

2024 VCE Specialist Mathematics Year 12 Trial Examination 2 Detailed Answers



Kilbaha Education

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SECTION A

ANSWERS

1		A		B		C		D
2		A		B		C		D
3		A		B		C		D
4		A		B		C		D
5		A		B		C		D
6		A		B		C		D
7		A		B		C		D
8		A		B		C		D
9		A		B		C		D
10		A		B		C		D
11		A		B		C		D
12		A		B		C		D
13		A		B		C		D
14		A		B		C		D
15		A		B		C		D
16		A		B		C		D
17		A		B		C		D
18		A		B		C		D
19		A		B		C		D
20		A		B		C		D

SECTION A

Question 1

Answer C

h : the heating is on

$\neg r$: the room is not cold

If the heating is on then the room is not cold $h \rightarrow \neg r$

the contrapositive is $r \rightarrow \neg h$ if the room is cold, then the heating is not on.

Question 2

Answer D

$f(x) = \frac{x+b}{x^2-a^2}$ has $y=0$ as a horizontal asymptote and $x=\pm a$ a vertical asymptotes,

so three straight line asymptotes provided $b \neq a$, Alan is correct

If $|b| = \sqrt{a^2}$ then $b = \pm\sqrt{a^2}$ consider when $b = \sqrt{a^2} = a$ $f(x) = \frac{x+a}{x^2-a^2} = \frac{1}{x-a}$ $x \neq -a$, $y=0$ as a horizontal asymptote and $x=a$ a vertical asymptote and $x=-a$ is a point of discontinuity,

or consider when $b = -\sqrt{a^2} = -a$ $f(x) = \frac{x-a}{x^2-a^2} = \frac{1}{x+a}$ $x \neq a$, $y=0$ as a horizontal asymptote and $x=-a$ a vertical asymptote and $x=a$ is a point of discontinuity, Ben is correct.

$f(0) = -\frac{b}{a^2}$ and $f(x) = 0 \Rightarrow x = -b$ so $\left(0, -\frac{b}{a^2}\right)$ is the y -intercept and $(-b, 0)$ is the x -intercept,

Colin is correct.

Question 3

Answer B

If $z = x + yi$, $\text{Arg}(z) = \tan^{-1}\left(\frac{y}{x}\right)$, is true in the first and fourth quadrant only, so only when $x > 0$

Question 4

Answer B

Line (1), $\vec{r}(t) = (3+2t)\vec{i} - (2+t)\vec{j} + (4+t)\vec{k}$ has direction $2\vec{i} - \vec{j} + \vec{k}$ and

parametric equations $x = 3+2t$, $y = -2-t$, $z = 4+t$.

Line (2), $\frac{x+3}{-2} = y-1 = 1-z$ rewrite as $\frac{x+3}{-2} = \frac{y-1}{1} = \frac{z-1}{-1} = s$ has direction $-2\vec{i} + \vec{j} - \vec{k}$

so the lines are parallel, and has the parametric form $x = -3-2s$, $y = 1+s$, $z = 1-s$

equate the x components $-3-2s = 3+2t$ so $s = -3-t$ then $y = 1+(-3-t) = -2-t$ and

$z = 1-s = 1-(-3-t) = 4+t$, so the two lines are in fact the same line.

Question 5

Answer A

Assume $\binom{2k}{k} < 2^{2k-2}$ replace k with $k+1$ and show that $\binom{2(k+1)}{k+1} = \binom{2k+2}{k+1} < 2^{(k+1)-2} = 2^{2k}$

Question 6 **Answer D**

$I_n = \int x^n e^{-kx} dx$ integration by parts, let

$$u = x^n \quad \frac{dv}{dx} = e^{-kx}$$

$$\frac{du}{dx} = nx^{n-1} \quad v = \int e^{-kx} dx = -\frac{1}{k} e^{-kx}$$

$$I_n = -\frac{x^n e^{-kx}}{k} + \frac{n}{k} \int x^{n-1} e^{-kx} dx = -\frac{x^n e^{-kx}}{k} + \frac{n}{k} I_{n-1} \quad \text{A. B. and C. are all incorrect.}$$

$I_n = \int x^n e^{-kx} dx$ integration by parts, let

$$u = e^{-kx} \quad \frac{dv}{dx} = x^n$$

$$\frac{du}{dx} = -ke^{-kx} \quad v = \int x^n dx = \frac{x^{n+1}}{n+1}$$

$$I_n = \frac{x^{n+1} e^{-kx}}{n+1} + \frac{k}{n+1} \int x^{n+1} e^{-kx} dx = \frac{x^{n+1} e^{-kx}}{n+1} + \frac{k}{n+1} I_{n+1}$$

Question 7 **Answer C**

The distance of the plane $ax + by + cz = d$ from the origin is $\frac{|d|}{\sqrt{a^2 + b^2 + c^2}}$

Plane (1) $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ and $\underline{n} = 4\underline{i} + 2\underline{j} - 4\underline{k}$ $\underline{r} \cdot \underline{n} = 4$ gives $4x + 2y - 4z = 4$ or

$$2x + y - 2z = 2 \text{ the distance from the origin is } \frac{2}{\sqrt{2^2 + 1^2 + (-2)^2}} = \frac{2}{3}$$

Plane (2) $-2x - y + 2z = 1$ the distance from the origin is $\frac{1}{\sqrt{(-2)^2 + (-1)^2 + 2^2}} = \frac{1}{3}$

But the normals of the two planes are in the opposite direction, so the planes are on opposite sides of the origin, so the distance between the planes is $\frac{1}{3} + \frac{2}{3} = 1$

Question 8 **Answer B**

A particle is travelling on a circle, so that $x^2 + y^2 = r^2$ using implicit differentiation

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0, \quad \frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt} = -\frac{x}{y} \times (y) = -x, \text{ so } y \text{ decreases as } x \text{ increases}$$

the particle moves clockwise around the circle.

Question 9

Answer A

The scalar resolute of $\underline{a}$ in the direction $\underline{b}$, is $\underline{a} \cdot \hat{\underline{b}} = \frac{\underline{a} \cdot \underline{b}}{|\underline{b}|} = p$.

Let $\underline{c} = n\underline{b}$, now $|\underline{c}| = n|\underline{b}|$

The scalar resolute of $m\underline{a}$ in the direction $\underline{c}$ is $m\underline{a} \cdot \hat{\underline{c}} = \frac{m\underline{a} \cdot \underline{c}}{|\underline{c}|} = \frac{m\underline{a} \cdot n\underline{b}}{n|\underline{b}|} = m \frac{\underline{a} \cdot \underline{b}}{|\underline{b}|} = mp$

Question 10

Answer D

$\underline{a} \times \underline{b} = |\underline{a}||\underline{b}|\sin\theta \hat{n}$ where $\hat{n} = \frac{\underline{a} \times \underline{b}}{|\underline{a} \times \underline{b}|}$ is a unit vector perpendicular to both $\underline{a}$ and $\underline{b}$, so $|\hat{n}| = 1$

(1) $|\underline{a} \times \underline{b}| = |\underline{a}||\underline{b}|\sin\theta$ and (2) $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos(\theta)$ $\frac{(1)}{(2)} \quad \frac{|\underline{a} \times \underline{b}|}{\underline{a} \cdot \underline{b}} = \tan(\theta)$ **A.** is true

$\hat{\underline{a}} \cdot \hat{\underline{b}} = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}||\underline{b}|} = \cos(\theta)$ **B.** is true

If $\underline{a}$ is parallel to $\underline{b}$, then $\underline{a} \times \underline{b} = \underline{0}$ and if $\underline{a}$ is perpendicular to $\underline{b}$, then $\underline{a} \cdot \underline{b} = 0$ **C.** is true

