

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 2



2024 Trial Examination

SOLUTIONS

SECTION A

Question 1

D

Explanation:

“For $x \in N$, there exists a prime number of the form: $x^2 + 3x + 2$ ” is false, since:

$x^2 + 3x + 2 = (x + 1)(x + 2)$, so $x^2 + 3x + 2$ has factors 1, $x + 1$, $x + 2$, $(x + 1)(x + 2)$ therefore cannot be prime.

Question 2

A

Explanation:

$E \Rightarrow C$

n is a natural number $\Rightarrow n^2 + n$ is an even number.

$n \in N$

If n is even

$$n^2 + n = (2m)^2 + 2m, m \in N$$

$$(2m)^2 + 2m = 2(2m^2 + m), \text{ which is even}$$

If n is odd

$$n^2 + n = (2m + 1)^2 + 2m + 1, m \in \mathbb{N}$$

$$(2m + 1)^2 + 2m + 1 = 2(2m^2 + 3m + 1), \text{ which is also even}$$

Question 3

B

Explanation:

$$f(x) = \frac{2x}{\sqrt{x^2 - 36}} + 2$$

$$\text{As } x \rightarrow \pm\infty, f(x) \rightarrow \frac{2x}{x} + 2 = 4$$

So, the equation of the horizontal asymptote is $y = 4$

Question 4

D

Explanation:

The table of values is:

a	b
2	7
3	20
4	44
5	81

Question 5

C

Explanation:

Consider the graphs of the functions:

Check the graph of $y = \sec 3\theta$ and $y = \operatorname{cosec} 2\theta$ on your CAS.

There are 4 intersection points every 2π

Question 6**B***Explanation:*

$$y = \cos^{-1}((1 - 2x) - \sin^{-1}(\frac{1}{2x}))$$

For $y = \cos^{-1}((1 - 2x) - \sin^{-1}(\frac{1}{2x}))$ to be defined, $-1 \leq 1 - 2x \leq 1$

$$-2 \leq -2x \leq 0$$

$$-1 \leq -x \leq 0$$

$$0 \leq x \leq 1$$

For $\sin^{-1}(\frac{1}{2x})$ to be defined, $-1 \leq \frac{1}{2x} \leq 1$

$$-2 \leq \frac{1}{x} \leq 2$$

$$x \leq -\frac{1}{2} \cup x \geq \frac{1}{2}$$

$$\{0 \leq x \leq 1\} \cap \left\{x \leq -\frac{1}{2} \cup x \geq \frac{1}{2}\right\} = \left\{\frac{1}{2} \leq x \leq 1\right\}$$

$$x \in \left[\frac{1}{2}, 1\right]$$

Question 7**B**

$$\cot a = b$$

$$\tan a = \frac{1}{b}$$

$$\sin a = \frac{1}{\sqrt{b^2 + 1}}$$

$$\cos a = \frac{b}{\sqrt{b^2 + 1}}$$

$$\sin 2a = 2 \sin a \cos a$$

$$= 2 \times \frac{1}{\sqrt{b^2 + 1}} \times \frac{b}{\sqrt{b^2 + 1}}$$

$$= \frac{2b}{1 + b^2}$$

Question 8**B***Explanation:*

$$\bar{z}_1 = 2\text{cis}\left(\frac{\pi}{6}\right)$$

$$\sqrt{z_2} = \sqrt{4}\text{cis}\left(\frac{\pi}{6}\right) = 2\text{cis}\left(\frac{\pi}{6}\right)$$

$$\bar{z}_1\sqrt{z_2} = 2\text{cis}\left(\frac{\pi}{6}\right) \times 2\text{cis}\left(\frac{\pi}{6}\right) = 4\text{cis}\left(\frac{\pi}{3}\right)$$

$$4\text{cis}\left(\frac{\pi}{3}\right) = 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

