

The Mathematical Association of Victoria

Trial Examination 2024

SPECIALIST MATHEMATICS

Written Examination 2 - SOLUTIONS

SECTION A: Multiple Choice

Question	Answer	Question	Answer
1	B	11	A
2	B	12	B
3	C	13	C
4	D	14	B
5	B	15	A
6	B	16	B
7	A	17	B
8	D	18	D
9	B	19	A
10	D	20	A

Question 1 Answer B

$2xy - y^2 = -3$, $x = 1$ for $y > 0$ gives $(1, 3)$

gradient to the curve $= \frac{3}{2}$

Edit Action Interactive

$2xy - y^2 = -3 \mid x=1$
 $-y^2 + 2 \cdot y = -3$
 $\text{solve}(-y^2 + 2 \cdot y = -3, y)$
 $\{y = -1, y = 3\}$
 $\text{impDiff}(2 \cdot x \cdot y - y^2 = -3, x, y)$
 $y' = \frac{-2 \cdot y}{2 \cdot x - 2 \cdot y}$
 $y' = \frac{-2 \cdot 3}{2 \cdot 1 - 2 \cdot 3}$
 $y' = \frac{3}{2}$

$\text{solve}(2 \cdot x \cdot y - y^2 = -3 \text{ and } x = 1 \text{ and } y > 0, x, y)$
 $x = 1 \text{ and } y = 3$
 $\text{impDiff}(2 \cdot x \cdot y - y^2 = -3, x, y)$
 $\frac{-y}{x - y}$
 $\frac{-y}{x - y} \mid x = 1 \text{ and } y = 3$
 $\frac{3}{2}$

Question 2 **Answer B**

$$y = \frac{ax^3}{x^2 + bx - 2}$$

two vertical asymptotes at $x=1$ and $x=-2$, oblique asymptote at $y=3x-3$.

Possible graph: $y = 3x - 3 + \frac{ax+c}{(x+2)(x-1)} = 3x - 3 + \frac{ax+c}{x^2+x-2}$ where c is a real constant.

Gives possible values $a=3, b=1$

Left pane output:

$$\frac{3x^3 + ax - 9x + c + 6}{x^2 + x - 2}$$

Right pane output:

$$\frac{3x^3 - 12x^2 + ax + 15x + c - 6}{x^2 - 3x + 2}$$
Question 3 **Answer C**

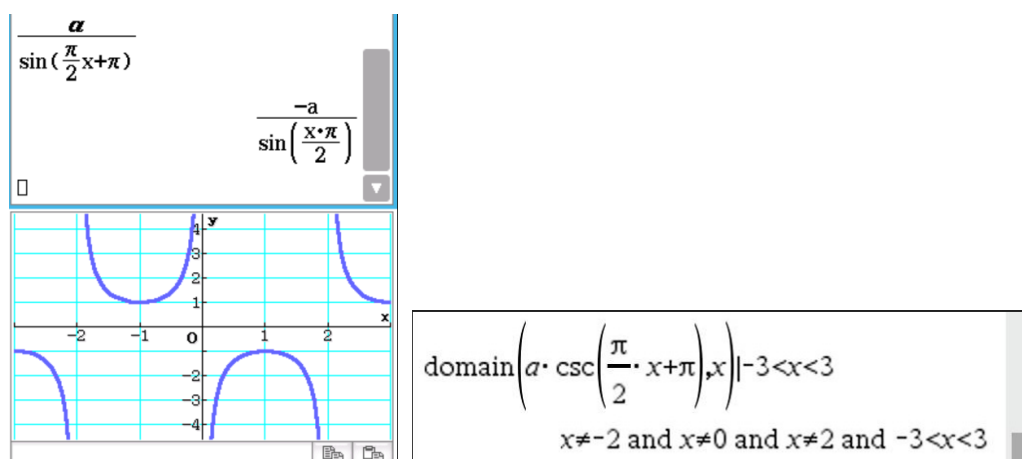
$$y = a \operatorname{cosec}\left(\frac{\pi}{2}x + \pi\right) = \frac{a}{\sin\left(\frac{\pi}{2}x + \pi\right)}$$

$$\text{Period} = \frac{2\pi}{\frac{\pi}{2}} = 4$$

So for cosec graph, asymptotes will be 2 units apart.

In the interval $-3 < x < 3$

Asymptotes at $x = -2, x = 0, x = 2$



Question 4 **Answer D**Let $z = x + yi$

$$\therefore \frac{z\bar{z}}{|z|} = \frac{(x+yi)(x-yi)}{|(x+yi)|} = \sqrt{13}$$

$$\frac{x^2 + y^2}{\sqrt{x^2 + y^2}} = \sqrt{13}$$

$$\text{gives } \sqrt{x^2 + y^2} = \sqrt{13}$$

$$x^2 + y^2 = 13$$

Possible $z = 2^2 + 3^2$ gives $z = 2 + 3i$

$$iz = i2 + 3i^2 = -3 + 2i$$



$$\frac{z \cdot \text{conj}(z)}{|z|} = \sqrt{13} \quad |z| = x + yi \quad i \quad \sqrt{x^2 + y^2} = \sqrt{13}$$

Question 5 **Answer B**

$$x(t) = (t+1)^3 \text{ and } y(t) = \frac{2}{t+1} \text{ where } t \geq 0.$$

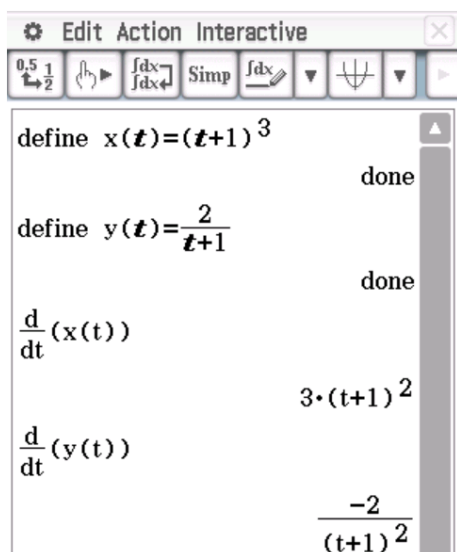
$$x'(t) = 3(t+1)^2$$

$$y'(t) = \frac{-2}{(t+1)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{-2}{(t+1)^2} \times \frac{1}{3(t+1)^2} = \frac{-2}{3(t+1)^4}$$

$$\text{Gradient of tangent at } t=1 \text{ is } \frac{-2}{3 \times 16} = -\frac{1}{24}$$

perpendicular to the tangent at $t=1$, gradient = 24


TI-Nspire calculator interface showing the definition of functions $x(t) = (t+1)^3$ and $y(t) = \frac{2}{t+1}$, and their derivatives:

```

define x(t)=(t+1)^3
done
define y(t)=2/(t+1)
done
d/dt (x(t))
3*(t+1)^2
d/dt (y(t))
-2/(t+1)^2

```

$$\frac{\frac{d}{dt}(y(t))}{\frac{d}{dt}(x(t))} \Big|_{t=1} = -\frac{1}{24}$$

$$\frac{\frac{d}{dt}\left(\frac{2}{t+1}\right) \cdot \left(\frac{d}{dt}\left((t+1)^3\right)\right)^{-1}}{\frac{d}{dt}\left(\frac{2}{t+1}\right) \cdot \left(\frac{d}{dt}\left((t+1)^3\right)\right)^{-1} \Big|_{t=1}} = \frac{-2}{3 \cdot (t+1)^4} \Big|_{t=1} = \frac{-1}{24}$$

Question 6 Answer B

The pseudocode ultimately is calculating $\sin^2(1^\circ) + \sin^2(2^\circ) + \sin^2(3^\circ) + \dots + \sin^2(89^\circ)$ or $\sum_{t=1}^{89} \sin^2(t^\circ)$.

CAS can be used to calculate the answer.

Or otherwise note that $\sin^2 x + \cos^2 x = 1$ and $\sin^2(90^\circ - n) = \cos^2(n)$.

This means $\sin^2 1^\circ + \sin^2 89^\circ = 1$ where there are 44 pairs in this algorithm.

$44 + \sin^2 45^\circ = 44.5$.

$$\sum_{t=1}^{89} (\sin(t^\circ))^2 = \frac{89}{2}$$

When convert the pseudocode into Python,

1.1	*Doc	RAD	1.1	1.2	*Doc	RAD
MAV2024.py		8/8	Python Shell		4/4	
<pre>from math import * a = 0 t = 1 while t < 90: a += sin(radians(t)) ** 2 t += 1 print(a)</pre>			<pre>>>>#Running MAV2024.py >>>from MAV2024 import * 44.5 >>> </pre>			

Question 7 Answer A

$$\int_{-1}^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$= \int_{-1}^2 \sqrt{\left(\frac{d}{dt}(2t)\right)^2 + \left(\frac{d}{dt}\left((t-1) - (1-2t)^2\right)\right)^2} dt$$

$$= \int_{-1}^2 \sqrt{(2)^2 + (5-8t)^2} dt$$

$$= 19.617 \approx 20$$

Handwritten solution for Question 8:

$$\int_{-1}^2 \text{norm} \left(\frac{d}{dt} \left(\begin{bmatrix} 2 \cdot t \\ t-1-(1-2 \cdot t)^2 \end{bmatrix} \right) \right) dt = 19.6175$$

$y(t) := t - 1 - (1 - 2 \cdot t)^2$ Done

$\text{arcLen} \left(y \left(\frac{t}{2} \right), t, -1 \cdot 2, 2 \cdot 2 \right) = 19.6175$

