

SPECIALIST MATHEMATICS

Written examination 2



2023 Trial Examination

SOLUTIONS

SECTION A

Question 1

B

Explanation:

$$\begin{aligned}\sin x + \cos x &= 0 \\ \tan x &= -1\end{aligned}$$

$$x = \frac{3\pi}{4} + n\pi, n \in \mathbb{Z}$$

Question 2

E

Explanation:

$y = e^{|x|}$ is not differentiable at the point $(0,1)$ due to the function being non-smooth at this point

The gradient of $y = e^{|x|}$ approaches -1 from the left and $+1$ from the right as x approaches 0 .

Question 3**C***Explanation:*

$$\sec \theta = -4$$

$$\cos \theta = -\frac{1}{4}$$

Using Pythagoras' theorem:

$$\sin \theta = \frac{\sqrt{15}}{4} \text{ (second quadrant) or } -\frac{\sqrt{15}}{4} \text{ (third quadrant).}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta = 2 \times \frac{\sqrt{15}}{4} \times \left(-\frac{1}{4}\right) = -\frac{\sqrt{15}}{8} \text{ or } 2 \times \left(-\frac{\sqrt{15}}{4}\right) \times \left(-\frac{1}{4}\right) = \frac{\sqrt{15}}{8}$$

$$\operatorname{cosec} \theta = \frac{8}{\sqrt{15}} \text{ or } -\frac{8}{\sqrt{15}}$$

Question 4**B***Explanation:*For $y = \cos^{-1}(\sqrt{x-2})$ to be defined, $-1 \leq \sqrt{x-2} \leq 1$ Check the graph of $y = \cos^{-1}(\sqrt{x-2})$ on your CAS.This will occur when $x \in [2,3]$ This will result in $y \in [1, \frac{\pi}{2}]$

Question 5**E***Explanation:*

$$g: [8, \infty) \rightarrow \mathbb{R} \text{ where } g(x) = 12 \sin^{-1}\left(\frac{4}{x}\right)$$

$$\text{Let } y = 12 \sin^{-1}\left(\frac{4}{x}\right)$$

$$\text{The inverse is } x = 12 \sin^{-1}\left(\frac{4}{y}\right)$$

$$\frac{4}{y} = \sin\left(\frac{x}{12}\right)$$

$$y = 4 \left(\sin\left(\frac{x}{12}\right) \right)^{-1}$$

Question 6**D***Explanation:*

$$P(2i) = (2i)^3 + (2-i)(2i)^2 + n(1-i)(2i) + 4 = 0$$

$$-8i - 4(2-i) + 2ni(1-i) + 4 = 0$$

$$-8i - 8 + 4i + 2ni + 2n + 4 = 0$$

$$-4i - 4 + 2ni + 2n = 0$$

$$n = -2$$

$$z^3 + (2-i)z^2 - 2(1-i)z + 4 = 0$$

$$(z-2i)(z^2 + iz + 2z + 2i) = 0$$

$$(z-2i)(z+2)(z+i) = 0$$

$$z = 2i, \quad z = -i, \quad z = -2$$

Question 7**D***Explanation:*

Which statement relating to $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ is untrue?

A. The magnitude of $\mathbf{a}$ is 3 units.

$$|\mathbf{a}| = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$$

B. The dot product of the two vectors is -9 .

$$\mathbf{a} \cdot \mathbf{b} = (2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \cdot (-\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = -2 - 6 - 1 = -9$$

C. The angle between the two vectors is closest to 155° .

$$\mathbf{a} \cdot \mathbf{b} = -9 = 3 \times \sqrt{11} \times \cos \theta, \theta \approx 155^\circ$$

D. The cross product of the two vectors is $-\mathbf{i} - \mathbf{j} + 4\mathbf{k}$.

$$\mathbf{a} \times \mathbf{b} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k} \text{ FALSE}$$

E. The two vectors are neither parallel nor perpendicular.

$$\mathbf{a} \neq k \times \mathbf{b}, \quad \mathbf{a} \cdot \mathbf{b} = -9 \neq 0$$