D. is false.

Question 11

Answer C

$$x = \log_e(kt) \quad y = \cos(kt)$$

$$\frac{dx}{dt} = \dot{x} = \frac{1}{t} \quad \frac{dy}{dt} = \dot{y} = -k \sin(kt)$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{\dot{y}}{\dot{x}} = -kt \sin(kt)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} (-kt \sin(kt)) \cdot t$$

$$\frac{d^2y}{dx^2} = t(-k^2 t \cos(kt) - k \sin(kt))$$

$$\frac{d^2y}{dx^2} = -kt(kt \cos(kt) + \sin(kt))$$

$$\frac{d^2}{dx^2}(\cos(e^x)) \quad -e^{2 \cdot x} \cdot \cos(e^x) - e^x \cdot \sin(e^x)$$

$$\frac{d^2}{dx^2}(\cos(e^x))|_{x=\ln(kt)}$$

$$-k^2 \cdot t^2 \cdot \cos(kt) - kt \cdot \sin(kt)$$

Question 12

Answer B

$$|z - a| = 2|z - ai|, \quad z = x + yi$$

$$|(x - a) + yi| = 2|x + (y - a)i|$$

$$\sqrt{(x - a)^2 + y^2} = 2\sqrt{x^2 + (y - a)^2}$$

$$(x - a)^2 + y^2 = 4(x^2 + (y - a)^2)$$

$$x^2 - 2xa + a^2 + y^2 = 4x^2 + 4y^2 - 8ya + 4a^2$$

$$3x^2 + 2xa + 3y^2 - 8ya = -3a^2$$

$$\left(x + \frac{a}{3}\right)^2 + \left(y - \frac{4a}{3}\right)^2 = \frac{8a^2}{9} \quad \text{a circle}$$

Question 13

Answer D

$$f(x) = (x^2 + bx + 2)e^{-2x}$$

$$f'(x) = (-2x^2 + (2 - 2b)x + (b - 4))e^{-2x}$$

for turning points

$$f'(x) = 0 \Rightarrow -2x^2 + (2 - 2b)x + (b - 4) = 0$$

$$x = \frac{-b + 1 \pm \sqrt{b^2 - 7}}{2}$$

so there are two turning points if $|b| > \sqrt{7}$

one turning point if $|b| = \sqrt{7}$

and no turning points if $|b| < \sqrt{7}$

$$f''(x) = (4x^2 + (4b - 8)x + (10 - 4b))e^{-2x}$$

for inflexion points

$$f''(x) = 0 \Rightarrow 4x^2 + (4b - 8)x + (10 - 4b) = 0$$

$$x = \frac{-b + 2 \pm \sqrt{b^2 - 6}}{2}$$

so there are two inflexion points if $|b| > \sqrt{6}$

one inflexion point if $|b| = \sqrt{6}$

and no inflexion points if $|b| < \sqrt{6}$

D. is false

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Define $f(x) = (x^2 + b \cdot x + 2) \cdot e^{-2 \cdot x}$ Done

$$\frac{d}{dx}(f(x)) \quad (-2 \cdot x^2 + (2 - 2 \cdot b) \cdot x + b - 4) \cdot e^{-2 \cdot x}$$

solve $\left(\frac{d}{dx}(f(x)) = 0, x\right)$

$$x = \frac{\sqrt{b^2 - 7} - b + 1}{2} \text{ or } x = \frac{-\left(\sqrt{b^2 - 7} + b - 1\right)}{2}$$

$$\frac{d^2}{dx^2}(f(x))$$

$$(4 \cdot x^2 + (4 \cdot b - 8) \cdot x - 4 \cdot b + 10) \cdot e^{-2 \cdot x}$$

solve $\left(\frac{d^2}{dx^2}(f(x)) = 0, x\right)$

$$x = \frac{\sqrt{b^2 - 6} - b + 2}{2} \text{ or } x = \frac{-\left(\sqrt{b^2 - 6} + b - 2\right)}{2}$$

Question 14 **Answer A**

$$\log_e(n) - \log_e(100 - n) = \frac{t}{5} - \log_e(9),$$

satisfies the differential equation

$$\frac{dn}{dt} = \frac{n(100 - n)}{500} = \frac{n}{5} \left(1 - \frac{n}{100}\right) = \frac{100n - n^2}{500}$$

A. is false, **B. C.** and **D.** are all true

The initial number of penguins was 10 and the number of penguins cannot exceed 100.

The solution of the differential equation is

$$n(t) = \frac{100}{1 + 9e^{-\frac{t}{5}}}$$

The penguin growth rate is increasing most rapidly

$$\text{when } \frac{d^2n}{dt^2} = \left(\frac{100 - 2n}{500}\right) \frac{dn}{dt} = 0 \Rightarrow n = 50$$

$$\frac{100}{1 + 9e^{-\frac{t}{5}}} = 50 \Rightarrow t = 5 \log_e(9) = 10 \log_e(3)$$

Question 15 **Answer A**

$$\frac{dy}{dx} = f(x) = \sin^2(3x) \text{ using Euler's Method}$$

$$h = \frac{\pi}{18}, x_0 = 0 \text{ and } y_0 = 3, x_1 = \frac{\pi}{18}, x_2 = \frac{\pi}{9}$$

$$y_1 = y_0 + h f(x_0) = 3 + \frac{\pi}{18} \sin^2(0) = 3$$

$$y_2 = y_1 + h f(x_1) = 3 + \frac{\pi}{18} \sin^2\left(\frac{\pi}{6}\right) = 3 + \frac{\pi}{18} \times \frac{1}{4} = 3 + \frac{\pi}{72}$$

$$y_3 = y_2 + h f(x_2) = 3 + \frac{\pi}{72} + \frac{\pi}{18} \sin^2\left(\frac{\pi}{3}\right) = 3 + \frac{\pi}{72} + \frac{\pi}{18} \times \frac{3}{4} = 3 + \frac{\pi}{18}$$

Question 16 **Answer C**

when $x = \pm 2$ the slope $m = 0$ and when $x = 0$ $y = 0$, the slope $m = -1$

$$\text{the slopes are never infinite, only satisfied by } \frac{dy}{dx} = \frac{x^2 - 4}{y^2 + 4}$$

$$\text{solve}\left(\ln(n) - \ln(100 - n) = \frac{t}{5} - \ln(9), n\right) \Big|_{t=0}$$

$n = 10$

$$\text{impDif}\left(\ln(n) - \ln(100 - n) = \frac{t}{5} - \ln(9), t, n\right)$$

$$\frac{-n \cdot (n - 100)}{500}$$

$$\text{deSolve}\left(n' = \frac{-n \cdot (n - 100)}{500} \text{ and } n(0) = 10, t, n\right)$$

$$n = \frac{100 \cdot e^{\frac{t}{5}}}{e^{\frac{t}{5}} + 9}$$

$$t = 10 \cdot \ln(3)$$

$$\text{solve}\left(\frac{100 \cdot e^{\frac{t}{5}}}{e^{\frac{t}{5}} + 9} = 50, t\right)$$

$$\text{euler}\left((\sin(3 \cdot x))^2, x, y, \left\{0, \frac{\pi}{6}\right\}, 3, \frac{\pi}{18}\right)$$