Question 8 **Answer D**

$$a = v(1+v)^2$$

$$v \frac{dv}{dx} = v(1+v)^2$$

$$\frac{dv}{dx} = (1+v)^2 \text{ where } v \neq 0$$

$$\frac{dx}{dv} = \frac{1}{(1+v)^2}$$

$$x = \int (1+v)^{-2} dv$$

$$x = -\frac{1}{1+v} + c$$

Given $x=1, v=1, 1 = -\frac{1}{1+1} + c \Rightarrow c = \frac{3}{2}$

$$x = -\frac{1}{1+v} + \frac{3}{2}$$

At $v=10, x = -\frac{1}{1+10} + \frac{3}{2}$

At $v=10, x = \frac{31}{22}$ metres

Handwritten solution for Question 8:

$\text{deSolve}(v, v' = v \cdot (1+v)^2 \text{ and } v(1)=1, x, v)$

$v = \frac{-2}{2 \cdot x - 3} - 1$

$\text{solve} \left(v = \frac{-2}{2 \cdot x - 3} - 1, x \right) | v=10 \quad x = \frac{31}{22}$

Question 9 **Answer B**

$$\underline{a} = m\underline{i} + 4\underline{j} + 5\underline{k}, \underline{b} = -\underline{i} - n\underline{j} \text{ and } \underline{c} = 2\underline{i} + p\underline{k}$$

For linearly **dependent**, allow $\underline{a} = E\underline{b} + F\underline{c}$ where E, F are real constants

$$m\underline{i} + 4\underline{j} + 5\underline{k} = E(-\underline{i} - n\underline{j} + 0\underline{k}) + F(2\underline{i} + 0\underline{j} + p\underline{k})$$

	$\underline{i}$	$\underline{j}$	$\underline{k}$
$\underline{a}$	m	4	5
$\underline{b}$	$-E$	$-En$	$0E$
$\underline{c}$	$2F$	$0F$	pF

For $\underline{a} = E\underline{b} + F\underline{c}$ we get the following equations

$$\begin{cases} m = -E + 2F \\ 4 = -En + 0F \\ 5 = 0E + pF \end{cases}$$

$$\begin{cases} m = -E + 2F \\ 4 = -En \Rightarrow E = -\frac{4}{n} \\ 5 = pF \Rightarrow F = \frac{5}{p} \end{cases}$$

Giving $m = -\left(-\frac{4}{n}\right) + 2\left(\frac{5}{p}\right)$

$$m = \left(\frac{4}{n}\right) + 2\left(\frac{5}{p}\right)$$

$$\therefore m = \frac{4}{n} + \frac{10}{p}$$

$\text{solve}\left(\det\begin{pmatrix} m & -1 & 2 \\ 4 & -n & 0 \\ 5 & 0 & p \end{pmatrix} = 0, m\right)$
 $m = \frac{2 \cdot (5 \cdot n + 2 \cdot p)}{n \cdot p}$
 $\text{expand}\left(m = \frac{2 \cdot (5 \cdot n + 2 \cdot p)}{n \cdot p}\right) \quad m = \frac{4}{n} + \frac{10}{p}$

Question 10 **Answer D**

$$\dot{\underline{r}}(t) = \sin(t)\cos(t)\underline{i} + \cos(2t)\underline{j}$$

$$\underline{r}(t) = \int (\sin(t)\cos(t)\underline{i} + \cos(2t)\underline{j}) dt$$

$$\underline{r}(t) = \int \left(\frac{1}{2} \sin(2t)\underline{i} + \cos(2t)\underline{j} \right) dt$$

$$\therefore \underline{r}(t) = -\frac{1}{4} \cos(2t)\underline{i} + \frac{1}{2} \sin(2t)\underline{j} + \underline{c}$$

Given $\underline{r}(\pi) = 2\underline{i} - 3\underline{j}$

$$2\hat{i} - 3\hat{j} = -\frac{1}{4}\cos(2\pi)\hat{i} + \frac{1}{2}\sin(2\pi) + \hat{c}$$

$$\hat{c} = \frac{1}{4}\cos(2\pi)\hat{i} - \frac{1}{2}\sin(2\pi) + 2\hat{i} - 3\hat{j}$$

$$\hat{c} = \frac{1}{4}\hat{i} + 2\hat{i} - 3\hat{j} = \frac{9}{4}\hat{i} - 3\hat{j}$$

Gives

$$\underline{r}(t) = -\frac{1}{4}\cos(2t)\hat{i} + \frac{1}{2}\sin(2t) + \frac{9}{4}\hat{i} - 3\hat{j}$$

displacement of the body at time t , $\underline{r}(t)$ is given by

$$\underline{r}(t) = \left(-\frac{1}{4}\cos(2t) + \frac{9}{4}\right)\hat{i} + \left(\frac{1}{2}\sin(2t) - 3\right)\hat{j}$$

$$\int_{\pi}^t \begin{bmatrix} \sin(t) \cdot \cos(t) \\ \cos(2 \cdot t) \end{bmatrix} dt + \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} \frac{9}{4} - \frac{\cos(2 \cdot t)}{4} \\ \frac{\sin(2 \cdot t)}{2} - 3 \end{bmatrix}$$

Question 11

Answer A

$\frac{dP}{dt} = P\left(6 - \frac{P}{8000}\right)$ with initial population, P , of 4000 bacteria.

Solution to DE is $P = \frac{48000(-\frac{1}{11})e^{6t}}{-\frac{1}{11}e^{6t} - 1}$

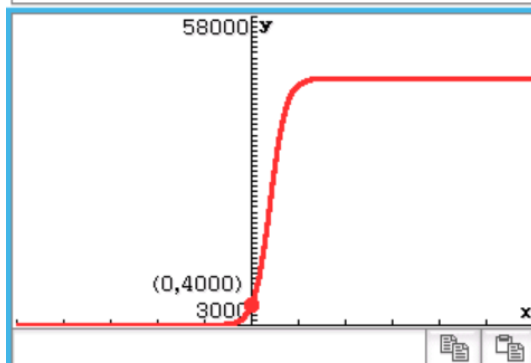
Giving horizontal asymptote $P = 48000$

$$\text{propFrac}\left(\frac{48000 \cdot (-\frac{1}{11}) \cdot e^{6 \cdot x}}{-\frac{1}{11} \cdot e^{6 \cdot x} - 1}\right)$$

$$\frac{48000 \cdot e^{6 \cdot x}}{e^{6 \cdot x} + 11}$$

solve(f(x)=48000, x)

No Solution



The image shows three screenshots of a TI-Nspire calculator interface, likely a TI-Nspire CX or similar model, used for solving a differential equation and finding constants.

Top Screenshot: The calculator is in the "Edit Action Interactive" mode. The input field shows the differential equation $\text{dSolve}\left(y' = y \cdot \left(6 - \frac{y}{8000}\right), x, y\right)$. Below it, the solution is displayed as $\left\{ \frac{\frac{1}{(|y|)^{\frac{1}{48000}}} - \frac{1}{(|y-48000|)^{\frac{1}{48000}}} = e^{\frac{x}{8000}} \cdot \text{const}(1)} \right\}$. Below that, the solution is further simplified to $\text{solve}\left(\left(\frac{y}{y-48000}\right)^{\frac{1}{48000}} = a \cdot e^{\frac{x}{8000}}, y\right)$. The calculator is set to "Alg" mode.

Middle Screenshot: The calculator is in the "Edit Action Interactive" mode. The input field shows the definition of a function $\text{define } f(x) = \frac{48000 \cdot a^{48000} \cdot e^{6 \cdot x}}{a^{48000} \cdot e^{6 \cdot x} - 1}$. Below it, the command $\text{solve}(f(0) = 4000, a)$ is entered. The solution is displayed as $\left\{ a^{48000} = -\frac{1}{11} \right\}$. The calculator is set to "Alg" mode.

Bottom Screenshot: The calculator is in the "Edit Action Interactive" mode. The input field shows the definition of a function $\text{define } f(x) = \frac{48000 \cdot \left(-\frac{1}{11}\right) \cdot e^{6 \cdot x}}{-\frac{1}{11} \cdot e^{6 \cdot x} - 1}$. Below it, the command $\text{solve}(f(x) = 1/11, x)$ is entered. The solution is displayed as $\left\{ x = \frac{-\ln(527999)}{6} + \frac{\ln(11)}{6} \right\}$. The calculator is set to "Alg" mode.

Question 12 Answer B

Option A is incorrect as 3 is rational but $\sqrt{3}$ is not;

Option C is incorrect as if $x = 0$ and $y = \sqrt{3}$ gives $xy = 0$ where it is rational;

Option D is incorrect as 0 is even and $0(0+1)$ is also even.

Proof of Option B:

If an integer n is odd then $n^2 + 2$ is odd.

Proof:

Let $n = 2\ell + 1$ where $\ell \in \mathbb{Z}$.

$$n^2 + 2 = (2\ell + 1)^2 + 2$$

$$= 4\ell^2 + 1 + 2 \times 2\ell + 2$$

$$= 2(2\ell^2 + 2\ell + 1) + 1$$

Thus $n^2 + 2$ is odd.

If $n^2 + 2$ is odd then n is odd where n is an integer.

Proof:

We proceed by proving the contrapositive is true. The contrapositive of the above statement is

If n is even, then $n^2 + 2$ is even where n is an integer.

Let $n = 2\ell$ where $\ell \in \mathbb{Z}$.

$$n^2 + 2 = (2\ell)^2 + 2$$

$$= 4\ell^2 + 2$$

$$= 2(2\ell^2 + 1)$$

Thus $n^2 + 2$ is even.

Therefore, if $n^2 + 2$ is odd then n is odd where n is an integer.