Question 8**E***Explanation:*

$$\mathbf{m} \cdot \mathbf{n} = u - 2 + 6 = 3 \times \sqrt{u^2 + 1 + 9} \times \cos 60^\circ$$

$$u + 4 = 3 \times \sqrt{u^2 + 10} \times \frac{1}{2}$$

$$2u + 8 = 3\sqrt{u^2 + 10}$$

Question 9**B***Explanation:* $P = (1, 1, 0), Q = (2, 1, -1), R = (1, 2, -1)$ is:

$$\overrightarrow{PQ} = \mathbf{i} + 0\mathbf{j} - \mathbf{k}, \quad \overrightarrow{PR} = 0\mathbf{i} + \mathbf{j} - \mathbf{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \mathbf{i} + \mathbf{j} + \mathbf{k}$$

Select any point in plane, say P

$$1(x - 1) + 1(y - 1) + 1(z - 0) = 0$$

$$x + y + z = 2$$

Question 10**B***Explanation:*

$$\frac{dF}{dt} = 1.04F - 10000$$

$$t = \frac{1}{1.04} \int \frac{1.04}{1.04F - 10000} dt$$

$$t = \frac{1}{1.04} \log_e(1.04F - 10000) + c$$

$$0 = \frac{1}{1.04} \log_e(1.04 \times 200000 - 10000) + c$$

$$c = -\frac{1}{1.04} \log_e(198000)$$

$$t = \frac{1}{1.04} \log_e \left(\frac{1.04F - 10000}{198000} \right)$$

$$F = \frac{1}{1.04} (198000e^{1.04t} + 10000)$$

Question 11

C

Explanation:

Proof by contradiction.

1. Assume the original statement is true. (In this case.)
2. That assumption leads to a mathematical inconsistency.
3. The original assumption must be false.

Question 12

E

Explanation:

Statement **B** is not the converse of statement **A**. The converse of statement **A** is:

Given θ is the internal angle at Q in the right-angled triangle PQR , then $\sin \theta = \frac{3}{5}$.

Question 13

D

Explanation:

$$f: [0,2] \rightarrow R \text{ where } f(x) = x^3$$

$$S = \int_0^2 2\pi y \sqrt{1 + (f'(x))^2}$$

$$= \int_0^2 2\pi(2x^3) \sqrt{1 + ((6x^2))^2}$$

Use CAS

$$\approx 806 U^2$$

Question 14**B***Explanation:*

$$\int_0^{\frac{\pi}{3}} \frac{e^{\tan x}}{\cos^2 x} dx$$

Let $u = \tan x$

$$\frac{du}{dx} = \sec^2 x$$

$$= \int_0^{\sqrt{3}} (e^{\tan x} \sec^2 x) dx$$

$$= \int_0^{\sqrt{3}} \left(e^u \frac{du}{dx} \right) dx$$

$$= \int_0^{\sqrt{3}} (e^u) du$$

Question 15**D***Explanation:*

$$a = 2 - x$$

$$\frac{d}{dx} \left(\frac{1}{2} v^2 \right) = 2 - x$$

$$\frac{1}{2} v^2 = \int (2 - x) dx$$

$$v^2 = -x^2 + 4x + c$$

When $x = 0, v = 4$

$$16 = -0^2 + 4(0) + c$$

$$c = 16$$

$$v^2 = -x^2 + 4x + 16$$

$$v^2 = -4^2 + 4(4) + 16$$

$$v^2 = 16$$

$$v = 4 \text{ ms}^{-1}$$

(Reject $v = -4$ since particle's initial velocity is positive and it has turned around.)

