

VCE Specialist Mathematics Units 3&4

Suggested Solutions

2024 Trial Examination 2

Section A – Multiple-choice questions

1	☐ A	☒ B	☐ C	☐ D
2	☐ A	☒ B	☐ C	☐ D
3	☐ A	☒ B	☐ C	☐ D
4	☒ A	☐ B	☐ C	☐ D
5	☐ A	☒ B	☐ C	☐ D
6	☒ A	☐ B	☐ C	☐ D
7	☐ A	☐ B	☐ C	☒ D
8	☐ A	☐ B	☒ C	☐ D
9	☐ A	☐ B	☐ C	☒ D
10	☐ A	☐ B	☐ C	☒ D
11	☐ A	☐ B	☐ C	☒ D
12	☐ A	☒ B	☐ C	☐ D
13	☒ A	☐ B	☐ C	☐ D
14	☐ A	☐ B	☒ C	☐ D
15	☐ A	☐ B	☐ C	☒ D
16	☒ A	☐ B	☐ C	☐ D
17	☐ A	☐ B	☒ C	☐ D
18	☐ A	☐ B	☐ C	☒ D
19	☒ A	☐ B	☐ C	☐ D
20	☐ A	☐ B	☒ C	☐ D

Question 1 B

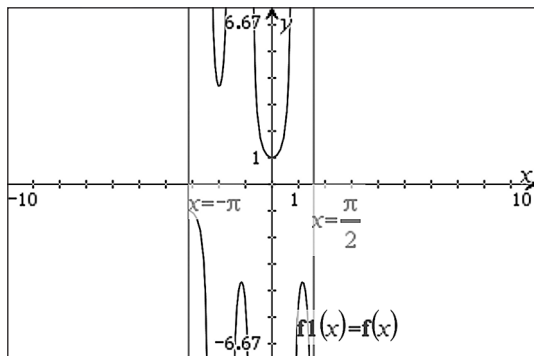
The given statement is $p \rightarrow q$, where p represents 'my plants are turning yellow' and q represents 'they are not getting enough water'.

The contrapositive of the statement is given by $\neg q \rightarrow \neg p$, where $\neg p$ represents 'my plants are not turning yellow' and $\neg q$ represents 'they are getting enough water'.

Therefore, the contrapositive statement is 'If they are getting enough water, then my plants are not turning yellow.'

Question 2 B

Using a CAS calculator to sketch the graph of f gives:



The asymptote at $x = \frac{\pi}{2}$ is excluded since the interval is $\left(-\pi, \frac{\pi}{2}\right)$. Thus, reading from the graph, there are four vertical asymptotes in the given interval.

Question 3 B

Using a CAS calculator gives:

$f(x) := \sin^{-1}(\ln(x))$	Done
$\text{solve}\left(\frac{d^2}{dx^2}(f(x)) < 0, x\right)$	$\frac{\sqrt{5}}{2} - \frac{1}{2}$
	$e^{-1} < x < e$

$$\therefore \frac{1}{e} < x < e^{\frac{\sqrt{5}-1}{2}}$$

Question 4 A

Using a CAS calculator gives:

$p(z) := 2 \cdot z^3 + b \cdot z^2 + c \cdot z + 100$	Done
$\text{solve}\left(\begin{cases} p(2+4i)=0 \\ p(2-4i)=0 \end{cases}, b, c\right)$	$b = -3, \text{ and } c = 20.$

$$\begin{aligned} \therefore b + c &= -3 + 20 \\ &= 17 \end{aligned}$$

Question 5 B

The subset is the perpendicular bisector of points $(-3, -1)$ and $(3, 3)$.

Thus, the line will pass through $\left(\frac{-3+3}{2}, \frac{-1+3}{2}\right) = (0, 1)$.

$$m = -\frac{1}{\frac{-1-3}{-3-3}}$$

$$= -\frac{3}{2}$$

Finding the equation of the path gives:

$$y - 1 = -\frac{3}{2}(x - 0)$$

$$y = -\frac{3}{2}x + 1$$

Through a process of elimination, it can be found that only $(-2, 4)$ satisfies this equation.

