfrac{\pi t}{3}\right) + 3\sqrt{3}\cos\left(\dfrac{\pi t}{3}\right)$, $0 \le t \le 3$. Find:

a when and where the particle comes to rest

b when and where the particle has zero acceleration.

39 A particle moves in a line so that its velocity is directly proportional to its displacement from a fixed point, O, on the line. The particle starts at a position 2 metres from O, with a velocity of $\dfrac{1}{4}$ m/s away from O. Find how far the particle is from O after 12 seconds.

40 A particle moves in a line with velocity, v m/s, where $v^2 = -2x^2 - 12x + 24$, and where x m is the displacement of the particle at time t seconds from a fixed point O on the line.

a If a m/s^2 is the acceleration at time t seconds, express a in terms of x.

b Prove that the motion only occurs for $-3 - \sqrt{21} \le x \le -3 + \sqrt{21}$.

c Find the maximum speed of the particle.

41 A car is travelling at 10 m/s and the brakes are applied. If the acceleration, a m/s^2, of the car, t seconds after the brakes are applied, is given by $a = \frac{-(900 - v^2)}{60}$, where v m/s is the speed

a express t in terms of v

b find the exact value of the time, in seconds, for the car to stop after the brakes are applied.

42 An object is dropped from a height. x metres is the distance fallen, v m/s is the velocity measured downwards and a m/s^2 is downwards acceleration of the object. Due to air resistance, $a = g - \frac{1}{100}v^2$.

a Using $a = v\frac{dv}{dx}$, express x in terms of v.

b Hence express v in terms of x.

43 Find the Cartesian equation for the graphs represented by the following vector equations.

a $\boldsymbol{r}(t) = (3 + 2t)\,\boldsymbol{i} + (1 - 3t^2)\,\boldsymbol{j}$

b $\boldsymbol{r}(t) = (1 - \cos(t))\,\boldsymbol{i} + (3 + \sin(t))\,\boldsymbol{j}$

44 The following vector equations each represent the position of a particle at time t, $t \geq 0$. For each equation

i find the corresponding Cartesian equation stating domain and range

ii sketch the path of the particle indicating the initial position and the initial direction of motion.

a $\boldsymbol{r}(t) = \sin\left(t - \frac{\pi}{3}\right)\boldsymbol{i} + \cos\left(t - \frac{\pi}{3}\right)\boldsymbol{j}$

b $\boldsymbol{r}(t) = (5 - 2t)\boldsymbol{i} + (t^2 - 4t)\boldsymbol{j}$

c $\boldsymbol{r}(t) = 2\cos(t)\,\boldsymbol{i} + 3\sin(t)\,\boldsymbol{j}$

45 The motion of two particles is given by the vector equations $\boldsymbol{r}_1(t) = (t + 1)\boldsymbol{i} + (t^2 - 3t + 2)\boldsymbol{j}$ and $\boldsymbol{r}_2(t) = (2t - 2)\boldsymbol{i} + (t - 1)\boldsymbol{j}$, where $t \geq 0$. Find

a the point at which the particles collide

b the points at which the paths cross

c the distance between the particles when $t = 2$.

46 For each of the following vector equations

i find the corresponding Cartesian equation stating domain and range

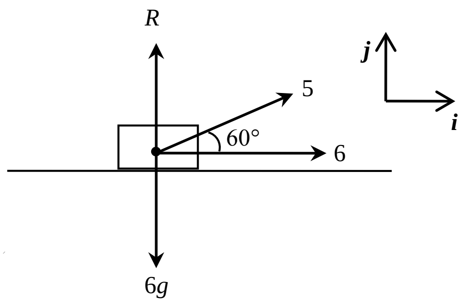
ii sketch the relation.

a $\boldsymbol{r}(t) = (t^2 + 1)\boldsymbol{i} + (3t - 1)\boldsymbol{j},\ t \in \mathbb{R}$

b $\boldsymbol{r}(t) = \tan(t)\,\boldsymbol{i} + \sec(t)\,\boldsymbol{j},\ t \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

c $\boldsymbol{r}(t) = \frac{1}{t+1}\boldsymbol{i} + \frac{1}{t+2}\boldsymbol{j},\ t \in [0, \infty)$

47 A block of mass 6 kg lies on a rough table. The coefficient of friction between the block and the table is 0.1. Forces of magnitude 5 N and 6 N are applied to the block as shown. Assume the acceleration due to gravity has a magnitude of 9.8 m/s^2.

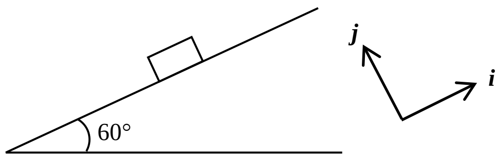


a Find the value of R.

b Find the acceleration of the block (correct to two decimal places).

c Find the velocity of the particle after 5 seconds (correct to two decimal places), given that initially the block is at rest.

