



Trial Examination 2022

VCE Specialist Mathematics Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (3 marks)

$$\begin{aligned}a &= \frac{|F|}{m} \\&= \frac{\sqrt{8^2 + 6^2}}{2} \\&= 5\end{aligned}$$

A1

$$\begin{aligned}s &= ut + \frac{1}{2}at^2 \\90 &= 0 + \frac{1}{2} \times 5t^2 \\t &= \sqrt{\frac{180}{5}} \\t &= 6 \text{ seconds}\end{aligned}$$

A1

$$\begin{aligned}\underline{v} &= \underline{a}t \\&= \frac{8\underline{i} + 6\underline{j}}{2} \times 6 \\&= 24\underline{i} + 18\underline{j} \text{ ms}^{-1}\end{aligned}$$

A1

Question 2 (3 marks)

Since $x \in \left(\pi, \frac{3\pi}{2}\right)$:

$$\begin{aligned}\sin(x) &= -\sqrt{1 - \cos^2 x} \\ &= -\sqrt{1 - \left(-\frac{2}{5}\right)^2} \\ &= -\sqrt{\frac{21}{25}} \\ &= -\frac{\sqrt{21}}{5}\end{aligned}$$

A1

$$\begin{aligned}\tan(x) &= \frac{\sin(x)}{\cos(x)} \\ &= \frac{-\frac{\sqrt{21}}{5}}{-\frac{2}{5}} \\ &= \frac{\sqrt{21}}{2}\end{aligned}$$

A1

$$\begin{aligned}\tan(2x) &= \frac{2 \tan(x)}{1 - \tan^2(x)} \\ &= \frac{\sqrt{21}}{1 - \frac{21}{4}} \\ &= -\frac{4\sqrt{21}}{17}\end{aligned}$$

A1

Note: Accept consequential result from incorrect values obtained for $\tan(x)$.

Question 3 (2 marks)

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos(\alpha)$$

$$\cos(\alpha) = \frac{1 - 1 + \sqrt{6}}{\sqrt{3} \times \sqrt{8}}$$

M1

$$\cos(\alpha) = \frac{\sqrt{6}}{\sqrt{24}}$$

$$\cos(\alpha) = \frac{1}{2}$$

$$\alpha = \frac{\pi}{3} \text{ OR } 60^\circ$$

A1

Question 4 (3 marks)

$$I = \int_0^{\frac{\pi}{2}} \sin(2x) \cos^3(x) dx = \int_0^{\frac{\pi}{2}} 2 \sin(x) \cos^4(x) dx \quad \text{M1}$$

$$u = \cos(x)$$

$$du = -\sin(x) dx$$

$$I = - \int_{\cos(0)}^{\cos(\frac{\pi}{2})} 2u^4 du = - \int_1^0 2u^4 du = \int_0^1 2u^4 du \quad \text{A1}$$

Note: Accept any of the expressions shown above for I.

$$\begin{aligned} &= \left[\frac{2u^5}{5} \right]_0^1 \\ &= \frac{2}{5} \quad \text{A1} \end{aligned}$$

Question 5 (4 marks)

Let D be the time it takes in hours to build a deck: $D \sim N(12, 4^2)$.

Let P be the time it takes in hours to build a pergola: $P \sim N(10, 3^2)$.

$$E(D - P) = 12 - 10 = 2 \quad \text{A1}$$

$$\text{var}(D - P) = 16 + 9 = 25 \quad \text{A1}$$

$$D - P \sim N(2, 5^2)$$

Let Z be the standard normal variable for $D - P$.

$$\Pr(D - P < 0) = \Pr\left(Z < \frac{0 - 2}{5}\right) \quad \text{M1}$$

$$\begin{aligned} \Pr(Z < -0.4) &= \Pr(Z > 0.4) \\ &= 1 - 0.66 \\ &= 0.34 \quad \text{A1} \end{aligned}$$

