Neap

Trial Examination 2022

VCE Specialist Mathematics Units 1&2

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

1	A	B	C	**D**	E	11	**A**	B	C	D	E
2	A	B	**C**	D	E	12	A	B	**C**	D	E
3	A	B	C	D	**E**	13	A	**B**	C	D	E
4	A	B	C	D	**E**	14	A	**B**	C	D	E
5	A	B	C	**D**	E	15	A	B	**C**	D	E
6	A	**B**	C	D	E	16	A	B	C	**D**	E
7	A	B	C	**D**	E	17	A	B	C	**D**	E
8	**A**	B	C	D	E	18	A	B	C	**D**	E
9	A	B	C	D	**E**	19	A	**B**	C	D	E
10	A	B	C	D	**E**	20	A	**B**	C	D	E

Question 1 D

$$\begin{aligned} 2\overrightarrow{BA} &= 2\left(\overrightarrow{BO} + \overrightarrow{OA}\right) \\ &= 2\left(\underset{\sim}{i} - 5\underset{\sim}{j} + 2\underset{\sim}{i} + 3\underset{\sim}{j}\right) \\ &= 2\left(3\underset{\sim}{i} - 2\underset{\sim}{j}\right) \\ &= 6\underset{\sim}{i} - 4\underset{\sim}{j} \end{aligned}$$

Question 2 C

$$\begin{aligned} t_6 - t_2 &= t_1 + 5d - (t_1 + d) \\ &= 4d \end{aligned}$$

$$4d = 16$$
$$d = 4$$

$$\begin{aligned} t_8 - t_5 &= 3d \\ &= 12 \end{aligned}$$

Question 3 E

$$\begin{aligned} \angle A &= 180 - 32 - 72 \\ &= 76° \end{aligned}$$

$$\frac{BC}{\sin(A)} = \frac{AB}{\sin(C)}$$

$$\frac{BC}{\sin 76} = \frac{8}{\sin 32}$$

$$\begin{aligned} BC &= 14.6482 \\ &\approx 15 \text{ cm} \end{aligned}$$

Question 4 E

Since $2x + 3$ is under the square root:

$$2x + 3 \geq 0$$
$$x \geq -\frac{3}{2}$$

Since $x^2 - 9x + 18$ is the denominator:

$$x^2 - 9x + 18 \neq 0$$
$$(x - 6)(x - 3) \neq 0$$
$$\therefore x \neq 6 \text{ and } x \neq 3$$

Therefore, the domain is $\left[-\frac{3}{2}, 3\right) \cup (3, 6) \cup (6, \infty)$.

Question 5 D

$$\begin{aligned} \underset{\sim}{a}+\underset{\sim}{b} &= -2\underset{\sim}{i}-3\underset{\sim}{j}+5\underset{\sim}{i}+k\underset{\sim}{j} \\ &= 3\underset{\sim}{i}+(k-3)\underset{\sim}{j} \end{aligned}$$

$\underset{\sim}{a}+\underset{\sim}{b}$ being perpendicular to y-axis implies that the $\underset{\sim}{j}$ component has a coefficient of 0.

$$\begin{aligned} k-3 &= 0 \\ k &= 3 \end{aligned}$$

Question 6 B

By the conjugate root theorem, the other root is $5 - 3i$.

$$\begin{aligned} x_1x_2 &= (5-3i)(5+3i) \\ &= 34 \end{aligned}$$

Question 7 D

D is correct.

$$\sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2}=1$$

Option **A**:

$$\begin{aligned} \sqrt{\left(\frac{\sqrt{2}}{2}\sin(\theta)\right)^2+\left(\frac{\sqrt{2}}{2}\cos(\theta)\right)^2} &= \sqrt{\frac{1}{2}\sin^2\theta+\frac{1}{2}\cos^2\theta} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

Option **B**:

$$\sqrt{(-1)^2+(-1)^2}=\sqrt{2}$$

Option **C**:

$$\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\frac{\sqrt{2}}{2}$$

Option **E**:

$$\sqrt{\left(\frac{5}{13}\right)^2+\left(\frac{11}{13}\right)^2}=\sqrt{\frac{146}{169}}$$

Question 8 A

The angle between a tangent and a chord drawn from the point of contact is equal to any angle in the alternate segment.