48 A particle of mass 10 kg is on a smooth plane inclined at 60° to the horizontal. There is a force of 20 N acting up the plane applied to the block. This force acts in the direction of the unit vector $\boldsymbol{i}$.



a On the diagram mark in all forces.

b Find the acceleration of the particle, correct to two decimal places. Assume the acceleration due to gravity has a magnitude of 9.8 m/s^2.

49 A constant force acts on a particle of mass 5 kg for 2 seconds causing the particle's velocity to change from $5\boldsymbol{i} + 12\boldsymbol{j}$ m/s to $4\boldsymbol{i} - 6\boldsymbol{j}$ m/s. Find

a the acceleration of the particle

b the magnitude of the constant force.

Extended-response questions

1 Let C_1 be the curve specified by the parametric equations $x = 2\cos t$, $y = \sin t$ for $0 < t < \frac{\pi}{2}$ and let L be the line through P on C_1 with a gradient of $2\tan t$. This line L intersects the x- and y-axes at the points X and Y respectively.

a Find, in terms of t, an expression for the area of the triangle OXY, where O is the origin.

b Find the maximum area of this triangle and the coordinates of P when this occurs.

c Find the Cartesian equation of the locus, C_2, of the midpoint of the interval XY.

d Sketch the graphs of C_1 and C_2 on the same set of axes.

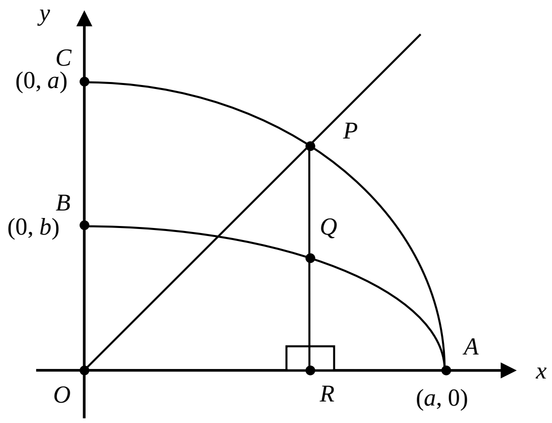
2 For the graph of $y = 4\cos\left(\frac{\pi}{6} - 2x\right) - 2$, $x \in [0, 2\pi]$, find

a the y-axis intercept

b the x-axis intercepts

c the coordinates of the stationary points.

3 In this diagram APC is a quarter of a circle, centre O, and $P(a \cos t, a \sin t)$ is a point on that curve. Also, AQB is a quarter of an ellipse, centre O, and Q is a point on that curve. Points P, Q and R are collinear, where R is the foot of the perpendicular on the x-axis.



a Find the Cartesian equation of the circle.

b Find the Cartesian equation of the ellipse.

c Find the coordinates of R.

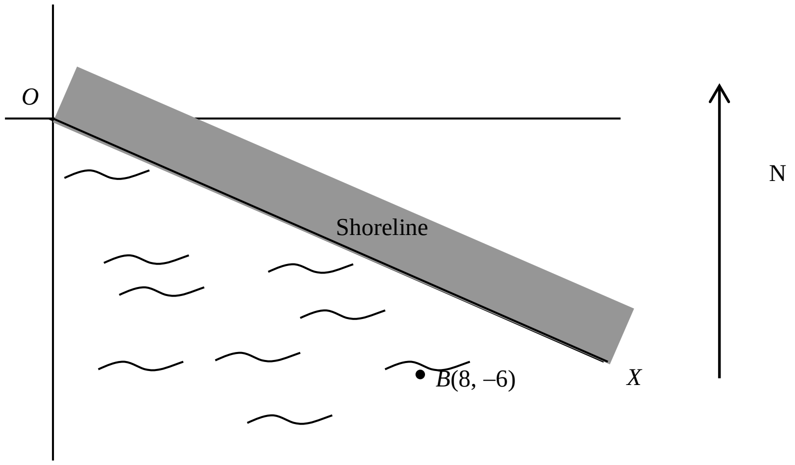
d Find the coordinates of Q.

e Find the ratio of the area of ΔOQR to the area of ΔOPQ.

f Find the coordinates of M, where M is the midpoint of PQ.

g Find the Cartesian equation of the locus of M, and identify the locus carefully.

4 In the diagram there is a yacht at a point B. There is a straight line, shoreline OX. The equation of the line through O and X is $y = \frac{-1}{4}x$. Let $\boldsymbol{i}$ and $\boldsymbol{j}$ be vectors in the north and east directions respectively. Let $P(4m, -m)$ be a point on the shoreline.

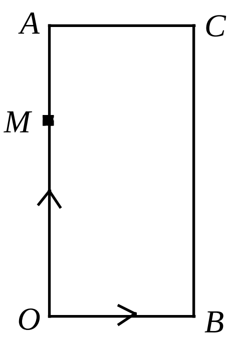


a Express the scalar product $\overrightarrow{OP} \cdot \overrightarrow{PB}$ in terms of m.

b **i** Hence or otherwise find the coordinates of the point on the shoreline closest to the yacht.

ii Find the distance from the yacht to this closest point.