Question 16

B

Explanation:

$$E(A) = 10, \text{Var}(A) = 3, E(B) = 12, \text{Var}(B) = 4, C = 2A + B$$

$$E(C) = E(2A + B) = 20 + 12 = 32$$

$$\text{Var}(C) = \text{Var}(2A + B) = 4 \times \text{Var}(A) + \text{Var}(B)$$

$$= 4 \times 3 + 4 = 16$$

$$\Pr(C < 30) = \Pr\left(z < \frac{30 - 32}{\sqrt{16}}\right)$$

$$= \Pr(z < -0.5) \approx 0.309$$

Question 17**C***Explanation:*

$$\Pr\left(\frac{-0.8}{\frac{\sigma}{\sqrt{30}}} < z < \frac{0.8}{\frac{\sigma}{\sqrt{30}}}\right) = 0.99$$

$$\Pr\left(z < \frac{0.8}{\frac{\sigma}{\sqrt{30}}}\right) = 0.995$$

$$\frac{0.8}{\frac{\sigma}{\sqrt{30}}} \approx 2.57584$$

$$\sigma \approx 1.701$$

Question 18**A***Explanation:*

$$\mathbf{v}(t) = 6\sqrt{t}\mathbf{i} - \left(\frac{1}{t+1}\right)\mathbf{j}$$

$$\mathbf{a}(t) = \frac{3}{\sqrt{t}}\mathbf{i} + \left(\frac{1}{(t+1)^2}\right)\mathbf{j}$$

$$\mathbf{x}(t) = 4t^{\frac{3}{2}}\mathbf{i} - \log_e(t+1)\mathbf{j} + \mathbf{c}$$

$$\mathbf{x}(t) = 4t^{\frac{3}{2}}\mathbf{i} - \log_e(t+1)\mathbf{j} + 2\mathbf{j}$$

$$|\mathbf{a}(t)| = \sqrt{\left(\frac{3}{\sqrt{t}}\right)^2 + \left(\frac{1}{(t+1)^2}\right)^2} = 2$$

Solve on CAS. $t \approx 2.255 \text{ s}$

$$\mathbf{x}(2.255) = 4(2.255)^{\frac{3}{2}}\mathbf{i} - \log_e(2.255+1)\mathbf{j} + 2\mathbf{j} \approx 13.55\mathbf{i} + 0.82\mathbf{j}$$

Question 19

D

Explanation:

$$h: \left[\frac{\pi}{2}, a\right] \rightarrow \mathbb{R} \text{ where } h(x) = \cos^{-1} x$$

$$V(y) = \pi \int_{\frac{\pi}{2}}^a (\cos y)^2 dy = 2$$

Solve on CAS.

$$a \approx 3$$

Question 20

A

Explanation:

$$\mathbf{a}(t) = -9.8\mathbf{j}$$

$$\mathbf{v}(t) = 30 \cos 50^\circ \mathbf{i} + (30 \sin 50^\circ - 9.8t)\mathbf{j}$$

$$\mathbf{x}(t) = (30 \cos 50^\circ t)\mathbf{i} + (30 \sin 50^\circ t - 4.9t^2 + 3.5)\mathbf{j}$$

$$30 \sin 50^\circ t - 4.9t^2 + 3.5 = 0$$

Solve on CAS.

$$t \approx 4.8377 \text{ s}$$

$$\mathbf{x}(4.8377) = (30 \cos 50^\circ \times 4.8377)\mathbf{i} + 0\mathbf{j}$$

$$\approx 93.3\mathbf{i} + 0\mathbf{j}$$

SECTION B

Question 1 (10 marks)

a. (1 mark)

Answer:

$$z_2 = 2 - i$$

1A

b. (2 marks)

Answer:

$$(z - 2 - i)(z - 2 + i)$$

$$= z^2 - 2z + iz - 2z - 2i + 4 - iz + 2i + 1$$

$$= z^2 - 4z + 5$$

1W

$$a = 1, b = -4, c = 5$$

1A

c. (2 marks)

Answer:

$$Q(z) = z^2 - 4i - 3 = 0$$

$$Q(z) = (z - 2 - i)(z - m - ni) = 0$$

$$(z - 2 - i)(z - m - ni) = z^2 - 4i - 3$$

1W

Equating coefficients:

