

YEAR 12 *Trial Exam Paper*
2023
SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 (3 marks)

Evaluate $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} (\sin(x) + \cos(x))^2 dx$

Question 2 (6 marks)

- a. Two position vectors relative to a fixed origin $O(0, 0, 0)$ are given by $\underline{a} = 2\underline{i} - 3\underline{j} + 4\underline{k}$ and $\underline{b} = \underline{i} + 2\underline{j} - 3\underline{k}$.

Find a unit vector perpendicular to both $\underline{a}$ and $\underline{b}$.

2 marks

- b.** Find the Cartesian equation of the plane containing vectors $\underline{a}$, $\underline{b}$ and $O(0,0,0)$.

2 marks

- c.** Find the distance between the plane containing vectors $\underline{a}$ and $\underline{b}$ and the point $P(-2, 2, 1)$.

2 marks

THIS PAGE IS BLANK

Question 3 (4 marks)

The rate of change of the area, A , of an oil spill measured in square metres t seconds after the spill was formed is described by $\frac{dA}{dt} = \frac{A^{\frac{3}{2}}}{t^2}, t > 0$.

- a.** Find a solution to the above differential equation giving your answer in the form

$A = f(t)$ given that $A = \frac{1}{4} \text{ m}^2$ one second after the spill was formed.

3 marks

- b.** Find the limiting value of A as t becomes very large.

1 mark

Question 4 (4 marks)

Solve the equation $2z^2 + \bar{z}^2 = 27 - 2\sqrt{10}i$ for $z \in \mathbb{C}$.

Give your answers in the form $z = a + bi$, $a, b \in \mathbb{R}$.

Question 5 (4 marks)

Find the equation of the line normal to the curve described by $x^2 + (y - x)^3 = 9$ at the point where $x = 1$.

CONTINUES OVER PAGE

Question 6 (4 marks)

The curve given by $y = x^3$ is rotated about the x -axis to form a solid of revolution. Find the surface area of the part of this solid where $y \in \left[0, \frac{\sqrt{3}}{9}\right]$.

Question 7 (3 marks)

A machine makes a serving of lime cordial by combining 7 units of water and 4 units of syrup. A unit of water is normally distributed with a mean of 25 mL and standard deviation of 1 mL. A unit of syrup is normally distributed with a mean of 5 mL and a standard deviation of 1.5 mL.

- a.** Find the mean and standard deviation for the volume of one serving of lime cordial.

2 marks

- b.** Find the probability that the machine dispenses a serving of cordial with a volume more than 202.84 mL. Give your answer correct to 3 decimal places.

1 mark

Question 8 (4 marks)

Two jets, A and B, are observed from a control tower at $O(0, 0, 0)$. Relative to O , their position vectors at time t hours after midday are given by

$\underline{r}_A(t) = (2+2t)\underline{i} + (9+3t)\underline{j} + (15+6t)\underline{k}$ and $\underline{r}_B(t) = (1+t)\underline{i} + (2+4t)\underline{j} + (2+8t)\underline{k}$ where displacements are measured in kilometres.

- a.** Find the distance between jets A and B at 3 pm.

2 marks

- b.** Find the time after midday when the distance between jet B and an observation tower located at $T(20, 20, 0)$ is a minimum.

2 marks

THIS PAGE IS BLANK

Question 9 (3 marks)

Prove by mathematical induction that the number $4^n + 6n - 1$ is divisible by 3 for all $n \in \mathbb{N}$.

Question 10 (5 marks)

Find the area bound by $f(x) = e^x \sin(x)$ and the x -axis for $x \in [0, \pi]$.

END OF QUESTION AND ANSWER BOOKLET