5 $OACB$ is a rectangle with = $a\boldsymbol{i}$. M divides OA in the ratio 2 : 1, with M closer to A.



a Find $\overrightarrow{OM}$ in terms of a.

b **i** Show that $\overrightarrow{MC} = a\boldsymbol{i} + \frac{2}{3}a\boldsymbol{j}$.

P is a point on MC such that $\overrightarrow{MP} = \lambda \overrightarrow{MC}$

ii Find $\overrightarrow{PB}$ in terms of λ and a.

iii Find the value of λ such that BP is perpendicular to MC.

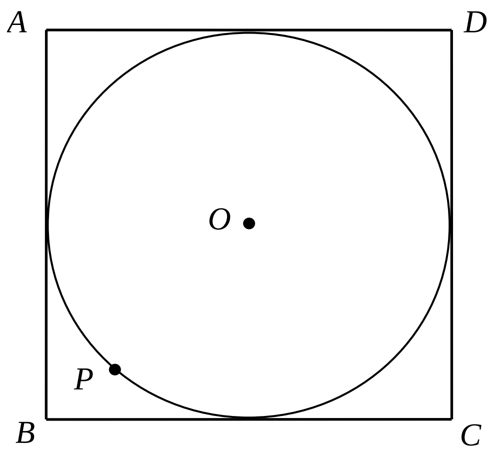
6 For vectors $\boldsymbol{a} = 3\boldsymbol{i} + 5\boldsymbol{j}$ and $\boldsymbol{b} = 4\boldsymbol{i} - 5\boldsymbol{j}$, describe through a Cartesian equation and sketch the graph of the set of points with position vector $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$ such that:

a $|\boldsymbol{r} - \boldsymbol{a}| = |\boldsymbol{r} - \boldsymbol{b}|$

b $|\boldsymbol{r} - \boldsymbol{a}| = 6$

c $\boldsymbol{r} \cdot (\boldsymbol{r} - \boldsymbol{a}) = 0$

7 In the figure, O is the centre of the circle, radius r. The circle is inscribed in a square $ABCD$, and P is any point on the circumference of the circle.



a Show that $\overrightarrow{AP} \cdot \overrightarrow{AP} = 3r^2 - 2\overrightarrow{OP} \cdot \overrightarrow{OA}$.

b Hence find $AP^2 + BP^2 + CP^2 + DP^2$ in terms of r.

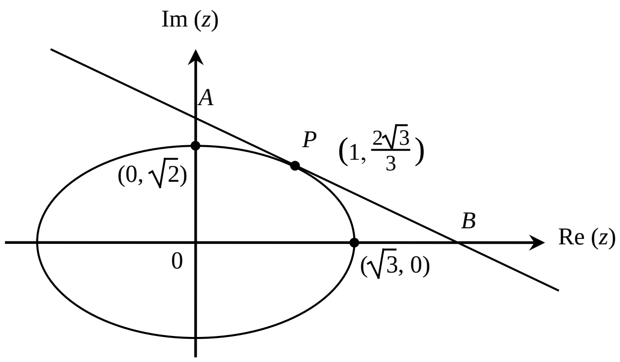
8 **a** Find the solutions of the equation $z^2 + 5z + 7 = 0$, expressing your answers in exact Cartesian form.

b Plot the solutions on an Argand diagram.

c Express the solutions in polar form.

d An equation of the form $z^4 + az^3 + bz^2 + cz + d = 0$, where $\{a, b, c, d\} \in \mathbb{R}$, has solutions $\pm i$ and the solutions of the quadratic equation in **a**. Find the values of a, b, c and d.

9



a Find the equation of the ellipse shown above.

b Show that P is a point on the ellipse.

c If $w = \sqrt{3} - i$, prove that $z\overline{w} - \overline{z}w = 6i$, where $z \in C$, describes AB, the tangent to the ellipse at P.

10 **a** Find the solutions of $z^2 - 4z + 7 = 0$ where z is a complex number, and hence find the sum and the product of the solutions.

b Let v and w be the solutions of the equation $z^2 + az + b = 0$ where a, b and z are complex numbers.

i Show that $v + w = -a$ and $vw = b$.

ii Hence show that if $v = p + qi$ where p and q are real numbers, and v and w are complex conjugates, then a and b are real.

c Find the quadratic equation which has solutions $3 + \sqrt{2}\, i$ and $3 - \sqrt{2}\, i$.

d Given that a quadratic equation has solutions v and w, and the sum and product of the solutions is -4 and 5 respectively, find a quadratic equation which has solutions $v + w$ and $v - w$.

11 Let $u = \text{cis}\left(\frac{5\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4} + \frac{\sqrt{6}+\sqrt{2}}{4}\, i$, and let $v = \text{cis}\ \frac{\pi}{12}$.

a Express v in Cartesian form.

b Find the exact value of Arg $(v - u)$.

c Find the exact value of Arg $(v + u)$.

d Let u and $\overline{u}$ be roots of the equation $z^2 + bz + c = 0$. Show that $c = 1$ and find the exact value of b.