Question 6 A

Since $\angle COA = 90^\circ$, OC can be obtained by rotating OA anticlockwise at an angle of 90° .

Since $OC = 2OA$, point C can be obtained by rotating point A anticlockwise at an angle of 90° and dilating by a factor of 2.

Therefore, point C can be represented as $2iz$.

As $OABC$ is a rectangle, it is also a parallelogram where OB is its diagonal.

Point B is the sum of points A and C .

Therefore, point B is represented by the complex number $z + 2iz$.

Question 7 D

$$\int_0^1 \sqrt{\left(\frac{d}{dt}e^{2t}\right)^2 + \left(\frac{d}{dt}te^t\right)^2} dt = \int_0^1 \sqrt{(2e^{2t})^2 + (e^t + te^t)^2} dt$$

$$= \int_0^1 \sqrt{4e^{4t} + (e^t + te^t)^2} dt$$

Question 8 C

$$u = x$$

$$du = dx$$

$$dv = (x+1)^9 dx$$

$$v = \frac{(x+1)^{10}}{10}$$

$$\int u dv = uv - \int v du$$

$$\int x(x+1)^9 dx = \frac{x(x+1)^{10}}{10} - \int \frac{(x+1)^{10}}{10} dx$$

$$= \frac{1}{10} x(x+1)^{10} - \int \frac{1}{10} (x+1)^{10} dx$$

Question 9 D

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$$= (y_1 z_2 - y_2 z_1) \hat{i} - (x_1 z_2 - x_2 z_1) \hat{j} + (x_1 y_2 - x_2 y_1) \hat{k}$$

The missing part of the algorithm is $-(x_1 z_2 - x_2 z_1) = x_2 z_1 - x_1 z_2$.

Question 10 D

Using a CAS calculator gives:

$\text{euler}\left(3 \cdot x^2 + x, x, y, \{1, 1.3\}, 0, 0.1\right)$
$\begin{bmatrix} 1. & 1.1 & 1.2 & 1.3 \\ 0. & 0.4 & 0.873 & 1.425 \end{bmatrix}$

$$y_3 = y_2 + 0.1f(x_2)$$

$$= 0.873 + 0.1f(1.2)$$

Question 11 D

Using a CAS calculator gives:

$n1 := [3 \quad -2 \quad 1]$	$[3 \quad -2 \quad 1]$
$n2 := [-2 \quad 4 \quad 3]$	$[-2 \quad 4 \quad 3]$
$\cos^{-1}\left(\frac{ \text{dotP}(n1, n2) }{\text{norm}(n1) \cdot \text{norm}(n2)}\right)$	56.9124

$$\therefore \angle \approx 57^\circ$$

Question 12 B

Using a CAS calculator gives:

$v(x) := \frac{2}{4+x^2}$	<i>Done</i>
$v(x) \cdot \frac{d}{dx}(v(x))$	$\frac{-8 \cdot x}{(x^2+4)^3}$

$$\therefore \text{acceleration} = \frac{-8x}{(x^2+4)^3}$$

Question 13 A

Using a CAS calculator gives:

$v0:=0$	0
$v1:=v0+2 \cdot 5$	10
$\text{solve}(0.5 \cdot 2 \cdot 5^2 + v1 \cdot t + 0.5 \cdot 1 \cdot t^2 = 60, t)$	$t = -23.0384$ or $t = 3.0384$
$\text{time}:=5+3.0384$	8.0384

$\therefore t \approx 8$ s

Question 14 C

Let x be the distance of the bottom of the ladder from the wall.

