

VCE Specialist Mathematics Units 1&2

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E

Question 1 B

$$\underline{a} = 2\underline{i} - \underline{j} \text{ and } \underline{b} = \underline{i} - 3\underline{j}.$$

$$\begin{aligned} 2\underline{b} - \underline{a} &= 2(\underline{i} - 3\underline{j}) - (2\underline{i} - \underline{j}) \\ &= 2\underline{i} - 6\underline{j} - 2\underline{i} + \underline{j} \\ &= -5\underline{j} \end{aligned}$$

Question 2 C

t_4 is the geometric mean of t_2 and t_6 .

$$\begin{aligned} t_4 &= \sqrt{6 \times 24} \\ &= 12 \end{aligned}$$

Question 3 D

Area of $\triangle ABD$:

$$\frac{1}{2} \times b \times h = 24$$

$$\frac{1}{2} \times 4 \times h = 24$$

$$h = 12$$

Area of trapezium $ABCD$:

$$\begin{aligned} \frac{1}{2}(a+b)h &= \frac{1}{2}(4+6) \times 12 \\ &= 60 \end{aligned}$$

Question 4 D

As the largest angle is opposite the longest side, use the cosine rule.

$$\begin{aligned} \theta &= \cos^{-1} \left(\frac{6^2 + 12^2 - 13^2}{2 \times 6 \times 12} \right) \\ &= 85.61897126 \end{aligned}$$

Sum of the two smallest angles:

$$\begin{aligned} 180 - 85.61897126 &= 94.38102874 \\ &\approx 94^\circ \end{aligned}$$

Question 5 D

$$\begin{aligned} y &= \frac{x}{x^2 + 3x + 2} \\ &= \frac{x}{(x+2)(x+1)} \end{aligned}$$

$$x \in \mathbb{R} \setminus \{-2, -1\}$$

Question 6 A

$$\begin{aligned} \underline{a} + \underline{b} &= \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ m \end{bmatrix} \\ &= \begin{bmatrix} 3 \\ 2+m \end{bmatrix} \end{aligned}$$

If $\underline{a} + \underline{b}$ is parallel to $6\underline{i} - \underline{j}$, then $\begin{bmatrix} 3 \\ 2+m \end{bmatrix} = k \begin{bmatrix} 6 \\ -1 \end{bmatrix}$.

$$\Rightarrow 3 = 6k$$

$$k = \frac{1}{2}$$

$$2 + m = \frac{1}{2} \times -1$$

$$m = -\frac{5}{2}$$

Question 7 A

Solving on a CAS calculator gives:

$(5+12 \cdot i)^2$	$-119+120 \cdot i$
$\text{real}((5+12 \cdot i)^2)$	-119

Question 8 C

$$\begin{aligned} |2(m\underline{i} + m)\underline{j}| &= \sqrt{(2m)^2 + (2m)^2} \\ &= \sqrt{8m^2} \end{aligned}$$

$$\text{Let } |2(m\underline{i} + m\underline{j})| = 1.$$

$$\sqrt{8m^2} = 1$$

$$8m^2 = 1$$

$$m = \frac{\sqrt{2}}{4} \quad (\text{as } m > 0)$$

Question 9 D

$$\overrightarrow{CA} + \overrightarrow{AB} = \underline{q}$$

$$2\underline{r} + \overrightarrow{AB} = \underline{q}$$

$$\overrightarrow{AB} = \underline{q} - 2\underline{r}$$

$\triangle ABC$ is right angled (inscribed in semi-circle).

