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VCE[®] Specialist Mathematics

Practice Written Examination 1

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Solution Pathway

Below are sample answers. Please consider the merit of alternative responses.

Specialist Mathematics Examination 1: Marking Scheme

Question 1

Part a.:

A function can intersect with either a horizontal asymptote or a diagonal (oblique, slant) asymptote:

$$\begin{array}{r}
 x-1 \\
 x^2+1 \overline{) x^3 - x^2 + kx + 1} \\
 \underline{-(x^3 + 0x^2 + x)} \\
 -x^2 + (k-1)x + 1 \\
 \underline{-(-x^2 + 0x - 1)} \\
 (k-1)x + 2
 \end{array}$$

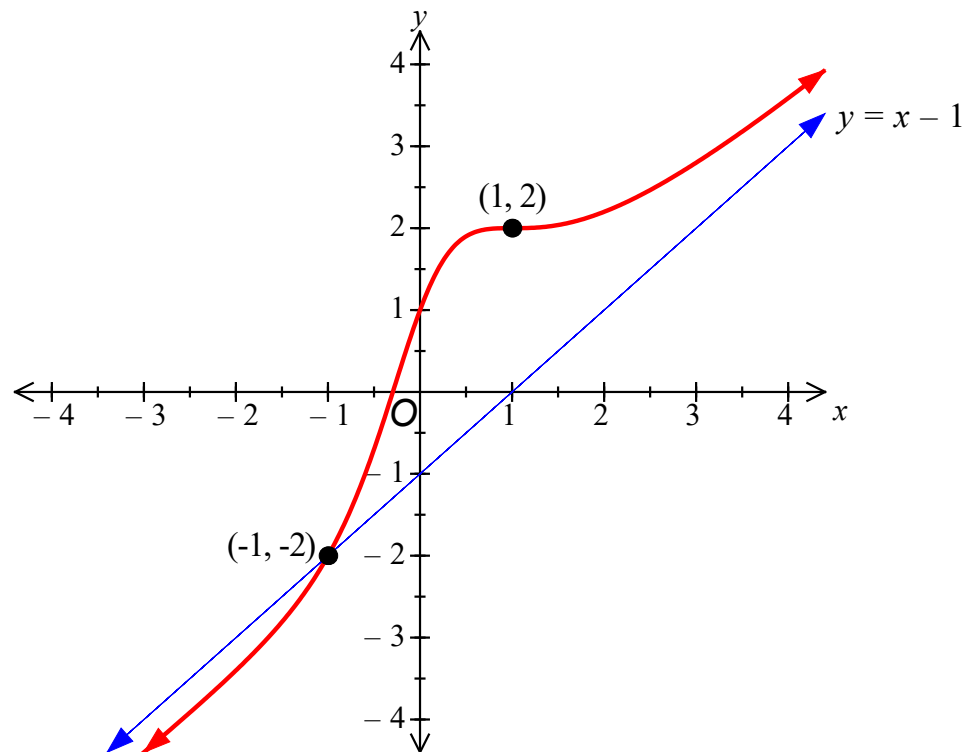
Therefore $f_k(x) = \frac{x^3 - x^2 + kx + 1}{x^2 + 1} = x - 1 + \frac{(k-1)x + 2}{x^2 + 1}$

Diagonal asymptote: $y = x - 1$	1 mark
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Solve $f_k(x) = x - 1 + \frac{(k-1)x + 2}{x^2 + 1} = x - 1$: $\frac{(k-1)x + 2}{x^2 + 1} = 0 \Rightarrow (k-1)x + 2 = 0$.

By inspection there is no solution when $k = 1$

Answer: $k = 1$	1 mark
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Part b.:**Answer:**

Shape: Must have a y -intercept at $(0, 1)$	1 mark
Turning point: $(1, 2)$	1 mark
Diagonal asymptote: $y = x - 1$ Graph must be consistent with axes scale. Intersection point: $(-1, -2)$	1 mark

Calculations:

- Diagonal asymptote: $y = x - 1$ (from **part a.**).