0.0000	0.1745	0.3491	0.5236
3.0000	3.0000	3.0436	3.1745

$3 + \frac{\pi}{18}$ 3.1745

Question 17 **Answer C**

The stone takes on the initial upwards speed of the balloon, but its acceleration is just due to gravity. Taking upwards as positive and downwards as negative,

$$s = -100 \quad u = 2 \quad a = -9.8 \quad t = ? \quad \text{using} \quad s = ut + \frac{1}{2}at^2$$

$$-100 = 2t - 4.9t^2 \quad \text{solving} \Rightarrow t = 4.73$$

Question 18 **Answer A**

$$(a, b) = \bar{x} \pm z \frac{\sigma}{\sqrt{n}} \quad 95\%, \quad z = 1.96$$

$$(1) \quad a = \bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} \quad (2) \quad b = \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}$$

$$\frac{1}{2}((1) + (2)) \quad \bar{x} = \frac{1}{2}(a + b)$$

$$\frac{1}{2}((2) - (1)) \quad 1.96 \frac{\sigma}{\sqrt{n}} = \frac{1}{2}(b - a), \quad \frac{\sigma}{\sqrt{n}} = \frac{1}{2 \times 1.96}(b - a)$$

$$99\%, \quad z = 2.575$$

$$\begin{aligned} CI &: \left(\bar{x} - 2.575 \frac{\sigma}{\sqrt{n}}, \bar{x} + 2.575 \frac{\sigma}{\sqrt{n}} \right) \\ &= \left(\frac{1}{2}(a + b) - \frac{2.575}{2 \times 1.96}(b - a), \frac{1}{2}(a + b) + \frac{2.575}{2 \times 1.96}(b - a) \right) \\ &= \left(\frac{a}{2} \left(1 + \frac{2.575}{1.96} \right) + \frac{b}{2} \left(1 - \frac{2.575}{1.96} \right), \frac{a}{2} \left(1 - \frac{2.575}{1.96} \right) + \frac{b}{2} \left(1 + \frac{2.575}{1.96} \right) \right) \\ &= (1.157a - 0.157b, -0.157a + 1.157b) \end{aligned}$$

Question 19 **Answer B**

$$\bar{B} \stackrel{d}{=} N\left(\mu = ?, \frac{6^2}{25}\right),$$

$$\Pr(\bar{B} < 250) = 0.2$$

$$\frac{250 - \mu}{\frac{6}{5}} = -0.8416$$

$$\mu = 251.01$$

invNorm(0.2,0,1)	-0.84162
solve $\left(\frac{250-m}{\frac{6}{5}} = -0.8416, m\right)$	$m = 251.01$

Question 20

Answer D

$$v(3) = \frac{6}{\pi} \tan^{-1}(1) = \frac{3}{2}, \quad v(7) = -2$$

$$b = \frac{\frac{3}{2} + 2}{3 - 7} = -\frac{7}{8}, \quad a = \frac{3}{2} - 3 \times \left(-\frac{7}{8}\right) = \frac{33}{8} \quad \text{A. is true}$$

The area of the triangle when $v > 0$ is $A_1 = \frac{1}{2} \times \frac{3}{2} \times \left(\frac{33}{7} - 3\right) = \frac{9}{7}$,

and when $v < 0$ is $A_2 = \frac{1}{2} \times 2 \times \left(7 - \frac{33}{7}\right) = -\frac{16}{7}$

The toy car moves a total distance of

$$\int_0^7 |v(t)| dt = \frac{6}{\pi} \int_0^3 \tan^{-1}\left(\frac{t}{3}\right) dt + \frac{9}{7} + \frac{16}{7} = \frac{6}{\pi} \int_0^3 \tan^{-1}\left(\frac{t}{3}\right) dt + \frac{25}{7} \text{ metres. B. is true}$$

The toy car has a displacement of

$$\int_0^7 v(t) dt = \frac{6}{\pi} \int_0^3 \tan^{-1}\left(\frac{t}{3}\right) dt + \frac{9}{7} - \frac{16}{7} = \frac{6}{\pi} \int_0^3 \tan^{-1}\left(\frac{t}{3}\right) dt - 1 \text{ metres. C. is true}$$

$$\int_3^7 \left| \frac{33}{8} - \frac{7 \cdot t}{8} \right| dt \quad \frac{25}{7}$$

$$\int_3^7 \left(\frac{33}{8} - \frac{7 \cdot t}{8} \right) dt \quad -1$$

D. is false the average speed is the total distance travelled over total time.

END OF SECTION A SUGGESTED ANSWERS

SECTION B

Question 1

a.
$$f(x) = \frac{x^2 - 2x - 3}{x^2 - x}$$

$$f(x) = \frac{(x-3)(x+1)}{x(x-1)} = 1 - \frac{x+3}{x(x-1)}$$

$x = 0$ and $x = 1$ are vertical asymptotes,
and $y = 1$ is a horizontal asymptote.

Note the graph will cross the horizontal asymptote at $x = -3$

b.
$$f'(x) = \frac{x^2 + 6x - 3}{x^2(x-1)^2}$$

for turning points $f'(x) = 0$

solving $x^2 + 6x - 3 = 0$

gives $x = -3 \pm 2\sqrt{3}$

$$(-3 + 2\sqrt{3}, 4\sqrt{3} + 8), (-3 - 2\sqrt{3}, -4\sqrt{3} + 8)$$

c. The graph crosses the x-axis at $x = 3$ and $x = -1$, $(-3, 0)$, $(-1, 0)$

Define $f1(x) = \frac{x^2 - 2 \cdot x - 3}{x^2 - x}$ Done

factor(f1(x)) $\frac{(x-3) \cdot (x+1)}{x \cdot (x-1)}$

propFrac(f1(x)) $1 - \frac{x+3}{x \cdot (x-1)}$

A1

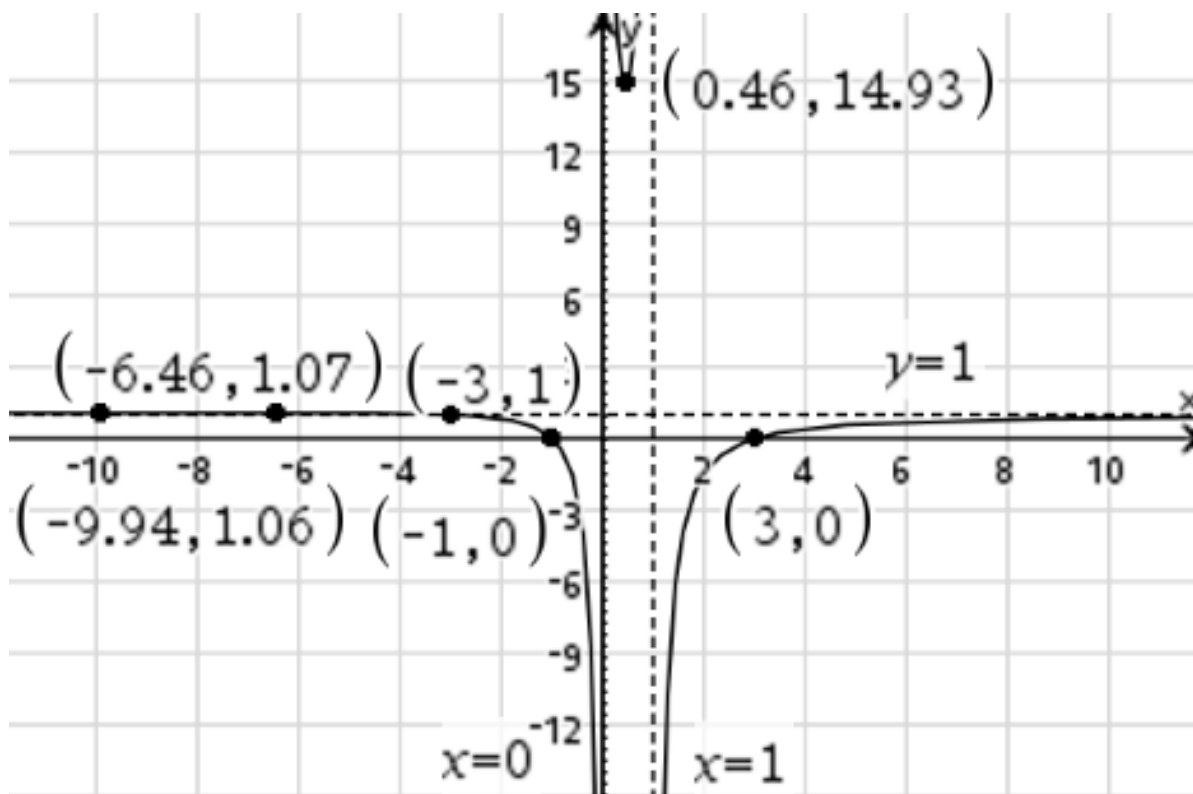
$\frac{d}{dx}(f1(x))$ $\frac{x^2 + 6 \cdot x - 3}{x^2 \cdot (x-1)^2}$

zeros($\frac{d}{dx}(f1(x)), x$) $\rightarrow$ xtps $\{-(2 \cdot \sqrt{3} + 3), 2 \cdot \sqrt{3} - 3\}$

f1(xtps) $\{-4 \cdot (\sqrt{3} - 2), 4 \cdot (\sqrt{3} + 2)\}$

A1

G3



d.i. when $k = -2$ $f_{-2}(x) = \frac{x^2 + 2x - 3}{x^2 - x} = \frac{(x-1)(x+3)}{x(x-1)} = \frac{x+3}{x} = 1 + \frac{3}{x}$ $x \neq 1$

when $k = -2$ the graph has a point of discontinuity and will have no turning points A1