1A

$$= z^2 - mz - niz - 2z + 2m + 2ni - iz + mi - n = z^2 - 4i - 3$$

$$m = -2, n = -1$$

1A

d. (2 marks)**i.**

Answer:

$$z^4 = p + qi, \quad p, q \in \mathbb{R}$$

$$(2 + i)^4 = p + qi = -7 + 24i$$

$$p = -7, q = 24$$

1A

The four roots lie symmetrically in a circle.

$$z_1 = 2 + i$$

$$z_4 = (2 + i)i = -1 + 2i$$

$$z_5 = (-1 + 2i)i = -2 - i$$

$$z_6 = (-2 - i)i = 1 - 2i$$

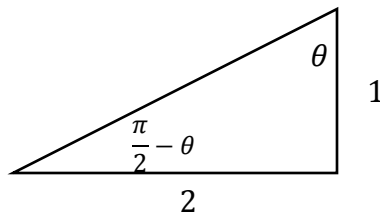
1A**ii.** (3 marks)

Answer:

Consider the first root $x_1 = 2 + i$

$$\theta = \tan^{-1} 2$$

$$\tan \theta = 2$$

Relevant angle in first quadrant is $\frac{\pi}{2} - \theta$ **1W**

$$x_1 = 2 + i = \sqrt{5} \operatorname{cis} \left(\frac{\pi}{2} - \theta \right)$$

1W

$$x_4 = (2 + i)i = -1 + 2i = \sqrt{5} \operatorname{cis}(\pi - \theta)$$

$$x_5 = (-1 + 2i)i = -2 - i = \sqrt{5} \operatorname{cis} \left(\frac{3\pi}{2} - \theta \right)$$

$$x_6 = (-2 - i)i = 1 - 2i = \sqrt{5} \operatorname{cis}(2\pi - \theta) = \sqrt{5} \operatorname{cis}(-\theta)$$

1A

Question 2 (9 marks)**a.** (3 marks)*Answer:*

$$\mathbf{a}(t) = 2 \cos t \mathbf{i} + \sin t \mathbf{j}$$

$$\mathbf{v}(t) = 2 \sin t \mathbf{i} - \cos t \mathbf{j} + \mathbf{c}$$

$$\mathbf{v}(0) = 2 \sin 0 \mathbf{i} - \cos 0 \mathbf{j} + \mathbf{c} = -1\mathbf{j}$$

$$\mathbf{c} = 0\mathbf{j}$$

1W

$$\mathbf{v}(t) = 2 \sin t \mathbf{i} - \cos t \mathbf{j}$$

$$\mathbf{r}(t) = -2 \cos t \mathbf{i} - \sin t \mathbf{j} + \mathbf{c}$$

$$\mathbf{r}(0) = -2 \cos 0 \mathbf{i} - 0 \mathbf{j} + \mathbf{c} = 2\mathbf{i}$$

$$\mathbf{c} = 4\mathbf{i}$$

$$\mathbf{r}(t) = (4 - 2 \cos t) \mathbf{i} - \sin t \mathbf{j}$$

1W

$$x = 4 - 2 \cos t, y = -\sin t$$

$$\cos^2 t + \sin^2 t = 1$$

$$\left(\frac{4-x}{2}\right)^2 + (-y)^2 = 1$$

$$\frac{(x-4)^2}{4} + y^2 = 1$$

1A**b.** (3 marks)*Answer:*

$$\mathbf{v}(t) = 2 \sin t \mathbf{i} - \cos t \mathbf{j}$$

$$|\mathbf{v}(t)| = \sqrt{(2 \sin t)^2 + (\cos t)^2}$$

$$= \sqrt{1 + 3 \sin^2 t}$$

1W

$$|v(t)|_{\max} = \sqrt{1+3} = 2 \text{ ms}^{-1}, t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \text{ so, } t = \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$$

$$|v(t)|_{\min} = \sqrt{1+0} = 1 \text{ ms}^{-1}, t = 0, \pi, 2\pi, \dots \text{ so, } t = n\pi, n \in \mathbb{Z}$$

