

The Mathematical Association of Victoria

Trial Exam 2023

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 18 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the space provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$ where $g = 9.8$.

Question 1 (3 marks)

- a. Find the vector (cross) product of the vectors $2\vec{i} - \vec{k}$ and $\vec{j} + 2\vec{k}$.

2 marks

The vectors $2\vec{i} - \vec{k}$ and $\vec{j} + 2\vec{k}$ are parallel to a plane that contains the point $(3, 2, -1)$.

- b. Find a cartesian equation of this plane.

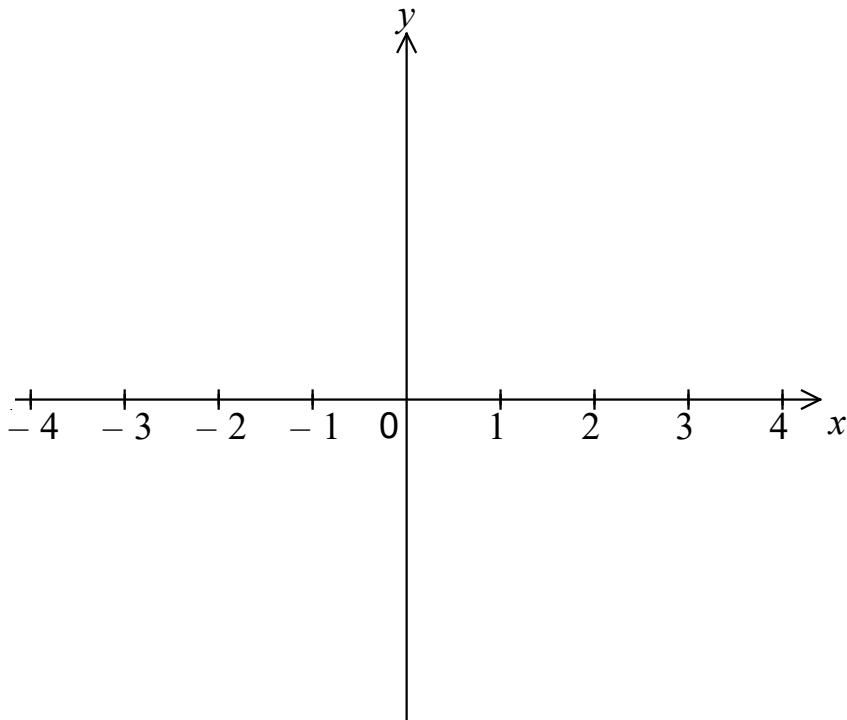
1 mark

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Question 2 (4 marks)

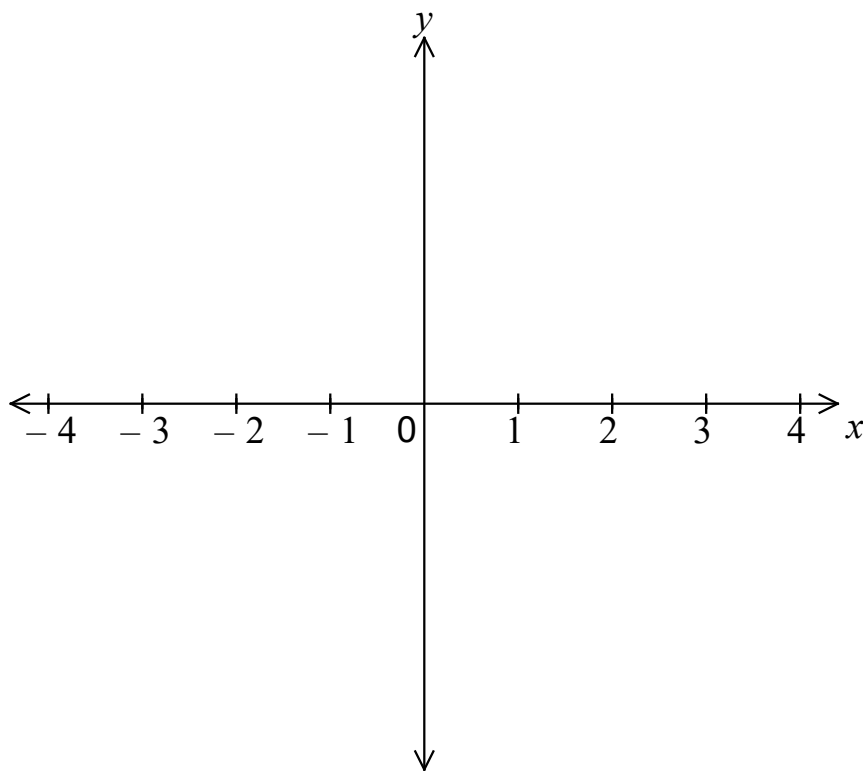
- a. Sketch the graph of $y = \frac{x^3 + 1}{x^2 - 4}$ on the axes provided below, labelling any asymptotes with their equations and any axial intercepts with their coordinates. You can assume that there are turning points at the points where $x = 0$ and $x \approx -0.314$.

3 marks

**Working space****Question 2 cont'd**

- b. Hence sketch the graph of $y = \frac{|x^3 + 1|}{x^2 - 4}$ on the axes provided below, labelling any asymptotes with their equations and any axial intercepts with their coordinates.

1 mark

**Working space****TURN OVER**

- 2 marks

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

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Question 4 (3 marks)

The *Gratuitous Citrus Farm* grows oranges. The mass, in grams, of the oranges is known to be normally distributed with a standard deviation of 5 grams. A bag of oranges is collected and used to calculate an approximate 95% confidence interval for the mean mass, measured in grams, of all oranges grown on the farm. The confidence interval was found to be (198.6, 201.4).

Find the total mass of oranges in the bag. Use $\Pr(-1.96 < Z < 1.96) = 0.95$ and give your answer correct to the nearest gram.

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Use proof by induction to prove that $3^{2^n} + 7$ is divisible by 8 $\forall n \in \mathbb{N}$.

[illegible]

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Question 7 (6 marks)

Consider the function f with rule $f(x) = \sin^{-1}(2x-1) + \frac{\pi}{2}$.

- a.** State the maximal domain and range of f . 1 mark

- b.** State the coordinates of the point of inflection of f . 1 mark

- c.** Find a rule for f^{-1} , the inverse of f . 1 mark

Question 7 cont'd

Question 8 (6 marks)

The velocity of an object at time t is given by $\vec{v}(t) = \cos(t) \vec{i} + (\sin(2t) - 1) \vec{j}$, where $t \geq 0$.

The initial position of the object is $\vec{i} + 2 \vec{j}$.

- a. Find the position vector of the object.

2 marks

Question 8 cont'd

- b.** Find the first time at which the scalar (dot) product of the velocity and acceleration of the object is equal to $-\frac{1}{2}$.

4 marks

[illegible]

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- b.** The acceleration of an object moving in a straight line is given by $x \arctan\left(\frac{1}{x}\right) \text{ ms}^{-2}$ where x is the displacement in meters of the object from an origin after t seconds.

Given that the speed of the object is 1 ms^{-1} when $x = 1$, find the speed of the object when $x = \sqrt{3}$.

Give your answer in the form $\sqrt{\sqrt{a} + \frac{\pi}{b}}$ where $a, b \in \mathbb{Z}^+$.

2 marks

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