Question 6 (6 marks)

a. $x = t + 1 \Rightarrow t = x - 1$

$$y = t^2 + 3t$$

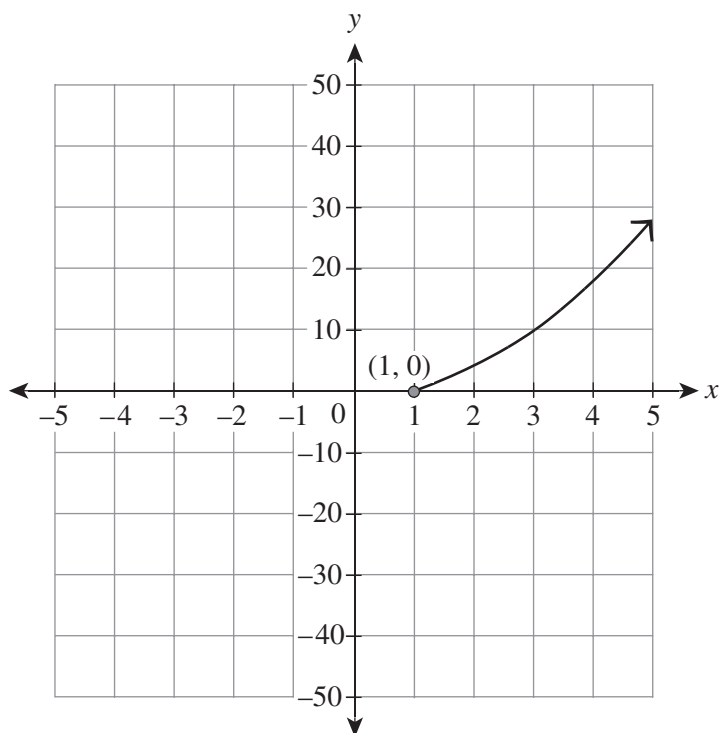
$$= (x - 1)^2 + 3(x - 1)$$

$$= x^2 - 2x + 1 + 3x - 3$$

$$= x^2 + x - 2$$

M1

b.



correct shape A1
correct endpoint and direction A1

c. $\text{speed} = |\dot{\mathbf{r}}|$

$$\dot{\mathbf{r}}(t) = \dot{\mathbf{i}} + (2t + 3)\dot{\mathbf{j}}$$

M1

$$|\dot{\mathbf{r}}| = \sqrt{(1)^2 + (2t + 3)^2}$$

$$= \sqrt{4t^2 + 12t + 10}$$

A1

$$|\dot{\mathbf{r}}| = 5\sqrt{2} \Rightarrow \sqrt{4t^2 + 12t + 10} = 5\sqrt{2}$$

$$4t^2 + 12t + 10 = 50$$

$$t^2 + 3t - 10 = 0$$

$$t = -5 \text{ or } t = 2$$

$$t = 2 \quad (\text{since } t \geq 0)$$

A1

Question 7 (5 marks)

- a. Considering the range of $\arctan(\sqrt{3}t)$ to find the domain gives:

$$\text{domain} = [\arctan(-\sqrt{3}), \arctan(\sqrt{3})] \quad (\text{since } t \in [-1, 1])$$

$$= \left[-\frac{\pi}{3}, \frac{\pi}{3} \right]$$

A1

Considering the range of $\arcsin(t)$ to find the range gives:

$$\text{range} = [\arcsin(-1), \arcsin(1)]$$

$$= \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \quad (\text{since } t \in [-1, 1])$$

A1

b. $\frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}}$

$$\frac{dx}{dt} = \frac{\sqrt{3}}{1+3t^2}$$

$$x = \frac{\pi}{4} \Rightarrow \sqrt{3}t = \tan\left(\frac{\pi}{4}\right) \Rightarrow t = \frac{1}{\sqrt{3}}$$

A1

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{\sqrt{1-t^2}}}{\frac{\sqrt{3}}{1+3t^2}}$$

M1

$$\begin{aligned} t = \frac{1}{\sqrt{3}} \Rightarrow \text{gradient} &= \frac{\frac{1}{\sqrt{1-\frac{1}{3}}}}{\frac{\sqrt{3}}{1+3 \times \frac{1}{3}}} \\ &= \frac{\sqrt{3}}{\sqrt{2}} \times \frac{2}{\sqrt{3}} \\ &= \sqrt{2} \end{aligned}$$

A1

Question 8 (5 marks)