1W, 1A**c. (3 marks)***Answer:*

$$\mathbf{a}(t) \cdot \mathbf{v}(t) = (2 \cos t \mathbf{i} + \sin t \mathbf{j}) \cdot (2 \sin t \mathbf{i} - \cos t \mathbf{j}) = 0$$

$$4 \cos t \sin t - \cos t \sin t = 0$$

1W

$$3 \cos t \sin t = 0$$

$$\cos t = 0 \text{ or } \sin t = 0$$

1W

$$t = 0, \frac{\pi}{2}, \pi, \dots t = \frac{n\pi}{2}, n \in \mathbb{Z}$$

1A**Question 3 (13 marks)****a. (1 mark)***Answer:*

$$450 \pm 1.96 \times 8$$

$$434 \text{ mm to } 466 \text{ mm}$$

1A**b. (3 marks)***Answer:*

$$\bar{x} = 450, \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{30}}$$

1W

$$450 \pm 2.576 \times \frac{8}{\sqrt{30}}$$

1W

$$446.2 \text{ mm to } 453.8 \text{ mm}$$

1A

c. (2 marks)

Answer:

$$\Pr(\bar{y} < 446) = \Pr\left(z < \frac{446 - 450}{\frac{8}{\sqrt{25}}}\right) \approx 0.006$$

1W, 1A

d. (3 marks)

Answer:

i.

$$H_0 \quad \mu = 450mm$$

$$H_1 \quad \mu < 450mm$$

1A

ii.

$$z = \frac{447 - 450}{\frac{8}{\sqrt{20}}} \approx -1.6771$$

$$\Pr(z < -1.6771) \approx 0.047$$

Since $0.047 < 0.05$ Reject H_0

1W, 1A

e. (3 marks)

Answer:

$$E(1.2X) = 1.2 \times 450 = 540mm$$

$$Var(1.2X) = 1.2^2 \times 64 = 92.16mm$$

$$SD(1.2X) = 9.6mm$$

1A

$$z = \frac{550 - 540}{9.6} \approx 1.0417$$

$$\Pr(z > 1.047) \approx 0.14878$$

Use CAS

Binomial PD with $n = 10, x = 2, p = 0.14878$

The chance that exactly 2 pinbolts are longer than 550mm is 0.2746

1W, 1A

Question 4 (12 marks)**a.** (2 marks)*Answer:*

$$a = 9.8, u = 0, t = 2.5$$

1W

$$s = \frac{1}{2}at^2 + ut$$

$$s = \frac{1}{2}(-9.8)(2.5)^2 + 0$$

$$s = -30.625 \text{ m}$$

The particle falls 30.625 m

1A**b.** (5 marks)*Answer:*

$$a = kt - 9.8$$

$$v = \int (kt - 9.8) dt$$

1W

$$v = \frac{1}{2}kt^2 - 9.8t + 0$$

$$v = \frac{1}{2}kt^2 - 9.8t$$

1W

$$x = \int \left(\frac{1}{2}kt^2 - 9.8t \right) dt$$

$$x = \frac{1}{6}kt^3 - 4.9t^2 + 0$$

$$x = \frac{1}{6}kt^3 - 4.9t^2$$

1W

$$-30.625 = \frac{1}{6}k(2.6)^3 - 4.9(2.6)^2$$

Solve on CAS.

$$k \approx 0.853$$

1A

$$v(2.6) = \frac{1}{2} \times 0.853 \times (2.6)^2 - 9.8 \times 2.6 \approx -22.60 \text{ ms}^{-1}$$

1A

c. (5 marks)

Answer:

$$a = -0.5v - 9.8$$

$$t = \int \left(\frac{1}{-0.5v - 9.8} \right) dt$$

1W

Solve on CAS.