Question 13 Answer C

$$A = |\underline{a} \times \underline{c}|$$

$$= \left| \underline{a} \times (\sqrt{3}\underline{a} + \sqrt{2}\underline{b}) \right|$$

$$= \left| \underline{a} \times \sqrt{3}\underline{a} + \underline{a} \times \sqrt{2}\underline{b} \right|$$

$$= \left| \sqrt{3}\underline{a} \times \underline{a} + \sqrt{2}\underline{a} \times \underline{b} \right|$$

$$= \left| \sqrt{3} \times 0 + \sqrt{2}\underline{a} \times \underline{b} \right|$$

$$= \sqrt{2} \|\underline{a}\| \cdot \|\underline{b}\| \cdot \sin(\theta) \cdot \hat{n}$$

From $\underline{a} \cdot \underline{b} = 3$, we know that $\|\underline{a}\| \cdot \|\underline{b}\| \cdot \cos(\theta) = 3 \Rightarrow \|\underline{a}\| \cdot \|\underline{b}\| = \frac{3}{\cos(\theta)}$.

$$A = |\underline{a} \times \underline{c}| = \sqrt{2} \|\underline{a}\| \cdot \|\underline{b}\| \cdot \sin(\theta)$$

$$= \sqrt{2} \times \left| \frac{3}{\cos(\theta)} \times \sin(\theta) \right|$$

$$= \sqrt{2} \times |3 \times \tan(\theta)|$$

$$= \sqrt{2} \times \left| 3 \times \frac{1}{3} \right|$$

$$= \sqrt{2}$$

Question 14 Answer B

$$\underline{b} \times (\underline{a} + \underline{b} + \underline{c}) = \underline{b} \times \underline{0}$$

$$\underline{b} \times \underline{a} + \underline{b} \times \underline{b} + \underline{b} \times \underline{c} = \underline{0}$$

$$-\underline{a} \times \underline{b} + \underline{0} + \underline{b} \times \underline{c} = \underline{0}$$

$$\underline{a} \times \underline{b} = \underline{b} \times \underline{c}$$

Question 15 Answer A

$$a(t) = e^{-2t} \Rightarrow t = \frac{-1}{2} \log_e a$$

$$v(t) = -\frac{1}{2}e^{-2t} + 3 \text{ and } x(t) = \frac{1}{4}e^{-2t} + 3t + 4$$

$$\Delta v = \frac{-1}{2} \cdot e^{-2 \cdot t + 3} \Big|_t = \frac{-\ln(a)}{2} \quad v = 3 - \frac{a}{2}$$

Question 16 Answer B

If two planes are perpendicular to each other then $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

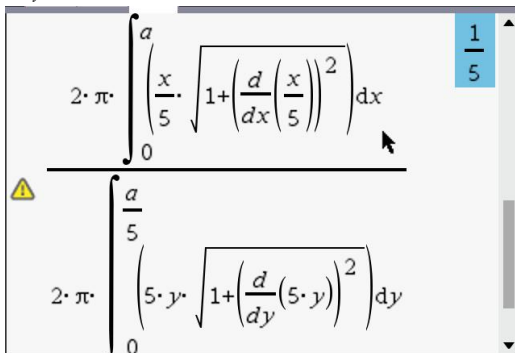
$$2 \times 1 + 2 \times 1 + \lambda \times 1 = 0 \Rightarrow \lambda = -4$$

Question 17 Answer B

$$A_x = 2 \cdot \pi \cdot \int_0^a \left(\frac{x}{5} \cdot \sqrt{1 + \left(\frac{d}{dx} \left(\frac{x}{5} \right) \right)^2} \right) dx = \frac{a^2 \cdot \pi \cdot \sqrt{26}}{25}$$

$$A_y = 2 \cdot \pi \cdot \int_0^{a/5} \left(5 \cdot y \cdot \sqrt{1 + \left(\frac{d}{dy} (5 \cdot y) \right)^2} \right) dy = \frac{a^2 \pi \sqrt{26}}{5}$$

$$\frac{A_x}{A_y} = \frac{a^2 \cdot \pi \cdot \sqrt{26}}{25} \times \frac{5}{a^2 \pi \sqrt{26}} = \frac{1}{5}$$



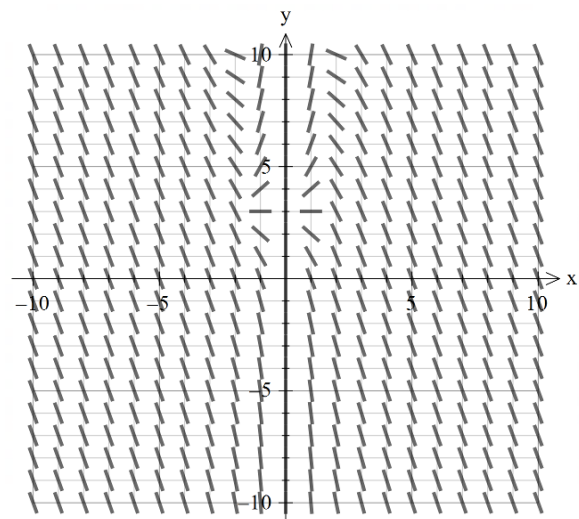
The image shows a handwritten solution for Question 17. It displays the ratio of the areas A_x and A_y as follows:

$$\frac{2 \cdot \pi \cdot \int_0^a \left(\frac{x}{5} \cdot \sqrt{1 + \left(\frac{d}{dx} \left(\frac{x}{5} \right) \right)^2} \right) dx}{2 \cdot \pi \cdot \int_0^{a/5} \left(5 \cdot y \cdot \sqrt{1 + \left(\frac{d}{dy} (5 \cdot y) \right)^2} \right) dy} = \frac{1}{5}$$

Question 18 Answer D

Fourth quadrant means x value is positive and y value is negative.

Therefore $\frac{y}{x^2}$ is negative and -3 will make it more negative.



Question 19

Answer A

$X - Y \sim N(0, 2\sigma^2)$

$$P(-1 < X - Y < 1) = P\left(\frac{-1}{\sqrt{2}\sigma} < \frac{X - Y}{\sqrt{2}\sigma} < \frac{1}{\sqrt{2}\sigma}\right)$$

Therefore, the probability is independent from μ but dependent on σ .

Question 20

Answer A

zInterval $\sqrt{2}$,15,36,0.99: stat.results

"Title"	"z Interval"
"CLower"	14.3929
"CUpper"	15.6071
" $\bar{x}$ "	15.
"ME"	0.607129
"n"	36.
" σ "	1.41421

C-Level0.99

σ 2^0.5

$\bar{x}$ 15

n36

<< Back

☐ Help

Next >>

Lower14.392871

Upper15.607129

$\bar{x}$ 15

n36

<< Back

☐ Help

OneSampleZIntOneSampleZInt

END OF MULTIPLE CHOICE SOLUTIONS

SECTION B

Question 1 (10 marks)

$$f(x) = \frac{ax^2 + 1}{x^2 - 3x + 2} \text{ where } a \in \mathbb{R} \setminus \{0\}.$$

a. $f(x) = \frac{ax^2 + 1}{x^2 - 3x + 2} = \frac{ax^2 + 1}{(x-1)(x-2)}$

Equations of the vertical asymptotes.

$$x = 1, x = 2$$

Equation of the horizontal asymptote.

$$y = a$$

1A

Edit Action Interactive
 $\text{propFrac}\left(\frac{ax^2+1}{x^2-3x+2}\right)$

$$a + \frac{3 \cdot a \cdot x}{x^2 - 3 \cdot x + 2} - \frac{2 \cdot a}{x^2 - 3 \cdot x + 2} + \frac{1}{x^2 - 3 \cdot x + 2}$$

Define $f(x) = \frac{a \cdot x^2 + 1}{x^2 - 3 \cdot x + 2}$ Done
 domain($f(x), x$) $x \neq 1$ and $x \neq 2$

b. Let $a = 1$. Local maximum stationary point $\left(\frac{1 + \sqrt{10}}{3}, -2\sqrt{10} - 6\right)$

1A

Edit Action Interactive
 $\text{define } f(x) = \frac{ax^2+1}{x^2-3x+2}$
 done
 $\frac{d}{dx}(f(x)) | a=1$

$$\frac{-(3 \cdot x^2 - 2 \cdot x - 3)}{(x^2 - 3 \cdot x + 2)^2}$$

 $\text{solve}\left(\frac{-(3 \cdot x^2 - 2 \cdot x - 3)}{(x^2 - 3 \cdot x + 2)^2} = 0, x\right)$

$$\left\{x = \frac{-\sqrt{10}}{3} + \frac{1}{3}, x = \frac{\sqrt{10}}{3} + \frac{1}{3}\right\}$$

 $\text{simplify}\left(f\left(\frac{\sqrt{10}}{3} + \frac{1}{3}\right) | a=1\right)$

$$-2 \cdot \sqrt{10} - 6$$

Define $f'(x) = \frac{d}{dx}(f(x))$

Done

Define $f''(x) = \frac{d^2}{dx^2}(f(x))$

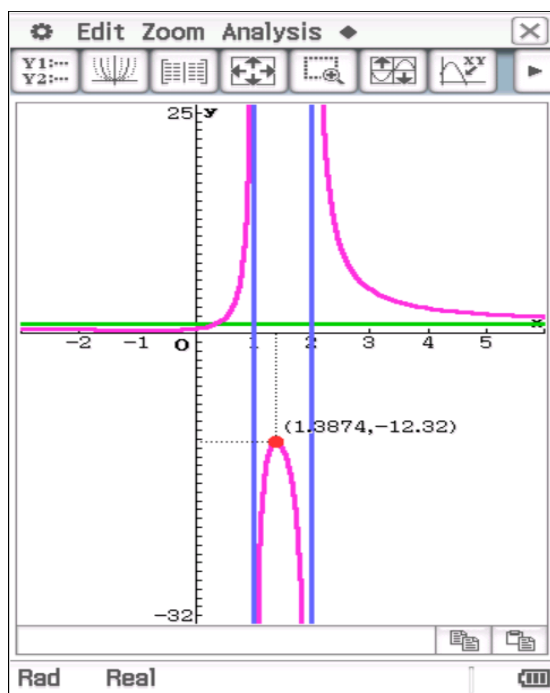
Done

$\text{solve}(f'(x)=0 \text{ and } f''(x)<0, x) | a=1$
 $x = \frac{\sqrt{10}+1}{3}$