$$\Rightarrow \overrightarrow{AB} \cdot \underline{q} = 0 \quad (\text{as } \overrightarrow{AB} \text{ is perpendicular to } \underline{q})$$

$$(\underline{q} - 2\underline{r}) \cdot \underline{q} = 0$$

$$\underline{q} \cdot \underline{q} - 2\underline{r} \cdot \underline{q} = 0$$

$$\underline{q} \cdot \underline{q} = 2\underline{r} \cdot \underline{q}$$

Question 10 B

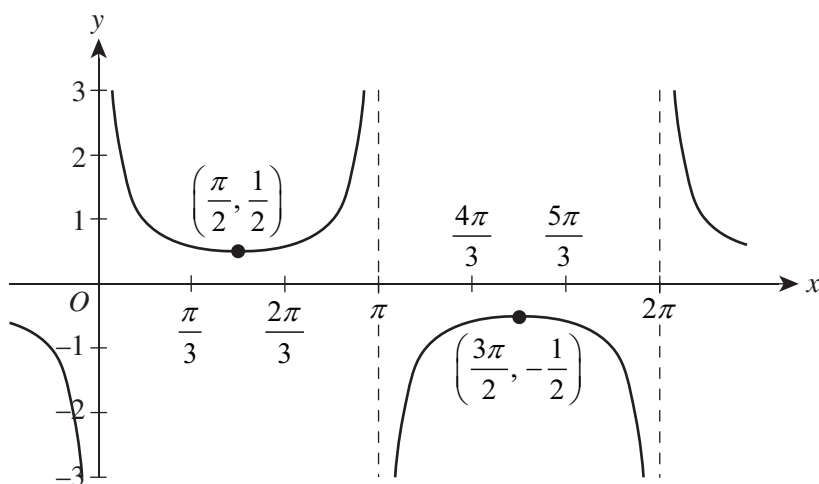
$$\frac{x+1}{x} = \frac{5}{4}$$

$$4x + 4 = 5x$$

$$x = 4 \text{ cm}$$

Question 11 E

E is correct. The maximum and minimum of $y = \frac{1}{2\sin(x)}$ occur at $\pm\frac{1}{2}$ and therefore a vertical translation of $k \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ will take the graph to the x -axis without creating an x -intercept.



A, B, C and D are incorrect. These graphs will have x -intercepts.

Question 12 B

B is correct. Sketching each option on a CAS calculator gives **B** as the only match. **A, C, D** and **E** are incorrect. These equations will not result in the graph shown.

Question 13 B

$$S_n = a \left(\frac{1-r^n}{1-r} \right)$$

$$S_8 = 2 \left(\frac{1-1.1^8}{1-1.1} \right)$$

$$= 20(1.1^8 - 1)$$

Question 14 C

$$z = 3 + 2i$$

$$\text{Im}(z) = 2$$

Question 15 D

$$b^2 - 4ac = 0$$

$$(-2ki)^2 - 4(-3k) = 0$$

$$-4k^2 + 12k = 0$$

$$-4k(k - 3) = 0$$

$$k = 0 \text{ or } 3$$

However, $k = 0$ gives a real solution. Therefore, $k = 3$ is the only solution.

$$\begin{array}{ll} \text{cSolve}(z^2 - 2 \cdot k \cdot z \cdot i - 3 \cdot k = 0, z) | k=0 & z=0 \\ \text{cSolve}(z^2 - 2 \cdot k \cdot z \cdot i - 3 \cdot k = 0, z) | k=3 & z=3 \cdot i \end{array}$$

Question 16 D

Triangles $\triangle ABC$ and $\triangle DEC$ are similar.

$$\begin{aligned} \text{area ratio} &= \left(\frac{BC}{EC} \right)^2 \\ &= \left(\frac{5}{3} \right)^2 \\ &= \frac{25}{9} \end{aligned}$$

Question 17 C

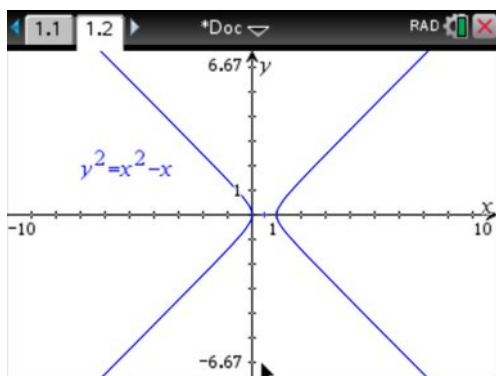
$$a \cdot b = 0$$

$$m^2 + n - n^2 = 0$$

$$m^2 = n^2 - n$$

As $m^2 \geq 0$, $0 < n < 1$ is not possible.

