

VCE Specialist Mathematics Units 3&4

Question and Answer Booklet

2024 Trial Examination 1

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Materials supplied

- Question and Answer Booklet of 11 pages
- Formula Sheet

Instructions

- Write your responses in English.
- Write **your name** and your **teacher's name** in the spaces above on this page.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents	pages
10 questions, 40 marks	2–11

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2024 VCE Specialist Mathematics Units 3&4 Examination.

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- Answer **all** questions in the spaces provided.
- Unless otherwise specified, an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, diagrams in this booklet are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$.

Find the equation of the line that is perpendicular to the graph of $x \tan^{-1}(y) + 2y = x^2 + 2$ at the point $(0, 1)$.

[illegible]

Question 2 (3 marks)

Evaluate $\int_0^{\pi} x \cos(x) dx$.

Question 3 (5 marks)

- a. Find $\left(2 - \frac{2}{\sqrt{3}}i\right)^4$. Express your answer in the form $x + iy$, where $x, y \in \mathbb{R}$. 4 marks

- b. Find the integer values of n for which $\left(2 - \frac{2}{\sqrt{3}}i\right)^n$ is real. 1 mark

Question 4 (4 marks)

Consider the vectors $\vec{a} = 2\vec{i} - 4\vec{j} + \vec{k}$ and $\vec{b} = -\vec{i} + 3\vec{j} - 5\vec{k}$.

- a. Find the vector resolute of $\vec{a}$ in the direction of $\vec{b}$.

2 marks

- b. The vectors $\vec{a}$ and $\vec{b}$ lie in a plane that passes through the point $(3, 1, -1)$.
Find the Cartesian equation of the plane.

2 marks

Question 5 (3 marks)

A factory produces bags of garden soil that vary in weight. The bags have a mean weight of 25 kg and a standard deviation of 1.8 kg. The factory sells the bags in pallets, which contain 36 bags.

Determine the maximum mean weight of the bags in a randomly selected pallet, such that 84% of the pallets have a mean weight that is less than this value. Give your answer in kilograms, correct to one decimal place.

Consider the graph of $f(x) = x^4 e^{2x}$.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

A curve with the equation $y = x^3$, where $x \in [0, 1]$, is rotated about the x-axis to form a solid of revolution.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

A particle's position at time t is given by the vector $\mathbf{r} = -e^t \mathbf{i} + (e^{2t} - 2e^t - 1) \mathbf{j}$, where $t \geq 0$.

- 2 marks

- 2 marks

- 2 marks

This image shows a single page of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page, leaving small margins at the top and bottom. There are no vertical margin lines, text, or other markings on the page.

Find $\int \frac{-2x^2 + 13x - 1}{(x + 5)(x^2 + 4)} dx$.

This image shows a single sheet of white paper with horizontal blue ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 10 (3 marks)

Use proof by contradiction to prove that $\sin(x) + \cos(x) \geq 1$ for all $x \in \left[0, \frac{\pi}{2}\right]$.

End of examination questions