12 A right circular cone, with base diameter 20 cm and height 20 cm, is held vertex down and completely filled with water. There is a hole at the vertex through which water leaks at a constant rate of 10 cm^3/s. The water leaked out is caught in a hemispherical bowl, of radius 10 cm, which is directly below the cone. At the moment that the depth of water in the bowl is 5 cm, calculate, correct to two decimal places:

a the depth of water in the cone

b the rate at which water level in the cone is falling

c the rate at which water level in the bowl is rising.

13 **a** Show that $\dfrac{d}{dx}\left(x\sqrt{9-x^2}+9\sin^{-1}\left(\dfrac{x}{3}\right)\right)=2\sqrt{9-x^2}$

b Sketch the curve with equation $x^2 + 9y^2 = 9$.

c Find the area of the region enclosed by the curve with equation $x^2 + 9y^2 = 9$.

d Find the equation of the tangent to the curve with equation $x^2 + 9y^2 = 9$ at the point $\left(\sqrt{5},\dfrac{2}{3}\right)$.

e Find the volume of the solid of revolution formed by rotating the curve with equation $x^2 + 9y^2 = 9$ about:

i the x-axis

ii the y-axis.

14 When filled to a depth of h metres, a water tank contains v litres of water, where

$v = 4000\pi\left(h^2-\dfrac{h^3}{3}\right)$, $0 \le h \le 1$.

a If water is pumped into the tank at 500 litres per hour, at what rate, in metres per hour, is the water level rising when the depth is 0.5 metres?

b When the tank is emptied for cleaning purposes, the intake flow is cut off and water is allowed to drain out from a tap at the bottom of the tank at a variable rate of $\dfrac{dv}{dt}=-1000\sqrt{h}$ litres per hour, where h metres is the depth of water in the tank.

i When water drains from the tank under these conditions, show that the differential equation which relates h to t is given by $\dfrac{dh}{dt}=\dfrac{-1}{4\pi\sqrt{h}(2-h)}$.

ii Express t in terms of h, given that initially the tank is full (that is, $h = 1$).

iii How long, to the nearest tenth of an hour, will it take to empty the tank if it is full initially?

c On one occasion when the tank was being prepared for cleaning, the intake flow of 500 litres per hour was accidentally allowed to continue while the tank was being drained.

i Find the differential equation which relates h to t in this situation.

ii Hence, without attempting to solve the differential equation, explain what happens to the water level in the tank given that initially the tank is full.

15 An amateur rocket scientist builds a small rocket. He determines after testing that it has an initial velocity of 50 m/s and that for the first 10 seconds, when launched vertically, the acceleration, a m/s^2 is given by $a = \dfrac{-v^2}{125}$, where v m/s is the velocity of the rocket at time t s. The rocket is launched from ground level. After this initial phase, the rocket's motion is subject to gravity and air resistance so that the retardation during the upwards motion is given by $a = -(0.01v + 9.8)$ m/s^2.

a For the 'powered' phase of the motion:

i prove that $v = \dfrac{250}{2t+5}$

ii prove that the height above ground level, y m, is given by

$$y = 125 \log_e\left(\frac{2t+5}{5}\right)$$

iii find the velocity and height of the rocket at the end of this phase.

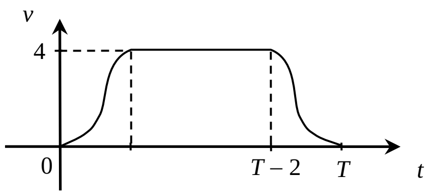
b For the second phase of the motion of the rocket, in the upwards section,

i express v in terms of t

ii express y in terms of t

iii find the maximum height reached by the rocket, correct to two decimal places.

16 The velocity–time graph below shows the velocity of a lift as it travels from the first floor to the twelfth floor of a tall building during the T seconds of its motion.



The velocity, v m/s, at time t s for $0 \le t \le 2$ is given by $v = t^2(3 - t)$. After the first 2 seconds, the lift moves with a constant velocity of 4 m/s for a time, and then decelerates to rest in the final 2 seconds. The acceleration of the lift is a m/s^2 at time t s and the velocity–time graph is symmetrical about $t = \frac{1}{2}T$.

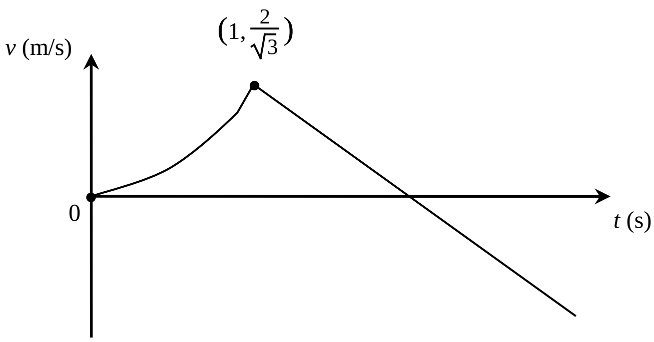
a **i** Express a in terms of t for the first 2 seconds of the motion of the lift.

ii Hence find the maximum acceleration of the lift during the first 2 seconds of its motion.

b Given that the total distance travelled by the lift during its 'journey' is 41 metres, use calculus to find the exact value of T.