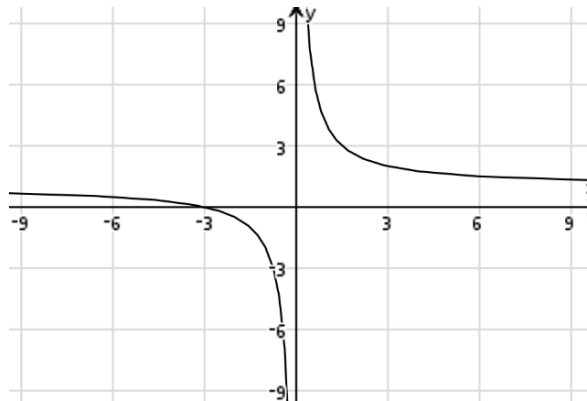
ii. $f_k(x) = \frac{x^2 - kx - 3}{x^2 - x}$
 $f_k(x) = 1 - \frac{(k-1)x + 3}{x^2 - x}$

Define $f_k(x) = \frac{x^2 - k \cdot x - 3}{x^2 - x}$ Done
 $\text{propFrac}(f_k(x))$ $1 - \frac{(k-1) \cdot x + 3}{x \cdot (x-1)}$

The graph will cross the horizontal asymptote $y = 1$ when $k \in \mathbb{R} \setminus \{1, -2\}$

A1

Define $f_k(x) = \frac{x^2 - k \cdot x - 3}{x^2 - x} | k = -2$ Done
 $f_k(x)$ $\frac{x+3}{x}$



iii. $f'(x) = \frac{(k-1)x^2 + 6x - 3}{x^2(x-1)^2}$
 for turning points $f'(x) = 0$
 solving $(k-1)x^2 + 6x - 3 = 0$
 $\Delta = 36 + 12(k-1) = 12(k+2)$
 gives $x = \frac{-3 \pm \sqrt{3(k+2)}}{k-1}$
 investigate the case when $k = 1$

$$f_1(x) = \frac{x^2 - x - 3}{x^2 - x}$$

$$f'_1(x) = \frac{3(2x-1)}{x^2(x-1)^2}$$

$\frac{d}{dx}(f_k(x))$ $\frac{(k-1) \cdot x^2 + 6 \cdot x - 3}{x^2 \cdot (x-1)^2}$

solve $((k-1) \cdot x^2 + 6 \cdot x - 3 = 0, x)$
 $x = \frac{-(\sqrt{3 \cdot (k+2)} + 3)}{k-1}$ or $x = \frac{\sqrt{3 \cdot (k+2)} - 3}{k-1}$

$f_k(x) | k = 1$ $\frac{x^2 - x - 3}{x \cdot (x-1)}$

$\frac{d}{dx}\left(\frac{x^2 - x - 3}{x \cdot (x-1)}\right)$ $\frac{3 \cdot (2 \cdot x - 1)}{x^2 \cdot (x-1)^2}$

solve $\left(\frac{3 \cdot (2 \cdot x - 1)}{x^2 \cdot (x-1)^2} = 0, x\right)$ $x = \frac{1}{2}$

values of k	the graph of f_k has
$k > -2, k \neq 1$ $= (-2, 1) \cup (1, \infty)$	two turning points
$k = 1$	one turning point
$k \leq -2 = (-\infty, -2]$	no turning points
$k = -2$ or $k = 1$	no points of inflection
$k \in \mathbb{R} \setminus \{-2, 1\}$	one point of inflection

A3

Use a slider on the graphs page for k for $f_k(x)$ and watch the inflection point appear and disappear or use trial and error for the inflection points.

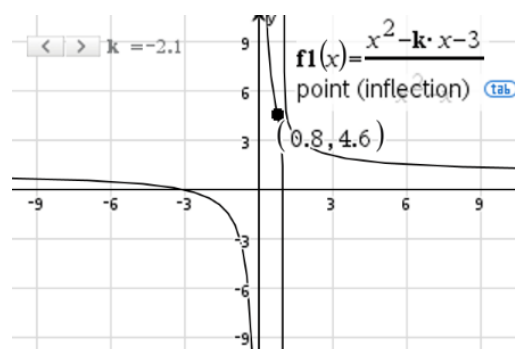
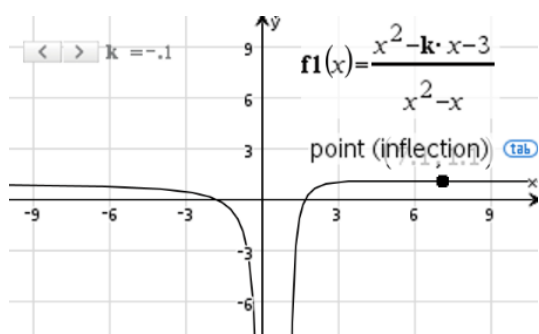
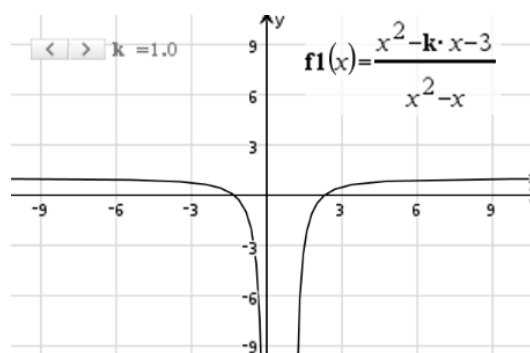
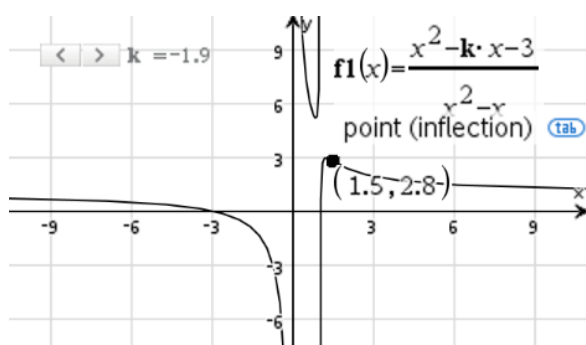
⚠ solve $\left(\frac{d^2}{dx^2}(f_k(x))=0, x\right) | k=-1$ $x=3.26117$

⚠ solve $\left(\frac{d^2}{dx^2}(f_k(x))=0, x\right) | k=-2$ false

⚠ solve $\left(\frac{d^2}{dx^2}(f_k(x))=0, x\right) | k=1$ false

⚠ solve $\left(\frac{d^2}{dx^2}(f_k(x))=0, x\right) | k=-1.9$ $x=1.47456$

⚠ solve $\left(\frac{d^2}{dx^2}(f_k(x))=0, x\right) | k=-2.1$ $x=0.75653$



Question 2

a.i. $z = x + yi$

$$|z - 2| = 2$$

$$|(x - 2) + yi| = 2$$

$$\sqrt{(x - 2)^2 + y^2} = 2$$

$$(x - 2)^2 + y^2 = 4$$

C_1 is a circle centre at

$(2, 0)$ radius 2.

$$|z - 4i| = 4$$

$$|x + (y - 4)i| = 4$$

$$\sqrt{x^2 + (y - 4)^2} = 4$$

$$x^2 + (y - 4)^2 = 16$$

C_2 is a circle centre at

$(0, 4)$ radius 4.

