TSSM
Creating VCE Success

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2024 Trial Examination

STUDENT NUMBER | | | | | | | | | Letter |

SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 13 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer **all** questions in pencil on the answer sheet provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the acceleration due to gravity to have magnitude $g\ ms^{-2}$, where $g = 9.8$.

Question 1 (4 marks)

Prove by mathematical induction that $3^n + 3, n \in N$ is divisible by 6.

4 marks

Question 2 (5 marks)

The quadratic equation: $z^2 + bz + c = 0$, with real coefficients, has a root of $z_1 = 2cis\left(\frac{\pi}{6}\right)$

a. Write down, in polar form, z_2, the second root to the quadratic equation.

_____

_____

1 mark

b. Find the values of coefficients b and c.

_____

_____

_____

_____

_____

2 marks

TURN OVER

c. The quadratic $z^2 + 2iz - 4 = 0$, has one of the roots from the equation in **part a.** and a second root z_3. Show that the product of the roots of $z^2 + 2iz - 4 = 0$ is a real number.

2 marks

1 + 2 + 2 = 5 marks

Question 3 (4 marks)

Find the equation of the normal to the curve $3x^2y - 2xy^2 + y = 1$ when $y = 1$ and $x > 0$.

4 marks

TURN OVER

Question 4 (3 marks)

The following approximations may be useful for this question.

Let Z be an observation from a standard normal distribution.

$\Pr(-1 \leq Z \leq 1) \approx 0.68, \Pr(-2 \leq Z \leq 2) \approx 0.95, \ \Pr(-3 \leq Z \leq 3) \approx 0.997$

GREENAFIELD produce strips of turf with an advertised length of $1m$. One of the machines cutting the strips is suspected of producing strips that are undersize. A sample of n strips are measured. The results are normally distributed with a mean length of $99cm$ and a standard deviation of $3cm$. It is correctly concluded that there is a 2.5% chance that the actual mean length produced by the machine is above $100cm$. Find n, the number of strips of turf analysed in the sample.

_____

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3 marks

Question 5 (4 marks)

Clarry and Tammy are best friends.

Clarry claims that: "The graph of a function can never cross one of its asymptotes."

Tammy disagrees. To support her opinion, Tammy graphs the function $y = \frac{x^3}{x^2-9}$

a. Find the equations of all asymptotes for $y = \frac{x^3}{x^2-9}$

_____

_____

_____

_____

_____

_____

2 marks

b. Find any intercepts for $y = \frac{x^3}{x^2-9}$

_____

_____

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_____

_____

_____

1 mark

TURN OVER

c. What conclusion can be drawn regarding Clarry and Tammy's disagreement?

_____

_____

_____

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_____

_____

1 mark

2 + 1 + 1 = 4 marks

Question 6 (4 marks)

A population of insects (P $thousands$), is growing according to the differential equation: $\frac{dP}{dt} = \frac{1}{8}P(4 - P)$ where t is in months. Initially, (t=0), 2000 insects are present.

a. Solve the differential equation, writing P as a function of t.

3 marks

b. Hence find the possible range of values for P.

1 mark

3 + 1 = 4 marks

Question 7 (8 marks)

Consider the function: $f: [0,1] \rightarrow R$ where $f(x) = \tan^{-1} x$

a. Complete: For a suitable domain, $y = \tan^{-1} x$ and $x =$ _____ are equivalent.

1 mark

b. Let $a = f(0)$ and $b = f(1)$. Evaluate a and b.

1 mark

c. By first considering the area bound by $x = \tan y$, the y axis and the line $y = b$, find the area bound by $f(x) = \tan^{-1} x$, the x axis and the line $x = 1$.

3 marks

d. Find the volume formed when the area bound by $f(x) = \tan^{-1} x$, the y axis and the line $y = b$ is rotated around the y axis.

3 marks

1 + 1 + 3 + 3 = 8 marks

Question 8 (8 marks)

Consider the points in three dimensional space: $A(1,1,-2), B(0,2,-1), C(2,-1,-5)$.

a. Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

2 marks

TURN OVER

b. Hence find $\overrightarrow{AB} \times \overrightarrow{AC}$, the cross product of $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

2 marks

c. Hence find a unit vector normal to the plane containg points A, B, C.

1 mark

d. Find the Cartesian equation of the plane containing the points A, B, C.

3 marks

2 + 2 + 1 + 3 = 8 marks

END OF QUESTION AND ANSWER BOOK