17 **a** Find an antiderivative of $\dfrac{2t}{\sqrt{4-t^2}}$.

b This is the velocity–time graph for a particle moving in a line.



The velocity, v m/s, at time t s is given by $v = \begin{cases} \dfrac{2t}{\sqrt{4-t^2}} & 0 \le t \le 1 \\ \dfrac{3-t}{\sqrt{3}} & t > 1 \end{cases}$

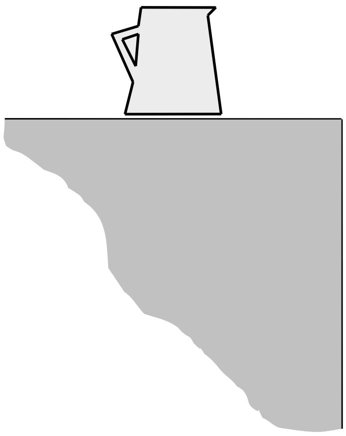
i Find the distance travelled in the first second.

ii Find when the particle comes to rest.

iii Find the distance travelled from its initial position until it comes to rest.

iv If a m/s^2 is the acceleration at time t seconds, express a in terms of t for $0 \le t < 1$.

18 A jug of mass 1 kilogram is pushed by a force $\dfrac{7+5t}{5}$ N along a rough platform towards the edge. The coefficient of friction between the platform and the jug is 0.1. (Acceleration due to gravity is 9.8 m/s^2.)



a On the diagram mark in all of the forces acting on the jug.

b Find the acceleration, in terms of t, of the jug at time t seconds, $t \ge 0$.

c Initially the jug is at rest. It reaches the edge of the platform in 2 seconds.

i Find the speed of the jug at this time.

ii How far has it moved along the platform in the 2 seconds?

As soon as it reaches the edge of the platform the force is no longer applied and it moves off the platform with only gravitational force acting.

d **i** Find the vector equation for the velocity of the jug at time t seconds after leaving the edge of the platform.

ii Find the vector equation for the position of the jug at time t. The edge of the platform, as shown in the diagram, is to be taken as the origin.

iii The table is 1.4 m high. How far from the bottom of the platform will the jug land? Give your answer to the nearest centimetre.

Answers to Additional exercises 2

Answers to multiple-choice questions

1	D	**22**	A
2	C	**23**	D
3	D	**24**	C
4	B	**25**	A
5	D	**26**	E
6	D	**27**	A
7	C	**28**	D
8	C	**29**	D
9	D	**30**	A
10	E	**31**	D
11	B	**32**	D
12	A	**33**	C
13	E	**34**	B
14	B	**35**	E
15	E	**36**	D
16	A	**37**	D
17	D	**38**	C
18	E	**39**	B
19	E	**40**	E
20	E	**41**	D
21	A	**42**	E

Answers to short-answer (technology-free) questions

1 **a** $50°$ **b** $50°$

c $100°$ **d** $50°$

2 **a** 30 **b** 3

3 31

4 $\dfrac{3\pi}{4}, \dfrac{7\pi}{4}, \dfrac{11\pi}{4}, \dfrac{\pi}{12}, \dfrac{13\pi}{12}, \dfrac{25\pi}{12}$

6 $9.5\boldsymbol{i} + \dfrac{3\sqrt{3}}{2}\boldsymbol{j}$

8 **a** $\overrightarrow{OF} = \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$, $\frac{1}{2}(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$

9 $\pm\sqrt{3}$

10 $m = 2$, $n = 1$

11 **a** $\overrightarrow{RQ} = \boldsymbol{q} - \boldsymbol{r}$, $\overrightarrow{RP} = -\boldsymbol{q} - \boldsymbol{r}$

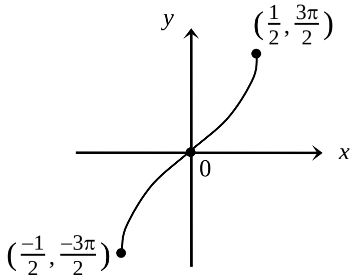
12 $\frac{x^2}{25} + \frac{y^2}{9} = 1$

13 **a** $\frac{5\pi}{6}$ **b** $\frac{\pi}{4}$

14 Dom = [2, 4], ran = $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

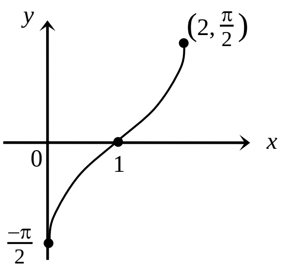
15 **a** $\left[\frac{-1}{2}, \frac{1}{2}\right]$