- Intersection with asymptote:

Substitute $k = 3$ into $\frac{(k-1)x+2}{x^2+1} = 0$ from **part a.**: $2x+2=0 \Rightarrow x=-1$. $y = f_3(-1) = -2$

- Approach towards diagonal asymptote:

$$y = x - 1 + \frac{2x+2}{x^2+1} \text{ (from **part a.**)} \quad \sim x + 0^+ \text{ as } x \rightarrow +\infty$$

therefore, the approach is from above.

$$y = x - 1 + \frac{2x+2}{x^2+1} \text{ (from **part a.**)} \quad \sim x + 0^- \text{ as } x \rightarrow -\infty$$

therefore, the approach is from below.

The approach can also be deduced from the fact that the graph must intersect the diagonal asymptote.

- Turning point: Solve $f'_3(x) = 0$.

$$f'_3(x) = \frac{(3x^2 - 2x + 3)(x^2 + 1) - 2x(x^3 - x^2 + 3x + 1)}{(x^2 + 1)^2} = \frac{x^4 - 4x + 3}{(x^2 + 1)^2}$$

$$\frac{x^4 - 4x + 3}{(x^2 + 1)^2} = 0 \quad \Rightarrow x^4 - 4x + 3 = 0 \quad \Rightarrow x = 1 \text{ (by inspection). } y = f_3(1) = 2$$

Question 2Substitute $z = x + iy$, $x, y \in \mathbb{R}$:

$$3(x^2 + 2xyi - y^2) - 2(x - iy) = |2(x + iy) - 1|$$

$$\Rightarrow 3x^2 + 6xyi - 3y^2 - 2x + 2yi = |(2x - 1) + 2yi|$$

$$\Rightarrow (3x^2 - 3y^2 - 2x) + i(6xy + 2y) = \underbrace{|(2x - 1) + 2yi|}_{\text{Real}} \quad (1)$$

$$\text{Equate real parts of (1): } 3x^2 - 3y^2 - 2x = |(2x - 1) + 2yi| \quad (2)$$

Equate imaginary parts of (1):

$$6xy + 2y = 0 \Rightarrow 2y(3x + 1) = 0 \text{ therefore } y = 0 \text{ or } x = -\frac{1}{3}$$

Case 1: $y = 0$. Substitute $y = 0$ into equation (2): $3x^2 - 2x = 2x - 1 $	1 mark
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$$\text{Case 1 (a): } 2x - 1 \geq 0 \Rightarrow x \geq \frac{1}{2}$$

$$3x^2 - 2x = 2x - 1 \Rightarrow 3x^2 - 4x + 1 = 0 \Rightarrow (3x - 1)(x - 1) = 0 \Rightarrow x = 1$$

$$(x = \frac{1}{3} \text{ is rejected because } x \geq \frac{1}{2})$$

Answer: $z = 1$	1 mark
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$$\text{Case 1 (b): } 2x - 1 < 0 \Rightarrow x < \frac{1}{2}$$

$$3x^2 - 2x = -(2x - 1) \Rightarrow 3x^2 = 1 \Rightarrow x = -\frac{1}{\sqrt{3}}$$

$$(x = \frac{1}{\sqrt{3}} \text{ is rejected because } x < \frac{1}{2}).$$

Answer: $z = -\frac{1}{\sqrt{3}}$	1 mark
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Case 2: $x = -\frac{1}{3}$ Substitute into equation (2) and simplify: $1 - 3y^2 = \left -\frac{5}{3} + 2yi \right \Rightarrow 1 - 3y^2 = \sqrt{\frac{25}{9} + 4y^2}$ There is no real solution (proof is not required).	1 mark
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The fact that there is no real solution can be seen by inspecting the graphs of

$f(y) = 1 - 3y^2$ and $g(y) = \sqrt{\frac{25}{9} + 4y^2}$ and noting that the graphs do not intersect each other.