Let y be the distance of the top of the ladder from the floor.

$$x^2 + y^2 = 100$$

$$y = \sqrt{100 - x^2}, y \geq 0$$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{100 - x^2}}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \Big|_{x=6} \times \frac{dx}{dt}$$

$$= \frac{-6}{8} \times 1$$

$$= -0.75$$

Question 15 D

D is correct. Using a CAS calculator gives:

$a := [1 \ -2 \ 2]$	$[1 \ -2 \ 2]$
$b := [-2 \ -4 \ 4]$	$[-2 \ -4 \ 4]$
$c := [4 \ 1 \ -1]$	$[4 \ 1 \ -1]$

$\text{solve}(a=k \cdot b, k)$	false
$\text{dotP}(a, b)$	14
$\text{norm}(a) = \text{norm}(b)$	false
$\text{solve}(a=m \cdot b + n \cdot c, m, n)$	$m = \frac{9}{14}$ and $n = \frac{4}{7}$

$$\underline{a} = m\underline{b} + n\underline{c} \text{ for } m = \frac{9}{14} \text{ and } n = \frac{4}{7}$$

A is incorrect. There is no $k \in \mathbb{R}$ such that $\underline{a} = k\underline{b}$.

B is incorrect. $\underline{a} \cdot \underline{b} = 14 \neq 0$.

C is incorrect. $|\underline{a}| \neq |\underline{b}|$.

Question 16 A

Using a CAS calculator gives:

$y(t) := h - \frac{1}{2} \cdot g \cdot t^2$	<i>Done</i>
$\text{solve}(y(t)=0, t)$	
$t = -\sqrt{\frac{2 \cdot h}{g}} \text{ and } \frac{h}{g} \geq 0 \text{ or } t = \sqrt{\frac{2 \cdot h}{g}} \text{ and } \frac{h}{g} \geq 0$	

$$\therefore T = \sqrt{\frac{2h}{g}}$$

Question 17 C

$$\underline{n} = 3\underline{i} - 7\underline{j} + 11\underline{k}$$

$$\underline{r}(t) = \underline{n}t + 2\underline{i} - 5\underline{j} + 0\underline{k}$$

$$= 3t\underline{i} - 7t\underline{j} + 11t\underline{k} + 2\underline{i} - 5\underline{j} + 0\underline{k}$$

$$= (3t + 2)\underline{i} + (-7t - 5)\underline{j} + 11t\underline{k}$$

Question 18 D

$$\mu = 4 \times 180 + 3 \times 200$$

$$= 1320 \text{ g}$$

$$\sigma = \sqrt{15 + 15 + 15 + 15 + 10 + 10 + 10}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10}$$

Question 19 A

$$\mu = 0.4E(w) + 0.2$$

$$= 0.4 \times 5 + 0.2$$

$$= 2.2$$

$$\text{Var}(T) = 0.4^2 \times \text{Var}(w)$$

$$= 0.16 \times 0.05^2$$

$$\sigma = \sqrt{0.16} \times \sqrt{0.05^2}$$

$$= 0.4 \times 0.05$$

$$= 0.02$$

Using a CAS calculator gives:

zInterval 0.02,2.2,50,0.95: stat.results	
"Title"	"z Interval"
"CLower"	2.19446
"CUpper"	2.20554
" $\bar{x}$ "	2.2
"ME"	0.005544
"n"	50.
" σ "	0.02

$\therefore$ 95% confidence interval = (2.19, 2.21)

Question 20 C

$$T_1 \sim N(20, 2^2)$$

$$T_2 \sim N(20, 2^2)$$

$$T_1 - T_2 \sim N(0, (\sqrt{8})^2)$$

Using a CAS calculator to find $\Pr(|T_1 - T_2| < 1)$ gives:

$1 - \text{normCdf}(-1, 1, 0, \sqrt{8})$	0.723674
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Section B

Question 1 (12 marks)

a. $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$
 $= -2\mathbf{i} + \mathbf{j} - 5\mathbf{k} + 0\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
 $= -2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ A1