$\therefore \angle C = 60°$

$$\begin{aligned}\text{area of } \Delta ABC &= \frac{1}{2} \times 7 \times 8 \times \sin 60 \\ &= 14\sqrt{3} \text{ cm}^2\end{aligned}$$

Question 9 E

Since $\angle B = \angle E$ and $\angle ACB = \angle DCE$, ΔACB is similar to ΔDCE.

$$\frac{CE}{BC} = \frac{DC}{AC}$$

$$\frac{5}{7.5} = \frac{DC}{6}$$

$$DC = 4 \text{ cm}$$

$$\begin{aligned}BD &= DC + CB \\ &= 4 + 7.5 \\ &= 11.5 \text{ cm}\end{aligned}$$

Question 10 E

$$x^2 - 2x + 2 = 0$$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= -4\end{aligned}$$

Therefore, $x^2 - 2x + 2$ has no real roots and the reciprocal function has no vertical asymptotes.

Question 11 A

$$\tan(\theta) + \frac{1}{\tan(\theta)} = 2$$

$$\tan^2(\theta) + 1 = 2\tan(\theta)$$

$$\frac{1}{\cos^2(\theta)} = \frac{2\sin(\theta)}{\cos(\theta)}$$

$$2\sin(\theta)\cos(\theta) = 1$$

$$(\sin(\theta) + \cos(\theta))^2 = \sin^2(\theta) + 2\sin(\theta)\cos(\theta) + \cos^2(\theta)$$

$$(\sin(\theta) + \cos(\theta))^2 = 2$$

$$\sin(\theta) + \cos(\theta) = \pm\sqrt{2}$$

Question 12 C
C is correct.

A is incorrect. The graph of this equation would have the first positive vertical asymptote at $x=\frac{3\pi}{2}$.

B is incorrect. The graph of this equation would have a vertical asymptote at $x=0$.

D is incorrect. The graph of this equation would have a vertical asymptote at $x=\frac{\pi}{2}$.

E is incorrect. The graph of this equation would have a vertical asymptote at $x=\frac{\pi}{2}$.

Question 13 B
The value for the power of i repeats itself every four terms.

$i=i$

$i^2=-1$

$i^3=-i$

$i^4=1$

The sum of these four terms is $i+i^2+i^3+i^4=0$.
The first 2019 terms consist of 504 repetitions plus three terms.

$$\begin{aligned}\therefore S_{2019}&=504\times 0+i+i^2+i^3\\&=-1\end{aligned}$$

Question 14 B
As the complex number is on the imaginary axis, the real part should be equal to 0.

$$\begin{aligned}m^2-m-2&=0\\(m-2)(m+1)&=0\\m&=2\text{ or }-1\end{aligned}$$

The imaginary part should not be equal to 0.

$$\begin{aligned}m^2-3m+2&\neq 0\\(m-2)(m-1)&\neq 0\\m&\neq 1\text{ or }2\end{aligned}$$

$\therefore m=-1$

Question 15 C

Since $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$, $\underset{\sim}{a} = k\underset{\sim}{b}$, where k is a constant.

Equating coefficients gives:

$1 - \sin\theta = \frac{1}{2}k$ and $1 = k(1 + \sin\theta)$

$$\begin{aligned} 2(1-\sin\theta) &= \frac{1}{(1+\sin\theta)} \\ (1+\sin\theta)(1-\sin\theta) &= \frac{1}{2} \\ \cos^2\theta &= \frac{1}{2} \\ \theta &= 45° \end{aligned}$$

Question 16 D

Let C have the coordinates (x, y).

$AC = \sqrt{(x+2)^2 + y^2}$

$AB = 4$

$BC = \sqrt{(x-2)^2 + y^2}$

Since AC, AB and BC form an arithmetic sequence:

$$\begin{aligned} AB - AC &= BC - AB \\ 4 - \sqrt{(x+2)^2+y^2} &= \sqrt{(x-2)^2+y^2} - 4 \\ -\sqrt{(x+2)^2+y^2} &= \sqrt{(x-2)^2+y^2} - 8 \\ (x+2)^2 + y^2 &= (x-2)^2 + y^2 - 16\sqrt{(x-2)^2+y^2} + 64 \\ x - 8 &= -2\sqrt{(x-2)^2+y^2} \\ x^2 - 16x + 64 &= 4x^2 - 16x + 16 + 4y^2 \\ 3x^2 + 4y^2 &= 48 \\ \frac{x^2}{16} + \frac{y^2}{12} &= 1 \end{aligned}$$

The shape of the locus is an ellipse.

Question 17 D

D is correct. In the fourth quadrant, sine is an increasing function and tangent is an increasing function. Therefore, $\tan(\alpha) > \tan(\beta)$.