$f(x)|_{x=\frac{\sqrt{10}+1}{3}} \text{ and } a=1$

$-2 \cdot (\sqrt{10}+3)$

c. Let $a=1$.



asymptotes $x=1, x=2, y=1$

3A

d. i. Stationary points

$$x = \frac{2a-1 \pm \sqrt{4a^2+5a+1}}{3a}$$

1A

$$\text{zeros}(f'(x), x)$$

$$\left\{ \frac{-\left(\sqrt{4 \cdot a^2+5 \cdot a+1}-2 \cdot a+1\right)}{3 \cdot a}, \frac{\sqrt{4 \cdot a^2+5 \cdot a+1}}{3 \cdot a} \right\}$$

ii. Two stationary points for $4a^2+5a+1 > 0$

$$a \in (-\infty, -1) \cup \left(-\frac{1}{4}, \infty\right)$$

1A

TI-Nspire CX CAS calculator screen showing the solution of a differential equation and an inequality.

Input: $\text{solve}\left(\frac{d}{dx}(f(x))=0, x\right)$

Output: $\left\{x=\frac{2\cdot a-\sqrt{4\cdot a^2+5\cdot a+1}-1}{3\cdot a}, x=\frac{2\cdot a+\sqrt{4\cdot a^2+5\cdot a+1}-1}{3\cdot a}\right\}$

Input: $\text{solve}(4\cdot a^2+5\cdot a+1>0, a)$

Output: $\{a<-1, -\frac{1}{4}<a\}$

TI-Nspire CX CAS calculator screen showing the solution of an inequality.

Input: $\text{solve}(4\cdot a^2+5\cdot a+1>0, a)$

Output: $a<-1 \text{ or } a>\frac{-1}{4}$

$h(x) = x$ and $g(x) = |f(x)|$ where $a = 1$.

- e. h and g intersect once at $x = 3.5115\dots$

1A

TI-Nspire CX CAS calculator screen showing the solution of an equation.

Input: $\text{solve}(|f(x)|=x | a=1, x)$

Output: $\{x=3.511547142\}$

TI-Nspire CX CAS calculator screen showing the definition of functions and the solution of an equation.

Define $h(x)=x$ Done

Define $g(x)=|f(x)| | a=1$ Done

count(zeros($h(x)-g(x), x$)) 1

solve($h(x)=g(x), x$) $x=3.51155$

- f. The region bounded by the curves of h and g and the line $x = 5$ is rotated around the x -axis.

$$\text{Vol} = \pi \int_{3.5115\dots}^5 (x^2 - (f(x))^2) dx$$

$$\text{Vol} = \pi \int_{3.5115\dots}^5 \left(x^2 - \left(\frac{x^2+1}{x^2-3x+2}\right)^2\right) dx$$

1M

Volume = 51.38 cubic units, to two decimal places 1A

TI-Nspire CX CAS calculator screen showing the calculation of the volume of a solid of revolution.

Input: $\text{solve}(h(x)=g(x), x)$ $x=3.51155$

Output: $\pi \int_{3.5115471416946}^5 \left(x^2 - \left(\frac{x^2+1}{x^2-3\cdot x+2}\right)^2\right) dx$

Result: 51.379

Question 2 (10 marks)

$$\frac{dx}{dt} = 2 \operatorname{cosec}(x) \sin^2(2t) \text{ where } x = \pi \text{ when } t = \frac{\pi}{4}, t \geq 0$$

a. Use $\sin^2(2t) = \frac{1}{2}(1 - \cos(4t))$

i. $\frac{dx}{dt} = 2 \operatorname{cosec}(x) \sin^2(2t)$

$$\frac{dx}{dt} = 2 \operatorname{cosec}(x) \left(\frac{1}{2}(1 - \cos(4t)) \right)$$

$$\int \frac{1}{2 \operatorname{cosec}(x)} dx = \int \left(\frac{1}{2}(1 - \cos(4t)) \right) dt$$

$$\int \frac{\sin(x)}{2} dx = \frac{1}{2} \int (1 - \cos(4t)) dt$$

$$\int \sin(x) dx = \int (1 - \cos(4t)) dt \text{ of the required form } \int f(x) dx = \int g(t) dt$$

1A

ii. Solve in the form $x = \cos^{-1}(at + b \sin(4t) + c)$

$$\int \sin(x) dx = \int (1 - \cos(4t)) dt$$

$$-\cos(x) = t - \frac{1}{4} \sin(4t) + c$$

1M

$$\cos(x) = -t + \frac{1}{4} \sin(4t) + c$$

$$\cos(\pi) = -\frac{\pi}{4} + \frac{1}{4} \sin(4 \cdot \frac{\pi}{4}) + c$$

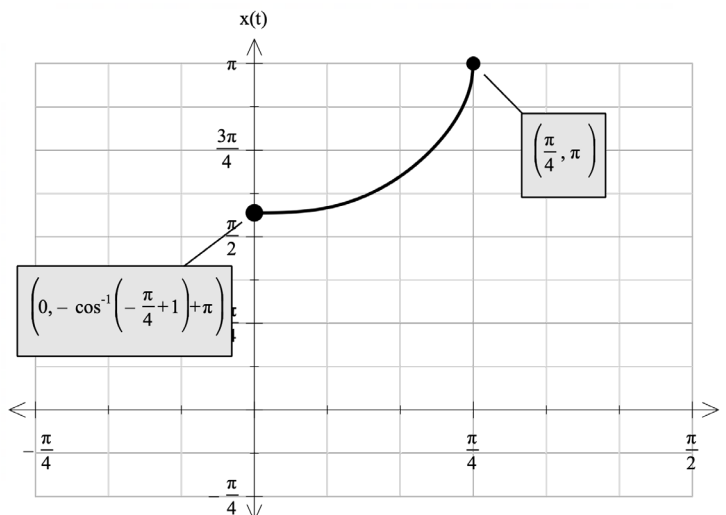
$$-1 = -\frac{\pi}{4} + c \therefore c = -1 + \frac{\pi}{4}$$

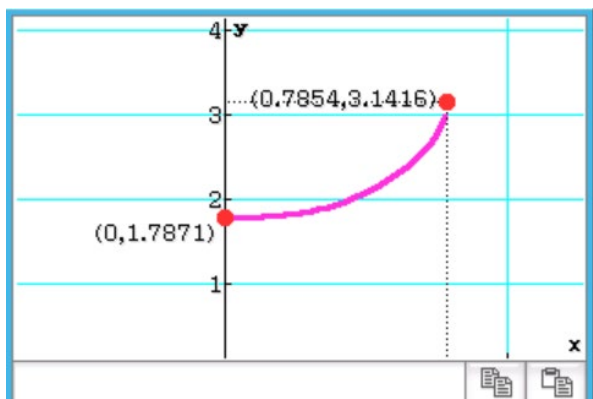
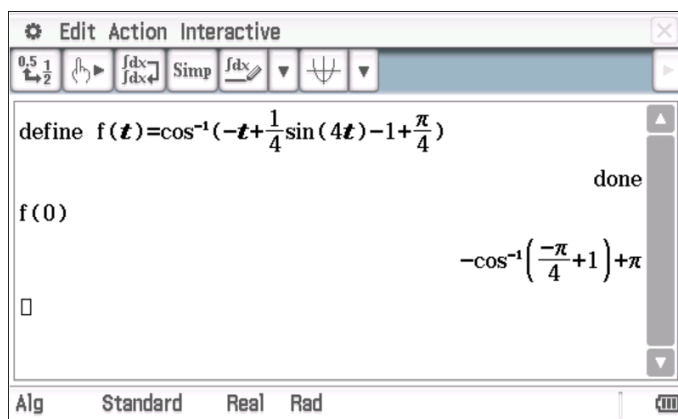
1M

$$x = \cos^{-1} \left(-t + \frac{1}{4} \sin(4t) - 1 + \frac{\pi}{4} \right)$$

1A

iii.





$$\text{Domain } \left[0, \frac{\pi}{4}\right]$$

$$\text{Range } \left[\pi - \cos^{-1}\left(1 - \frac{\pi}{4}\right), \pi\right]$$

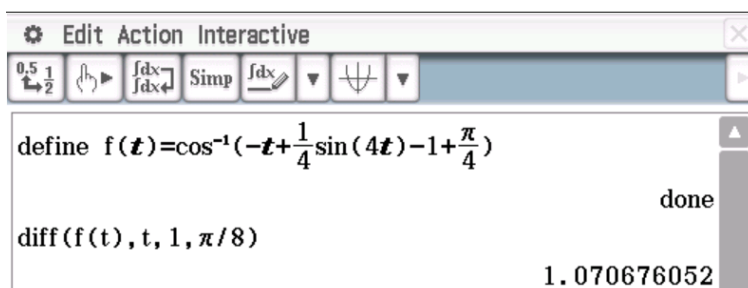
2A

$$\text{b. } x(t) = \cos^{-1}\left(-t + \frac{1}{4}\sin(4t) - 1 + \frac{\pi}{4}\right)$$