Graphing the relation $y^2 = x^2 - x$ may be useful to visualise solutions to $m^2 = n^2 - n$.



Question 18 D

$$\begin{aligned}\sqrt{(x-1)^2 + (y-2)^2} &= \sqrt{(x-0)^2 + (y-(-3))^2} \\ x^2 - 2x + 1 + y^2 - 4y + 4 &= x^2 + y^2 + 6y + 9 \\ -2x - 4 &= 10y \\ y &= -\frac{1}{5}x - \frac{2}{5}\end{aligned}$$

Question 19 D

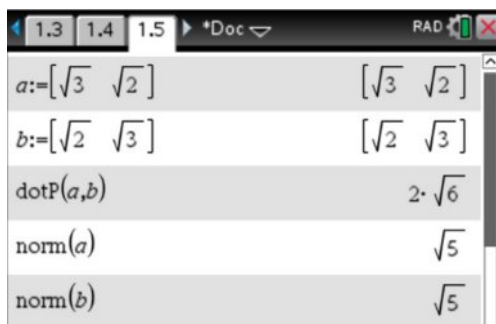
$$\begin{aligned}2(x^2 + 2ax) + y^2 - 2by + c &= 0 \\ 2((x+a)^2 - a^2) + (y-b)^2 - b^2 + c &= 0 \\ 2(x+a)^2 + (y-b)^2 &= 2a^2 + b^2 - c\end{aligned}$$

As the centre of the ellipse is (2, 1), $a = -2$, $b = -1$.

$$\begin{aligned}2a^2 + b^2 - c &= 1 \\ 2(-2)^2 + (-1)^2 - c &= 1 \\ c &= 8\end{aligned}$$

Question 20 C

$$\cos(\theta) = \frac{a \cdot b}{|a||b|}$$



$a := [\sqrt{3} \quad \sqrt{2}]$	$[\sqrt{3} \quad \sqrt{2}]$
$b := [\sqrt{2} \quad \sqrt{3}]$	$[\sqrt{2} \quad \sqrt{3}]$
$\text{dotP}(a, b)$	$2 \cdot \sqrt{6}$
$\text{norm}(a)$	$\sqrt{5}$
$\text{norm}(b)$	$\sqrt{5}$

$$\begin{aligned}\cos(\theta) &= \frac{2\sqrt{6}}{\sqrt{5} \times \sqrt{5}} \\ &= \frac{2\sqrt{6}}{5}\end{aligned}$$

$$\begin{aligned}\sin^2(\theta) + \cos^2(\theta) &= 1 \\ \Rightarrow \tan^2(\theta) &= \sec^2(\theta) - 1\end{aligned}$$

$$\begin{aligned}\tan^2(\theta) &= \left(\frac{5}{2\sqrt{6}}\right)^2 - 1 \\ &= \frac{1}{24} \\ \tan(\theta) &= \frac{1}{\sqrt{24}} \\ &= \frac{\sqrt{6}}{12}\end{aligned}$$

SECTION B**Question 1** (5 marks)