$\overrightarrow{AC} = -\overrightarrow{OA} + \overrightarrow{OC}$
 $= -2\mathbf{i} + \mathbf{j} - 5\mathbf{k} - 2\mathbf{i} + \mathbf{j} + \mathbf{k}$
 $= -4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ A1

b. There is no $\lambda \in R$ such that $\overrightarrow{AB} = \lambda \overrightarrow{AC}$. M1
 Therefore, A , B and C are not collinear.

c. $\text{area} = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$
 $= \sqrt{65}$ A1

$ab := [-2 \ 4 \ -3]$	$[-2 \ 4 \ -3]$
$ac := [-4 \ 2 \ -4]$	$[-4 \ 2 \ -4]$
$\frac{1}{2} \cdot \text{norm}(\text{crossP}(ab, ac))$	$\sqrt{65}$

Note: Consequential on answer to Question 1a.

d. $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 4 & -3 \\ -4 & 2 & -4 \end{vmatrix}$
 $= -10\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$ A1

$\Pi_1: -10x + 4y + 12z = -10 \times 0 + 4 \times 3 + 12 \times 2 = 36$

$\Pi_1: -5x + 2y + 6z = 18$ M1

$ab := [-2 \ 4 \ -3]$	$[-2 \ 4 \ -3]$
$ac := [-4 \ 2 \ -4]$	$[-4 \ 2 \ -4]$
$\text{crossP}(ab, ac)$	$[-10 \ 4 \ 12]$

Note: Consequential on answer to Question 1a.

e. Letting $z = 0$ gives:

M1

$$\begin{cases} -5x + 2y = 18 \\ x - 3y = -1 \end{cases} \Rightarrow x = -4, y = -1$$

Hence, point $P(-4, -1, 0)$ lies on line L .

M1

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 2 & 6 \\ 1 & -3 & 1 \end{vmatrix}$$

$$= 20\hat{i} + 11\hat{j} + 13\hat{k}$$

A1

$$L: \vec{r}(t) = (-4 + 20t)\hat{i} + (-1 + 11t)\hat{j} + 13t\hat{k}$$

A1

$\text{solve}\left(\begin{cases} -5 \cdot x + 2 \cdot y = 18 \\ x - 3 \cdot y = -1 \end{cases}, x, y\right)$	$x = -4$ and $y = -1$
$\text{crossP}([-5 \ 2 \ 6], [1 \ -3 \ 1]) \quad [20 \ 11 \ 13]$	

Note: Accept responses that use alternate z-coordinates to find point P.

f. $\overrightarrow{AP} = -6\hat{i} - 5\hat{k}$

M1

$$\vec{n} = \hat{i} - 3\hat{j} + \hat{k}$$

$$d = \frac{|\overrightarrow{AP} \cdot \vec{n}|}{|\vec{n}|}$$

$$= \sqrt{11}$$

A1

Note: Consequential on answer to Question 1a. The value of $\overrightarrow{AP}$ is consequential on point P.

Question 2 (10 marks)

a. $u = 2\sqrt{2} \operatorname{cis}\left(\frac{\pi}{3}\right)$

A1

$$v = 2\sqrt{2} \operatorname{cis}\left(-\frac{\pi}{3}\right)$$

A1

$u := \sqrt{2} + \sqrt{6} \cdot i$	$\sqrt{2} + \sqrt{6} \cdot i$
$v := \operatorname{conj}(u)$	$\sqrt{2} - \sqrt{6} \cdot i$
$ u $	$2 \cdot \sqrt{2}$
$\operatorname{angle}(u)$	$\frac{\pi}{3}$
$\operatorname{angle}(v)$	$-\frac{\pi}{3}$

$$\begin{aligned}
 \text{b. } u^3 &= 8\sqrt{8} \operatorname{cis}(\pi) \\
 &= -16\sqrt{2} \\
 v^3 &= 8\sqrt{8} \operatorname{cis}(-\pi) \\
 &= -16\sqrt{2}
 \end{aligned}$$

solving u^3 or v^3 M1

$$\text{Therefore, } u^3 + 16\sqrt{2} = v^3 + 16\sqrt{2} = 0.$$

M1

$$\text{c. } z^3 + 16\sqrt{2} = (z - u)(z - v)(z - a), \text{ where } a \in \mathbb{R}$$