Question 3

$$v = \frac{dx}{dt} = 2 - e^{-x} \quad \Rightarrow \quad \frac{dt}{dx} = \frac{1}{2 - e^{-x}}$$

$\Rightarrow t = \int \frac{1}{2 - e^{-x}} dx$	1 mark
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$$= \int \frac{e^x}{2e^x - 1} dx.$$

Substitute $u = e^x$:

$$t = \int \frac{1}{2u - 1} du = \frac{1}{2} \log_e |2u - 1| + c, \text{ where } c \in R$$

$t = \frac{1}{2} \log_e 2e^x - 1 + c$	1 mark
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$$\Rightarrow e^{2(t-c)} = |2e^x - 1| \quad \Rightarrow e^{-2c} e^{2t} = |2e^x - 1| \quad \Rightarrow \pm e^{-2c} e^{2t} = 2e^x - 1$$

$$\Rightarrow Ae^{2t} = 2e^x - 1, \text{ where } A = \pm e^{-2c}.$$

Substitute $x = 0$ when $t = 0$: $A = 1$.

$e^{2t} = 2e^x - 1$	1 mark
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Substitute $t = \log_e(7)$:

$$2e^x - 1 = e^{2\log_e(7)} = e^{\log_e(49)} = 49$$

$$\Rightarrow 2e^x = 50 \quad \Rightarrow e^x = 25$$

Answer: $x = \log_e(25)$ Accept $x = 2\log_e(5)$	1 mark
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Question 4

The value of $a = \frac{d^2x}{dt^2}$ when $t=1$ and $x=1$ is required.

Use implicit differentiation:

Differentiate $t^3 + 2x^3 - xt = 2$ with respect to t : $3t^2 + 6x^2 \frac{dx}{dt} - x - t \frac{dx}{dt} = 0$
Chain Rule Product Rule

Result of implicit differentiation: $3t^2 + 6x^2 \frac{dx}{dt} - x - t \frac{dx}{dt} = 0$	1 mark
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Substitute $t=1$ and $x=1$: $3 + 6 \frac{dx}{dt} - 1 - \frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = -\frac{2}{5}$.

Differentiate $3t^2 + 6x^2 \frac{dx}{dt} - x - t \frac{dx}{dt} = 0$ with respect to t :

$$6t + \left(12x \frac{dx}{dt} \right) \frac{dx}{dt} + 6x^2 \frac{d^2x}{dt^2} - \frac{dx}{dt} - \frac{dx}{dt} - t \frac{d^2x}{dt^2} = 0$$

Chain Rule Product Rule Product Rule

Result of implicit differentiation:	1 mark
$6t + 12x \frac{dx}{dt} \times \frac{dx}{dt} + 6x^2 \frac{d^2x}{dt^2} - \frac{dx}{dt} - \frac{dx}{dt} - t \frac{d^2x}{dt^2} = 0$.	

Substitute $t=1$ and $x=1$ and $\frac{dx}{dt} = -\frac{2}{5}$:

$$6 + 12 \left(-\frac{2}{5} \right) \left(-\frac{2}{5} \right) + 6 \frac{d^2x}{dt^2} - \left(-\frac{2}{5} \right) - \left(-\frac{2}{5} \right) - \frac{d^2x}{dt^2} = 0 \Rightarrow 6 + \frac{48}{25} + 5 \frac{d^2x}{dt^2} + \frac{4}{5} = 0$$

Answer: $a = -\frac{218}{125} \text{ ms}^{-2}$	1 mark
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Question 5

Use integration by parts to calculate $\int x \tan^{-1}\left(\frac{x}{2}\right) dx$:

$$\int u \, dv = uv - \int v \, du$$

$$u = \tan^{-1}\left(\frac{x}{2}\right) \quad \Rightarrow \quad du = \frac{2}{x^2 + 4} dx$$

$$dv = x \, dx \quad \Rightarrow \quad v = \frac{1}{2}x^2$$

$\int x \tan^{-1}\left(\frac{x}{2}\right) dx = \frac{1}{2}x^2 \tan^{-1}\left(\frac{x}{2}\right) - \int \frac{x^2}{x^2 + 4} dx$	1 mark
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$$= \frac{1}{2}x^2 \tan^{-1}\left(\frac{x}{2}\right) - \int \frac{(x^2 + 4) - 4}{x^2 + 4} dx = \frac{1}{2}x^2 \tan^{-1}\left(\frac{x}{2}\right) - \int 1 - \frac{4}{x^2 + 4} dx$$

