



Trial Examination 2022

VCE Specialist Mathematics Units 1&2

Written Examination 1

Suggested Solutions

Question 1 (4 marks)

a. $\hat{p} = \frac{33\,500}{200}$
 $= 167.5 \text{ cm}$ A1

b. $200 \times 60\% = 120 \text{ students}$ A1

c. $\Pr(X \geq 1) = 1 - \Pr(X < 1)$ M1
 $= 1 - \Pr(X = 0)$
 $= 1 - \binom{200}{0} (0.6)^{200} (0.4)^0$
 $= 1 - \binom{200}{0} (0.6)^{200}$
 $= 1 - (0.6)^{200}$ A1

Question 2 (4 marks)

a. In triangle ABC :
 $\cos(\alpha) = \frac{8^2 + 7^2 - (\sqrt{57})^2}{2 \times 8 \times 7}$
 $\cos(\alpha) = \frac{56}{112}$
 $\cos(\alpha) = \frac{1}{2}$ M1
 $\alpha = \cos^{-1}\left(\frac{1}{2}\right)$
 $= \frac{\pi}{3}$ A1

b. In triangle ACD :
 $DC^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos(\alpha)$
 $DC^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \frac{1}{2}$
 $DC^2 = 39$ M1
 $DC = \sqrt{39}$ A1

Note: Consequential on working to Question 2a.

Question 3 (6 marks)

a. $t_3 = r^2 t_1 = 45$

$$t_7 = r^6 t_1 = 3645$$

$$\frac{t_7}{t_3} = \frac{r^6 t_1}{r^2 t_1} = \frac{3645}{45}$$

$$r^4 = 81$$

$$r = \pm 3$$

Since the sequence is positive:

$$\therefore r = 3$$

A1

$$t_9 = r^2 t_7$$

$$= 9 \times 3645$$

$$= 32\,805$$

A1

b. i. $t_1 = 1$

$$t_2 = 2t_1 + 3 = 5$$

$$t_3 = 2t_2 + 3 = 13$$

$$t_4 = 2t_3 + 3 = 29$$

A1

ii. $S_1 = t_1 = 1$

$$S_2 = t_1 + t_2 = 1 + 5 = 6$$

$$S_3 = t_1 + t_2 + t_3 = 1 + 5 + 13 = 19$$

$$S_4 = t_1 + t_2 + t_3 + t_4 = 1 + 5 + 13 + 29 = 48$$

A1

Note: Consequential on answer to Question 3b.i.

iii. $S_n = t_1 + t_2 + t_3 + \dots + t_{n-2} + t_{n-1} + t_n$

$$= t_1 + (2t_1 + 3) + (2t_2 + 3) + (2t_3 + 3) + \dots + (2t_{n-3} + 3) + (2t_{n-2} + 3) + (2t_{n-1} + 3)$$

M1

$$= t_1 + (2t_1 + 2t_2 + 2t_3 + \dots + 2t_{n-3} + 2t_{n-2} + 2t_{n-1}) + 3(n-1)$$

$$= 2S_{n-1} + t_1 + 3n - 3$$

$$= 2S_{n-1} + 3n - 2$$

A1

Question 4 (9 marks)

a. i. $z + w = 6 + 8i + 5 + 12i$
 $= 11 + 20i$

A1

ii. $zw = (6 + 8i)(5 + 12i)$
 $= 30 + 72i + 40i - 96$
 $= -66 + 112i$

A1

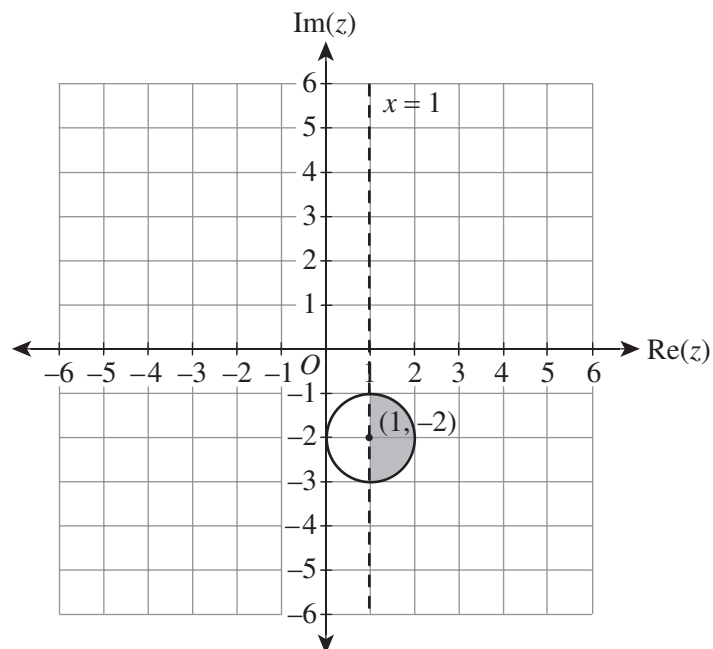
iii. $\frac{\bar{z}}{w} = \frac{6 - 8i}{5 + 12i}$
 $= \frac{(6 - 8i)(5 - 12i)}{(5 + 12i)(5 - 12i)}$
 $= \frac{-66 - 112i}{169}$
 $= -\frac{66}{169} - \frac{112}{169}i$