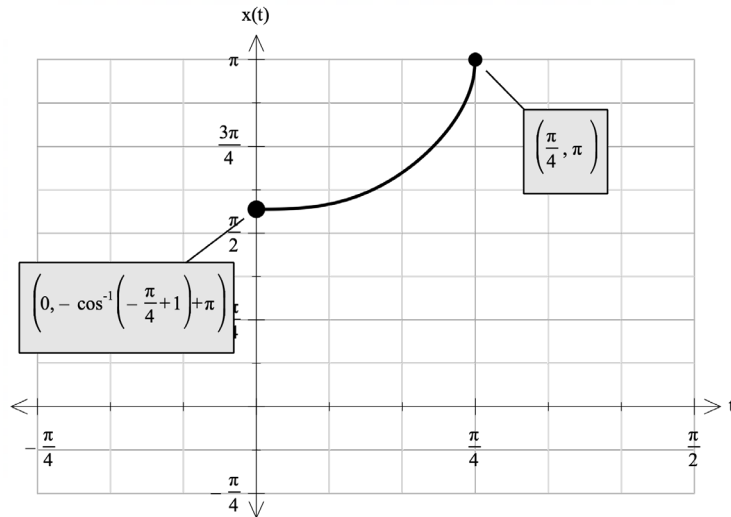
$$x'\left(\frac{\pi}{8}\right) = \cos^{-1}\left(-\frac{\pi}{8} + \frac{1}{4}\sin\left(\frac{\pi}{2}\right) - 1 + \frac{\pi}{4}\right)$$

$$\text{Speed} = \left|x'\left(\frac{\pi}{8}\right)\right| = 1.07 \text{ ms}^{-1} \text{ to two decimal places}$$

1A



c.



3A

1 shape, 2 endpoints

Question 3 (11 marks)

- a. Use appropriate trigonometric formulas to show that $\text{cis}\left(\frac{5\pi}{12}\right)$ can be expressed in the form $A + Bi$,

where $A, B \in \mathbb{R}$ as $\text{cis}\left(\frac{5\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4} + \frac{\sqrt{6}+\sqrt{2}}{4}i$.

$$\text{cis}\left(\frac{5\pi}{12}\right) = \text{cis}\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \text{cis}\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

$$\text{cis}\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

Taking Real components

$$\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$

$$\therefore \cos\left(\frac{5\pi}{12}\right) = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

and taking Imaginary components

$$i \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = i \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) + i \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$

1M

$$\therefore i \sin\left(\frac{5\pi}{12}\right) = i \frac{1}{2} \times \frac{\sqrt{2}}{2} + i \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}i$$

Giving

$$\text{cis}\left(\frac{5\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4} + \frac{\sqrt{6}+\sqrt{2}}{4}i \text{ where } A = \frac{\sqrt{6}-\sqrt{2}}{4}, B = \frac{\sqrt{6}+\sqrt{2}}{4}$$

1M

- b. Express $\left\{ z : \left| z - 3 \operatorname{cis} \left(\frac{5\pi}{12} \right) \right| = \left| z + 3 \operatorname{cis} \left(\frac{5\pi}{12} \right) \right|, z \in C \right\}$ in the form $y = ax + b$, where $a, b \in R$.

$$\left\{ z : \left| z - 3 \operatorname{cis} \left(\frac{5\pi}{12} \right) \right| = \left| z + 3 \operatorname{cis} \left(\frac{5\pi}{12} \right) \right|, z \in C \right\}$$

From **part a**).

$$\text{Let } 3 \operatorname{cis} \left(\frac{5\pi}{12} \right) = \operatorname{cis} \left(\frac{5\pi}{12} \right) = \frac{3\sqrt{6} - 3\sqrt{2}}{4} + i \frac{3\sqrt{6} + 3\sqrt{2}}{4} \quad \mathbf{1M}$$

$$\text{So } \left\{ z : \left| z - 3 \operatorname{cis} \left(\frac{5\pi}{12} \right) \right| = \left| z + 3 \operatorname{cis} \left(\frac{5\pi}{12} \right) \right|, z \in C \right\}$$

$$\left| z - \left(\frac{3\sqrt{6} - 3\sqrt{2}}{4} + i \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right) \right| = \left| z + \frac{3\sqrt{6} - 3\sqrt{2}}{4} + i \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right|$$

Let $z = x + iy$

$$\left| x + iy - \left(\frac{3\sqrt{6} - 3\sqrt{2}}{4} + i \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right) \right| = \left| x + iy + \frac{3\sqrt{6} - 3\sqrt{2}}{4} + i \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right|$$

$$\sqrt{\left(x - \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left(y - \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2} = \sqrt{\left(x + \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left(y + \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2}$$

$$\left(x - \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left(y - \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2 = \left(x + \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left(y + \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2 \quad \mathbf{1M}$$

Giving

$$y = -x \left(\frac{3 - \sqrt{3}}{3 + \sqrt{3}} \right)$$

In rationalised form

$$y = (\sqrt{3} - 2)x \quad \mathbf{1A}$$

$\text{solve} \left(\left(x - \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left(y - \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2 = \left(x + \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left(y + \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2, y \right)$
 $\left\{ y = \frac{\sqrt{3} \cdot (x - y) - 3 \cdot x}{3} \right\}$
 $\text{solve} \left(y = \frac{\sqrt{3} \cdot (x - y) - 3 \cdot x}{3}, y \right)$
 $\left\{ y = \frac{-x \cdot (-\sqrt{3} + 3)}{\sqrt{3} + 3} \right\}$

$$\left(\frac{\sqrt{6}+3\sqrt{2}}{4}\right)^2 = \left(x + \frac{3\sqrt{6}-3\sqrt{2}}{4}\right)^2 + \left(y + \frac{3\sqrt{6}+3\sqrt{2}}{4}\right)^2, y$$

$$\left\{ y = \frac{\sqrt{3} \cdot (x-y) - 3 \cdot x}{3} \right\}$$

$$\text{solve}\left(y = \frac{\sqrt{3} \cdot (x-y) - 3 \cdot x}{3}, y\right)$$

$$\left\{ y = \frac{-x \cdot (-\sqrt{3}+3)}{\sqrt{3}+3} \right\}$$

Alg Standard Real Rad

Define $\text{cis}(t) = \cos(t) + i \cdot \sin(t)$ Done

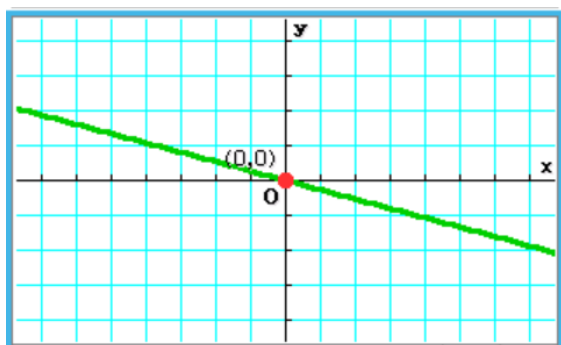
$$\left| z - 3 \cdot \text{cis}\left(\frac{5 \cdot \pi}{12}\right) \right| = \left| z + 3 \cdot \text{cis}\left(\frac{5 \cdot \pi}{12}\right) \right|, z = x + y \cdot i$$

$$\frac{\sqrt{2} \cdot (2 \cdot x^2 - 3 \cdot (\sqrt{3} - 1) \cdot \sqrt{2} \cdot x + 2 \cdot y^2 - 3 \cdot (\sqrt{3} - 1) \cdot y)}{2}$$

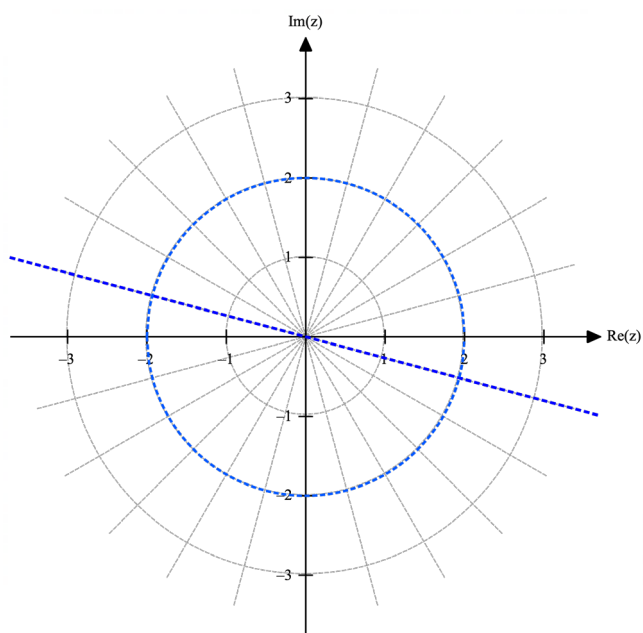
$$\text{solve}\left(\frac{\sqrt{2} \cdot (2 \cdot x^2 - 3 \cdot (\sqrt{3} - 1) \cdot \sqrt{2} \cdot x + 2 \cdot y^2 - 3 \cdot (\sqrt{3} - 1) \cdot y)}{2}, y\right)$$

$$y = (\sqrt{3} - 2) \cdot x$$

- c. Sketch the graph of $\left\{ z : \left| z - 3 \text{cis}\left(\frac{5\pi}{12}\right) \right| = \left| z + 3 \text{cis}\left(\frac{5\pi}{12}\right) \right|, z \in C \right\}$ in the form $y = ax + b$, where $a, b \in R$, 1 mark