A is incorrect. In the first quadrant, sine is an increasing function and cosine is a decreasing function. Therefore, $\cos(\alpha) < \cos(\beta)$.

B is incorrect. In the second quadrant, sine is a decreasing function and tangent is an increasing function. Therefore, $\tan(\alpha) < \tan(\beta)$.

C is incorrect. In the third quadrant, sine is a decreasing function and cosine is an increasing function. Therefore, $\cos\alpha < \cos\beta$.

E is incorrect. There is enough information provided in the question to determine the values of cosine and tangent based on the value of sine.

Question 18 D

$$\begin{aligned}\text{scalar resolute} &= \vec{\underset{\sim}{a}} \cdot \hat{\underset{\sim}{b}} \\ &= \frac{\left(3\underset{\sim}{i} + 4\underset{\sim}{j}\right)\left(2\underset{\sim}{i} + 2\underset{\sim}{j}\right)}{\sqrt{2^2 + 2^2}} \\ &= \frac{7\sqrt{2}}{2}\end{aligned}$$

Question 19 B

By circle geometry, $\angle BOC = 60°$.

$$\begin{aligned}\text{area of } ABOC &= \text{area of } \Delta ABC - \text{area of } \Delta AOC \\ &= \frac{1}{2} \times 7 \times 5 \times \sin 30° - \frac{1}{2} \times 2 \times 2 \times \sin 60° \\ &= 8.75 - \sqrt{3}\end{aligned}$$

Question 20 B

By the conjugate root theorem:

$x_1 = a + bi,\ x_2 = a - bi$

By the properties of quadratic equations, $ax^2 + bx + c = 0$.

The product of two roots is $\frac{c}{a}$, and the sum of two roots is $-\frac{b}{a}$. Therefore:

$x_1 + x_2 = -\frac{m}{2},\ x_1 x_x = \frac{m^2 - m}{2}$

$$\begin{aligned}x_1 x_x &= (a + bi)(a - bi) \\ &= a^2 + b^2 \\ &= 1\end{aligned}$$

$\therefore \frac{m^2 - m}{2} = 1$

$$\begin{aligned}m^2 - m - 2 &= 0 \\ m &= -1 \text{ or } 2\end{aligned}$$

When $m = 2$:

- the discriminant of the function is $\Delta = 6^2 - 4 \times 2 \times 2 = 20 > 0$
- the equation has real solutions.

Therefore, $m = -1$.

SECTION B

Question 1 (5 marks)

a. **i.** Since x, y and z are proportional, the right-hand sides of the equations are 1.

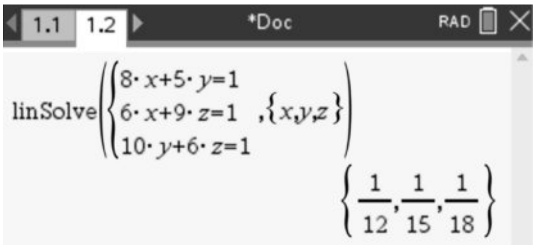
$$\begin{cases} 8x + 5y = 1 \\ 6x + 9z = 1 \\ 10y + 6z = 1 \end{cases}$$

correct left-hand side of each equation A1
correct right-hand side of each equation A1

ii. $$\begin{bmatrix} 8 & 5 & 0 \\ 6 & 0 & 9 \\ 0 & 10 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ A1

Note: Consequential on answer to ***Question 1a.i****.*

b. Using CAS with matrix or simultaneous equations gives the following.



$x = \frac{1}{12}, y = \frac{1}{18}, z = \frac{1}{15}$ A1

It would take Anna 12 days, Betty 18 days and Charles 15 days. A1

Note: Consequential on answer to ***Question 1a.i****.*

Question 2 (13 marks)

a. $t_5 = t_0 + 5 \times 288$
$= \$11\ 440$ A1

b. $t_5 = 1.021^5 t_0$
$= \$11\ 095.04$ A1

c. By solving the equation $10\,000 + 288x = 10\,000 \times 1.021^x$, it can be found that bank accounts A and B will have the same amount of return after 29.86 years. M1

solve$\left(10000+288\cdot x=(1.021)^x\cdot 10000,x\right)$
$x=0.000000000001$ or $x=29.8586152534$