M1

$$\begin{aligned}
 (z - u)(z - v) &= z^2 - (u + v)z + uv \\
 &= z^2 - 2\sqrt{2}z + 8
 \end{aligned}$$

$$(z^2 - 2\sqrt{2}z + 8)(z - a) = z^3 + 16\sqrt{2}$$

$$a = -2\sqrt{2}$$

$$z^3 + 16\sqrt{2} = (z^2 - 2\sqrt{2}z + 8)(z + 2\sqrt{2})$$

M1

*Note: All working must be shown to obtain full marks.*d. Let X be a point on the positive real axis.

$$\angle COX = \angle AOX - \angle AOC$$

$$= \frac{\pi}{3} - \frac{2\pi}{9}$$

$$= \frac{\pi}{9}$$

$$\angle COB = \angle COX + \angle BOX$$

$$= \frac{\pi}{9} + \frac{\pi}{3}$$

$$= \frac{4\pi}{9}$$

M1

$$\text{e. } \operatorname{Arg}(z) = \frac{\pi}{9}$$

A1

$$\text{f. } A = \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} \times \sin\left(\frac{2\pi}{9}\right) + \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} \times \sin\left(\frac{4\pi}{9}\right)$$

M1

$$= 4 \sin\left(\frac{2\pi}{9}\right) + 4 \times 2 \sin\left(\frac{2\pi}{9}\right) \cos\left(\frac{2\pi}{9}\right)$$

$$= 4 \sin\left(\frac{2\pi}{9}\right) \left(1 + 2 \cos\left(\frac{2\pi}{9}\right)\right)$$

A1

Question 3 (11 marks)

a. Finding the area of the inner surface gives:

$$x = \sqrt{y}$$

$$\frac{dx}{dy} = \frac{1}{2\sqrt{y}}$$

$$S_1 = \int_0^8 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$= 98.7353 \dots$$

A1

Finding the area of the outer surface gives:

$$x = \sqrt{2y}$$

$$\frac{dx}{dy} = \frac{1}{\sqrt{2y}}$$

$$S_2 = \int_0^8 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$= 144.708 \dots$$

A1

Finding the area of the top ring gives:

$$r_1 = 2\sqrt{2}$$

$$r_2 = 4$$

$$S_3 = \pi(r_2^2 - r_1^2)$$

$$= 8\pi$$

A1

$$\text{total area} = S_1 + S_2 + S_3$$

$$= 268.58 \text{ m}^2$$

A1

$$s1 := \int_0^8 \left(2 \cdot \pi \cdot \sqrt{2 \cdot y} \cdot \sqrt{1 + \left(\frac{1}{\sqrt{2 \cdot y}} \right)^2} \right) dy$$

$$\frac{2 \cdot \pi \cdot (17 \cdot \sqrt{17} - 1)}{3}$$

$$s2 := \int_0^8 \left(2 \cdot \pi \cdot \sqrt{y} \cdot \sqrt{1 + \left(\frac{1}{2 \cdot \sqrt{y}} \right)^2} \right) dy$$

$$\pi \cdot \left(\frac{11 \cdot \sqrt{33}}{2} - \frac{1}{6} \right)$$

$\text{solve}(8 = x^2, x) x > 0$	$x = 2 \cdot \sqrt{2}$
$\text{solve}\left(8 = \frac{1}{2} \cdot x^2, x\right) x > 0$	$x = 4$
$s1 + s2 + 8 \cdot \pi$	268.576