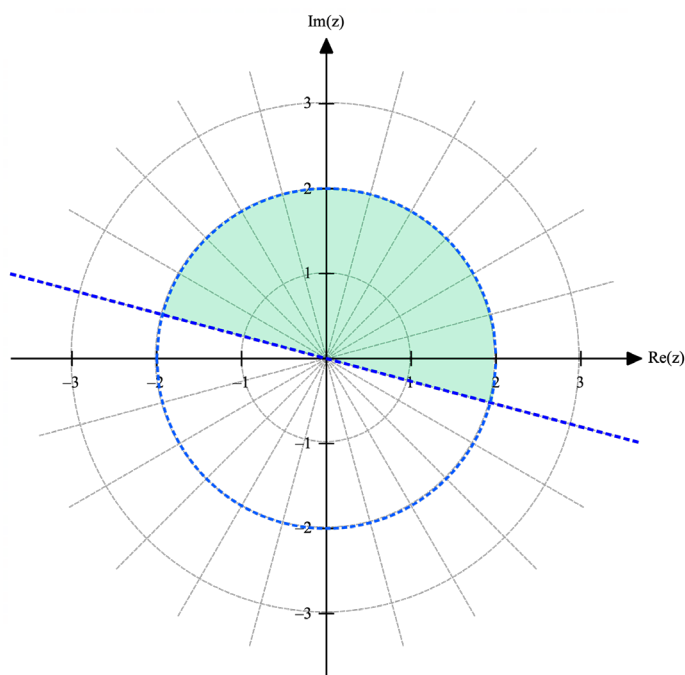
- d. On the Argand diagram below sketch and label $A = \left\{ z : z \bar{z} = 4, z \in C \right\}$ and
- $$B = \left\{ z : \left| z - 3 \text{cis}\left(\frac{5\pi}{12}\right) \right| = \left| z + 3 \text{cis}\left(\frac{5\pi}{12}\right) \right|, z \in C \right\}.$$



2A

- e. i. Shade of region defined by $\left\{z : z \bar{z} \leq 4, z \in C\right\} \cap \left\{z : \left|z - 3 \operatorname{cis}\left(\frac{5\pi}{12}\right)\right| \leq \left|z + 3 \operatorname{cis}\left(\frac{5\pi}{12}\right)\right|, z \in C\right\}$

1A



ii Intersection points on graph above

$$\left\{z : z \bar{z} \leq 4, z \in C\right\} \cap \left\{z : \left|z - 3 \operatorname{cis}\left(\frac{5\pi}{12}\right)\right| \leq \left|z + 3 \operatorname{cis}\left(\frac{5\pi}{12}\right)\right|, z \in C\right\}$$

Intersection between line $y = (\sqrt{3} - 2)x$ and circle $x^2 + y^2 = 4$

$$x = \frac{2}{\sqrt{6}-\sqrt{2}}, y = \frac{-(\sqrt{6}-\sqrt{2})}{2}$$
$$x = \frac{-2}{\sqrt{6}-\sqrt{2}}, y = \frac{\sqrt{6}-\sqrt{2}}{2}$$

1A

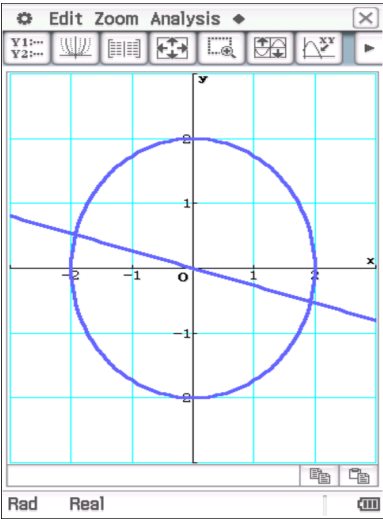
$z \cdot \text{conj}(z) = 4 \mid z = x + y \cdot i$

$x^2 + y^2 = 4$

solve($x^2 + y^2 = 4$ and $\frac{\sqrt{2} \cdot (2 \cdot x^2 - 3 \cdot (\sqrt{3} - 1) \cdot \sqrt{2})}{2}$

$y = (\sqrt{3} - 2) \cdot \sqrt{\sqrt{3} + 2}$ and $x = \sqrt{\sqrt{3} + 2}$ or $y = -(\sqrt{3} - 2) \cdot \sqrt{\sqrt{3} + 2}$ and $x = \sqrt{\sqrt{3} + 2}$ or $y = -(\sqrt{3} - 2) \cdot \sqrt{\sqrt{3} + 2}$ and $x = -\sqrt{\sqrt{3} + 2}$ or $y = (\sqrt{3} - 2) \cdot \sqrt{\sqrt{3} + 2}$ and $x = -\sqrt{\sqrt{3} + 2}$

$x = \frac{-\sqrt{6}}{2} - \frac{\sqrt{2}}{2}$ and $y = \frac{(\sqrt{3} - 1) \cdot \sqrt{2}}{2}$ or $x = \frac{\sqrt{6}}{2}$



1A

$\text{solve}(x^2 + y^2 = 4, y)$

$\{y = -\sqrt{-x^2 + 4}, y = \sqrt{-x^2 + 4}\}$

$\text{define } f(x) = (\sqrt{3} - 2) \cdot x$

done

$\text{solve}(-\sqrt{-x^2 + 4} = f(x), x)$

$\{x = \frac{2}{\sqrt{6} - \sqrt{2}}\}$

$\text{solve}(\sqrt{-x^2 + 4} = f(x), x)$

$\{x = \frac{-2}{\sqrt{6} - \sqrt{2}}\}$

$\text{simplify}(f(\frac{2}{\sqrt{6} - \sqrt{2}}))$

$\frac{-(\sqrt{6} - \sqrt{2})}{2}$

$\text{simplify}(f(\frac{2}{\sqrt{6} - \sqrt{2}}))$

$\frac{-(\sqrt{6} - \sqrt{2})}{2}$

$\text{simplify}(f(\frac{-2}{\sqrt{6} - \sqrt{2}}))$

$\frac{\sqrt{6} - \sqrt{2}}{2}$

- f. Find the area of the shaded region in **part e**. Line goes through centre of circle radius of 2.

$$\begin{aligned}\text{Area} &= \frac{1}{2} \pi 2^2 \\ &= 2\pi\end{aligned}$$

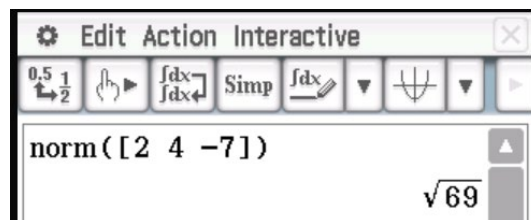
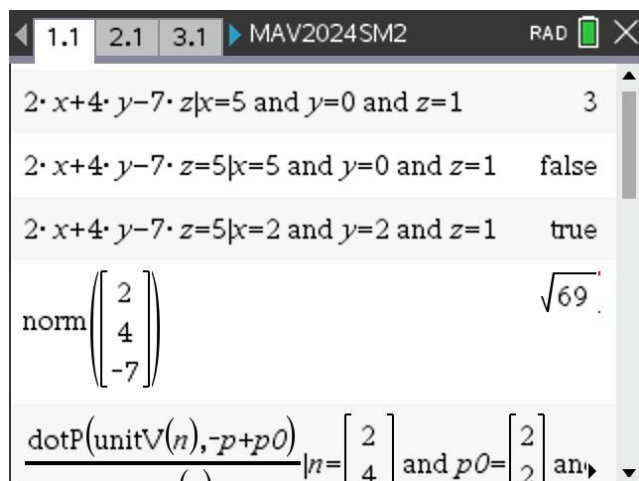
1A

Question 4

- a. Substituting $x = 5$, $y = 0$ and $z = 1$ into the plane equation gives

$$2x + 4y - 7z = 2 \times 5 + 4 \times 0 - 7 \times 1 = 3 \neq 5 \quad \mathbf{1M}$$

Therefore, $P(5, 0, 1)$ does not lie on the plane. $\mathbf{1A}$



- b. The vector perpendicular to the plane is given by the coefficients of x , y and z thus it could be $2\hat{i} + 4\hat{j} - 7\hat{k}$. $\mathbf{1A}$

- c. Let $\vec{n} = 2\hat{i} + 4\hat{j} - 7\hat{k}$ where $\vec{n}$ is a normal vector to the plane.

$$|\vec{n}| = \sqrt{2^2 + 4^2 + (-7)^2} = \sqrt{69}$$

Find any point P_0 that lies on the plane.

$P_0(2, 2, 1)$ where P_0 is a point on the plane

$$\overrightarrow{OP_0} = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$\overrightarrow{PP_0} = \overrightarrow{PO} + \overrightarrow{OP_0} = -(5\hat{i} + \hat{k}) + (2\hat{i} + 2\hat{j} + \hat{k}) = -3\hat{i} + 2\hat{j}$$

$$\vec{n} \cdot \overrightarrow{PP_0} = (2\hat{i} + 4\hat{j} - 7\hat{k}) \cdot (-3\hat{i} + 2\hat{j}) = 2 \quad \mathbf{1M}$$

$$D = \frac{\vec{n} \cdot \overrightarrow{PP_0}}{|\vec{n}|} = \frac{1}{\sqrt{69}} \times 2 = \frac{2}{\sqrt{69}} = \frac{2\sqrt{69}}{69}.$$

Therefore, the shortest distance from $P(5, 0, 1)$ to Π_1 is $\frac{2}{\sqrt{69}}$ (or $\frac{2\sqrt{69}}{69}$). $\mathbf{1A}$