Hence, from the thirtieth year, bank account B will outperform bank account A.
Therefore, from a long-term perspective, Juanita should choose bank account B. A1

d. **i.** After 1 year: $10\,000 + 288 \times 0.8 = \$10\,230.40$

After 2 years: $10\,230.4 + 288 \times 0.8 = \$10\,460.80$

A1

ii. After 1 year: $10\,000 + 0.021 \times 10\,000 \times 0.8 = \$10\,168$

After 2 years: $10\,168 + 0.021 \times 10\,168 \times 0.8 = \$10\,338.82$

A1

e. bank account A:

$$t_n = t_{n-1} + 288 \times 0.8$$
$$= t_{n-1} + 230.4$$
A1

bank account B:

$$t_n = t_{n-1} + 0.021 \times t_{n-1} \times 0.8$$
$$= 1.0168 t_{n-1}$$
A1

f.
$$S_n = a_1(1+r)^{n-1} + a_2(1+r)^{n-2} + a_3(1+r)^{n-3} + \ldots + a_{n-1}(1+r) + a_n$$
$$S_{n-1} = a_1(1+r)^{n-2} + a_2(1+r)^{n-3} + a_3(1+r)^{n-4} + \ldots + a_{n-1}$$
$$\therefore S_n = (1+r)S_{n-1} + a_n$$
A1

g. $S_n = a_1(1+r)^{n-1} + a_2(1+r)^{n-2} + a_3(1+r)^{n-3} + \ldots + a_{n-1}(1+r) + a_n$

Multiplying both sides of the equation by 1 + r gives:

$(1+r)S_n = a_1(1+r)^n + a_2(1+r)^{n-1} + a_3(1+r)^{n-2} + \ldots + a_{n-1}(1+r)^2 + a_n(1+r)$ M1

Subtracting S_n from the equation gives:

$rS_n = a_1(1+r)^{n-1} + d\left[(1+r)^{n-1} + (1+r)^{n-2} + \ldots + (1+r)\right] - a_n$

where d is the common difference for the sequence a_1, a_2, a_3 …

It can be noted that $1+r,\ (1+r)^2,\ (1+r)^3, \ldots\ (1+r)^{n-1}$ forms a geometric sequence with the first term being 1 + r and the common ratio being 1 + r. Therefore, the sum of this geometric sequence is $\dfrac{(1+r)^n - 1 - r}{r}$.

$rS_n = a_1(1+r)^{n-1} + \dfrac{d}{r}\left[(1+r)^n - 1 - r\right] - a_n$ M1

Dividing both sides by r and expressing a_n as $a_1 + (n-1)d$ gives:

$S_n = \dfrac{a_1 r + d}{r^2}(1+r)^n - \dfrac{d}{r}n - \dfrac{a_1 r + d}{r^2}$

It can be seen that $\dfrac{a_1 r + d}{r^2}(1+r)^n$ is a geometric sequence with the first term being

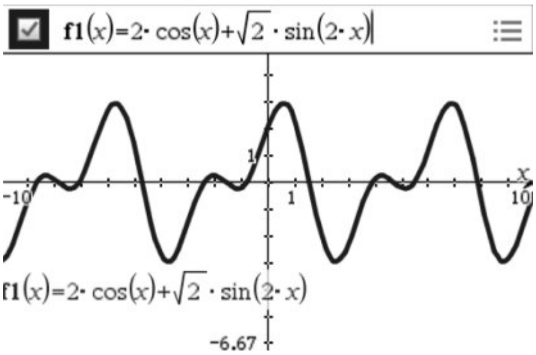
$\dfrac{a_1 r + d}{r^2}(1+r)$ and the common ratio being 1 + r. A1

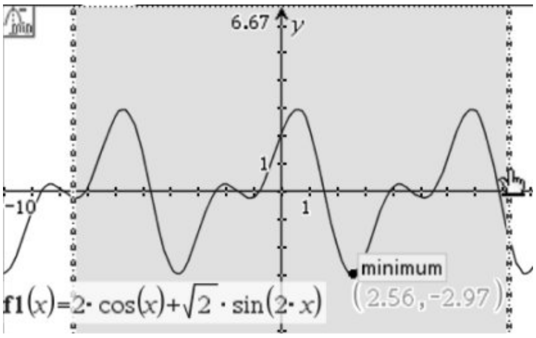
It can be seen that $-\dfrac{d}{r}n - \dfrac{a_1 r + d}{r^2}$ is an arithmetic sequence with the first term being

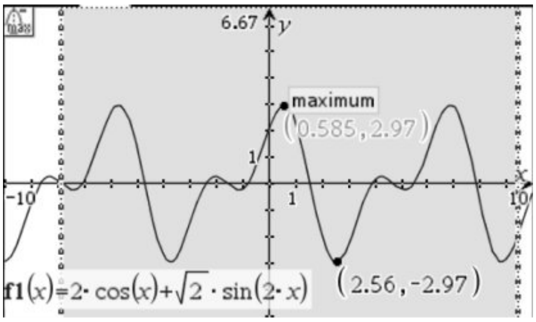
$-\dfrac{d}{r} - \dfrac{a_1 r + d}{r^2}$ and the common difference being $\dfrac{d}{r}$. A1

Question 3 (7 marks)

a.