b



c $\left[\frac{-3\pi}{2}, \frac{3\pi}{2}\right]$

16

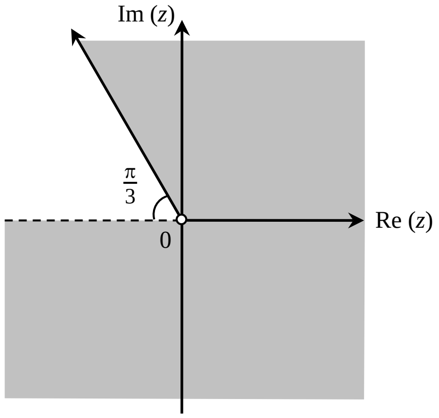


17 $1 - \sqrt{2}$

18 **a** $\frac{\pi}{4}$ **b** $\frac{-3\pi}{4}$

c $\frac{-\pi}{4}$

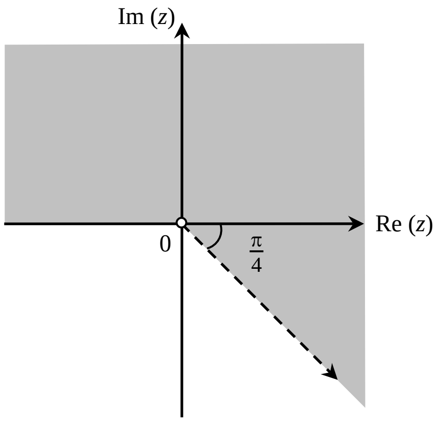
21



22 $1.15 + 1.29i$, $0.542 - 1.65i$, $-1.70 + 0.353i$

23 $2 \text{ cis} \frac{\pi}{6}$, $2 \text{ cis} \frac{5\pi}{6}$, $2 \text{ cis}\left(\frac{-\pi}{2}\right)$

24



25 $\frac{3 \pm \sqrt{5}}{6}$

26 **a** $\frac{y^2}{3 - 2xy}$ **b** 1

27 **a** $\frac{4}{15}$ **b** π

c $\frac{1}{2} \log_e \left(\frac{5}{4}\right)$ **d** $\frac{-2}{9}$

e $\frac{\pi}{2}$ **f** $2\sqrt{3}\,(\sqrt{5} - 1)$

28 **b** $2\sqrt{3}\tan^{-1}\left(\dfrac{x+3}{\sqrt{3}}\right)+c$ **c** $\dfrac{2\pi\sqrt{3}}{3}$

29 $\dfrac{\pi(3e^2-3+2e^4)}{6}$ cubic units

30 **a** 1593 years **b** 99.1%

31 $y = x^3 + 3x^2 - 4$

32 **a** $t = 600(\sqrt{5}+\sqrt{6})(\sqrt{6}-\sqrt{h}),\ 0 \le h \le 6$

b 11 minutes 4 seconds

33 $c = \dfrac{-1}{2}$

34 $\dfrac{dQ}{dt} = \dfrac{1}{4} - \dfrac{3Q}{500+2t}$ where at $t = 0$, $Q = 10$

35 $m = -6$, $n = 9$

36 $y = \dfrac{2x}{1-x}$

37 **a** $v = \dfrac{4\pi}{75}h^3$ **b** $\dfrac{dh}{dt} = \dfrac{-5}{4\pi h^2}$

38 **a** At $t = \dfrac{1}{2}$ when $x = 6$ **b** At $t = 2$ when $x = 0$

39 $2e^{\frac{3}{2}}$ metres

40 **a** $a = -2x - 6$ **c** $\sqrt{42}$ m/s

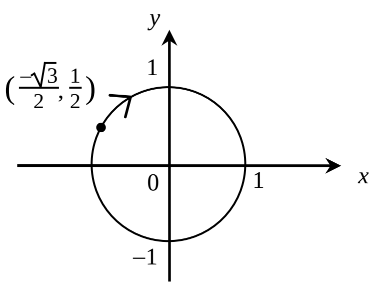
41 **a** $t = \log_e\left(\dfrac{2(30-v)}{30+v}\right)$ **b** $\log_e 2$ seconds

42 **a** $x = 50\log_e\left(\dfrac{100g}{100g-v^2}\right)$ **b** $v = 10\sqrt{g\left(1-e^{-\frac{x}{50}}\right)}$

43 **a** $y = \dfrac{-3}{4}x^2 + \dfrac{9}{2}x - \dfrac{23}{4}$ **b** $(x-1)^2 + (y-3)^2 = 1$

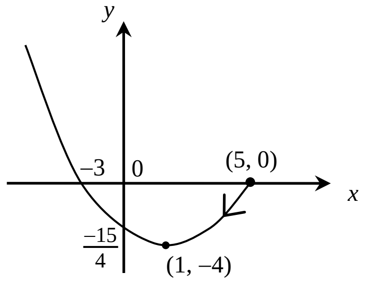
44 **a** **i** $x^2 + y^2 = 1$, dom: $[-1, 1]$, ran: $[-1, 1]$

ii



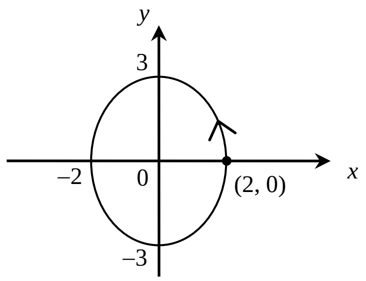
b **i** $y = \frac{1}{4}(x - 1)^2 - 4$, dom: $(-\infty, 5]$, ran: $[-4, \infty)$

