

The Mathematical Association of Victoria

Trial Exam 2021

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers,
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 15 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the space provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$ where $g = 9.8$.

Question 1 (2 marks)

Consider the curve defined by the relation $axy^3 + bx^3y = 1$ where a and b are real constants.

Find the values of a and b if the gradient of the curve at the point $(1, 1)$ is equal to -1 .

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Question 2 (6 marks)

A 20 kg parcel sits on a set of bathroom scales in a stationary lift. The lift begins moving upwards from rest with an acceleration $\frac{1}{3}\sqrt{25 - v^2}$ where $v \text{ ms}^{-1}$ is its speed t seconds after it starts moving.

- a.** State the maximum speed of the lift.

1 mark

Once the lift reaches its maximum speed the acceleration changes to $-\frac{1}{3}\sqrt{10v - v^2} \text{ ms}^{-2}$ and the lift slows down.

- b.** Find, in terms of g and in units of Newton, the magnitude of the force exerted on the scales by the parcel when the lift is slowing down and has a speed of 2 ms^{-1} .

2 marks

- c. Find, in terms of g and in units of Newton, the magnitude of the force exerted on the scales by the parcel $\frac{\pi}{2}$ seconds after the lift starts slowing down.

3 marks

[illegible]

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Question 3 (3 marks)

Let $z^4 + az^3 + bz^2 - 4z + 6 = 0$, $z \in C$, where a and b are real constants.

Given that $z = i$ is a solution to the equation, find all other solutions.

[illegible]

Question 5 (5 marks)

Let the function $f(x) = \frac{1}{2 + e^{-x}}$.

- a.** Find the equations of all asymptotes of the function.

2 marks

- b.** Prove that the function has a point of inflection and find the x -coordinate of that point.

3 marks

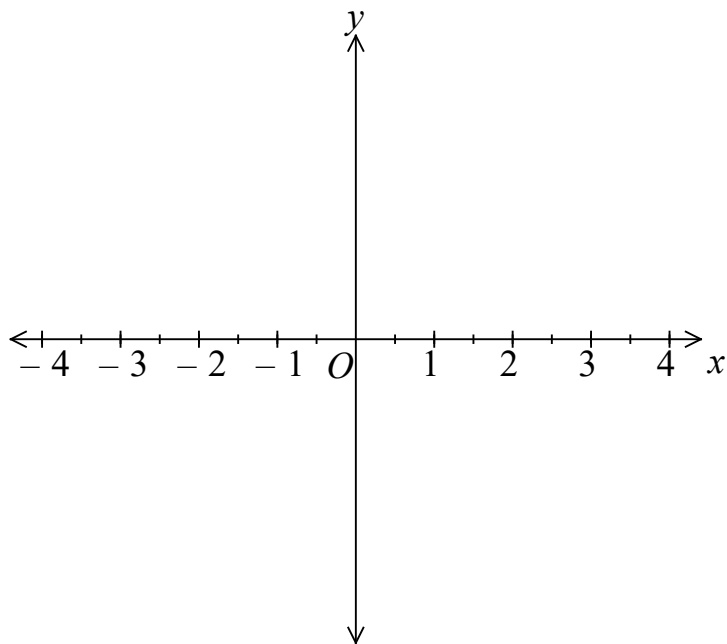
[illegible]

Question 7 (3 marks)

- a. Let the function $f : (-3, 3) \rightarrow \mathbb{R}$, $f(x) = \operatorname{cosec}\left(\frac{\pi x}{3}\right) + 1$.

Sketch the graph of $y = f(x)$ on the axes provided below, labelling any intercepts and endpoints with their coordinates and any asymptotes with their equation.

1 mark

**Working space**

b. Solve the equation $f(x) = \frac{|x+2|}{x+2}$ for x .

2 marks

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