Using CAS:

maximum = 2.97 A1

minimum = –2.97 A1

b.

$$2\cos(t)+\sqrt{2}\sin(2t)=0$$

$$2\cos(t)+2\sqrt{2}\sin(t)\cos(t)=0 \quad \text{M1}$$

$$2\cos(t)\left(1+\sqrt{2}\sin(t)\right)=0$$

$$\cos(t)=0 \text{ or } \sin(t)=-\frac{\sqrt{2}}{2} \quad \text{A1}$$

c. **i.** $r\sin(2t-\alpha) = r\sin(2t)\cos(\alpha) - r\cos(2t)\sin(\alpha)$

$$\begin{cases} r\cos(\alpha) = \sqrt{3} \\ r\sin(\alpha) = 3 \end{cases}$$

$\tan\alpha = \sqrt{3}$

$r > 0$, therefore, $\sin\alpha > 0, \cos\alpha > 0$.

$\alpha = \dfrac{\pi}{3}$ A1

Substituting $\alpha = \dfrac{\pi}{3}$ into one of the equations above gives $r = 2\sqrt{3}$.

$S = 2\sqrt{3}\sin\left(2t - \dfrac{\pi}{3}\right)$ A1

ii. $2\sqrt{3}\sin\left(2t - \dfrac{\pi}{3}\right) = 0$

$\sin\left(2t - \dfrac{\pi}{3}\right) = 0$

$t = \dfrac{\pi}{6}, \dfrac{2\pi}{3}, \dfrac{6\pi}{7}$ A1

Note: Consequential on answer to ***Question 3c.i.***

Question 4 (15 marks)

a. $z = \dfrac{-2\sqrt{2}i \pm \sqrt{\left(2\sqrt{2}i\right)^2 - 4\times 1\times(-5)}}{2}$ M1

$= \dfrac{-2\sqrt{2}i \pm 2\sqrt{3}}{2}$

$z_1 = -\sqrt{3} - \sqrt{2}i, z_2 = \sqrt{3} - \sqrt{2}i$ A1

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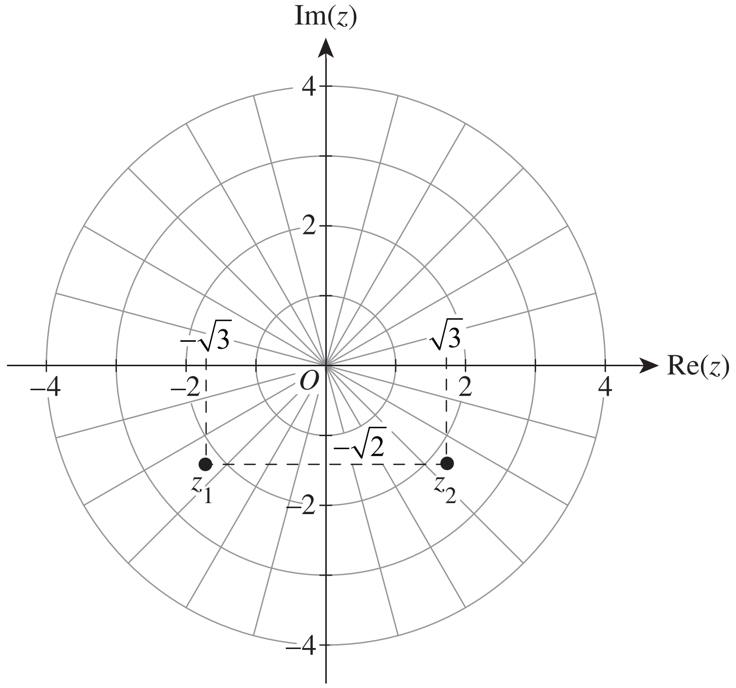
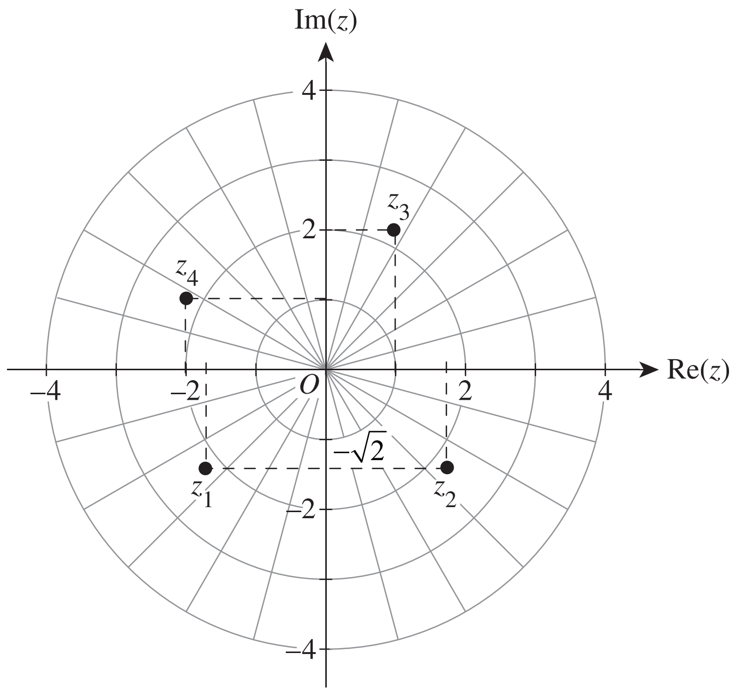
b.