A2

ii. solving $(x - 2)^2 + y^2 = 4$ with $x^2 + (y - 4)^2 = 16$ gives

$$(0, 0), \left(\frac{16}{5}, \frac{8}{5}\right), u = \frac{16}{5} + \frac{8}{5}i \quad a = \frac{16}{5}, b = \frac{8}{5}$$

A1

iii. the line through $(0, 4)$, $\left(\frac{16}{5}, \frac{8}{5}\right)$ has a gradient $\frac{\frac{8}{5} - 4}{\frac{16}{5}} = -\frac{3}{4}$

the ray is $y - 4 = -\frac{3x}{4}$, $y = -\frac{3x}{4} + 4$ for $x < \frac{16}{3}$ draw with open circle at $\left(\frac{16}{3}, 0\right)$

A1

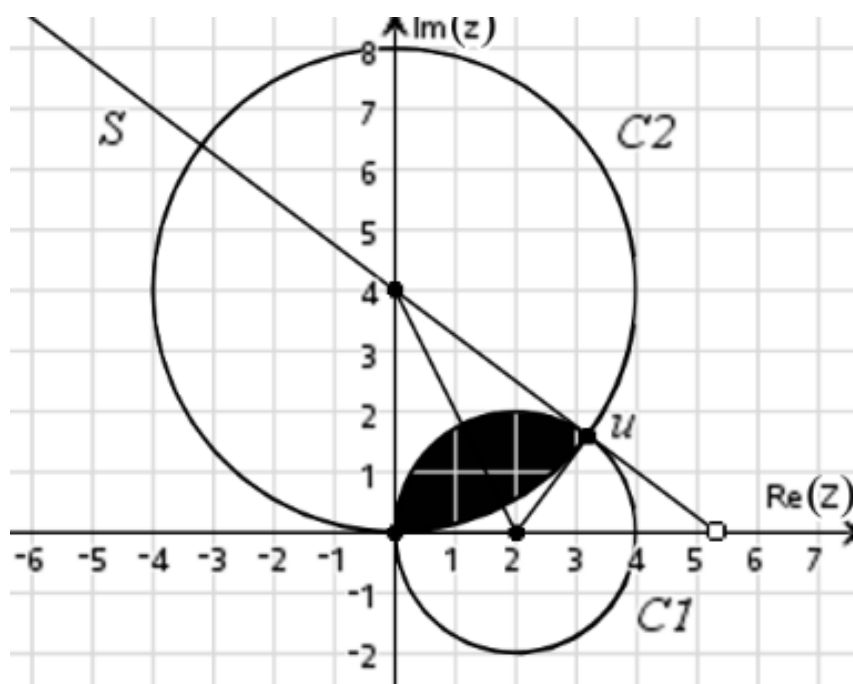
now the angle the ray makes with the positive x -axis is $\theta = \pi + \tan^{-1}\left(-\frac{3}{4}\right)$

$$\text{Arg}\left(z - \frac{16}{3}\right) = \pi + \tan^{-1}\left(-\frac{3}{4}\right), \quad d = \frac{16}{3}, \quad p = -\frac{3}{4}$$

A1

b.

G2



c. Consider three areas

Area 1: the area of the sector of radius 2, with an angle of 2α where $\alpha = \tan^{-1}(2)$

$$A_1 = \frac{1}{2}r^2 2\alpha = \frac{1}{2} \times 2^2 \times 2 \tan^{-1}(2) = 4 \tan^{-1}(2) \quad \text{A1}$$

Area 2: the area of the sector of radius 4, with an angle of 2β but $\alpha + \beta = \frac{\pi}{2}$ so that

$$\beta = \tan^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2} - \tan^{-1}(2)$$

$$A_2 = \frac{1}{2}r^2 2\beta = \frac{1}{2} \times 4^2 \times 2 \left(\frac{\pi}{2} - \tan^{-1}(2) \right) = 8\pi - 16 \tan^{-1}(2) \quad \text{A1}$$

Area 3: the area of two right angle triangles with side lengths 2 and 4, the total area of both triangles is $2 \times \frac{1}{2}bh = 2 \times \frac{1}{2} \times 2 \times 4 = 8$

The required area is

$$A = A_1 + A_2 - A_3 = 4 \tan^{-1}(2) + (8\pi - 16 \tan^{-1}(2)) - 8$$

$$A = 8\pi - 12 \tan^{-1}(2) - 8 \quad \text{A1}$$

Alternatively using calculus

$$(x-2)^2 + y^2 = 4 \text{ the top half of the circle is } y_1 = \sqrt{4 - (x-2)^2}$$

$$x^2 + (y-4)^2 = 16 \text{ the bottom half of the circle is } y_2 = 4 - \sqrt{16 - x^2} \quad \text{A1}$$

The required area is $A = \int_0^a (y_1 - y_2) dx$

$$A = \int_0^{\frac{16}{5}} \left(\sqrt{4 - (x-2)^2} - 4 + \sqrt{16 - x^2} \right) dx \quad \text{A1}$$

$$A = 4 \cos^{-1}\left(\frac{\sqrt{5}}{5}\right) + 8 \cos^{-1}\left(\frac{3}{5}\right) - 8 \quad \text{A1}$$

Which is equivalent to $A = 8\pi - 12 \tan^{-1}(2) - 8$

Define $f2(x) = 4 - \sqrt{16 - x^2}$ Done

Define $f3(x) = \sqrt{4 - (x-2)^2}$ Done

$$\int_0^{\frac{16}{5}} (f3(x) - f2(x)) dx$$

$$4 \cdot \cos^{-1}\left(\frac{\sqrt{5}}{5}\right) + 8 \cdot \cos^{-1}\left(\frac{3}{5}\right) - 8$$

$$\int_0^{\frac{16}{5}} (f3(x) - f2(x)) dx \quad 3.8470$$

$$8 \cdot \pi - 12 \cdot \tan^{-1}(2) - 8 \quad 3.8470$$

Question 3

a.i. $x = \sqrt{y(y^2 - 12y + 40)}$ solving when $x = 8$, $y = 8$, $H = 8$ A1

$$x^2 = y(y^2 - 12y + 40)$$

$$V = \pi \int_0^8 x^2 dy$$

$$V = \pi \int_0^8 (y^3 - 12y^2 + 40y) dy$$

$$V = 256\pi \text{ cm}^3$$

$$x^2 = y \cdot (y^2 - 12 \cdot y + 40) \quad x^2 = y \cdot (y^2 - 12 \cdot y + 40)$$

$$\text{solve}(x^2 = y \cdot (y^2 - 12 \cdot y + 40), y) | x=8 \quad y=8$$

$$\pi \cdot \int_0^8 (y \cdot (y^2 - 12 \cdot y + 40)) dy | h=8 \quad 256 \cdot \pi$$

A1

ii. $V = \pi \int_0^h (y^3 - 12y^2 + 40y) dy$

$$V = \pi \left[\frac{y^4}{4} - 4y^3 + 20y^2 \right]_0^h = \pi \left(\frac{h^4}{4} - 4h^3 + 20h^2 \right)$$

$$V = \frac{\pi h^2}{4} (h^2 - 16h + 80), \quad 0 \leq h \leq 8$$

A1

b.i. $S = 2\pi \int_{y_1}^{y_2} x \sqrt{1 + \left(\frac{dx}{dy} \right)^2} dy$

$$x = \sqrt{y(y^2 - 12y + 40)}, \quad \frac{dx}{dy} = \frac{3y^2 - 24y + 40}{2\sqrt{y(y^2 - 12y + 40)}}$$