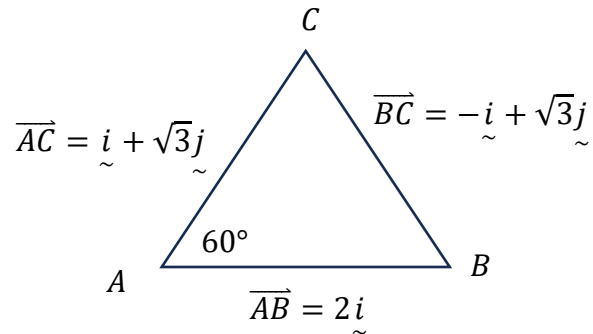
$$= 2 + 2\sqrt{3}i$$

Question 9**C***Explanation:*

$$\overrightarrow{AB} = 2\tilde{i}, \overrightarrow{BC} = -\tilde{i} + m\tilde{j} = -\tilde{i} + \sqrt{3}\tilde{j}$$

$$m = \sqrt{3}$$

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = (2\tilde{i}) \cdot (-\tilde{i} + \sqrt{3}\tilde{j}) = -2$$

**Question 10****A***Explanation:*

$$\tilde{a} \parallel \tilde{b} = -\frac{1}{3}\left(\frac{1}{3}(-2\tilde{i} - 2\tilde{j} + \tilde{k})\right) = -\frac{1}{9}(-2\tilde{i} - 2\tilde{j} + \tilde{k})$$

$$\tilde{a} \perp \tilde{b} = \tilde{a} - (\tilde{a} \parallel \tilde{b})$$

$$= (\tilde{i} + \tilde{k}) + \frac{1}{9}(-2\tilde{i} - 2\tilde{j} + \tilde{k})$$

$$= \frac{1}{9}(7\tilde{i} - 2\tilde{j} + 10\tilde{k}) = \frac{\tilde{c}}{9}$$

$$\tilde{c} = 7\tilde{i} - 2\tilde{j} + 10\tilde{k}$$

Question 11**D***Explanation:*

$$\Pi_1 = \tilde{r} \cdot \left(-\tilde{i} + \tilde{j} - 2\tilde{k} \right) = 2 \text{ and } \Pi_2 = \tilde{r} \cdot \left(2\tilde{i} - \tilde{j} + 2\tilde{k} \right) = 4$$

Cartesian equation of Π_1 : $-x + y - 2z = 2$ Cartesian equation of Π_2 : $2x - y + 2z = 4$

Add the equations:

$$x = 6$$

$$\text{So, } -6 + y - 2z = 2$$

$$y - 2z = 8$$

$$\text{Let } y = \lambda, \lambda \in R$$

$$\lambda - 2z = 8$$

$$z = \frac{\lambda - 8}{2}$$

$$x = 6, y = \lambda, z = \frac{\lambda - 8}{2}, \lambda \in R$$

$$\tilde{r} = 6\tilde{i} - 4\tilde{k} + \lambda \left(\tilde{j} + \frac{1}{2}\tilde{k} \right), \lambda \in R$$

Question 12**A***Explanation:*

$$\int (\tan^{-1} x) dx \text{ where } u = \tan^{-1} x \text{ and } \frac{dv}{dx} = 1$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$u = \tan^{-1} x, \frac{dv}{dx} = 1$$

$$\int (\tan^{-1} x) dx = x \tan^{-1} x - \int x \left(\frac{1}{1+x^2} \right) dx$$

