SPECIALIST MATHEMATICS

Unit 1 Written examination 2

2023 Trial Examination

SOLUTIONS

Section A – Multiple Choice Questions

Question 1
Answer: **B**

Explanation:
2 is an even prime number.

Question 2
Answer: **D**

Explanation:
$A \Leftrightarrow B$

Question 3
Answer: **B**

Explanation:
Since there are 366 different birth dates available, then any number greater than 366 must necessarily mean that at least one calander date is shared by 2 people. ($n + 1$ items to be allocated to n slots).

Question 4
Answer: **A**

Explanation:
$X = \{1, 2, 3, 4, 5\}, Y = \{2, 5, 6\}, Z = \{1, 7\}$
Sets Y and Z have no common elements.
$Y \cap Z = \emptyset$

Question 5
Answer: **D**

Explanation:
A regular graph has $\frac{nr}{2}$ edges.
$\frac{nr}{2} = \frac{12 \times 5}{2} = 30$

Question 6
Answer: **B**

Explanation:
A complete graph has $\frac{n(n-1)}{2}$ edges.
$\frac{n(n-1)}{2} = 45$
$n(n-1) = 90$
$n = 10$

Question 7
Answer: **C**

Explanation:

$1_2 = 1 = 2^1 - 1, 11_2 = 3 = 2^2 - 1,\ 111_2 = 7 = 2^3 - 1, 1111_2 = 15 = 2^4 - 1,$

$111 \ldots 11_2 = 2^n - 1$ where there are n 1's.

Question 8
Answer: **E**

Explanation:
De Morgan's law states that: $\neg(M \wedge N) = \neg M \vee \neg N$

Question 9
Answer: **B**

Explanation:
$200 = 8 \times 25 = 2^3 \times 5^2$

Question 10
Answer: **D**

Explanation:
$20 + 10 + 5 + 2.5 + \cdots =$
Geometric series.
$\mathrm{a} = 20, \mathrm{r} = 0.5$
$S_\infty = \frac{a}{1-r} = \frac{20}{0.5} = 40$

Question 11
Answer: **D**

Explanation:
$P_{n+1} = 1.04P_n - 5000, P_0 = 200000$

Question 12
Answer: **A**

Explanation:
$t_{n+1} = 3t_n + 1, t_1 = 4$

$n = 1,\ t_2 = 3t_1 + 1 = 12 + 1 = 13$

$n = 2,\ t_3 = 3t_2 + 1 = 39 + 1 = 40$

$n = 3,\ t_4 = 3t_3 + 1 = 120 + 1 = 121$

Question 13

Answer: **B**

Explanation:
$7 + 11 + 15 + 19 + \cdots t_n > 10^5$
Arithmetic series.
$\mathrm{a} = 7, \mathrm{d} = 4$
$\frac{n}{2}(2a + (n-1)d) > 10^5$
$\frac{n}{2}(14 + 4n - 4) > 10^5$
Use CAS.

Question 14
Answer: **A**

Explanation:
If $a < 0, b \geq 0$
$\frac{a}{a-\sqrt{b}} = \frac{negative\ number}{negative\ number - positive\ number} = \frac{negative\ number}{negative\ number} = positive\ number$

Question 15
Answer: **D**

Explanation:
In a connected graph, the number of edges is half the sum of all the vertex degrees.

Question 16
Answer: **E**

Explanation:
$\binom{5}{3} \times \binom{4}{3} + \binom{5}{4} \times \binom{4}{2} + \binom{5}{5} \times \binom{4}{1}$
$= 10 \times 4 + 5 \times 6 + 1 \times 4 = 74$

Question 17
Answer: **B**

Explanation:
$\frac{10!}{2!2!} = 907200$

Question 18

Answer: **E**

Explanation:

Since matrix A is a 2×3, it can be subtracted from a 2×3 matrix.

Question 19

Answer: **C**

Explanation:

Use CAS.

$$B^2 = \begin{bmatrix} 3 & 6 & -3 \\ -1 & -2 & -1 \\ -1 & 4 & -1 \end{bmatrix} = -\begin{bmatrix} -3 & -6 & 3 \\ 1 & 2 & 1 \\ 1 & -4 & 1 \end{bmatrix}$$

Question 20

Answer: **D**

Explanation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \times \begin{bmatrix} e \\ f \end{bmatrix}$$

Section B – Extended Response Questions

Question 1 (9 marks)

a. $3, 9, 27, 81$

2 marks

b. $y \in Q$
That is, y is a rational number.

1 mark

c. $x = 1, 3^1 = 3$ which is odd.
Let 3^k be an odd number, where $k \in N$
So, $3^k = 2n + 1$, where $n \in N$
Now, $3^{k+1} = 3 \times 3^k$, where $k \in N$
$= 3 \times (2n + 1)$
$= 6n + 3$
$= 2 \times (3n + 1) + 1$
This one greater than the even number $2 \times (3n + 1)$ so it must be odd.
So, 3^{k+1} where $k \in N$ is an odd number.
So all numbers of the form $3^x, x \in N$ are odd.