a. $-\frac{x}{2} = \cos(t) \quad \frac{y}{3} = \sin(t)$

M1

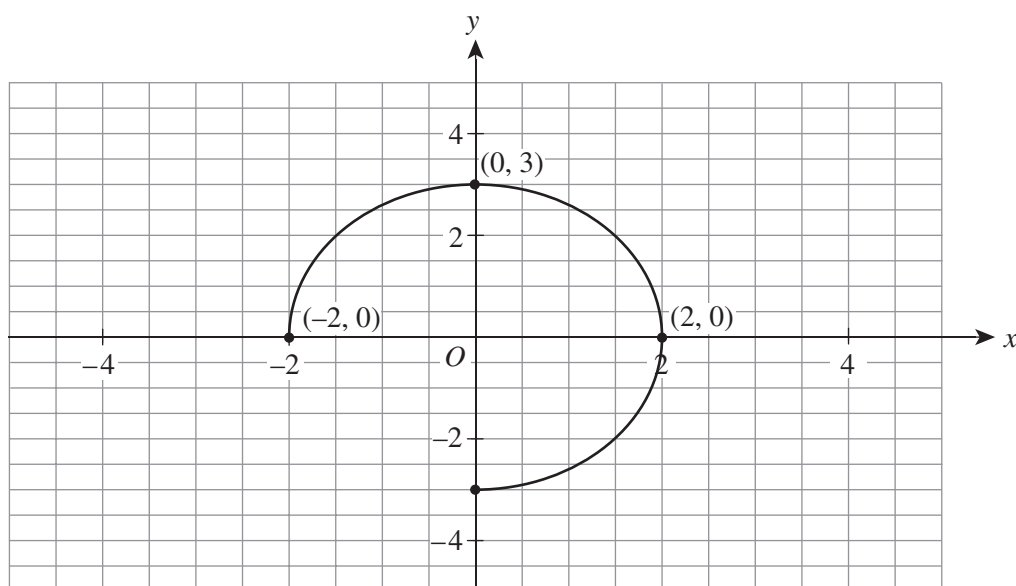
$$\sin^2(t) + \cos^2(t) = 1$$

$$\left(-\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

A1

b.



correct intercepts A1

correct endpoints A1

correct shape A1

*Note: Consequential on answer to **Question 1a**.***Question 2** (13 marks)

a. i. $\overrightarrow{AB} = -30\mathbf{i} + 50\mathbf{j}$

A1

ii. $|\overrightarrow{AB}| = \sqrt{(-30)^2 + (50)^2}$
 $= 10\sqrt{34} \text{ m}$

A1

- b.** $\overrightarrow{AB} \cdot \overrightarrow{OA} = 80 \times -30 + 40 \times 50$
 $= -400$ A1
 $\cos(\theta) = \frac{\overrightarrow{AB} \cdot \overrightarrow{OA}}{|\overrightarrow{AB}| |\overrightarrow{OA}|}$
 $|\overrightarrow{AB}| = 10\sqrt{34} \quad |\overrightarrow{OA}| = 40\sqrt{5}$
 $\cos(\theta) = \frac{-400}{10\sqrt{34} \times 40\sqrt{5}}$
 $= -\frac{1}{\sqrt{170}}$ M1
 $\theta = \cos^{-1}\left(-\frac{1}{\sqrt{170}}\right)$
 $\approx 94^\circ$
 $\angle OAB = 180^\circ - 94^\circ$
 $= 86^\circ$ A1
- c.** $\overrightarrow{OC} = 60\mathbf{i} + 30\mathbf{j} \quad \overrightarrow{CA} = 20\mathbf{i} + 10\mathbf{j}$ M1
 As $\overrightarrow{OC} = 3\overrightarrow{CA}$, O , C and A are all collinear. Hence, the student is standing on the footpath. A1
- d.** $\overrightarrow{BP} = (2p - 50)\mathbf{i} + (p - 90)\mathbf{j}$ A1
- e. i.** Minimum distance occurs when $\overrightarrow{BP}$ is perpendicular to $\overrightarrow{OA}$.
 Let $\overrightarrow{BP} \cdot \overrightarrow{OA} = 0$. M1
 $(2p - 50) \times 80 + (p - 90) \times 40 = 0$
 $p = 38$ A1
 The coordinates are $P(76, 38)$. A1
- ii.** $\overrightarrow{BP} = (2 \times 38 - 50)\mathbf{i} + (38 - 90)\mathbf{j}$ M1
 $= 26\mathbf{i} - 52\mathbf{j}$
 $|\overrightarrow{BP}| + |\overrightarrow{OP}| = \sqrt{(26)^2 + (-52)^2} + \sqrt{(76)^2 + (38)^2}$
 $= 64\sqrt{5} \text{ m}$ A1

Question 3 (9 marks)