ii



c **i** $\frac{x^2}{4} + \frac{y^2}{9} = 1$, dom: $[-2, 2]$, ran: $[-3, 3]$

ii

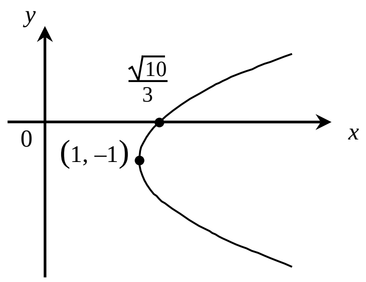


45 **a** (4, 2) **b** $(4, 2), \left(\frac{3}{2}, \frac{3}{4}\right)$

c $\sqrt{2}$ units

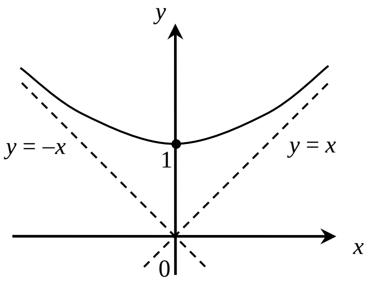
46 **a** **i** $9x = (y + 1)^2 + 9$, dom: $[1, \infty)$, ran: $\mathbb{R}$

ii



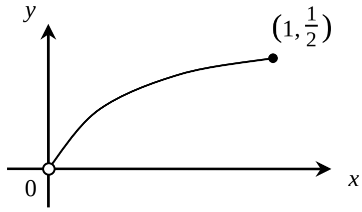
b **i** $x^2 + 1 = y^2$, dom: $\mathbb{R}$, ran: $[1, \infty)$

ii



c **i** $y = \dfrac{x}{x+1}$, dom: $(0, 1]$, ran: $\left(0, \dfrac{1}{2}\right]$

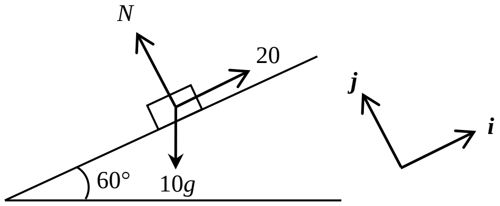
ii



47 **a** $58.8 - \dfrac{5\sqrt{3}}{2}$ **b** 0.51 m/s^2

c 2.54 m/s

48 **a**



b $-6.49\boldsymbol{i}$ m/s^2

49 **a** $\dfrac{-1}{2}\boldsymbol{i} - 9\boldsymbol{j}$ m/s^2 **b** $\dfrac{25\sqrt{13}}{2}$ N

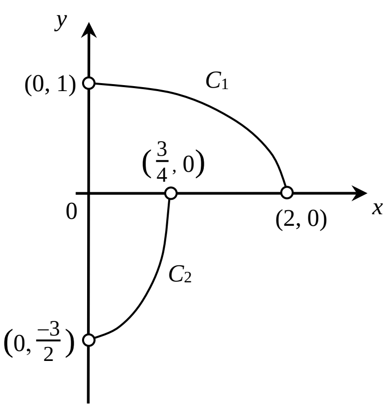
Answers to extended-response questions

1 **a** $\dfrac{9\sin t\cos t}{4}$

b Maximum area is 1.125 square units, $P = \left(\sqrt{2}, \dfrac{\sqrt{2}}{2}\right)$

c $\dfrac{16x^2}{9} + \dfrac{4y^2}{9} = 1,\ 0 < x < \dfrac{3}{4}, \dfrac{-3}{2} < y < 0$

d



2 **a** $2\sqrt{3} - 2$ **b** $\frac{\pi}{4}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{23\pi}{12}$

c $\left(\frac{\pi}{12}, 2\right), \left(\frac{7\pi}{12}, -6\right), \left(\frac{13\pi}{12}, 2\right), \left(\frac{19\pi}{12}, -6\right)$

3 **a** $x^2 + y^2 = a^2, 0 \le x \le a, 0 \le y \le a$ **b** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, 0 \le x \le a, 0 \le y \le b$

c $(a\cos t, 0)$ **d** $(a\cos t, b\sin t)$

e $\frac{b}{a-b}$ **f** $\left(a\cos t, \frac{a\sin t + b\sin t}{2}\right)$

g $\frac{x^2}{a^2} + \frac{4y^2}{(a+b)^2} = 1$. The locus of M is a quarter of an ellipse.