$$t = -2(\ln|v + 19.6| + 1.6094) + c$$

$$v = 0, t = 0$$

$$0 = -2(\ln|0 + 19.6| + 1.6094) + c$$

$$c \approx 9.16986$$

$$t = -2(\ln|v + 19.6| + 1.6094) + 9.16986$$

1W

$$v = e^{\left(\left(\frac{t-9.16986}{-2}\right) - 1.6094\right)} - 19.6$$

$$v = e^{((-0.5t+2.9755))} - 19.6$$

$$x = \int (e^{((-0.5t+2.9755))} - 19.6) dt$$

$$x = -2e^{((-0.5t+2.9755))} - 19.6t + c$$

$$x = 0, t = 0$$

$$0 = -2e^{(2.9755)} + c$$

$$c = 39.2$$

$$x = -2e^{((-0.5t+2.9755))} - 19.6t + 39.2$$

1W

$$-30.625 = -2e^{((-0.5t+2.9755))} - 19.6t + 39.2$$

Solve on CAS. $t \approx 3.1481 \text{ s}$

1W

$$v(3.1481) = e^{((-0.5 \times 3.1481 + 2.9755))} - 19.6 \approx 15.54 \text{ ms}^{-1}$$

1A

Question 5 (16 marks)**a.** (1 mark)*Answer:*

$$f: A \rightarrow R \text{ where } f(x) = \frac{x+1}{\sqrt{4-x^2}}$$

$$4 - x^2 > 0$$

$$x \in (-2, 2)$$

1A

b. (2 marks)*Answer:* x – intercept

$$\frac{x+1}{\sqrt{4-x^2}} = 0$$

$$x = -1$$

$$(-1, 0)$$

1A

 y – intercept

$$f(0) = \frac{0+1}{\sqrt{4-0^2}} = \frac{1}{2} \quad \left(0, \frac{1}{2}\right)$$

1A

c. (2 marks)

Answer:

$$f(x) = \frac{x+1}{\sqrt{4-x^2}}$$

$$f'(x) = \frac{x+4}{\sqrt{(4-x^2)^3}}$$

1W

For $x \in (-2, 2)$, $x+4 > 0$ and $\sqrt{(4-x^2)^3} > 0$

$$\text{So, } \frac{x+4}{\sqrt{(4-x^2)^3}} > 0$$

1A

d. (2 marks)

Answer:

This will occur at the inflection point.

Use CAS.

$$f(x) = \frac{x+1}{\sqrt{4-x^2}}$$

$$f'(x) = \frac{x+4}{\sqrt{(4-x^2)^3}}$$

$$f''(x) = \frac{-2(x^2+6x+2)}{(x^2-4)\sqrt{(4-x^2)^3}} = 0$$

$$x^2 + 6x + 2 = 0$$

1W

$$(x+3-\sqrt{7})(x+3+\sqrt{7}) = 0$$

Reject $-3 - \sqrt{7}$

$$x = -3 + \sqrt{7} \approx -0.3542$$

$$ff(-0.3542) \approx 0.33$$

1A

e. (3 marks)*Answer:*

$$\int_{-1}^0 \left(\frac{x+1}{\sqrt{4-x^2}} \right) dx$$

$$= \int_{-1}^0 \left(\frac{x}{\sqrt{4-x^2}} \right) dx + \int_{-1}^0 \left(\frac{1}{\sqrt{4-x^2}} \right) dx$$

$$\text{Let } u = 4 - x^2 \quad \frac{du}{dx} = -2x = \int_3^4 \left(\frac{-\frac{1}{2} \times \frac{du}{dx}}{\sqrt{u}} \right) dx + \int_{-1}^0 \left(\frac{1}{\sqrt{4-x^2}} \right) dx$$

$$= -\frac{1}{2} \int_3^4 \left(u^{-\frac{1}{2}} \right) du + \int_{-1}^0 \left(\frac{1}{\sqrt{4-x^2}} \right) dx$$

