



2023 Trial Examination

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT
NUMBER

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Letter

SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 11 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer **all** questions in pencil on the answer sheet provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 (6 marks)

Consider the vectors $\underline{a} = -\underline{i} + c\underline{j} + 3\underline{k}$ and $\underline{b} = 3\underline{i} + 6\underline{j} + d\underline{k}$ where $c, d \in \mathbb{R}$

- a.** Find the value(s) of c given the magnitude of $\underline{a}$ is $\sqrt{14}$.

1 mark

- b.** If instead, the two vectors are parallel, find the values of c and d .

2 marks

- c. Find the cross product of $\underline{a} = -\underline{i} + c\underline{j} + 3\underline{k}$ and $\underline{b} = 3\underline{i} + 6\underline{j} + d\underline{k}$ hence find the values of c, d and f , (different to either **part a.** or **part b.**), so that $\underline{a}$ and $\underline{b}$ are both perpendicular to $\underline{e} = -2\underline{i} + \underline{j} + f\underline{k}$

3 marks

1 + 2 + 3 = 6 marks

TURN OVER

Question 2 (3 marks)

Given $f(x) = x \tan^{-1}\left(\frac{x}{2}\right)$ find $f'(-2)$, writing your answer in the form $m\pi - n$ where m, n are simplified fractions.

3 marks

Question 3 (3 marks)

Prove, by mathematical induction, that $x^2 + x$ is an even integer for $x \in \mathbb{N}$.

3 marks

Question 4 (6 marks)

Given $\sec^2(2x) = 2$, where x is in the first quadrant, evaluate:

a. x

b. $\operatorname{cosec}(4x)$

c. $\cot(4x)$

2 + 2 + 2 = 6 marks

TURN OVER

Question 5 (6 marks)

Evaluate the following integrals, leaving your answer in exact, simplified form.

a. $\int_0^1 (x \tan^{-1} x) \, dx$

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3 marks

b. $\int_0^{\frac{1}{2}} \left(\frac{2x+1}{\sqrt{1-x^2}} \right) dx$

[illegible]

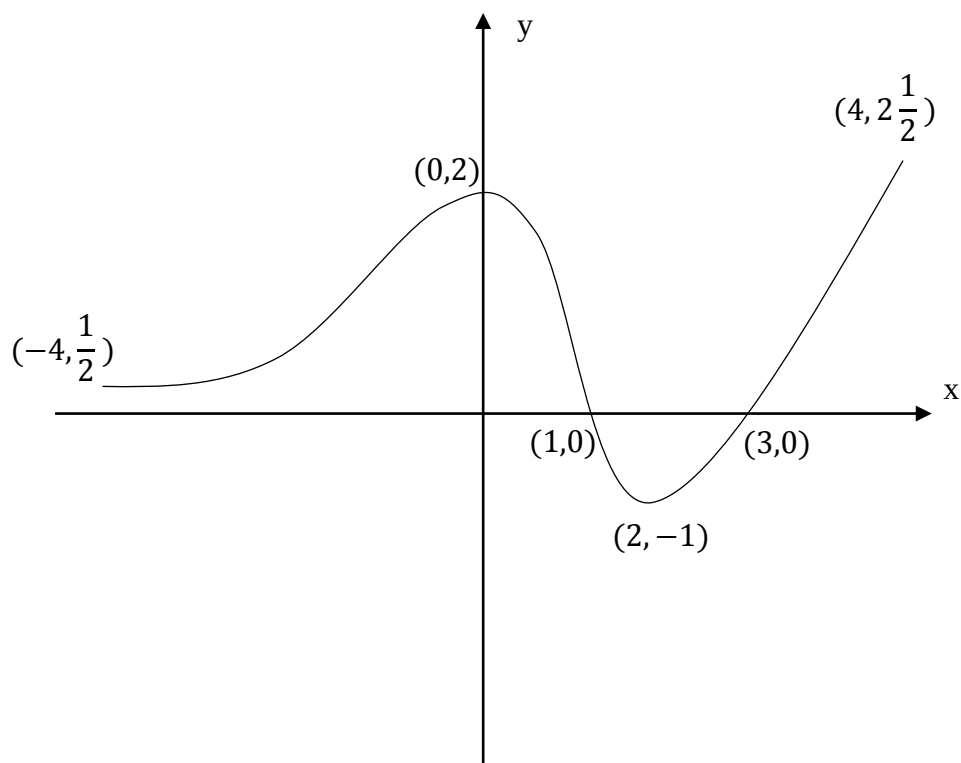
3 marks

3 + 3 = 6 marks

TURN OVER

Question 6 (3 marks)

Sketch on the same axes the graph of $y = \frac{1}{f(x)}$ given the graph of $y = f(x)$ shown below. Label key features of your sketch



3 marks

Question 7 (6 marks)

- a.** Write $z = -2\sqrt{2} + 2\sqrt{2}i$ in polar form where $z \in \mathbb{C}$.

2 marks

- b.** Hence find two solutions to $z^2 = -2\sqrt{2} + 2\sqrt{2}i$ in polar form.
(Call these solutions z_1 and z_2).

2 marks

- c.** Use the identity $\cos 2\theta = 2\cos^2\theta - 1$ to rewrite the solution to z_1 found in **part b.** into the form $z_1 = \sqrt{a + \sqrt{b}} + \sqrt{a - \sqrt{b}}i$, where $a, b, c \in \mathbb{R}$

TURN OVER

2 marks

2 + 2 + 2 = 6 marks

Question 8 (4 marks)

Find the gradients of the tangents to $2x^3 - xy^2 = y$ when $x = -1$.

4 marks

Question 9 (3 marks)

The following may be useful for question 9.

Let Z be an observation from a standard normal distribution.

$\Pr(Z \leq 1.96) \approx 0.975$, $\Pr(Z \leq 2.58) \approx 0.995$, $\Pr(Z \leq 3.00) \approx 0.999$

Suppose that the volume of lemonade delivered to a cup by a vending machine (X) is normally distributed with a mean of 200 mL and a standard deviation of 2.5 mL . Find the percentage chance that a sample of 25 drinks will produce a mean volume of greater than 201.5 mL .

3 marks

END OF QUESTION AND ANSWER BOOK