- a.** $L_{2021} = 1200$
 $L_{2022} = 0.85 \times L_{2021} + 400$
 $= 0.85 \times 1200 + 400$
 $= 1420$
 area = 1420 km^2 A1

b. $K_{2022} = 1.5 \times K_{2021}$
 $= 1.5 \times 4000$
 $= 6000 \text{ km}^2$ A1

Overpopulation ratio:

$$\frac{K_{2022}}{L_{2022}} = \frac{6000}{1420}$$

$$= 4.23 \text{ kangaroos km}^{-2}$$

$$4.23 > 4$$
 A1

Thus, the kangaroo habitat will be overpopulated in 2022.

c. The maximum land available occurs when $L_{n+1} \rightarrow L_n$.

$$L = 0.85L + 400$$

$$L_{\max} = 2666\frac{2}{3}$$
 M1

$$\text{maximum number of kangaroos} = 2666\frac{2}{3} \times 4$$

$$= 10666$$
 A1

d. $4000 = 1.5 \times 4000 - k$
 $= 2000$ A1

e. $L_{2023} = 0.85 \times L_{2022} + 400$
 $= 0.85 \times 1420 + 400$
 $= 1607 \text{ km}^2$
 $\text{maximum kangaroos} = 4 \times 1607$
 $= 6428 \text{ kangaroos}$ A1

$$P_{2021} = 4000$$

$$P_{2022} = 1.5 \times P_{2021} - k$$

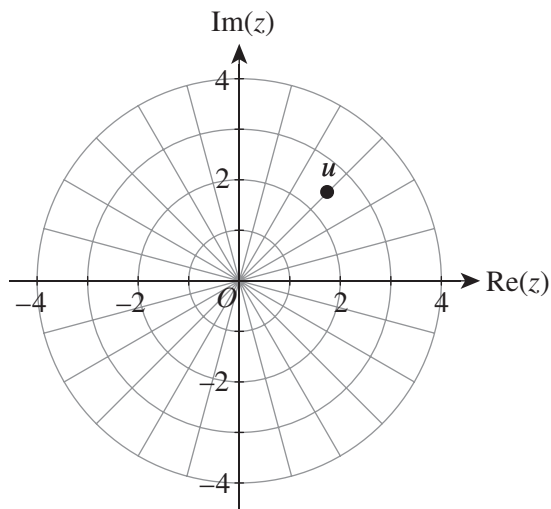
$$P_{2023} = 1.5 \times P_{2022} - k$$

$$6428 = 1.5(1.5 \times 4000 - k) - k$$
 M1

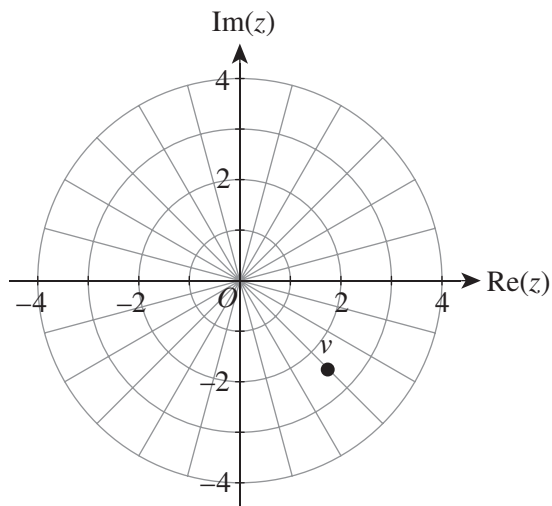
$$k = 1028.8$$

$$\approx 1029$$

Therefore, the minimum number of kangaroos that must be moved to other wildlife sanctuaries at the end of each year is 1029. A1