1.1 2.1 3.1 MAV2024SM2 RAD

$$\text{norm}\left(\begin{bmatrix} 2 \\ 4 \\ -7 \end{bmatrix}\right) \sqrt{69}$$

$$\frac{\text{dotP}(\text{unitV}(n), -p+p0)}{\text{norm}(n)} \mid n = \begin{bmatrix} 2 \\ 4 \\ -7 \end{bmatrix} \text{ and } p0 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \text{ an}$$

$$\frac{2}{69}$$

Edit Action Interactive

$$\text{norm}([2 \ 4 \ -7]) \sqrt{69}$$

$$[2 \ 4 \ -7] \Rightarrow n$$

$$[2 \ 2 \ 1] \Rightarrow p$$

$$\text{unitV}(n) \Rightarrow v$$

$$\left[\frac{2 \cdot \sqrt{69}}{69} \quad \frac{4 \cdot \sqrt{69}}{69} \quad \frac{-7 \cdot \sqrt{69}}{69} \right]$$

$$\frac{\text{dotP}(v, p - [5 \ 0 \ 1])}{\text{norm}([2 \ 4 \ -7])} \frac{2}{69}$$

d. $\overline{AB} = (2\hat{i} - \hat{j} + 4\hat{k}) - (\alpha\hat{i} + \beta\hat{j} + \hat{k}) = (2 - \alpha)\hat{i} + (-1 - \beta)\hat{j} + 3\hat{k}$
 $\overline{AC} = (5\hat{i} + \hat{j} + \hat{k}) - (\alpha\hat{i} + \beta\hat{j} + \hat{k}) = (5 - \alpha)\hat{i} + (1 - \beta)\hat{j}$ **1A**

e. Note that $\hat{r} \cdot (3\hat{i} + 6\hat{j} + 7\hat{k}) = 28 \equiv 3x + 6y + 7z = 28$

A normal vector to the plane is $\overline{AB} \times \overline{AC} = (3\beta - 3)\hat{i} + (15 - 3\alpha)\hat{j} + (-2\alpha + 3\beta + 7)\hat{k}$. **1M**

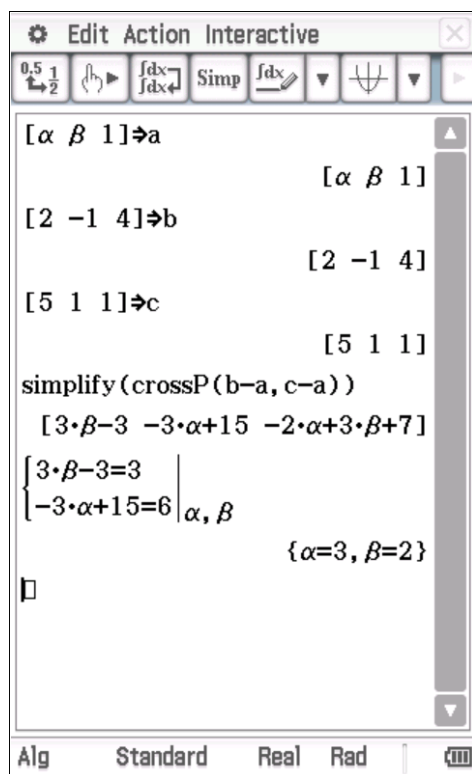
Equating $3\hat{i} + 6\hat{j} + 7\hat{k} = (3\beta - 3)\hat{i} + (15 - 3\alpha)\hat{j} + (-2\alpha + 3\beta + 7)\hat{k}$ will find $\alpha = 3$ and $\beta = 2$.

Therefore, the coordinate of A is $(3, 2, 1)$. **1A**

1.1 2.1 3.1 MAV2024SM2 RAD

$$\text{crossP}(-a+b, -a+c) \mid a = \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \text{ and } c = \begin{bmatrix} 3 \cdot (\beta - 1) \\ -3 \cdot (\alpha - 5) \\ -2 \cdot \alpha + 3 \cdot \beta + 7 \end{bmatrix}$$

$$\text{solve}\left(\begin{bmatrix} 3 \cdot (\beta - 1) \\ -3 \cdot (\alpha - 5) \\ -2 \cdot \alpha + 3 \cdot \beta + 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 7 \end{bmatrix}, \alpha, \beta\right) \alpha = 3 \text{ and } \beta = 2$$



f. A normal to plane Π_1 is $\underline{n}_1 = 2\hat{i} + 4\hat{j} - 7\hat{k}$ and a normal to the plane Π_2 is $\underline{n}_2 = 3\hat{i} + 6\hat{j} + 7\hat{k}$.
Let θ be the angle between the two planes.

$$\underline{n}_1 \cdot \underline{n}_2 = |\underline{n}_1| |\underline{n}_2| \cos(\theta)$$

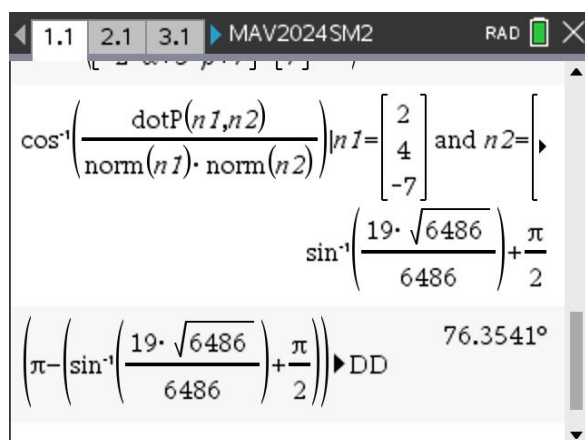
$$-19 = \sqrt{69} \cdot \sqrt{94} \cdot \cos(\theta)$$

$$\cos(\theta) = \frac{-19}{\sqrt{69} \cdot \sqrt{94}} \quad \mathbf{1M}$$

$$\theta = 1.809 = 103.646^\circ$$

The angle between $\underline{n}_1$ and $\underline{n}_2$ is $76.354^\circ = 76.35^\circ$.

Therefore, the acute angle between the two planes is 76.35° . $\mathbf{1A}$



g. Solve $2x + 4y - 7z = 5$ and $3x + 6y + 7z = 28$

would have $x = -2y + \frac{33}{5}$ (or $y = -\frac{x}{2} + \frac{33}{10}$) and $z = \frac{41}{35}$.

Let $y = t$. **1M**

We would have $x = -2t + \frac{33}{5}$, $y = t$ and $z = \frac{41}{35}$.

Therefore, $\underline{\ell} = \left(\frac{33}{5}\underline{i} + 0\underline{j} + \frac{41}{35}\underline{k} \right) + t(-2\underline{i} + \underline{j} + 0\underline{k})$. **1A**

$$\text{linSolve}\left(\left\{\begin{array}{l} 2 \cdot x + 4 \cdot y - 7 \cdot z = 5 \\ 3 \cdot x + 6 \cdot y + 7 \cdot z = 28 \end{array}, \{x, y, z\}\right\}\right)$$

$$\left\{ \frac{-(10 \cdot \mathbf{c1} - 33)}{5}, \mathbf{c1}, \frac{41}{35} \right\}$$

$$\text{expand}\left(\text{linSolve}\left(\left\{\begin{array}{l} 2 \cdot x + 4 \cdot y - 7 \cdot z = 5 \\ 3 \cdot x + 6 \cdot y + 7 \cdot z = 28 \end{array}, \{x, y, z\}\right\}\right)\right)$$

$$\left\{ \frac{33}{5} - 2 \cdot \mathbf{c2}, \mathbf{c2}, \frac{41}{35} \right\}$$

OR

$$\underline{u} = \underline{n}_1 \times \underline{n}_2 = (2\underline{i} + 4\underline{j} - 7\underline{k}) \times (3\underline{i} + 6\underline{j} + 7\underline{k}) = 70\underline{i} - 35\underline{j} \quad \mathbf{1M}$$

Let $x = 0$, $2 \times 0 + 4y - 7z = 5$ and $3 \times 0 + 6y + 7z = 28$.

Therefore, the line of intersection is

$$\begin{aligned} & \left(0\underline{i} + \frac{33}{10}\underline{j} + \frac{41}{35}\underline{k} \right) + t(70\underline{i} - 35\underline{j} + 0\underline{k}) \\ & \equiv \left(0\underline{i} + \frac{33}{10}\underline{j} + \frac{41}{35}\underline{k} \right) + \left(\frac{33}{5} \div 70 \right) (70\underline{i} - 35\underline{j} + 0\underline{k}) + \frac{1}{-35} \times t(70\underline{i} - 35\underline{j} + 0\underline{k}) \\ & = \left(0\underline{i} + \frac{33}{10}\underline{j} + \frac{41}{35}\underline{k} \right) + \left(\frac{33}{350} \right) (70\underline{i} - 35\underline{j} + 0\underline{k}) + t(-2\underline{i} + 1\underline{j} + 0\underline{k}) \\ & = \left(\frac{33}{5}\underline{i} + 0\underline{j} + \frac{41}{35}\underline{k} \right) + t(-2\underline{i} + 1\underline{j} + 0\underline{k}) \end{aligned}$$

1A

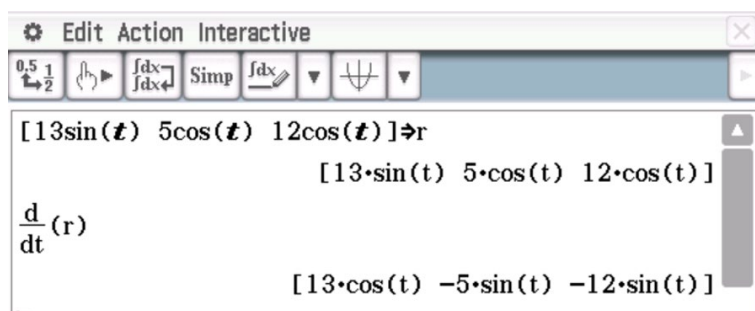
Question 5

a. $\dot{\underline{r}}(t) = 13 \cos(t)\underline{i} - 5 \sin(t)\underline{j} - 12 \sin(t)\underline{k}$

$$\underline{r1}(t) := \begin{bmatrix} 13 \cdot \sin(t) \\ 5 \cdot \cos(t) \\ 12 \cdot \cos(t) \end{bmatrix} \quad \text{Done}$$

$$\underline{r1}'(t) := \frac{d}{dt}(\underline{r1}(t)) \quad \text{Done}$$

$$\underline{r1}'(t) = \begin{bmatrix} 13 \cdot \cos(t) \\ -5 \cdot \sin(t) \\ -12 \cdot \sin(t) \end{bmatrix}$$



b.