b. $\pi \int_0^h ((\sqrt{2y})^2 - (\sqrt{y})^2) dy$ M1

$$= \frac{\pi h^2}{2}$$
 A1

c. i. $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ M1

$$\sqrt{h} = \pi h \times \frac{dh}{dt}$$

$$dt = \pi \sqrt{h} dh$$

$$\int_0^t dt = \pi \int_0^8 \sqrt{h} dh$$

$$t = 47.4 \text{ minutes}$$
 A1

$$\text{solve} \left(\int_0^t 1 \, dt = \pi \cdot \int_0^8 \sqrt{h} \, dh, t \right) \quad t=47.3908$$

ii. $\int_0^t dt = \pi \int_0^h \sqrt{h} dh$ M1

$$h = \frac{1}{2} \sqrt[3]{\frac{18t^2}{\pi^2}}$$
 A1

$$V = \frac{\pi h^2}{2}$$

$$= \frac{3\pi}{8} \sqrt[3]{\frac{12t^4}{\pi^4}}$$

$$= \sqrt[3]{\frac{81t^4}{128\pi}}$$
 A1

$$\text{solve} \left(\int_0^t 1 \, dt = \pi \cdot \int_0^h \sqrt{h} \, dh, h \right)$$

$$h = \frac{\frac{1}{2} \cdot \frac{2}{3} \cdot (3 \cdot t)^3}{2 \cdot \pi^{\frac{3}{2}}} \text{ and } t \geq 0$$

$$v(h) := \frac{1}{2} \cdot \pi \cdot h^2$$
 Done
$$v \left(\frac{\frac{2}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot t^3}{2 \cdot \pi^{\frac{3}{2}}} \right) = \frac{\frac{1}{3} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot t^3}{8 \cdot \pi^{\frac{3}{2}}}$$

Note: Consequential on answers to Questions 3b. and 3c.i.

Question 4 (9 marks)**a.**

$i(n) := \int_0^1 (1-x^2)^{\frac{n}{2}} dx$	<i>Done</i>
$i(1)$	$\frac{\pi}{4}$

$$I_1 = \frac{\pi}{4}$$

A1

b. $I_n = \int_0^1 (1-x^2)^{\frac{n}{2}} dx$

$$u = (1-x^2)^{\frac{n}{2}} \quad du = -nx(1-x^2)^{\frac{n}{2}-1} dx$$

$$dv = dx \quad v = x$$

M1

$$I_n = \underbrace{\left[x(1-x^2)^{\frac{n}{2}} \right]_0^1}_0 + n \int_0^1 x^2 (1-x^2)^{\frac{n}{2}-1} dx$$

A1

$$I_n = n \int_0^1 (x^2 - 1 + 1)(1-x^2)^{\frac{n-2}{2}} dx$$

$$= n \int_0^1 -(1-x^2-1)(1-x^2)^{\frac{n-2}{2}} dx$$

M1

$$= -n \int_0^1 \left[(1-x^2)(1-x^2)^{\frac{n-2}{2}} - (1-x^2)^{\frac{n-2}{2}} \right] dx$$

$$= -n \int_0^1 (1-x^2)^{\frac{n}{2}} dx + n \int_0^1 (1-x^2)^{\frac{n-2}{2}} dx$$

$$= -nI_n + nI_{n-2}$$

$$= \frac{n}{n+1} I_{n-2}$$

M1

c. $I_5 = \frac{5}{6} I_3$

$$= \frac{5}{6} \times \frac{3}{4} I_1$$

$$= \frac{5}{6} \times \frac{3}{4} \times \frac{\pi}{4}$$

$$= \frac{5\pi}{32}$$

M1

A1

d. $\pi \int_0^1 y^2 dx = \pi \int_0^1 (1-x^2)^n dx$

$$= \pi I_{2n}$$

$\therefore p = \pi, q = 2$

A1

A1

Question 5 (8 marks)