Question 4 (18 marks)**a.***correctly plotted point* A1**b. i.** $v = 2 - 2i$

A1

ii.*correctly plotted point* A1**iii.** $(z - (2 + 2i))(z - (2 - 2i)) = 0$

M1

$$z^2 - 4z + 8 = 0$$

$$\text{expand}((z - (2 + 2i)) \cdot (z - (2 - 2i)))$$

$$z^2 - 4z + 8$$

$$b = -4 \text{ and } c = 8$$

A1

c. i. $\text{RHS} = iv$

$$= i(2 - 2i)$$

$$= 2i - 2i^2$$

$$= 2 + 2i$$

$$= u$$

M1

ii. $n = 1 + 4k, k \in \mathbb{Z}, \text{ as } i^1 = i^5 = i^n.$

A1

$$\text{d. } u = 2\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right) \quad \text{A1}$$

$$v = 2\sqrt{2}\text{cis}\left(-\frac{\pi}{4}\right) \quad \text{A1}$$

$$u = 2 + 2 \cdot i \quad 2 + 2 \cdot i$$

$$v = 2 - 2 \cdot i \quad 2 - 2 \cdot i$$

$$u \rightarrow \text{Polar} \quad e^{\frac{i \cdot \pi}{4}} \cdot 2 \cdot \sqrt{2}$$

$$v \rightarrow \text{Polar} \quad e^{\frac{-i \cdot \pi}{4}} \cdot 2 \cdot \sqrt{2}$$

$$\text{e. i. } u - v = 4i \quad \text{A1}$$

$$|u - v| = 4$$

$$\text{ii. length of the diagonal of the square} \quad \text{A1}$$

$$\text{iii. } w = 4 + 0i \quad \text{A1}$$

$$w = 4\text{cis}(0) \quad \text{A1}$$

$$\text{f. } z(z - 4)(z^2 - 4z + 8) = 0 \quad \text{M1}$$

$$z^4 - 8z^3 + 24z^2 - 32z = 0$$

$$q = -8, r = 24, s = -32 \quad \text{A1}$$

$$\text{expand}(z \cdot (z - 4) \cdot (z^2 - 4z + 8))$$

$$z^4 - 8z^3 + 24z^2 - 32z$$

$$\text{g. The solutions are } z = 0, z = k, z = \frac{k}{2} + \frac{k}{2}i \text{ and } z = \frac{k}{2} - \frac{k}{2}i. \quad \text{M1}$$

$$z(z - k)\left(z - \left(\frac{k}{2} + \frac{k}{2}i\right)\right)\left(z - \left(\frac{k}{2} - \frac{k}{2}i\right)\right) = 0 \quad \text{M1}$$

$$(z^2 - kz)\left(z^2 - kz + \frac{k^2}{2}\right) = 0 \quad \text{A1}$$

$$z^4 - 2kz^3 + \frac{3k^2}{2}z^2 - \frac{k^3}{2}z = 0$$

Question 5 (7 marks)

a. $\angle SPQ = \angle SQP$ (isosceles triangle) M1

$\angle OPQ = \angle OQP$ (isosceles triangle)

$\angle SQO = \angle ORS$ (isosceles triangle)

$\angle OPS = \angle SPQ - \angle OPQ$ M1

$\angle ORS = \angle SQP - \angle OQP$ M1

As $\angle OPS = \angle ORS$, $PRSO$ is a cyclic quadrilateral with $\angle OPS$ and $\angle ORS$ and equal angles subtended on the same arc of the circle with the points $PRSO$ on the circumference. A1

b. Area of $\triangle OSP$:

$$A = \frac{1}{2}ab \sin(\theta)$$

$$= \frac{1}{2}OS \times OP \times \sin(\angle OSP)$$

$$OP = r$$

$$OS = \frac{1}{2}OP$$

$$= \frac{r}{2}$$

$\angle OSP = 180^\circ - \angle SRP$ M1

$$= 180^\circ - 60^\circ$$

$$= 120^\circ$$

$$A = \frac{1}{2} \times r \times \frac{r}{2} \times \sin(120^\circ)$$

$$= \frac{\sqrt{3}r^2}{8}$$
 A1

area of triangle : area of circle

$$\frac{\sqrt{3}r^2}{8} : \pi r^2$$

$$1 : \frac{8\sqrt{3}}{3}\pi$$
 A1

Question 6 (8 marks)