- a. **Method 1:**

$$\frac{1}{T-10} dT = k dt$$

$$\int_{100}^{70} \frac{1}{T-10} dT = \int_0^5 k dt$$

M1

$$[\log_e |T-10|]_{100}^{70} = [kt]_0^5$$

A1

$$\log_e(60) - \log_e(90) = 5k$$

$$k = \frac{1}{5} \log_e \left(\frac{60}{90} \right)$$

M1

$$= \frac{1}{5} \log_e \left(\frac{2}{3} \right)$$

Method 2:

$$\int \frac{1}{T-10} dT = \int k dt$$

$$\log_e |T-10| = kt + c$$

M1

$$\log_e (T-10) = kt + c \quad (\text{since } T \geq 10)$$

$$\begin{cases} t=0 \\ T=100 \end{cases} \Rightarrow \log_e (100-10) = 0 + c$$

$$c = \log_e (90)$$

$$\log_e (T-10) = kt + \log_e (90)$$

$$T = e^{kt + \log_e (90)} + 10$$

$$= 90e^{kt} + 10$$

A1

$$\begin{cases} t=5 \\ T=70 \end{cases} \Rightarrow 70 = 90e^{5k} + 10$$

$$e^{5k} = \frac{70-10}{90}$$

$$5k = \log_e \left(\frac{60}{90} \right)$$

M1

$$k = \frac{1}{5} \log_e \left(\frac{2}{3} \right)$$

b. Method 1:

$$\int_{70}^T \frac{1}{T-10} dT = \int_0^5 \frac{1}{5} \log_e \left(\frac{2}{3} \right) dt$$

M1

$$[\log_e |T-10|]_{70}^T = \left[\frac{1}{5} \log_e \left(\frac{2}{3} \right) \cdot t \right]_0^5$$

$$\log_e (T-10) - \log_e (60) = \log_e \left(\frac{2}{3} \right)$$

$$\frac{T-10}{60} = \frac{2}{3}$$

$$T = 50^\circ\text{C}$$

A1

Method 2:

$$T = 90e^{kt} + 10$$

$$t=10 \Rightarrow T = 90e^{\frac{1}{5} \log_e \left(\frac{2}{3} \right) \times 10} + 10$$

M1

$$= 90e^{\log_e \left(\frac{4}{9} \right)} + 10$$

$$= 90 \times \frac{4}{9} + 10$$

$$= 50^\circ\text{C}$$

A1

Question 9 (5 marks)

a. Let $z = a + bi$, where $a, b \in \mathbb{R}$.

If $z^2 = -12 + 16i$, then:

$$a^2 + 2abi + b^2i^2 = -12 + 16i$$

M1

Comparing real and imaginary parts:

$$\begin{cases} a^2 - b^2 = -12 \\ 2ab = 16 \end{cases}$$

$$b = \frac{8}{a} \Rightarrow a^2 - \left(\frac{8}{a}\right)^2 = -12$$

$$a^4 + 12a^2 - 64 = 0$$

M1

$$(a^2 - 4)(a^2 + 16) = 0$$

$$a = -2 \text{ or } a = 2 \quad (\text{since } a \in \mathbb{R})$$

Therefore, the square roots are $-2 - 4i$ and $2 + 4i$.

A1

b. $z_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4(4 - 4i)}}{2}$

M1

$$= \frac{-2 \pm \sqrt{-12 + 16i}}{2} \quad (\text{from part a., } \sqrt{-12 + 16i} = 2 + 4i \text{ or } -2 - 4i)$$

$$= \frac{-2 \pm (2 + 4i)}{2} = -1 \pm (1 + 2i)$$

$$= 2i \text{ or } -2 - 2i$$

A1

Note: Consequential on answer to Question 9a.

Question 10 (4 marks)

The region bounded is between two semi-circles for $x \in [1 - 2, 1 + 2] = [-1, 3]$.

M1

$$(y - 3)^2 = 4 - (x - 1)^2$$

$$y = 3 \pm \sqrt{4 - (x - 1)^2} = 3 \pm \sqrt{3 + 2x - x^2}$$

M1

$$\text{volume} = \pi \int_{-1}^3 \left[(3 + \sqrt{3 + 2x - x^2})^2 - (3 - \sqrt{3 + 2x - x^2})^2 \right] dx$$

A1

Using the identity $a^2 - b^2 = (a + b)(a - b)$ gives:

$$\text{volume} = \pi \int_{-1}^3 \left(3 + \sqrt{3 + 2x - x^2} + 3 - \sqrt{3 + 2x - x^2} \right) \left(3 + \sqrt{3 + 2x - x^2} - 3 + \sqrt{3 + 2x - x^2} \right) dx$$

$$= \pi \int_{-1}^3 6 \times 2\sqrt{3 + 2x - x^2} dx$$

M1

$$= 12\pi \int_{-1}^3 \sqrt{3 + 2x - x^2} dx$$