$$= x \tan^{-1} x - \frac{1}{2} \log_e(1+x^2) + c$$

Question 13**B***Explanation:*

$$a = \frac{dv}{dt} = (0.5v - 1)ms^{-2}.$$

$$t = 2 \int \frac{0.5}{0.5v - 1} dv$$

$$t = 2\log_e(0.5v - 1) + c$$

$$0 = 2\log_e(0.5 \times 6 - 1) + c$$

$$c = -2\log_e(2)$$

$$t = 2\log_e(0.5v - 1) - 2\log_e(2)$$

$$t = 2\log_e\left(\frac{0.5v - 1}{2}\right)$$

$$0.5v - 1 = 2e^{0.5t}$$

$$v = 4e^{0.5t} + 2$$

$$x = 8e^{0.5t} + 2t - 8$$

$$x(2) = (8e - 4)m \approx 17.75m$$

Question 14**D***Explanation:*

$$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = (-36x)ms^{-2}.$$

$$\frac{1}{2}v^2 = \int (-36x)dx$$

$$v^2 = -36x^2 + c$$

$$(12)^2 = -36(0)^2 + c$$

$$c = 144$$

$$v^2 = 144 - 36x^2$$

$$v^2 = 36(4 - x^2)$$

$$v = \pm 6\sqrt{(4 - x^2)}$$

$$x = 0, v = 6$$

$$v = 6\sqrt{(4 - x^2)}$$

$$\frac{dx}{dt} = 6\sqrt{(4-x^2)}$$

$$\frac{dt}{dx} = \frac{1}{6\sqrt{(4-x^2)}}$$

$$t = \frac{1}{6} \sin^{-1}\left(\frac{x}{2}\right)$$

$$x = 2 \sin 6t$$

$$v = 12 \cos 6t$$

$$a = 72 \sin 6t$$

$$a(2.5) = 72 \sin(15) \approx 47 \text{ cm s}^{-2}$$

Question 15**D***Explanation:*

$$E(X) = 24, \text{Var}(X) = 1, E(Y) = 30, \text{Var}(Y) = 2.$$

$$C = 4X - 3Y$$

$$E(C) = 4E(X) - 3E(Y)$$

$$= 4 \times 24 - 3 \times 30 = 6$$

$$\text{Var}(C) = 16\text{Var}(X) + 9\text{Var}(Y)$$

$$= 16 \times 1 + 9 \times 2 = 34$$

$$\Pr(C < 0) = \Pr\left(z < \frac{0 - 6}{\sqrt{34}}\right) \approx 0.15$$

Question 16**B***Explanation:*

$$\Pr\left(\frac{-2}{\frac{5}{\sqrt{n}}} < z < \frac{2}{\frac{5}{\sqrt{n}}}\right) = 0.90$$

$$\Pr\left(z < \frac{2}{\frac{5}{\sqrt{n}}}\right) = 0.95$$

$$\frac{2}{\frac{5}{\sqrt{n}}} \approx 1.645$$

$$\frac{5}{\sqrt{n}} \approx 1.2159$$

$$n \approx 16.9$$

So, 17 days.

Question 17**C***Explanation:*

The chance that sample statistic is **more** extreme than the one observed is known the p – value.

Question 18**C***Explanation:*

$$\vec{a}(t) = 2\sin 2t \vec{i} - 4\cos 2t \vec{j}$$

$$\vec{v}(t) = -\cos 2t \vec{i} - 2\sin 2t \vec{j} + \vec{c}$$

$$\vec{v}(0) = -\cos 0 \vec{i} - 2\sin 0 \vec{j} + \vec{c} = \vec{i} + 2\vec{j}$$

$$-\vec{i} + \vec{c} = \vec{i} + 2\vec{j}$$

$$\vec{c} = 2\vec{i} + 2\vec{j}$$

$$\vec{v}(t) = (2 - \cos 2t)\vec{i} + (2 - 2\sin 2t)\vec{j}$$

$$\vec{v}(\pi) = (2 - \cos 2\pi)\vec{i} + (2 - 2\sin 2\pi)\vec{j}$$

$$\vec{v}(\pi) = \vec{i} + 2\vec{j}$$

$$|\vec{v}(\pi)| = \sqrt{1 + 4} = \sqrt{5} \text{ ms}^{-1}$$

Question 19**B***Explanation:*

$$L(t) = \int_0^{\frac{\pi}{6}} \sqrt{\left(\frac{8}{(\cos 2t)^2}\right)^2 + (-\sin t)^2} dt \approx 6.93$$