A1

$$1 + \left(\frac{dx}{dy} \right)^2 = 1 + \frac{(3y^2 - 24y + 40)^2}{4(y(y^2 - 12y + 40))}$$

$$= \frac{9y^4 - 140y^3 + 768y^2 - 1760y + 1600}{4(y(y^2 - 12y + 40))}$$

$$S = 2\pi \int_0^8 \sqrt{y(y^2 - 12y + 40)} \times \frac{\sqrt{9y^4 - 140y^3 + 768y^2 - 1760y + 1600}}{2\sqrt{y(y^2 - 12y + 40)}} dy$$

M1

$$S = \pi \int_0^8 \sqrt{9y^4 - 140y^3 + 768y^2 - 1760y + 1600} dy$$

A1

$$b = 8, \quad p = 9, \quad q = -140, \quad r = 768, \quad s = -1760, \quad t = 1600$$

ii. $S = 453.24 \text{ cm}^2$

A1

Define $x(y) = \sqrt{y \cdot (y^2 - 12 \cdot y + 40)}$ *Done*

$\text{comDenom}\left(\frac{d}{dy}(x(y))\right)$

$$\frac{3 \cdot y^2 - 24 \cdot y + 40}{2 \cdot \sqrt{y \cdot (y^2 - 12 \cdot y + 40)}}$$

$\text{comDenom}\left(1 + \left(\frac{3 \cdot y^2 - 24 \cdot y + 40}{2 \cdot \sqrt{y \cdot (y^2 - 12 \cdot y + 40)}}$

$$\frac{9 \cdot y^4 - 140 \cdot y^3 + 768 \cdot y^2 - 1760 \cdot y + 1600}{4 \cdot y^3 - 48 \cdot y^2 + 160 \cdot y}$$

$$2 \cdot \pi \cdot x(y) \cdot \sqrt{\frac{9 \cdot y^4 - 140 \cdot y^3 + 768 \cdot y^2 - 1760 \cdot y}{4 \cdot y^3 - 48 \cdot y^2 + 160 \cdot y}}$$

$$\pi \cdot \sqrt{9 \cdot y^4 - 140 \cdot y^3 + 768 \cdot y^2 - 1760 \cdot y + 1600}$$

$$\int_0^8 \left(\pi \cdot \sqrt{9 \cdot y^4 - 140 \cdot y^3 + 768 \cdot y^2 - 1760 \cdot y + 1600} \right)$$

453.2444

c. $V = \frac{\pi h^2}{4}(h^2 - 16h + 80), \quad \frac{dV}{dh} = \pi h(h^2 - 12h + 40)$ A1

$$\frac{dV}{dt} = -k\sqrt{h}$$

$$\frac{dt}{dh} = \frac{dt}{dV} \cdot \frac{dV}{dh} = \frac{\pi h(h^2 - 12h + 40)}{-k\sqrt{h}}$$
M1

$$T = 120 = \frac{\pi}{-k} \int_8^0 \sqrt{h}(h^2 - 12h + 40) dh$$

$$T = 120 = \frac{\pi}{k} \int_0^8 \sqrt{h}(h^2 - 12h + 40) dh$$

solving gives $k = \frac{1376\pi\sqrt{2}}{1575}$ A1

Define $v(h) = \frac{h^2 \cdot (h^2 - 16 \cdot h + 80) \cdot \pi}{4}$ *Done*

$$\frac{d}{dh}(v(h)) \quad h \cdot (h^2 - 12 \cdot h + 40) \cdot \pi$$

$$\int_0^8 \frac{\pi \cdot h \cdot (h^2 - 12 \cdot h + 40)}{k \cdot \sqrt{h}} dh \quad \frac{11008 \cdot \pi \cdot \sqrt{2}}{105 \cdot k}$$

solve $\left(\frac{11008 \cdot \pi \cdot \sqrt{2}}{105 \cdot k} = 120, k \right)$

$$k = \frac{1376 \cdot \pi \cdot \sqrt{2}}{1575}$$

Question 4

a. $r_p(t) = 2t\mathbf{i} + 3\sqrt{2}t\mathbf{j} + 6t\mathbf{k}$

$v_p(t) = 2\mathbf{i} + 3\sqrt{2}\mathbf{j} + 6\mathbf{k}, \quad |v_p(t)| = \sqrt{4+18+36} = \sqrt{58}$

$A(0, 12\sqrt{2}, 32), B(6, 15\sqrt{2}, 34)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 6\mathbf{i} + 3\sqrt{2}\mathbf{j} + 2\mathbf{k},$

$|\overrightarrow{AB}| = |v_p(t)| = \sqrt{58}$ A1

$|v_Q(t)| = 2|v_p(t)|$

$v_Q(t) = 2\overrightarrow{AB} = 12\mathbf{i} + 6\sqrt{2}\mathbf{j} + 4\mathbf{k}$

$r_Q(t) = \int (12\mathbf{i} + 6\sqrt{2}\mathbf{j} + 4\mathbf{k}) dt = 12t\mathbf{i} + 6\sqrt{2}t\mathbf{j} + 4t\mathbf{k} + \mathbf{c}$

$r_Q(0) = \mathbf{c} = \overrightarrow{OA} = 12\sqrt{2}\mathbf{j} + 32\mathbf{k}$ M1

$r_Q(t) = 12t\mathbf{i} + (6\sqrt{2}t + 12\sqrt{2})\mathbf{j} + (4t + 32)\mathbf{k}$

$r_Q(t) = r_p(s) \quad r_p(s) = 2s\mathbf{i} + 3\sqrt{2}s\mathbf{j} + 6s\mathbf{k}$

$\mathbf{i}: (1) \quad 12t = 2s, \Rightarrow s = 6t$ M1

$\mathbf{j}: (2) \quad 6\sqrt{2}t + 12\sqrt{2} = 3\sqrt{2}s, \quad s = 2t + 4$

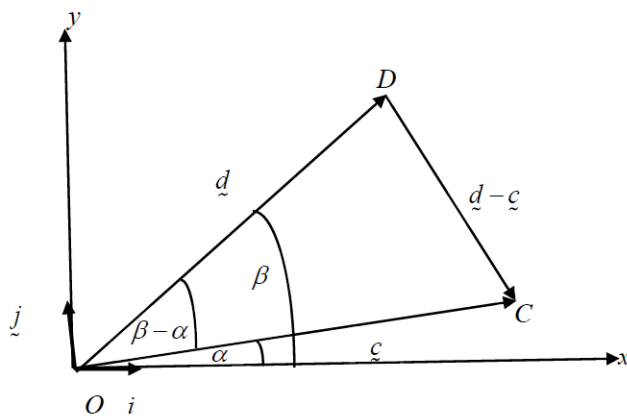
$6t = 2t + 4, \quad t = 1, \quad s = 6$

need to check in $\mathbf{k}: (3) \quad 4t + 32 = 6s = 36$ yes A1

so the time between release is $6 - 1 = 5$ seconds and the point of collision is

$(12, 18\sqrt{2}, 36)$ A1

b.



$\underline{c} = \overrightarrow{OC} = c(\cos(\alpha)\underline{i} + \sin(\alpha)\underline{j})$ the vector $\underline{c}$ has a length of c so $|\underline{c}| = c$ and makes an angle of α with the positive x -axis.