4 marks

d. Now, $9^x = 3^{2x} = 3^x \times 3^x$
Since 3^x is an odd number (proved in **part c.**)
$9^x = (2n + 1)^2 = 4n^2 + 4n + 1, n \in N$
$4n^2 + 4n + 1 = 2(2n^2 + 2n) + 1$
This one greater than the even number $2(2n^2 + 2n)$ so it must be odd.
So, 9^x where $x \in N$ is an odd number.

2 marks

Question 2 (8 marks)

a. $8! = 40320$

1 mark

b. $2! \times 2! \times 2! \times 2! \times 4! = 192$

2 marks

c. $(8 - 1)! = 5040$

1 mark

d. $2! \times 2! \times 2! \times 2! \times (4-1)! = 48$

2 marks

e. $2! \times 7! - 2! \times 2! \times 6! = 10080 - 2880 = 7200$

2 marks

Question 3 (9 marks)

a. $A_{n+1} = 1.05A_n + 3000, A_0 = 50000, n \geq 0$
$a = 1.05, b = 3000, c = 50000$

3 marks

b.

$A_{n+1} = aA_n + b, A_0 = c, n \geq 0$

$A_{n+1} = 1.05^n \times 50000 + \dfrac{3000(1.05^n - 1)}{0.05} = 1.05^n \times 50000 + 60000(1.05^n - 1)$

$A_{20} = 1.05^{20} \times 50000 + 60000(1.05^{20} - 1) \approx \$231863$

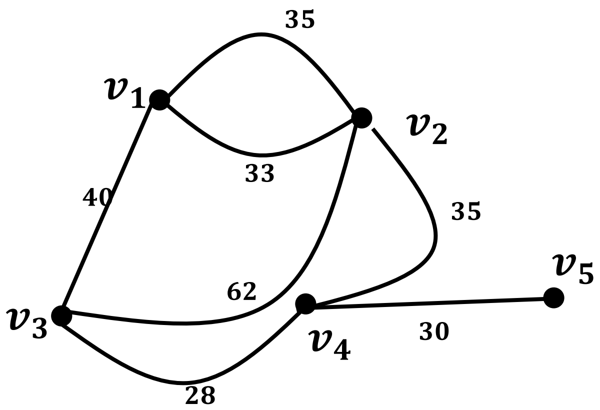
3 marks

c. $A_0 = \$50000$
$n = 0, A_1 = 1.05A_0 + 3000 = 1.05 \times 50000 + 3000 = \$55500$
$n = 1, A_2 = 1.05A_1 + 3000 = 1.05 \times 55500 + 3000 = \$61275$
$n = 2, A_3 = 1.05A_2 + 3000 = 1.05 \times 61275 + 3000 = \$67338.75$

$A_{x+1} = 1.03A_x , A_3 = 67338.75, x \geq 3$

3 marks

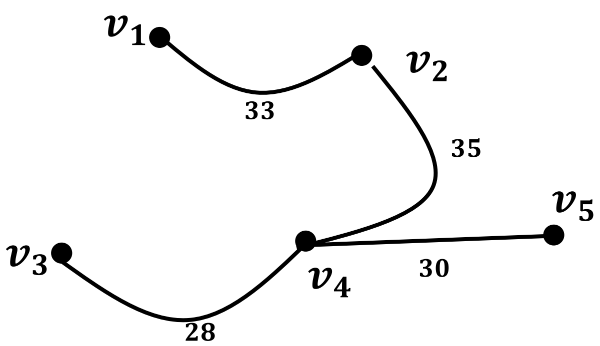
Question 4 (10 marks)



a. $v + f - e = 2$
$5 + 4 - 7 = 2$

1 mark

b. $MST = 33 + 35 + 28 + 30 = 126$



It will take at least 126 minutes to clear 4 roads so that the 5 towns are connected.

2 marks

c.

$$A = \begin{bmatrix} 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

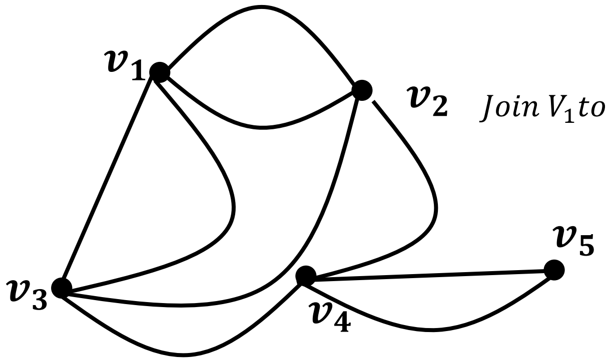
2 marks

d. $$A^2 = \begin{bmatrix} 5 & 1 & 2 & 3 & 0 \\ 1 & 6 & 2 & 1 & 1 \\ 2 & 3 & 2 & 1 & 1 \\ 2 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