$= \frac{1}{2}x^2 \tan^{-1}\left(\frac{x}{2}\right) - x + 2 \tan^{-1}\left(\frac{x}{2}\right)$	1 mark
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Therefore:

$$\int_0^{2\sqrt{3}} x \tan^{-1}\left(\frac{x}{2}\right) dx = \frac{1}{2}(12) \tan^{-1}(\sqrt{3}) - 2\sqrt{3} + 2 \tan^{-1}(\sqrt{3}) - 0 = 6\left(\frac{\pi}{3}\right) - 2\sqrt{3} + 2\left(\frac{\pi}{3}\right)$$

Answer: $\frac{8\pi}{3} - 2\sqrt{3}$ Accept $\frac{8\pi - 6\sqrt{3}}{3}$	1 mark
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Question 6

Confidence interval endpoints of a $C\%$ confidence interval:

$$\bar{x} \pm z_c \frac{\sigma}{\sqrt{n}}$$

where z_c is such that $\Pr(-z_c < Z < z_c) = \Pr(|Z| < z_c) = \frac{C}{100}$

Part a.:

Compare the endpoints of the given confidence interval (201.2, 202.6) with $\bar{x} \pm z_c \frac{\sigma}{\sqrt{n}}$

It follows that:

$$202.6 - 201.2 = 2z_c \frac{\sigma}{\sqrt{n}} \quad \Rightarrow 1.4 = 2k \frac{5}{\sqrt{64}} = \frac{5}{4}k \quad \Rightarrow k = \frac{4(1.4)}{5} = \frac{28}{25}$$

Answer: $k = \frac{28}{25}$ Accept 1.12	1 mark
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Part b.:**Method 1:**

Compare the endpoints of the given confidence interval (201.2, 202.6) with $\bar{x} \pm z_c \frac{\sigma}{\sqrt{n}}$

It follows that:

$$202.6 + 201.2 = 2\bar{x} \quad \Rightarrow 403.8 = 2\bar{x} \quad \Rightarrow \bar{x} = 201.9$$

Substitute $\bar{x} = 201.9$, $n = 4 \times 64$, $\sigma = 5$ and $z_c = k = 1.12$ into $\left(\bar{x} - z_c \frac{\sigma}{\sqrt{n}}, \bar{x} + z_c \frac{\sigma}{\sqrt{n}} \right)$

Method 2:

Let $n' = 4n$. Width of new confidence interval:

$$2k \frac{\sigma}{\sqrt{n'}} = 2k \frac{\sigma}{\sqrt{4n}} = \frac{1}{2} \left(\begin{array}{c} 2k \frac{\sigma}{\sqrt{n}} \\ 1.4243 \\ \text{Width of old} \\ \text{confidence interval} \end{array} \right) = \frac{1}{2}(1.4) = 0.7$$

$\bar{x}$ remains the same therefore the new confidence interval is

$$\left(201.9 - \frac{1}{2}(0.7), 201.9 + \frac{1}{2}(0.7) \right)$$

Answer: (201.55, 202.25)

1 mark

Part c.:

Let X be the random variable “Mass of a genetically modified apple”.

X is normally distributed with a mean of 144 grams and a standard deviation of 8 grams.

$$\Pr\left(X_1 + X_2 < \frac{7}{4}X_3\right) = \Pr\left(X_1 + X_2 - \frac{7}{4}X_3 < 0\right) \text{ is required}$$

where X_1, X_2, X_3 are independent copies of X .

Let Y be the random variable $Y = X_1 + X_2 - \frac{7}{4}X_3$. Then $\Pr(Y < 0)$ is required.