Question 20**D***Explanation:*

$$\tilde{x}(t) = 20t\tilde{i} + (-4.9t^2 + 20t + 2)\tilde{j}$$

$$\tilde{v}(t) = 20\tilde{i} + (-9.8t + 20)\tilde{j}$$

Projectile lands when $-4.9t^2 + 20t + 2 = 0$

$$t \approx 4.179s$$

$$\tilde{v}(4.179) \approx 20\tilde{i} + (-20.957)\tilde{j}$$

$$\tan \theta \approx \frac{20.957}{20}$$

$$\theta \approx 46^\circ 20'$$

SECTION B**Question 1 (12 marks)****a.** (1 mark)*Answer:*

$$V = 400 - 5t$$

1A**b.** (1 mark)*Answer:*

$$S = 400 \times 4 = 1600g$$

1A**c.** (2 marks)*Answer:*

$$dS/dt = \text{Salt in} - \text{Salt out}$$

$$dS/dt = 0 - \frac{S}{400-5t} \times 10$$

1W

$$dS/dt = \frac{-10S}{400-5t} = \frac{2S}{t-80}$$

1A

d. (3 marks)*Answer:*

$$\frac{dS}{dt} = \frac{2S}{t-80}$$

$$\int \frac{1}{S} dS = \int \frac{2}{t-80} dt$$

$$\log_e S = 2\log_e(|t-80|) + c$$

1W

$$t = 0, S = 1600$$

$$\log_e 1600 = 2\log_e(|0-80|) + c$$

$$c = \log_e 1600 - 2\log_e 80$$

$$c = \log_e \left(\frac{1600}{6400} \right) = \log_e \left(\frac{1}{4} \right)$$

1W

$$\log_e S = 2\log_e(|t-80|) + \log_e \left(\frac{1}{4} \right)$$

$$\log_e S = \log_e \left(\frac{(t-80)^2}{4} \right)$$

$$S = \frac{(t-80)^2}{4} = \frac{1}{4}(t-80)^2$$

1A**e.** (3 marks)*Answer:*

$$S = \frac{1}{4}(t-80)^2$$

$$\frac{dS}{dt} = \frac{1}{2}(t-80) = -30$$

$$t = 20$$

1W

$$S(20) = \frac{1}{4}(20-80)^2 = 900g$$

$$V(20) = 400 - 100 = 300L$$

1W

$$\text{Concentration of salt} = \frac{900}{300} = 3gL^{-1}$$

1A

f. (2 marks)

Answer:

$$\frac{dS}{dt} = -30, \quad \frac{dV}{dt} = -5$$

1W

$$\frac{dV}{dS} = \frac{dt}{dS} \times \frac{dV}{dt} = -\frac{1}{30} \times -5 = \frac{1}{6} \text{ Lg}^{-1}$$

1A

Question 2 (10 marks)

$$\overrightarrow{OA} = \underline{\underline{a}} = -\underline{\underline{i}} + 2\underline{\underline{j}} - \underline{\underline{k}}, \quad \overrightarrow{OB} = \underline{\underline{b}} = -2\underline{\underline{i}} + \underline{\underline{j}} + 2\underline{\underline{k}}$$

a. (1 mark)

Answer:

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = (\underline{\underline{i}} - 2\underline{\underline{j}} + \underline{\underline{k}}) + (-2\underline{\underline{i}} + \underline{\underline{j}} + 2\underline{\underline{k}}) = -\underline{\underline{i}} - \underline{\underline{j}} + 3\underline{\underline{k}}$$

1A

b. (2 marks)

Answer:

$$\underline{\underline{r}}(t) = \underline{\underline{a}} + t(\underline{\underline{b}} - \underline{\underline{a}}), t \in \mathbb{R}$$

$$\underline{\underline{r}}(t) = (-\underline{\underline{i}} + 2\underline{\underline{j}} - \underline{\underline{k}}) + t(-\underline{\underline{i}} - \underline{\underline{j}} + 3\underline{\underline{k}})$$

1W

When $t = 2$

$$\underline{\underline{r}}(2) = (-\underline{\underline{i}} + 2\underline{\underline{j}} - \underline{\underline{k}}) + 2(-\underline{\underline{i}} - \underline{\underline{j}} + 3\underline{\underline{k}}) = -3\underline{\underline{i}} + 0\underline{\underline{j}} + 5\underline{\underline{k}}$$

So, $C = (-3, 0, 5)$ is on the line containing the points A and B .