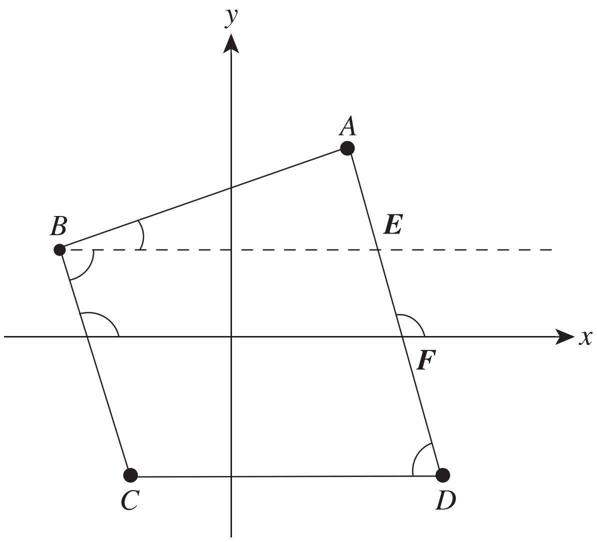
correct z_1 and label A1
correct z_2 and label A1

c.



correct z_3 and label A1
correct z_4 and label A1

d.



From the question, it is known $A(1, 2)$, $B(-2, 1)$, $C\left(-\sqrt{3}, -\sqrt{2}\right)$, $D\left(\sqrt{3}, -\sqrt{2}\right)$.

From the graph above, it can be seen that $\angle ABC = \angle ABE + \angle CBE$.

$\tan\angle ABE$ is the gradient of AB.

$$\tan\angle ABE = m_{AB} = \frac{2-1}{1+2} = \frac{1}{3}$$

$\tan\angle CBE$ is the negative of the gradient of BC, since co-interior angles add to 180° and $\tan(\pi - \theta) = -\tan(\theta)$.

$$\tan\angle CBE = -m_{BC} = \frac{1+\sqrt{2}}{1-\sqrt{3}}$$ M1

Using the addition formula for the tangent $\angle ABC$ gives:

$$\tan\angle ABC = \tan(\angle ABE + \angle CBE) = \frac{\frac{1}{3} + \frac{1+\sqrt{2}}{2-\sqrt{3}}}{1 - \frac{1}{3} \times \frac{1+\sqrt{2}}{2-\sqrt{3}}}$$ M1

Similarly, we can find that $\tan\angle ADC$ is the negative of the gradient of AD.

$$m_{AD} = \frac{2+\sqrt{2}}{1-\sqrt{3}}$$

$$\tan\angle ADC = \frac{2+\sqrt{2}}{\sqrt{3}-1}$$

Using the addition formula for the tangent gives:

$$\tan(\angle ABC + \angle ADC) = 0$$ M1

$$\angle ABC + \angle ADC > 0$$

$$\angle ABC + \angle ADC = \pi$$ A1

*Note: Consequential on answer to **Question 4c.***

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e. **i.** $z_5 = a^2 + 2ai - 1$