A1

b. $|z - 1 + 2i| \leq 1$
 $|x + yi - 1 + 2i| \leq 1$
 $\sqrt{(x - 1)^2 + (y + 2)^2} \leq 1$
 $(x - 1)^2 + (y + 2)^2 \leq 1$

M1

The area described by the inequality is the area in and on the circle centred at $(1, -2)$ with a radius of 1. The combined area is shaded in the following graph.



correct line and circle A1

correct shaded area A1

c. $z = -2 - 2i$

$$r = \sqrt{(-2)^2 + (-2)^2}$$

$$= 2\sqrt{2}$$

M1

$$\tan(\theta) = \frac{-2}{-2}$$

$$= 1$$

Since both real and imaginary components are negative, z is in the third quadrant.

$$\therefore \theta = -\frac{3\pi}{4}$$

Hence, $-2 - 2i = 2\sqrt{2}\text{cis}\left(-\frac{3\pi}{4}\right)$.

M1

$$(-2 - 2i)^{15} = (2\sqrt{2})^{15} \text{cis}\left(-\frac{3\pi}{4} \times 15\right)$$

$$= 2^{22}\sqrt{2}\text{cis}\left(-\frac{45\pi}{4}\right)$$

$$= 2^{22}\sqrt{2}\text{cis}\left(\frac{3\pi}{4}\right)$$

A1

Question 5 (4 marks)

a. $\overrightarrow{OA} = 2\mathbf{i} + 13\mathbf{j}$

$$\overrightarrow{OB} = 12\mathbf{i} + 8\mathbf{j}$$

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

$$= -\overrightarrow{OA} + \overrightarrow{OB}$$

$$= -2\mathbf{i} - 13\mathbf{j} + 12\mathbf{i} + 8\mathbf{j}$$

$$= 10\mathbf{i} - 5\mathbf{j}$$

A1

b. Let $\overrightarrow{OC} = x\mathbf{i} + y\mathbf{j}$.

Since $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{AB}$:

$$\overrightarrow{OC} \cdot \overrightarrow{AB} = 0$$

$$10x - 5y = 0$$

$$|\overrightarrow{OC}| = \frac{28}{\sqrt{5}}:$$

$$\sqrt{x^2 + y^2} = \frac{28}{\sqrt{5}}$$

$$x^2 + y^2 = \frac{28^2}{5}$$

Combining the two equations about x and y gives:

$$\begin{cases} 10x - 5y = 0 \\ x^2 + y^2 = \frac{28^2}{5} \end{cases}$$

Rearranging the first equation gives $2x = y$.

Substituting $2x = y$ into the second equation gives:

$$x^2 + (2x)^2 = \frac{28^2}{5}$$

$$5x^2 = \frac{28^2}{5}$$

$$x = \pm \frac{28}{5}$$

Since C is on AB and both A and B are in the first quadrant:

$$\therefore x = \frac{28}{5}$$

M1

$$y = 2x = \frac{56}{5}$$

The coordinates of C are $\left(\frac{28}{5}, \frac{56}{5}\right)$.

A1

Note: Consequential on answer to Question 5a.

c. $A = \frac{1}{2}|\overrightarrow{AB}||\overrightarrow{OC}|$

$$= \frac{1}{2} \times \sqrt{10^2 + (-5)^2} \times \frac{28\sqrt{5}}{5}$$

$$= 70 \text{ units}^2$$

A1

Note: Consequential on answer to Question 5a and working to Question 5b.