a. $x = \left(\frac{1}{\cos t} + a \cos t \right) \cos t$

$$= 1 + a \cos^2 t$$

$$y = \left(\frac{1}{\cos t} + a \cos t \right) \sin t$$

$$= \frac{1 + a \cos^2 t}{\cos t} \times \sin t$$

$$= (1 + a \cos^2 t) \tan t$$

M1

$$y = x \tan t$$

$$x^2 + y^2 = x^2 + x^2 \tan^2 t$$

$$= x^2 (1 + \tan^2 t)$$

$$= x^2 \sec^2 t$$

M1

$$x - 1 = a \cos^2 t$$

$$\therefore (x - 1)(x^2 + y^2) = (a \cos^2 t)(x^2 \sec^2 t)$$

$$= ax^2$$

M1

b. Since $a < -1$ and $x = 1 + a \cos t < 0$:

$$(x - 1)(x^2 + y^2) = ax^2$$

$$x - 1 = \frac{ax^2}{x^2 + y^2}$$

$$\neq 0$$

Therefore, $x = 1$ is an asymptote.

A1

c. $\frac{d}{dt} y(t) = 0$

M1

Since the graph indicates that there are two stationary points, the first two t -values should be used.

$$t = 0.452278 \dots, t = 2.68931 \dots$$

Substituting these values into the parametric equations gives:

$$\text{maximum} = (-0.62, 0.30), \text{minimum} = (-0.62, -0.30)$$

A1

$y(t) := (\sec(t) - 2 \cdot \cos(t)) \cdot \sin(t)$	Done
Δ solve $\left(\frac{d}{dt}(y(t)) = 0, t \right) t > 0$ $t = 0.452278$ or $t = 2.68931$ or $t = 3.59387$ or $\rightarrow$	
$x(t) := (\sec(t) - 2 \cdot \cos(t)) \cdot \cos(t)$	Done

$\{x(0.452278), y(0.452278)\}$
$\{-0.618035, -0.300283\}$
$\{x(2.68931), y(2.68931)\}$
$\{-0.618027, 0.300283\}$

d. $2 \int_0^{\frac{\pi}{4}} \sqrt{\left(\frac{d}{dt} x(t) \right)^2 + \left(\frac{d}{dt} y(t) \right)^2} dt$

$$= 2.49$$

A1

A1

solve $(y(t) = 0, t) 0 \leq t \leq 2 \cdot \pi$	
$t = 0$ or $t = \frac{\pi}{4}$ or $t = \frac{3 \cdot \pi}{4}$ or $t = \pi$ or $t = \frac{5 \cdot \pi}{4}$ or $t = \frac{7 \cdot \pi}{4}$	
$2 \cdot \int_0^{\frac{\pi}{4}} \sqrt{\left(\frac{d}{dt}(x(t)) \right)^2 + \left(\frac{d}{dt}(y(t)) \right)^2} dt$	2.4896

Question 6 (10 marks)**a.** 0.2685

A1

$\text{normCdf}(3000, \infty, 42 \cdot 70, 15 \cdot \sqrt{42})$ 0.268547

b. 39

A1

$f(n) := \text{normCdf}(3000, \infty, n \cdot 70, 15 \cdot \sqrt{n})$	Done
$f(40)$	0.017507
$f(39)$	0.001974

c. X – travel time (excluding waiting times)

$$E(X) = 55 - 6 \times 2$$

$$= 43$$

A1

$$SD(X) = \sqrt{3 + 6 \times \frac{24}{60}}$$

$$= \sqrt{\frac{27}{5}} \text{ OR } \frac{3\sqrt{15}}{5}$$

A1

d. 0.0271

A1

$\text{normCdf}\left(-\infty, 41, 43, \frac{\sqrt{\frac{27}{5}}}{\sqrt{5}}\right)$ 0.027146
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*Note: Consequential on answer to Question 6c.***e.** $H_0 : \mu = 55$ $H_1 : \mu > 55$

A1

f. p -value = 0.0570

A1

As p -value > 0.05, accept H_0 .

A1

$\text{normCdf}\left(57, \infty, 55, \frac{4}{\sqrt{10}}\right)$ 0.056923
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g. 57.081 minutes

A2

$\text{invNorm}\left(0.95, 55, \frac{4}{\sqrt{10}}\right)$ 57.0806