$= (a^2 - 1) + 2ai$

If z_5 is on the imaginary axis, $a^2 - 1 = 0$. M1

$a = \pm 1$ A1

ii. z_5 being in the fourth quadrant implies $a^2 - 1 > 0$ and $2a < 0$. M1

$a^2 - 1 > 0$

$a > 1$ or $a < -1$

And:

$2a < 0$

$a < 0$ A1

Combining the results of the two inequalities gives $a < -1$. A1

Question 5 (11 marks)

a. $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\underset{\sim}{i} + \underset{\sim}{j}$ A1

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -\underset{\sim}{i} + 2\underset{\sim}{i} + 5\underset{\sim}{j}$

$= \underset{\sim}{i} + 5\underset{\sim}{j}$ A1

b. $\left|2\overrightarrow{AB} + \overrightarrow{AC}\right| = \left|-2\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{i} + 5\underset{\sim}{j}\right|$

$= \left|-\underset{\sim}{i} + 7\underset{\sim}{j}\right|$ M1

$= \sqrt{(-1)^2 + 7^2}$

$= 5\sqrt{2}$ A1

Note: Consequential on answer to ***Question 5a.***

c. $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$= 2\underset{\sim}{i} + 4\underset{\sim}{j}$

Let the unit vector be $x\underset{\sim}{i} + y\underset{\sim}{j}$.

$2x + 4y = 0$

$x^2 + y^2 = 1$ M1

Solving the simultaneous equations gives:

$$\begin{cases} x = \dfrac{2\sqrt{5}}{5} \\ y = -\dfrac{\sqrt{5}}{5} \end{cases} \text{ or } \begin{cases} x = -\dfrac{2\sqrt{5}}{5} \\ y = \dfrac{\sqrt{5}}{5} \end{cases}$$

Therefore, the unit vector is $\dfrac{2\sqrt{5}}{5}\underset{\sim}{i} - \dfrac{\sqrt{5}}{5}\underset{\sim}{j}$ or $-\dfrac{2\sqrt{5}}{5}\underset{\sim}{i} + \dfrac{\sqrt{5}}{5}\underset{\sim}{j}$. A1

d. i. $\underset{\sim}{x} = \sqrt{3}\underset{\sim}{i} - \underset{\sim}{j} + (t^2 - 3)\left(\frac{1}{2}\underset{\sim}{i} + \frac{\sqrt{3}}{2}\underset{\sim}{j}\right)$

$$= \left(\sqrt{3} + \frac{t^2 - 3}{2}\right)\underset{\sim}{i} + \left(\frac{\sqrt{3}(t^2 - 3)}{2} - 1\right)\underset{\sim}{j}$$

$$\underset{\sim}{y} = -k\left(\sqrt{3}\underset{\sim}{i} - \underset{\sim}{j}\right) + t\left(\frac{1}{2}\underset{\sim}{i} + \frac{\sqrt{3}}{2}\underset{\sim}{j}\right)$$

$$= \left(\frac{t}{2} - k\sqrt{3}\right)\underset{\sim}{i} + \left(k + \frac{\sqrt{3}t}{2}\right)\underset{\sim}{j}$$

Since $\underset{\sim}{x}$ is perpendicular to $\underset{\sim}{y}$:

$$\underset{\sim}{x} \cdot \underset{\sim}{y} = 0$$

$$\left(\sqrt{3} + \frac{t^2 - 3}{2}\right)\left(\frac{t}{2} - k\sqrt{3}\right) + \left(\frac{\sqrt{3}(t^2 - 3)}{2} - 1\right)\left(k + \frac{\sqrt{3}t}{2}\right) = 0$$ M1

$$-4k + (t^2 - 3)t = 0$$

$$k = \frac{(t^2 - 3)t}{4}$$ A1

ii. $k = \frac{(t^2 - 3)t}{4}$

$$\frac{k}{t} = \frac{(t^2 - 3)}{4}$$

$$\frac{k}{t} + t = \frac{(t^2 - 3)}{4} + t$$ M1

$$\frac{k + t^2}{t} = \frac{t^2 - 3}{4} + t$$

$$= \frac{1}{4}(t + 2)^2 - \frac{7}{4}$$ A1

Therefore, the minimum value of $\frac{k + t^2}{t}$ is $-\frac{7}{4}$ when $t = -2$. A1

*Note: Consequential on answer to **Question 5d.i.***

Question 6 (9 marks)