Note: Using the random variable $X + X - \frac{7}{4}X = \frac{1}{4}X$ is incorrect: $X_1 + X_2 - \frac{7}{4}X_3 \neq \frac{1}{4}X$

Y follows a normal distribution since X_1, X_2, X_3 are independent normal random variables.

$$E(Y) = \mu_Y = \mu_{X_1} + \mu_{X_2} - \frac{7}{4}\mu_{X_3} = \frac{1}{4}\mu_X = \frac{1}{4}(144) = 36$$

$$\text{Var}(Y) = 1^2 \text{Var}(X_1) + 1^2 \text{Var}(X_2) + \left(\frac{7}{4}\right)^2 \text{Var}(X_3) = 2\text{Var}(X) + \frac{49}{16} \text{Var}(X) = \frac{81}{16} \text{Var}(X)$$

$$\Rightarrow \text{sd}(Y) = \sigma_Y = \frac{9}{4} \text{sd}(X) = \frac{9}{4}(8) = 18$$

$Y = X_1 + X_2 - \frac{7}{4}X_3$ $Y \sim \text{Normal}(\mu_Y = 36, \sigma_Y = 18)$ $\Pr(Y < 0)$	1 mark: Definition. Distribution. Probability.
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$$Z = \frac{Y - \mu_Y}{\sigma_Y} = \frac{0 - 36}{18} = -2 \quad \text{therefore } \Pr(Y < 0) = \Pr(Z < -2).$$

$$\Pr(|Z| \leq 2) = 0.9545 \Rightarrow \Pr(Z < -2) = \frac{1}{2}(1 - 0.9545) = \frac{1}{2}(0.0455).$$

$Z = -2$. Answer: 0.023	1 mark: Calculation of Z . Answer.
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Question 7

$$S = 2\pi \int_0^{\frac{\pi}{2}} x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Substitute $\frac{dx}{dt} = -3\cos^2(t)\sin(t)$ and $\frac{dy}{dt} = 3\sin^2(t)\cos(t)$:

$$\begin{aligned} \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= 9\cos^4(t)\sin^2(t) + 9\sin^4(t)\cos^2(t) = 9\cos^2(t)\sin^2(t)(\cos^2(t) + \sin^2(t)) \\ &= 9\cos^2(t)\sin^2(t) \end{aligned}$$

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 9\cos^2(t)\sin^2(t)$	1 mark
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$S = 2\pi \int_0^{\frac{\pi}{2}} \cos^3(t) \sqrt{9\cos^2(t)\sin^2(t)} dt$	1 mark
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$$= 6\pi \int_0^{\frac{\pi}{2}} \cos^3(t) |\cos(t)\sin(t)| dt = 6\pi \int_0^{\frac{\pi}{2}} \cos^3(t) \cos(t)\sin(t) dt$$

Since $\cos(t)\sin(t) \geq 0$ for $0 \leq t \leq \frac{\pi}{2}$ $= 6\pi \int_0^{\frac{\pi}{2}} \cos^4(t) \sin(t) dt$	1 mark: Justification for dropping modulus. Integral.
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Substitute $u = \cos(t)$:

$$S = -6\pi \int_1^0 u^4 du = 6\pi \int_0^1 u^4 du = \frac{6\pi}{5} \left[u^5 \right]_0^1.$$

Answer: $\frac{6\pi}{5}$ square units.	1 mark
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Question 8