1A

c. (2 marks)

Answer:

$$D = (1, -2, 1)$$

$$\vec{r}(t) = \left(\vec{i} - 2\vec{j} + \vec{k} \right) + t \left(-3\vec{i} + 0\vec{j} + 5\vec{k} \right), t \in \mathbb{R}$$

1W, 1A

d. (2 marks)

Answer:

$$mx + y + z = n, \quad m, n \in \mathbb{R}$$

$$A = (-1, 2, -1)$$

$$-m + 2 - 1 = n \dots (1)$$

$$m + n = 1 \dots (1)$$

$$B = (-2, 1, 2)$$

$$-2m + 1 + 2 = n \dots (2)$$

$$2m + n = 3 \dots (2)$$

1W

$$(1) - (2)$$

$$-m = -2$$

$$m = 2$$

$$n = -1$$

1A

e. (3 marks)

Answer:

$$E = (-1, 0, 1)$$

Use the cross product to find the normal to the plane containing points A and B .

$$\vec{OA} = \vec{a} = -\vec{i} + 2\vec{j} - \vec{k}, \quad \vec{OB} = \vec{b} = -2\vec{i} + \vec{j} + 2\vec{k}$$

$$\vec{OA} \times \vec{OB} = (4 + 1)\vec{i} - (-2 - 2)\vec{j} + (-1 + 4)\vec{k}$$

$$\vec{n} = 5\vec{i} + 4\vec{j} + 3\vec{k}$$

1W

$$\hat{n} = \frac{1}{\sqrt{50}}(5\hat{i} + 4\hat{j} + 3\hat{k}) = \frac{\sqrt{2}}{10}(5\hat{i} + 4\hat{j} + 3\hat{k})$$

$$\overrightarrow{EA} = \overrightarrow{EO} + \overrightarrow{OA} = (\hat{i} + 0\hat{j} - \hat{k}) + (-\hat{i} + 2\hat{j} - \hat{k}) = 0\hat{i} + 2\hat{j} - 2\hat{k}$$

1W

$$\overrightarrow{EA} \parallel \hat{n} = \frac{\sqrt{2}}{5}(\hat{n})$$

$$|\overrightarrow{EA} \parallel \hat{n}| = \frac{\sqrt{2}}{5}$$

1A

Question 3 (11 marks)**a.** (2 marks)*Answer:*

$$\bar{x} = 61, \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{32}}$$

1W

$$61 \pm 1.96 \times \frac{6}{\sqrt{32}}$$

58.9 to 63.1 months

1A

b. (2 marks)*Answer:*

$$\Pr(\mu > 63) = \Pr\left(z > \frac{63 - 61}{\frac{6}{\sqrt{32}}}\right) = \Pr(z > 1.886) \approx 0.03$$

1W, 1A

c. (1 mark)*Answer:*

$$H_0 \mu = 60$$

$$H_1 \mu > 60$$

1A

d. (2 marks)

$$z = \frac{61 - 60}{\frac{6}{\sqrt{32}}} \approx 0.9428$$

1W

$$\Pr(z > 0.9428) \approx 0.173$$

$$p - \text{value} \approx 0.173$$

1A**e.** (1 mark)*Answer:*Since $0.173 > 0.05$ Accept H_0 **1A****f.** (3 marks)*Answer:*

$$\Pr(\mu > 62) = \Pr\left(z > \frac{62 - 60}{\frac{6}{\sqrt{n}}}\right) = 0.01$$