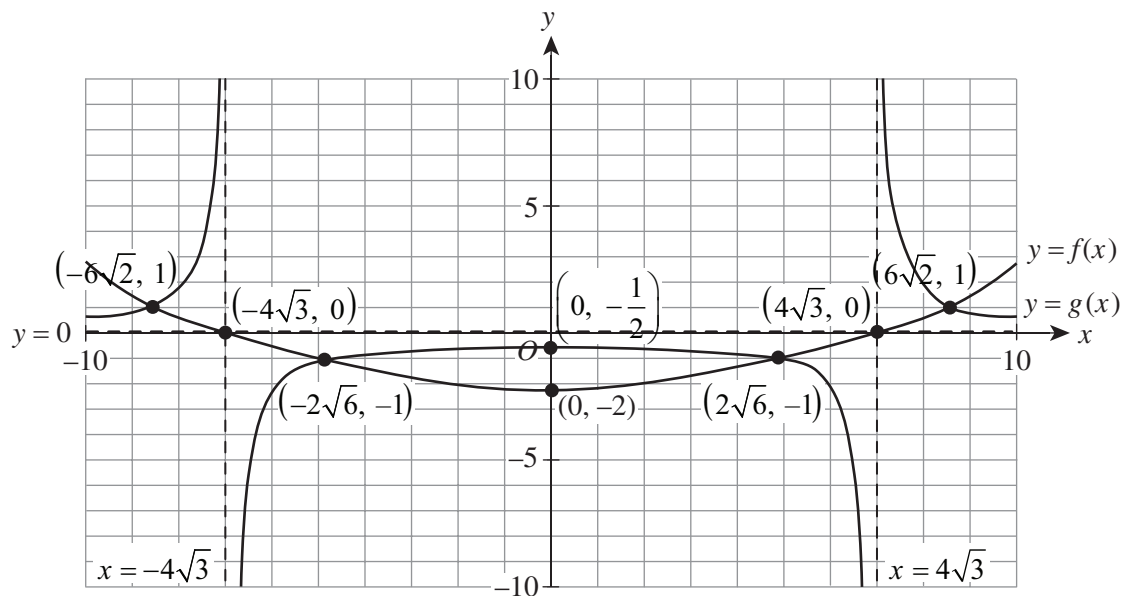
a. $\sqrt{x^2 + (y - 4)^2} = \sqrt{0^2 + (y - (-8))^2}$ M1

$$x^2 + y^2 - 8y + 16 = y^2 + 16y + 64$$

$$24y = x^2 - 48$$
 M1

$$f(x) = \frac{x^2}{24} - 2$$

b.

graph of $y = f(x)$ with correct shape and intercepts A1graph of $y = g(x)$ with correct shape and intercept A1

correct asymptotes with equations A1

four correct points of intersection A1

c.

$$\sqrt{x^2 + (y - k)^2} = \sqrt{0^2 + (y - (-2k))^2}$$

$$x^2 + y^2 - 2ky + k^2 = y^2 + 4ky + 4k^2$$

$$6ky = x^2 - 3k^2$$

$$h(x) = \frac{x^2}{6k} - \frac{k}{2}$$

M1

$$\text{Let } h(x) = \frac{1}{h(x)}.$$

As $x^2 \geq 0$ for four solutions, $3k^2 + 6k \geq 0$ and $3k^2 - 6k \geq 0$. $\Rightarrow k \in (-\infty, -2] \cup [2, \infty)$ for four solutions

$$\frac{x^2 - 3k^2}{6k} = \frac{6k}{x^2 - 3k^2}$$

$$(x^2 - 3k^2)^2 - 36k^2 = 0$$

$$(x^2 - 3k^2 - 6k)(x^2 - 3k^2 + 6k) = 0$$

$$x^2 = 3k^2 + 6k \text{ or } 3k^2 - 6k$$

Therefore, two solutions occur when $k \in (-2, 2) \setminus \{0\}$.

A1