- Let $S(n)$ be the conjecture $\frac{(f_1(x)L f_n(x))'}{f_1(x)L f_n(x)} = \frac{f_1'(x)}{f_1(x)} + K + \frac{f_n'(x)}{f_n(x)}$ - Check $n = 2$: $\text{LHS} = \frac{(f_1(x)f_2(x))'}{f_1(x)f_2(x)} = \frac{f_1'(x)f_2(x) + f_1(x)f_2'(x)}{f_1(x)f_2(x)} = \frac{f_1'(x)}{f_1(x)} + \frac{f_2'(x)}{f_2(x)} = \text{RHS}$ Therefore $S(2)$ is true. - Assume $S(k)$ is true for some $k > 2 \in N$: $\frac{(f_1(x)L f_k(x))'}{f_1(x)L f_k(x)} = \frac{f_1'(x)}{f_1(x)} + K + \frac{f_k'(x)}{f_k(x)}$	1 mark: Statement. Base case. Inductive hypothesis.
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Show that if $S(k)$ is true then it follows that $S(k+1)$ is true: $\frac{(f_1(x)L f_k(x)f_{k+1}(x))'}{f_1(x)L f_k(x)f_{k+1}(x)} = \frac{([f_1(x)L f_k(x)]f_{k+1}(x))'}{f_1(x)L f_k(x)f_{k+1}(x)}$ $= \frac{(f_1(x)L f_k(x))' f_{k+1}(x) + (f_1(x)L f_k(x)) f_{k+1}'(x)}{f_1(x)L f_k(x)f_{k+1}(x)}$	1 mark: Statement. Recognition that $f_1(x)K f_k(x)f_{k+1}(x)$ is a product of the two functions $f_1(x)K f_k(x)$ and $f_{k+1}(x)$.
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$$= \frac{\left(\frac{f'_1(x)}{f_1(x)} + K + \frac{f'_k(x)}{f_k(x)} \right) (f_1(x) \prod_{k=1}^k f_k(x)) f_{k+1}(x) + (f_1(x) \prod_{k=1}^k f_k(x)) f'_{k+1}(x)}{f_1(x) \prod_{k=1}^k f_k(x) f_{k+1}(x)}$$

Using the inductive hypothesis

$$= \frac{\left(\frac{f'_1(x)}{f_1(x)} + K + \frac{f'_k(x)}{f_k(x)} \right) f_1(x) \prod_{k=1}^k f_k(x) f_{k+1}(x)}{f_1(x) \prod_{k=1}^k f_k(x) f_{k+1}(x)} + \frac{f_1(x) \prod_{k=1}^k f_k(x) f'_{k+1}(x)}{f_1(x) \prod_{k=1}^k f_k(x) f_{k+1}(x)}$$

$$= \left(\frac{f'_1(x)}{f_1(x)} + K + \frac{f'_k(x)}{f_k(x)} \right) + \frac{f'_{k+1}(x)}{f_{k+1}(x)}$$

Which is equivalent to $S(k+1)$ true.

- Since $S(2)$ is true and it follows that if $S(k)$ is true then $S(k+1)$ is true, it follows from the principle of mathematical induction that $S(n)$ is true for $n \geq 2$.

1 mark:

Use of inductive hypothesis.

$S(k) \Rightarrow S(k+1)$.

Conclusion.

Question 9**Part a.:**

$$\Pi_1: \vec{n}_1 = 3\vec{i} - 2\vec{j} - m\vec{k}$$

$$\Pi_2: \vec{n}_2 = \vec{i} + m\vec{j} + 3m\vec{k}$$

Π_1 and Π_2 are perpendicular when $\vec{n}_1 \cdot \vec{n}_2 = 0$:

$3 - 2m - 3m^2 = 0$	1 mark
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$$\Rightarrow 3m^2 + 2m - 3 = 0 \quad \Rightarrow m = \frac{-2 \pm \sqrt{4 + 36}}{6} = \frac{-2 \pm 2\sqrt{10}}{6} = \frac{-1 \pm \sqrt{10}}{3}$$

Answer: $m = \frac{-1 \pm \sqrt{10}}{3}$	1 mark
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Part b.:

$$\Pi_1: \quad \underset{\sim}{n}_1 = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$$

Parametric equations of the line $x+1 = \frac{2-y}{2} = \frac{z-3}{2}$:

$$x+1=t \quad \Rightarrow x = -1+t$$

$$\frac{2-y}{2}=t \quad \Rightarrow y = 2-2t$$

$$\frac{z-3}{2}=t \quad \Rightarrow z = 3+2t$$

Vector in direction of the line (by inspection of the coefficients of t in the parametric equations):