4 **a** $38m - 17m^2$

b **i** $\left(\frac{152}{17}, \frac{-38}{17}\right)$

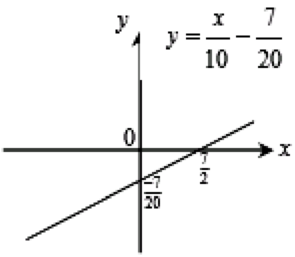
ii $\frac{16}{\sqrt{17}}$

5 **a** $\frac{4a}{3}\boldsymbol{j}$

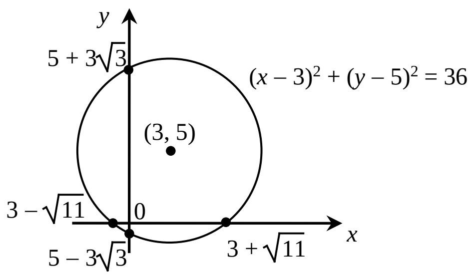
b **ii** $(1 - \lambda)a\,\boldsymbol{i} - \frac{2a}{3}(2 + \lambda)\,\boldsymbol{j}$

iii $\lambda = \frac{1}{13}$

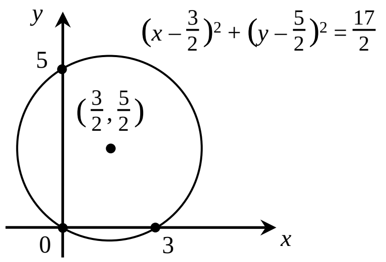
6 **a**



b



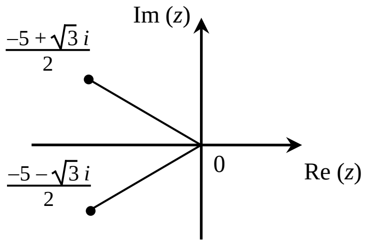
c



7 **b** $12r^2$

8 **a** $\dfrac{-5\pm\sqrt{3}i}{2}$

b



c $\sqrt{7}$ cis (2.808), $\sqrt{7}$ cis (–2.803)

d $a = 5$, $b = 8$, $c = 5$, $d = 7$

9 **a** $\dfrac{x^2}{3}+\dfrac{y^2}{2}=1$

10 **a** $2\pm\sqrt{3}\ i$, sum = 4, product = 7 **c** $z^2 - 6z + 11 = 0$

d $z^2 + (4 \pm 2i)z \pm 8i = 0$

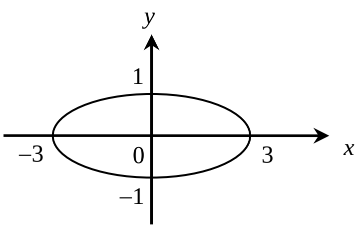
11 **a** $v=\dfrac{\sqrt{6}+\sqrt{2}}{4}+\dfrac{\sqrt{6}-\sqrt{2}}{4}\,i$ **b** $\dfrac{-\pi}{4}$

c $\dfrac{\pi}{4}$ **d** $-\left(\dfrac{\sqrt{6}-\sqrt{2}}{2}\right)$

12 **a** 17.65 cm **b** 0.04 cm/s

c 0.04 cm/s

13 **b**



c 3π

d $6y = -\sqrt{5}x + 9$

e **i** 4π cubic units

ii 12π cubic units

14 **a** $\frac{1}{6\pi}$ m/h

b **ii** $t = \frac{8\pi}{5}h^{\frac{5}{2}} - \frac{16\pi}{3}h^{\frac{3}{2}} + \frac{56\pi}{15}$

iii 11.7 hours

c **i** $\frac{dh}{dt} = \frac{1-2\sqrt{h}}{8\pi h(2-h)}$

ii The water does not drain out completely. It stabilises at a depth of 0.25 metres.

15 **a** **iii** $v = 10$, $y = 125\log_e 5$

b **i** $v = 990e^{\frac{10-t}{100}} - 980$

ii $y = -99\,000e^{\frac{1}{10}-\frac{t}{100}} - 980t + 125\log_e 5 + 108\,800$

iii 206.25 metres

16 **a** **i** $a = 6t - 3t^2$

ii 3 m/s^2

b $T = 12.25$

17 **a** $-2\sqrt{4-t^2}$

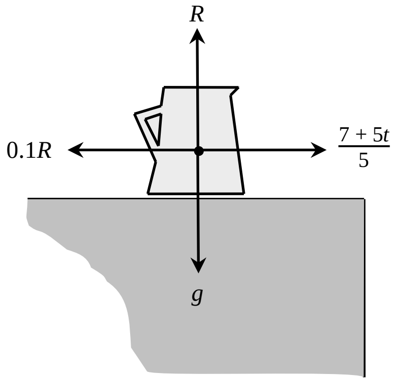
b **i** $4 - 2\sqrt{3}$ metres

ii At $t = 3$

iii $\frac{12-4\sqrt{3}}{3}$ metres

iv $a = \frac{8}{(4-t^2)^{\frac{3}{2}}}$

18 **a**



b $\dfrac{21+50t}{50}$ m/s^2

c **i** $\dfrac{71}{25}$ m/s

ii $\dfrac{163}{75}$ m

d **i** $2.84\boldsymbol{i} - 9.8t\boldsymbol{j}$

ii $2.84t\boldsymbol{i} - 4.9t^2\boldsymbol{j}$

iii 152 cm