$$\begin{aligned}
 |\dot{r}_1(t)| &= |13\cos(t)\underline{i} - 5\sin(t)\underline{j} - 12\sin(t)\underline{k}| \\
 &= \sqrt{(13\cos(t))^2 + (-5\sin(t))^2 + (-12\sin(t))^2} \\
 &= \sqrt{13^2\cos^2(t) + 25\sin^2(t) + 144\sin^2(t)} \\
 &= \sqrt{13^2\cos^2(t) + 169\sin^2(t)} \quad \mathbf{1A} \\
 &= \sqrt{13^2(\cos^2(t) + \sin^2(t))} \\
 &= \sqrt{13^2 \cdot 1} \\
 &= 13
 \end{aligned}$$

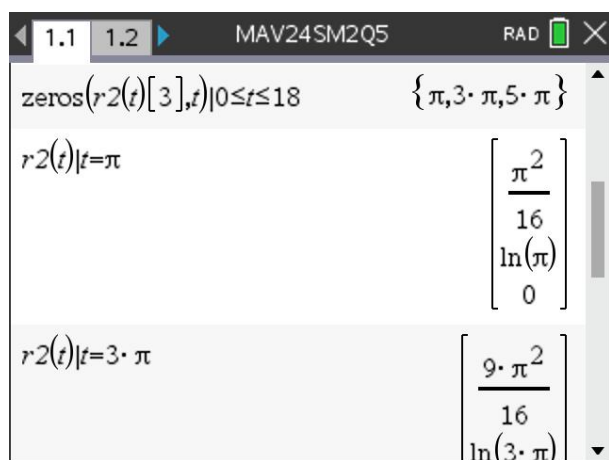
As the speed of this dragon is always 13 and is not dependent on time, thus the speed of this dragon is constant.

1A

c. The dragon passes through the xy plane when $z = 0$.

When $\cos\left(\frac{t}{2}\right), t = \pi$. **1M**

$$r_2(\pi) = \left(\frac{\pi^2}{16}, \log_e \pi, 0\right) \quad \mathbf{1A}$$

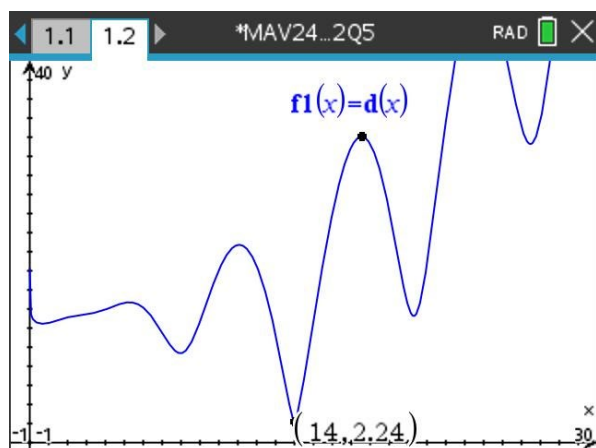
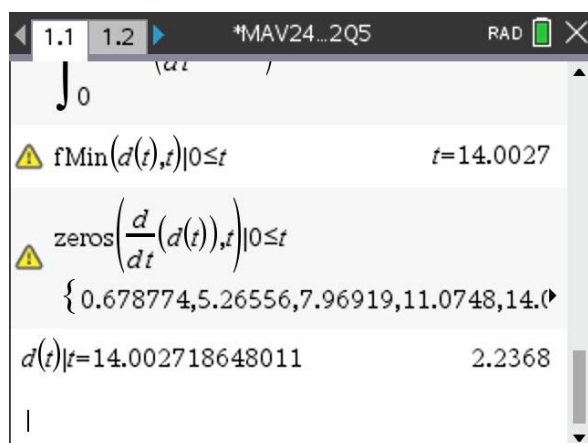


d. $D(t) = |r_2(t) - r_1(t)|$ **1M**

$$= \sqrt{\begin{aligned} &256 \cos^2\left(\frac{t}{2}\right) - 6144 \cos(t) \cos\left(\frac{t}{2}\right) \\ &- 1280 \left(\log_e(t) - \log_e\left(\frac{1}{t}\right) \right) \cos(t) \\ &- 416t^2 \sin(t) - 256 \log_e\left(\frac{1}{t}\right) \log_e(t) \\ &+ t^4 + 43264 \end{aligned}} \div 16 \quad (\text{students are not expected to copy this})$$

$D(t)$ is minimum when $t = 14.002718648011 \approx 14.003$. **1A**

Thus the minimum distance is $D(14.0027...) = 2.2368024196637 \approx 2.237$ **1A**



e. $\dot{\mathbf{r}}_2(t) = (-2 \sin(t) \cos(t) - 6 \sin(t)) \mathbf{i} + (-2 \cos^2(t) + 6 \cos(t) + 1) \mathbf{j} + \left(\frac{1}{t^2 + 1}\right) \mathbf{k}$ **1M**

The distance travelled by the second dragon is given by

$$\int_1^3 |\dot{\mathbf{r}}_2(t)| dt = \int_1^3 \sqrt{\left(\frac{t}{8}\right)^2 + \left(\frac{1}{t}\right)^2 + \left(\frac{-\sin \frac{t}{2}}{2}\right)^2} dt$$

1A

$$= 1.50019 \approx 1.500$$

$$\int_1^3 \text{norm}\left(\frac{d}{dt}(r2(t))\right) dt$$

1.50019

Question 6

a. Let the sales figure be K on that particular day.
 $E(K) = 3000 + (-10) \times 25 + 5 \times 200 = 3000 - 250 + 1000 = 3750$ 1A

$Var(K) = 25^2 \times 5^2 + 5^2 \times 15^2 = 21250$

$SD(K) = \sqrt{Var(K)} = \sqrt{21250} = 25\sqrt{34}$ 1A

Thus, $K \sim N(3750, 21250)$

b. $Pr(K > 3800) = 0.36580 = 0.3658$ 1A

Edit Action Interactive

0.5 1 2

$\int dx$

$\int dx$

$\int dx$

$\int dx$

Simp

$\int dx$

$\frac{d}{dx}$

$\frac{d}{dx}$

$\frac{d}{dx}$

$\frac{d}{dx}$

$\frac{d}{dx}$

$\frac{d}{dx}$

$\frac{d}{dx}$

$\frac{d}{dx}$

$\frac{d}{dx}$

$\frac{d}{dx}$

$\frac{d}{dx}$

$\frac{d}{dx}$

normCDF(3800, ∞, 25·√34, 3750)

0.3658002945

c. (3622.226, 3877.774) 1A

1.1 *Doc RAD

zInterval 25·√34, 3750, 5, 0.95: stat.results

"Title"	"z Interval"
"CLower"	3622.23
"CUpper"	3877.77
"x̄"	3750.
"ME"	127.774
"n"	5.
"σ"	145.774

C-Level 0.95

σ 25(34^0.5)

x̄ 3750

n 5

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OneSampleZInt

Lower 3622.226

Upper 3877.774

x̄ 3750

n 5

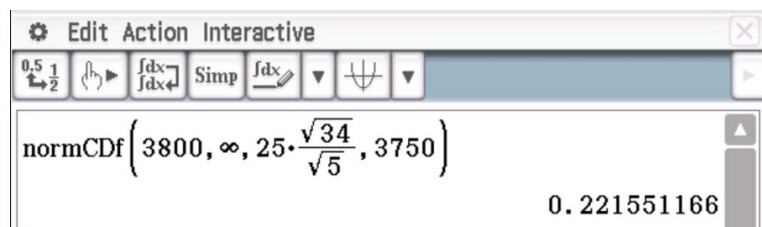
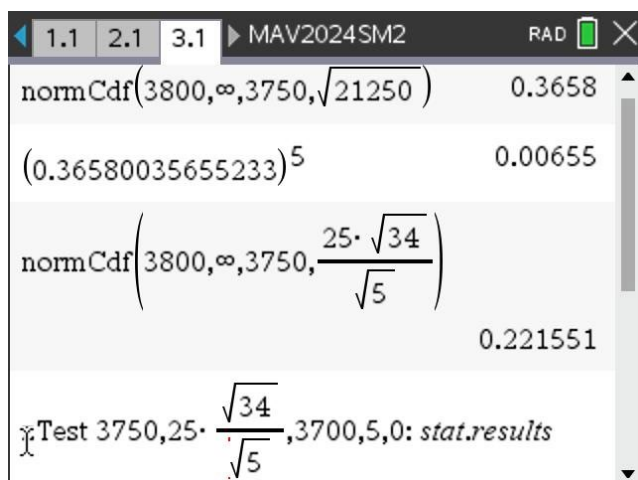
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OneSampleZInt

d. $\bar{K} \sim N\left(3750, \left(\frac{25\sqrt{34}}{\sqrt{5}}\right)^2\right)$ **1M**

$\Pr(\text{ave. Saturdays exceed } 3800) = \Pr(\bar{K} > 3800) = 0.2216$ **1A**



e.

$H_0: \mu = 3700$

$H_1: \mu < 3700$ **1A**

f. p -value $\Pr(k < 3700 | \mu = 3750) = 0.2215$ **1M**

We thus do not reject the null hypothesis. There could be some truth in owner's claim. **1A**

END OF QUESTION AND ANSWER BOOK SOLUTIONS