$\underline{d} = \overrightarrow{OD} = d(\cos(\beta)\underline{i} + \sin(\beta)\underline{j})$ the vector $\underline{d}$ has a length of d so $|\underline{d}| = d$ and makes an angle of β with the positive x -axis

$$\underline{d} - \underline{c} = d(\cos(\beta)\underline{i} + \sin(\beta)\underline{j}) - c(\cos(\alpha)\underline{i} + \sin(\alpha)\underline{j})$$

$$\underline{d} - \underline{c} = (d \cos(\beta) - c \cos(\alpha))\underline{i} + (d \sin(\beta) - c \sin(\alpha))\underline{j}$$

$$\begin{aligned} |\underline{d} - \underline{c}| &= \sqrt{(d \cos(\beta) - c \cos(\alpha))^2 + (d \sin(\beta) - c \sin(\alpha))^2} \\ &= \sqrt{d^2(\cos^2(\beta) + \sin^2(\beta)) + c^2(\cos^2(\alpha) + \sin^2(\alpha)) - 2dc(\cos(\beta)\cos(\alpha) + \sin(\beta)\sin(\alpha))} \\ &= \sqrt{d^2 + c^2 - 2dc \cos(\beta - \alpha)} \end{aligned}$$

Alternatively by the cosine rule, since $\beta - \alpha$ is the angle between the vectors $\underline{d}$ and $\underline{c}$

$$\text{so that } |\underline{d} - \underline{c}| = \sqrt{d^2 + c^2 - 2dc \cos(\beta - \alpha)} \quad \text{A1}$$

To prove that $|\underline{c}| + |\underline{d}| > |\underline{d} - \underline{c}|$ is equivalent to proving

$$c + d > \sqrt{d^2 + c^2 - 2dc \cos(\beta - \alpha)} \quad \text{A1}$$

To prove this use a proof by contradiction, assume that

$$c + d \leq \sqrt{d^2 + c^2 - 2dc \cos(\beta - \alpha)} \quad \text{now square both sides} \quad \text{A1}$$

$$c^2 + 2dc + d^2 \leq d^2 + c^2 - 2dc \cos(\beta - \alpha)$$

$$2dc(1 + \cos(\beta - \alpha)) \leq 0$$

but since $c > 0$ and $d > 0$, $0 < \beta - \alpha < \frac{\pi}{2}$, $\cos(\beta - \alpha) > 0$ so $2dc(1 + \cos(\beta - \alpha)) > 0$

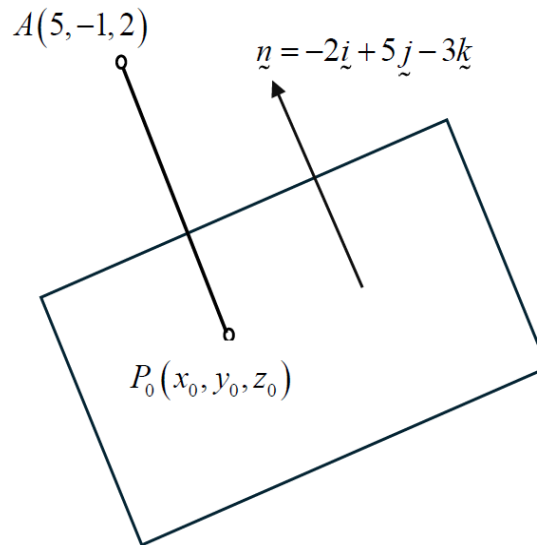
hence we have a contradiction, so the original statement A1

$$c + d \leq \sqrt{d^2 + c^2 - 2dc \cos(\beta - \alpha)} \quad \text{must be false,}$$

$$\text{hence } c + d > |\underline{d} - \underline{c}| = \sqrt{d^2 + c^2 - 2dc \cos(\beta - \alpha)}$$

Question 5

a.



The plane $-2x + 5y - 3z = 17$ has a normal $\vec{n} = -2\vec{i} + 5\vec{j} - 3\vec{k}$ let the point

$P_0(x_0, y_0, z_0)$ be the point on the plane closest to the point $A(5, -1, 2)$.

The vector $\overrightarrow{AP_0} = (x_0 - 5)\vec{i} + (y_0 + 1)\vec{j} + (z_0 - 2)\vec{k}$ is parallel to the normal to the plane,

so that $\overrightarrow{AP_0} = \lambda \vec{n}$, then

M1

$$\vec{i} \quad (1) \quad x_0 - 5 = -2\lambda,$$

$$\vec{j} \quad (2) \quad y_0 + 1 = 5\lambda,$$

$$\vec{k} \quad (3) \quad z_0 - 2 = -3\lambda$$

$$\text{rearranging } x_0 = 5 - 2\lambda, \quad y_0 = -1 + 5\lambda, \quad z_0 = 2 - 3\lambda$$

but since $P_0(x_0, y_0, z_0)$ lies on the plane $-2x_0 + 5y_0 - 3z_0 = 17$

M1

$$-2(5 - 2\lambda) + 5(-1 + 5\lambda) - 3(2 - 3\lambda) = 17$$

$$-10 + 4\lambda - 5 + 25\lambda - 6 + 9\lambda = 17$$

$$38\lambda = 38$$

solving $\lambda = 1$

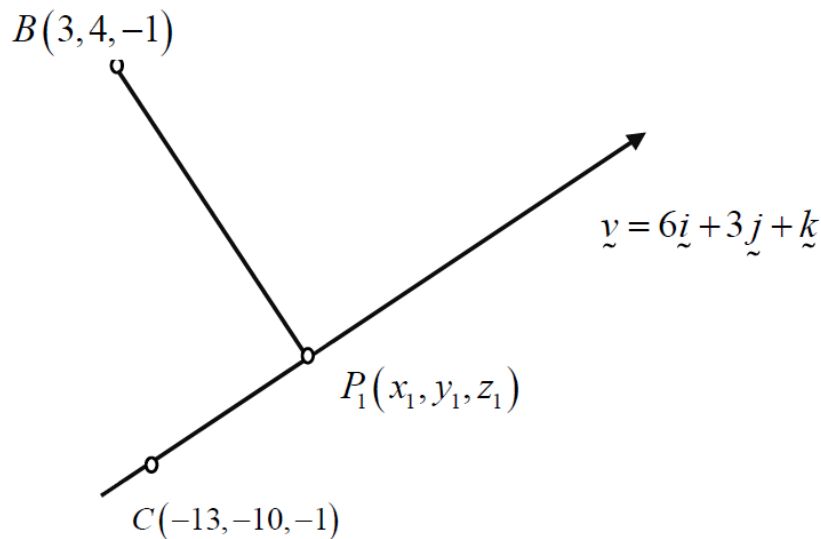
substituting $\lambda = 1$ gives $P_0(3, 4, -1)$ and

$$\overrightarrow{AP_0} = -2\vec{i} + 5\vec{j} - 3\vec{k},$$

A1

$$|\overrightarrow{AP_0}| = \sqrt{4 + 25 + 9} = \sqrt{38}$$

b.



The line $\vec{r}(t) = (-13+6t)\vec{i} + (-10+3t)\vec{j} + (-1+t)\vec{k}$ has direction $\vec{v} = 6\vec{i} + 3\vec{j} + \vec{k}$ and in parametric form is $x = -13+6t$, $y = -10+3t$, $z = -1+t$

let the point $P_1(x_1, y_1, z_1)$ be the point on the line which is closest to the point $B(3, 4, -1)$

The vector $\overrightarrow{BP_1} = (x_1-3)\vec{i} + (y_1-4)\vec{j} + (z_1+1)\vec{k}$ is perpendicular to the direction of the line,

so that $\overrightarrow{BP_1} \cdot \vec{v} = 0$, so that (4) $6(x_1-3) + 3(y_1-4) + (z_1+1) = 0$ M1

The point $C(-13, -10, -1)$ also lies on the line and the vector

$\overrightarrow{CP_1} = (x_1+13)\vec{i} + (y_1+10)\vec{j} + (z_1+1)\vec{k}$ is parallel to the direction of the line $\vec{v}$ so that

$\overrightarrow{CP_1} = \mu\vec{v}$ and (5) $x_1+13=6\mu$, (6) $y_1+10=3\mu$, (7) $z_1+1=\mu$ A1

rearranging $x_1 = -13+6\mu$, $y_1 = -10+3\mu$, $z_1 = -1+\mu$ substitute into (4)

$$(4) \quad 6(6\mu-16) + 3(3\mu-14) + \mu = 0$$

$$36\mu - 96 + 9\mu - 42 + \mu = 0$$

$$46\mu = 138$$
 M1

solving $\mu = 3$

substituting $\mu = 3$ gives $P_1(5, -1, 2)$ and

$$\overrightarrow{BP_1} = 2\vec{i} - 5\vec{j} + 3\vec{k},$$

$$|\overrightarrow{P_1B}| = \sqrt{4+25+9} = \sqrt{38}$$
 A1

- c. $\underline{r}(t) = (-13 + 6t)\underline{i} + (-10 + 3t)\underline{j} + (-1 + t)\underline{k}$, and the plane $-2x + 5y - 3z = 17$.