1W

$$\frac{2}{\frac{6}{\sqrt{n}}} = 2.326$$

$$\frac{2}{\frac{6}{\sqrt{n}}} = 2.326$$

1W

$$\sqrt{n} = 6.98$$

 $n = 49$ batteries.**1A**

Question 4 (12 marks)**a.** (3 marks)*Answer:*

$$z^4 = -64 = 64\text{cis}(\pi)$$

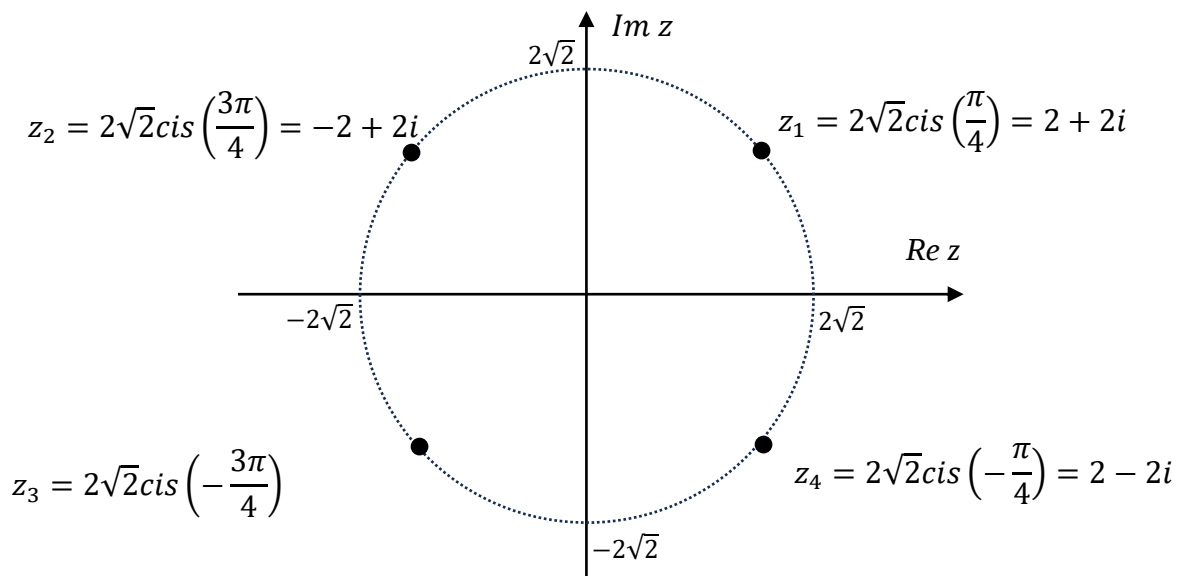
1W

$$z_1 = 64^{\frac{1}{4}}\text{cis}\left(\frac{\pi}{4}\right) = 2\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right) = 2 + 2i$$

$$z_2 = 2\sqrt{2}\text{cis}\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = 2\sqrt{2}\text{cis}\left(\frac{3\pi}{4}\right) = -2 + 2i$$

$$z_3 = 2\sqrt{2}\text{cis}\left(\frac{3\pi}{4} + \frac{\pi}{2}\right) = 2\sqrt{2}\text{cis}\left(\frac{5\pi}{4}\right) = 2\sqrt{2}\text{cis}\left(-\frac{3\pi}{4}\right) = -2 - 2i$$

$$z_4 = 2\sqrt{2}\text{cis}\left(\frac{5\pi}{4} + \frac{\pi}{2}\right) = 2\sqrt{2}\text{cis}\left(\frac{7\pi}{4}\right) = 2\sqrt{2}\text{cis}\left(-\frac{\pi}{4}\right) = 2 - 2i$$

1W, 1A**b.** (3 marks)*Answer:***2W, 1A**

c. (2 marks)

Answer:

Equation 1:

$$\begin{aligned} z_1 &= 2 + 2i, & z_4 &= 2 - 2i \\ (z - 2 - 2i)(z - 2 + 2i) &= 0 \\ z^2 - 4z + 8 &= 0 \end{aligned}$$

1A

Equation 2:

$$\begin{aligned} z_2 &= -2 + 2i, & z_3 &= -2 - 2i \\ (z + 2 - 2i)(z + 2 + 2i) &= 0 \\ z^2 + 4z + 8 &= 0 \end{aligned}$$

1A

d. (2 marks)

Answer:

$$x^2 + y^2 = 8$$

1A

$$|z| = 2\sqrt{2}$$

1A

e. (2 marks)