Question 6 (9 marks)

- a. This is a reciprocal function of $x^2 + 7x + 2k$. If it has two vertical asymptotes, the original function must have two x -intercepts.

$$\Delta > 0$$

M1

$$7^2 - 4 \times 1 \times 2k > 0$$

$$k < \frac{49}{8}$$

A1

- b. $5x^2 - 20x + 6y^2 + 72y + 206 = 0$

$$5(x^2 - 4x) + 6(y^2 + 12y) + 206 = 0$$

$$5(x^2 - 4x + 4 - 4) + 6(y^2 + 12y + 36 - 36) + 206 = 0$$

M1

$$5[(x-2)^2 - 4] + 6[(y+6)^2 - 36] + 206 = 0$$

$$5(x-2)^2 + 6(y+6)^2 = 30$$

$$\frac{(x-2)^2}{6} + \frac{(y+6)^2}{5} = 1$$

A1

The shape is an ellipse centred at (2, -6).

A1

- c. $AP = \sqrt{x^2 + (y-5)^2}$

$$BP = \sqrt{x^2 + (y+5)^2}$$

$$|AP - BP| = 2 \text{ implies } AP - BP = 2 \text{ or } BP - AP = 2.$$

For $AP - BP = 2$:

$$\sqrt{x^2 + (y-5)^2} - \sqrt{x^2 + (y+5)^2} = 2$$

$$\sqrt{x^2 + (y-5)^2} = 2 + \sqrt{x^2 + (y+5)^2}$$

$$x^2 + (y-5)^2 = 4 + 4\sqrt{x^2 + (y+5)^2} + x^2 + (y+5)^2$$

$$(y-5)^2 - (y+5)^2 = 4 + 4\sqrt{x^2 + (y+5)^2}$$

M1

$$-20y = 4 + 4\sqrt{x^2 + (y+5)^2}$$

$$-5y - 1 = \sqrt{x^2 + (y+5)^2}$$

M1

This only holds true if $-5y - 1 \geq 0$ and $y \leq -\frac{1}{5}$.

$$(-5y - 1)^2 = x^2 + (y+5)^2$$

M1

$$24y^2 - x^2 = 24$$

$$y^2 - \frac{x^2}{24} = 1$$

For $BP - AP = 2$, it can be determined that it is the other branch of the hyperbola

with $y \geq \frac{1}{5}$.

Therefore, the shape described by the locus definition is a hyperbola with the

$$\text{equation } y^2 - \frac{x^2}{24} = 1.$$

A1

Question 7 (4 marks)

- a. Expanding the expression by addition formula gives:

$$r \sin(4x + \alpha) = r \sin(4x) \cos(\alpha) + r \cos(4x) \sin(\alpha) \quad \text{M1}$$

$$r \sin(4x) \cos(\alpha) + r \cos(4x) \sin(\alpha) = \frac{3}{2} \sin(4x) + \frac{\sqrt{3}}{2} \cos(4x)$$

$$\begin{cases} r \cos(\alpha) = \frac{3}{2} \\ r \sin(\alpha) = \frac{\sqrt{3}}{2} \end{cases}$$

Dividing the second equation by the first equation gives:

$$\tan(\alpha) = \frac{\sqrt{3}}{3}$$

Since $r \cos(\alpha) > 0$, $r \sin(\alpha) > 0$, $r > 0$, α is in the first quadrant.

$$\alpha = \frac{\pi}{6} \quad \text{A1}$$

$$\begin{aligned} r \sin\left(\frac{\pi}{6}\right) &= \frac{\sqrt{3}}{2} \\ r &= \sqrt{3} \end{aligned}$$

$$\therefore \frac{3}{2} \sin(4x) + \frac{\sqrt{3}}{2} \cos(4x) = \sqrt{3} \sin\left(4x + \frac{\pi}{6}\right) \quad \text{A1}$$

$$\begin{aligned} \text{b. } \frac{3}{2} \sin(4x) + \frac{\sqrt{3}}{2} \cos(4x) &= \sqrt{3} \sin\left(4x + \frac{\pi}{6}\right) \\ &= \sqrt{3} \sin\left[2\left(2x + \frac{\pi}{12}\right)\right] \\ &= 2\sqrt{3} \sin\left(2x + \frac{\pi}{12}\right) \cos\left(2x + \frac{\pi}{12}\right) \quad \text{A1} \end{aligned}$$

Note: Consequential on answer to Question 7a.