Now the direction of the line $\underline{v} = 6\underline{i} + 3\underline{j} + \underline{k}$ and the normal to the plane

$\underline{n} = -2\underline{i} + 5\underline{j} - 3\underline{k}$ are perpendicular since $\underline{v} \cdot \underline{n} = -12 + 15 - 3 = 0$,

therefore the line and the plane do not intersect.

A1

The point $A(5, -1, 2)$ is the same point as $P_1(5, -1, 2)$ and lies on the line and

The point $B(3, 4, -1)$ is the same point as $P_0(3, 4, -1)$ and lies on the plane.

Since from **a.** the closest distance from the plane to the point A was $\sqrt{38}$ and

from **b.** the closest distance from the line to the point B was also $\sqrt{38}$,

it follows that the distance between the line and the plane is also $\sqrt{38}$

A1

- d. the line $\frac{x+13}{6} = \frac{y+10}{b} = z+1 = t$

$x = -13 + 6t$, $y = -10 + bt$, $z = -1 + t$ substitute into the plane

$$-2x + 5y - 3z = d$$

$$-2(-13 + 6t) + 5(-10 + bt) - 3(-1 + t) = d$$

$$26 - 12t - 50 + 5bt + 3 - 3t = d$$

$$(5b - 15)t = d + 21$$

M1

- i. for a unique point of intersection

$$b \neq 3, \quad d \in \mathbb{R}$$

- ii. an infinite number of points of intersection

$$b = 3, \quad d = -21$$

A1

Question 6

a.i. $x = 20 \cos(t), \quad y = 10 \sin(t)$

rotate around the x-axis

$$S_x = 2\pi \int_{t_1}^{t_2} y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$\frac{dx}{dt} = -20 \sin(t), \quad \frac{dy}{dt} = 10 \cos(t)$$

by symmetry

$$S_x = 4\pi \int_0^{\frac{\pi}{2}} 10 \sin(t) \sqrt{400 \sin^2(t) + 100 \cos^2(t)} dt = 2147.84$$

A1

ii. $V_x = \pi \int_{y_1}^{y_2} x^2 dy$

$$\frac{x^2}{400} + \frac{y^2}{100} = 1$$

$$x^2 = 400 \left(1 - \frac{y^2}{100}\right)$$

by symmetry

$$V_x = 2\pi \int_0^{10} 400 \left(1 - \frac{y^2}{100}\right) dy = 16755.16$$

A1

b. watermelons $W \stackrel{d}{=} N(3, 0.4^2)$

cantaloupes $C \stackrel{d}{=} N(1.5, 0.2^2)$

total of three independent watermelons and two independent cantaloupes

$$T = W_1 + W_2 + W_3 + C_1 + C_2 \quad E(T) = 3E(W) + 2E(C) = 3 \times 3 + 2 \times 1.5 = 12$$

$$\text{Var}(T) = 3\text{Var}(W) + 2\text{Var}(C) = 3 \times 0.4^2 + 2 \times 0.2^2 = 0.56, \quad T \stackrel{d}{=} N(12, 0.56)$$

$$\Pr(T > 11) = 0.9093$$

A1

c. $CI: \bar{x} \pm z \frac{\sigma}{\sqrt{n}} = (1.45, 1.55)$

$$\bar{x} = 1.5, \quad \sigma = 0.2, \quad 95\%, \quad z = 1.96$$

$$\text{solving } \frac{1.96 \times 0.2}{\sqrt{n}} = 0.05$$

gives $n = 61.46$ so accept $n = 61$ or 62

A1

Define $x1(t) = 20 \cdot \cos(t)$ Done

Define $y1(t) = 10 \cdot \sin(t)$ Done

$$4 \cdot \pi \cdot \int_0^{\frac{\pi}{2}} \left(y1(t) \cdot \sqrt{\left(\frac{d}{dt}(x1(t))\right)^2 + \left(\frac{d}{dt}(y1(t))\right)^2} \right) dt$$

2147.84353

$$\pi \cdot \int_{-10}^{10} \left(400 \cdot \left(1 - \frac{y^2}{100} \right) \right) dy$$

16755.16082

$$2 \cdot \pi \cdot \int_0^{10} \left(400 \cdot \left(1 - \frac{y^2}{100} \right) \right) dy$$

16755.16082

$$3 \cdot (0.4)^2 + 2 \cdot (0.2)^2$$

0.5600

$$\text{normCdf}(11, \infty, 12, \sqrt{0.56})$$

0.9093

$$\text{solve}\left(\frac{1.96 \cdot 0.2}{\sqrt{n}} = 0.05, n\right)$$

$n = 61.4656$

zInterval 0.2, 1.5, 62, 0.95: stat.results

"Title"	"z Interval"
"CLower"	1.4502
"CUpper"	1.5498
"x"	1.5000
"ME"	0.0498
"n"	62.0000

d. $H_0: \mu = 3$
 $H_1: \mu < 3$ one-sided test

A1

e. $n = 36, \bar{W} \stackrel{d}{=} N\left(3, \frac{0.4^2}{36}\right), \sigma_{\bar{W}} = \frac{0.4}{6}$

$p \text{ value} = \Pr(\bar{W} < 2.9 | \mu = 3)$
 $= 0.0668$

Since the p value $0.0668 > 0.05$
there is no evidence to support the
alternative hypothesis

$$\text{normCdf}\left(-\infty, 2.9, 3, \frac{0.4}{6}\right) = 0.0668$$

zTest 3,0.4,2.9,36,-1: stat.results

"Title"	"z Test"
"Alternate Hyp"	" $\mu < \mu_0$ "
"z"	-1.5000
"PVal"	0.0668
" $\bar{x}$ "	2.9000
"n"	36.0000
" σ "	0.4000

A1

A1

f. $\Pr(W < w^*) = 0.05$

$Z \stackrel{d}{=} N(0,1)$

$\Pr(Z < -0.05) = -1.6449$

$-1.6449 = \frac{w^* - 3}{\frac{0.4}{6}}$

$w^* = 2.89034$

so round up $w^* \geq 2.8904$

$$\text{invNorm}(0.05, 0, 1) = -1.64485$$

$$\text{solve}\left(\frac{w-3}{\frac{0.4}{6}} = -1.64485, w\right) = w = 2.89034$$

$$\text{invNorm}\left(0.05, 3, \frac{0.4}{6}\right) = 2.89034$$

$$\text{normCdf}\left(-\infty, 2.8903, 3, \frac{0.4}{6}\right) = 0.04993$$

$$\text{normCdf}\left(-\infty, 2.8904, 3, \frac{0.4}{6}\right) = 0.05009$$

A1

g. $n = 36, \bar{S} \stackrel{d}{=} N\left(2.8, \frac{0.4^2}{36}\right), \sigma_{\bar{S}} = \frac{0.4}{6}$

$$\text{normCdf}\left(2.89034, \infty, 2.8, \frac{0.4}{6}\right) = 0.08769$$

$\beta = \Pr(\bar{S} > 2.89034 | \mu = 2.8) = 0.08769$

8.8%

A1

End of detailed answers for the
2024 Kilbaha VCE Specialist Mathematics Trial Examination 2

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