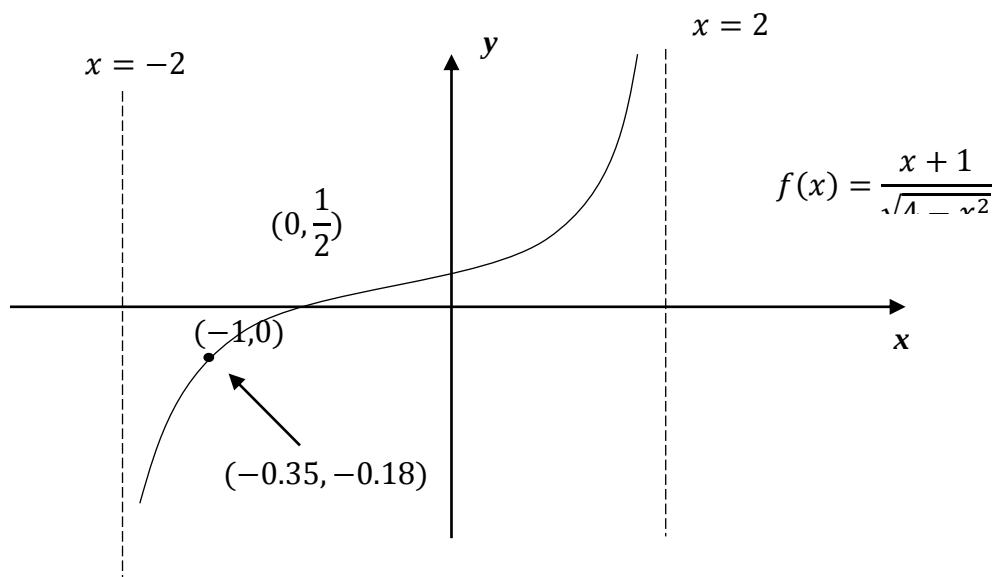
1W

$$= \left[-u^{\frac{1}{2}} \right]_3^4 + \left[\sin^{-1} \left(\frac{x}{2} \right) \right]_{-1}^0$$

1W

$$= -2 + \sqrt{3} + \sin^{-1}(0) - \sin^{-1} \left(-\frac{1}{2} \right)$$

$$= -2 + \sqrt{3} + \frac{\pi}{6}$$

1A**f.** (3 marks)*Answer:***Shape 1A, Intercepts 1A, Asymptotes 1A,**

g. (1 mark)*Answer:*

$$\frac{5}{4-x^2} = \frac{5}{(2-x)(2+x)} = \frac{5}{4(2-x)} + \frac{5}{4(2+x)}$$

1A**h.** (2 marks)*Answer:*

$$V_x = \pi \int_{-1}^0 \left(\frac{x+1}{\sqrt{4-x^2}} \right)^2 dx$$

$$= \pi \int_{-1}^0 \frac{(x+1)^2}{4-x^2} dx$$

$$= \pi \int_{-1}^0 \frac{x^2 + 2x + 1}{4-x^2} dx$$

$$= \pi \int_{-1}^0 \left(\frac{2x+5}{4-x^2} - 1 \right) dx$$

$$= \pi \int_{-1}^0 \left(\frac{2x}{4-x^2} + \frac{5}{4(2-x)} + \frac{5}{4(2+x)} - 1 \right) dx$$

1W

$$= \pi \left[-\log_e(4-x^2) + \frac{5}{4} \log_e \left(\frac{2+x}{2-x} \right) - x \right]_{-1}^0$$

$$= \pi \left(-\log_e(4) + \frac{5}{4} \log_e \left(\frac{2}{2} \right) - 0 - (-\log_e(3) + \frac{5}{4} \log_e \left(\frac{1}{3} \right) - (-1)) \right)$$

$$= \pi \left(\log_e \left(\frac{3}{4} \right) + \frac{5}{4} \log_e(3) + 1 \right)$$

1A

1 + 2 + 2 + 2 + 3 + 3 + 1 + 2 = 16 marks