Answer:

$$(z - 2 + 2i)^4 = -64$$

1A

$$z_1 = 4 + 0i$$

$$z_2 = 0 + 0i$$

$$z_3 = 0 - 4i$$

$$z_4 = 4 - 4i$$

1A

Question 5 (15 marks)**a.** (2 marks)

$$f: R \rightarrow R \text{ where } f(x) = \frac{x-4}{x^2+4}$$

 x –intercept: (4,0) y –intercept: (0,−1)**2A****b.** (2 marks)

$$f(x) = \frac{x-4}{x^2+4}$$

$$\text{Stationary points: } f'(x) = \frac{-(x^2-8x-4)}{(x^2+4)^2} = 0$$

$$x^2 - 8x - 4 = 0$$

$$x = 4 \pm 2\sqrt{5}$$

$$\left(4 - 2\sqrt{5}, \frac{-\sqrt{5}-2}{4}\right), \left(4 + 2\sqrt{5}, \frac{\sqrt{5}-2}{4}\right)$$

1W

$$\text{Range of } f = \left[\frac{-\sqrt{5}-2}{4}, \frac{\sqrt{5}-2}{4}\right]$$

1A**c.** (2 marks)

$$f''(x) = \frac{-(x^3 - 12x^2 - 12x + 16)}{(x^2 + 4)^3} = 0$$

$$x \approx 0.7735$$

1W

$$f'(0.7735) \approx 0.45$$

1A

d. (2 marks)

$$\int \left(\frac{x-4}{x^2+4} \right) dx = \int \left(\frac{x}{x^2+4} \right) dx - \int \left(\frac{4}{x^2+4} \right) dx$$

$$= \frac{1}{2} \int \left(\frac{2x}{x^2+4} \right) dx - 2 \int \left(\frac{2}{x^2+4} \right) dx$$

$$= \frac{1}{2} \log_e(x^2+4) - 2 \tan^{-1} \left(\frac{x}{2} \right) + c$$

1W**1A****e.** (2 marks)

$x - 5y = 4$ intersects $y = f(x)$ at $(-1, -1)$ and $(4, 0)$ and $(1, -\frac{3}{5})$

Area bound:

$$= \left[\left(\frac{x^2}{10} - \frac{4x}{5} \right) - \left(\frac{1}{2} \log_e(x^2+4) - 2 \tan^{-1} \left(\frac{x}{2} \right) \right) \right]_{-1}^1$$

$$+ \left[\left(\frac{1}{2} \log_e(x^2+4) - 2 \tan^{-1} \left(\frac{x}{2} \right) \right) - \left(\frac{x^2}{10} - \frac{4x}{5} \right) \right]_1^4$$

1W

$$= \left(\frac{1}{10} - \frac{4}{5} \right) - \left(\frac{1}{2} \log_e(5) - 2 \tan^{-1} \left(\frac{1}{2} \right) \right) - \left(\left(\frac{1}{10} + \frac{4}{5} \right) - \left(\frac{1}{2} \log_e(5) - 2 \tan^{-1} \left(-\frac{1}{2} \right) \right) \right)$$