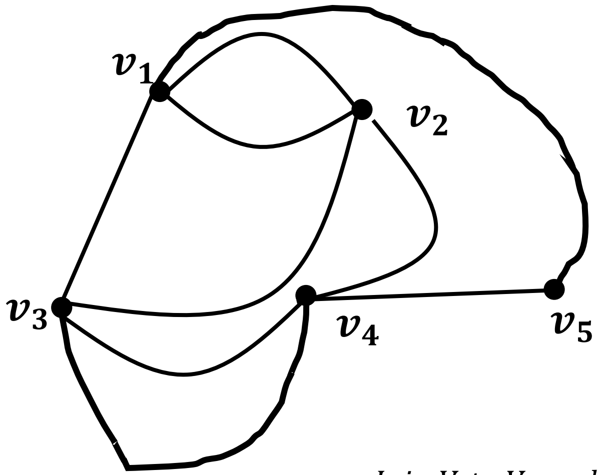
There are 5 ways to go from A to A via 2 edges.
There is 1 way to go from A to B via 2 edges.
And so forth.

2 marks

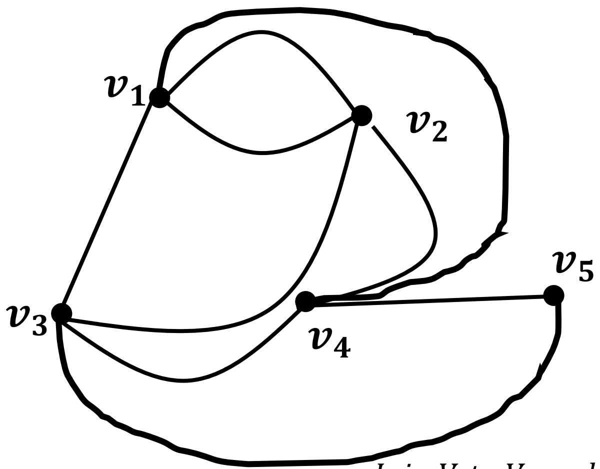
e. Three possibilities.



Join V_1 to V_3 and Join V_4 to V_5



Join V_1 to V_5 and Join V_3 to V_4



Join V_1 to V_4 and Join V_3 to V_5

3 marks

Question 5 (7 marks)

a.

$|M| + |S| - |M \cap S| = |M \cup S|$

$14 + 10 - |M \cap S| = 20 - 2$

$|M \cap S| = 24 - 18 = 6$

2 marks

b.

$|M| + |S| + |B| - |M \cap S| - |M \cap B| - |S \cap B| - |M \cap S| + |M \cap S \cap B| = |M \cup S \cup B|$

$14 + 10 + 9 - 6 - 5 - |S \cap B| + 2 = 20 - 1$

$|S \cap B| = 24 - 19 = 5$

2 marks

c. $\binom{20}{5} = 15504$

1 mark

d. $\binom{10}{0} \times \binom{4}{3} \times \binom{6}{1} + \binom{10}{0} \times \binom{4}{4} \times \binom{6}{0} = 24 + 1 = 25$

2 marks

Question 6 (8 marks)

a.

$C = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, $C^{-1} = \frac{1}{5}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

2 marks

b.

$AC - BC = D$

$BC = AC - D$

$BCC^{-1} = (AC - D)C^{-1}$

$B = (AC - D)C^{-1}$

$B = ACC^{-1} - DC^{-1}$

$B = A - DC^{-1}$

2 marks

c.
All four matrices have 2 columns.

1 mark

d.

$B = A - DC^{-1}$

$$B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 0 & 3 \end{bmatrix} - 3 \times \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 0 & 3 \end{bmatrix} \times \frac{1}{5}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 0 & 3 \end{bmatrix} - \frac{3}{5}\begin{bmatrix} 2 & 1 \\ -9 & 8 \\ -3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 0 & 3 \end{bmatrix} - \frac{3}{5}\begin{bmatrix} 2 & 1 \\ -9 & 8 \\ -3 & 6 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} -1 & 7 \\ 17 & -19 \\ 9 & -3 \end{bmatrix}$$

3 marks

Question 7 (6 marks)

a.

$A = X \wedge Y', \; B = (X \wedge Y) \vee (X' \wedge Z), \; C = X' \wedge Z'$

3 marks

b.

X	Y	Z	A	B	C
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	0	0
1	1	0	0	1	0
1	1	1	0	1	0

$A \vee B \vee C = 1$ One of the 3 outcomes will be true.

3 marks

Question 8 (3 marks)

More than one correct set of steps. Coding needs to reject either a or r being zero.

input a, r
sum $\leftarrow 0$
if $a \times r = 0$ *then*
print 'INVALID NUMBER(S)'
for i *from* 0 *to 5*
sum $\leftarrow$ *sum* $+ a \times r^i$
end for
print sum

3 marks