$\underset{\sim}{l} = \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$	1 mark:
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Vector perpendicular to $\underset{\sim}{n}_1 = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{l} = \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$:

$$\underset{\sim}{n}_1 \times \underset{\sim}{l} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 3 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix}$$

$= \underset{\sim}{i} \begin{vmatrix} -2 & 1 \\ -2 & 2 \end{vmatrix} - \underset{\sim}{j} \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} + \underset{\sim}{k} \begin{vmatrix} 3 & -2 \\ 1 & -2 \end{vmatrix}$	1 mark
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Answer: $-2\underset{\sim}{i} - 5\underset{\sim}{j} - 4\underset{\sim}{k}$.	1 mark
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Question 10**Part a.:**

$$\vec{r}(t) = \int \sin^2(t) dt \vec{i} + \int \sin(t) dt \vec{k} \quad \vec{r}(t) = \frac{1}{2} \int 1 - \cos(2t) dt \vec{i} + \int \sin(t) dt \vec{k}$$

$= \left(\frac{t}{2} - \frac{1}{4} \sin(2t) \right) \vec{i} - \cos(t) \vec{k} + \vec{c}$	1 mark
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where $\vec{c}$ is an arbitrary constant vector.

The particle is initially at the point (0, 1, 0) therefore $\vec{r}(0) = 0 \vec{i} + 1 \vec{j} + 0 \vec{k}$.

$$\text{But } \vec{r}(0) = \left(0 - \frac{1}{4} \sin(0) \right) \vec{i} - \cos(0) \vec{k} + \vec{c} = -\vec{k} + \vec{c}$$

$$\text{Therefore: } 0 \vec{i} + 1 \vec{j} + 0 \vec{k} = -\vec{k} + \vec{c} \quad \Rightarrow \vec{c} = \vec{j} + \vec{k}$$

Answer: $\vec{r}(t) = \left(\frac{t}{2} - \frac{1}{4} \sin(2t) \right) \vec{i} + \vec{j} + (1 - \cos(t)) \vec{k}$	1 mark
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Part b.:

The particles are moving in the same direction when their velocity vectors are equal:

Particle 1: $\vec{v}(t) = \sin^2(t)\vec{i} + \sin(t)\vec{k}$

Particle 2: $\frac{ds}{dt} = \vec{v}_2(t) = -2\sin(2t)\vec{i} + (2 - 2\cos(2t))\vec{k}$

Equate velocity components:

$\sin^2(t) = -2\sin(2t)$	(1)	1 mark: Both equations.
$\sin(t) = 2 - 2\cos(2t)$	(2)	

Simultaneous solutions to equations (1) and (2) are required. From equation (1):

$$\sin^2(t) + 2\sin(2t) = 0 \quad \Rightarrow \quad \sin^2(t) + 4\sin(t)\cos(t) = 0 \quad \Rightarrow \quad \sin(t)(\sin(t) + 4\cos(t)) = 0$$

Case 1 $\sin(t) = 0 \quad \Rightarrow t = n\pi, n \in \mathbb{Z}^+ \cup \{0\}$. Note: $t \geq 0$	1 mark: Both cases.
Case 2: $\sin(t) + 4\cos(t) = 0 \quad \Rightarrow \tan(t) = -4$	

Substitute **case 1** solutions into (2): $\sin(n\pi) = 2 - 2\cos(2n\pi) \quad \Rightarrow 0 = 0$ true.

Therefore accept **case 1** solutions.

Substitute **case 2** solutions into (2):

$$\tan(t) = -4 \text{ therefore } \sin(t) = \pm \frac{4}{\sqrt{17}} \text{ and } \cos(2t) = 1 - 2\sin^2(t) = 1 - \frac{8}{17}$$

$$\pm \frac{4}{\sqrt{17}} = 2 - 2\left(1 - \frac{8}{17}\right) = \frac{16}{17} \text{ false.}$$

Therefore reject **case 2**.

Answer: $t = n\pi, n \in \mathbb{Z}^+ \cup \{0\}$	1 mark: Answer. Explicit rejection of case 2.
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