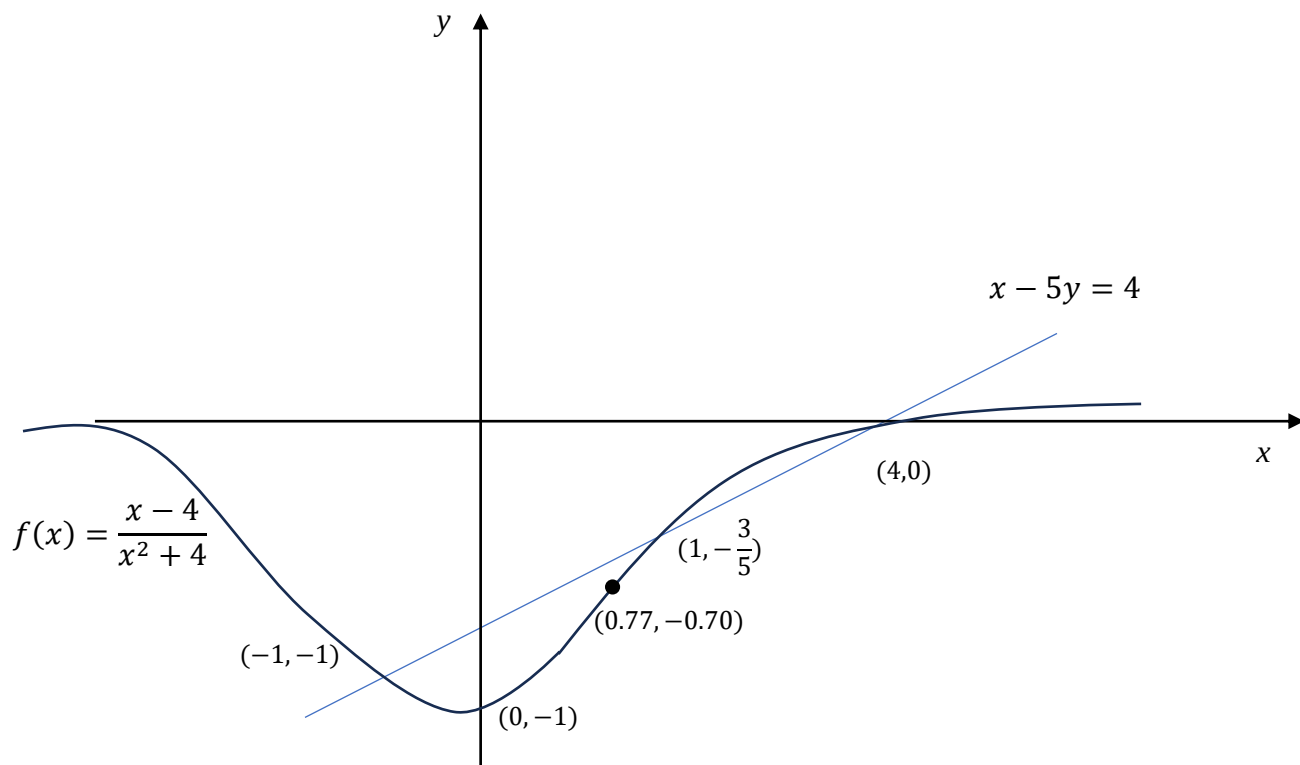
$$+ \left(\frac{1}{2} \log_e(20) - 2 \tan^{-1}(2) \right) - \left(\frac{8}{5} - \frac{16}{5} \right) - \left(\left(\frac{1}{2} \log_e(5) - 2 \tan^{-1} \left(\frac{1}{2} \right) \right) - \left(\frac{1}{10} - \frac{4}{5} \right) \right)$$

$$= \left(-\frac{16}{10} + 4 \tan^{-1} \left(\frac{1}{2} \right) \right) + \left(\frac{1}{2} \log_e(4) + 4 \tan^{-1} \left(\frac{1}{2} \right) - \pi + \frac{9}{10} \right)$$

$$= \log_e(2) + 8 \tan^{-1} \left(\frac{1}{2} \right) - \pi - \frac{7}{10}$$

1A

f. (3 marks)



1 shape, 1 POI, 1 straight line with points of intersection

3 marks

g. (2 marks)

$$V_x = \pi \int_0^a \left(\frac{x-4}{x^2+4} \right)^2 dx = \frac{5\pi^2}{16} - \frac{\pi}{8}$$

$$V_x = \pi \int_0^a \left(\frac{x-4}{x^2+4} \right)^2 dx = \frac{\left(5 \tan^{-1} \left(\frac{a}{2} \right) \right) \pi}{4} + \frac{(3a+8)\pi}{2(a^2+4)} - \pi$$

1W

$$\frac{\left(5 \tan^{-1} \left(\frac{a}{2} \right) \right) \pi}{4} + \frac{(3a+8)\pi}{2(a^2+4)} - \pi = \frac{5\pi^2}{16} - \frac{\pi}{8}$$

$$a = 2$$

1A