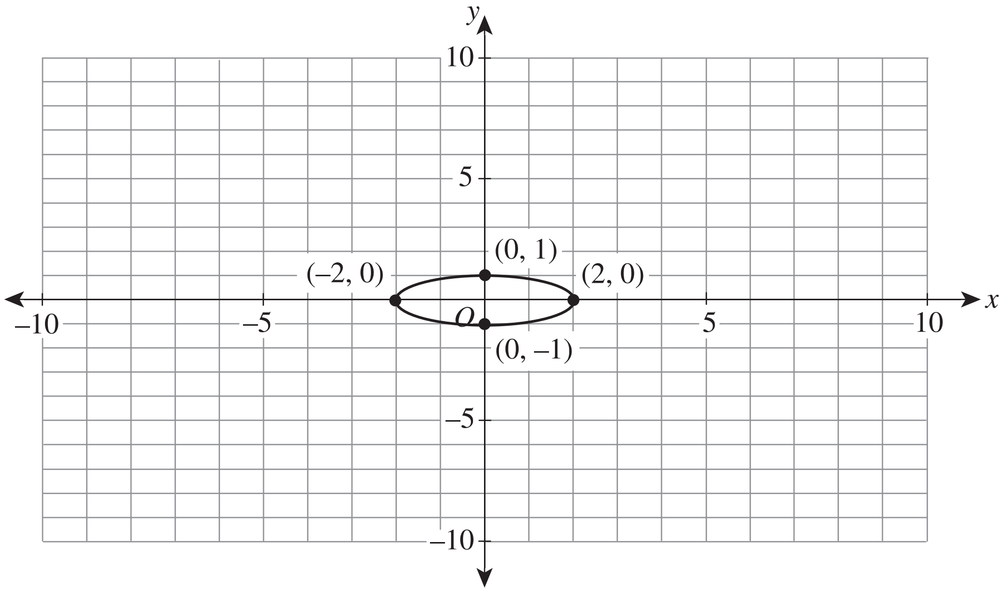
a.
$x^2 = 4\cos^2(\theta)$

$y^2 = \sin^2(\theta)$ M1

$$\frac{x^2}{4} + y^2 = \cos^2(\theta) + \sin^2(\theta)$$
$$= 1$$

The shape is an ellipse centred at (0, 0). A1

b.



correct shape A1
correct x- and y-axis intercepts A1
Note: Consequential on answer to ***Question 6a****.*

c. Let P be (x, y) and M be (m, n).

Since M is the midpoint of PD, $m = x$ and $n = \frac{y}{2}$. M1

$$x^2 + y^2 = 4$$
$$\frac{x^2}{4} + \frac{y^2}{4} = 1$$
$$\frac{x^2}{4} + \left(\frac{y}{2}\right)^2 = 1$$
$$\frac{m^2}{4} + n^2 = 1$$ A1

d. C has the coordinates (x, y) and is the midpoint of AB.

$$x = \frac{x_1 + x_2}{2}, y = \frac{(y_1 + y_2)}{2}$$ M1

Since A and B are on the shape found in **part a.**:

$\frac{x_1^2}{4} + y_1^2 = 1$ and $\frac{x_2^2}{4} + y_2^2 = 1$

Adding the two equations together gives:

$$\frac{x_1^2 + x_2^2}{4} + y_1^2 + y_2^2 = 2$$

Completing the square to $(x_1^2 + x_2^2)$ and $(y_1^2 + y_2^2)$ gives:

$$\frac{x_1^2 + x_2^2 + 2x_1x_2}{4} - \frac{x_1x_2}{2} + y_1^2 + y_2^2 + 2y_1y_2 - 2y_1y_2 = 2$$ M1

$$\frac{(x_1 + x_2)^2}{4} + (y_1 + y_2)^2 - \left(\frac{x_1x_2}{2} + 2y_1y_2\right) = 2$$

Since $y_1y_2 = -\frac{1}{4}x_1x_2$:

$$\frac{x_1x_2}{2} + 2y_1y_2 = 0$$

$$\frac{(x_1 + x_2)^2}{4} + (y_1 + y_2)^2 = 2$$

$$\left(\frac{x_1 + x_2}{2}\right)^2 + 4 \times \left(\frac{y_1 + y_2}{2}\right)^2 = 2$$

$$x^2 + 4y^2 = 2$$

$$\frac{x^2}{2} + 2y^2 = 1$$ A1

*Note: Consequential on answer to **Question 6a**.*