

Victorian Certificate of Education
2020

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

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SPECIALIST MATHEMATICS
Written examination 1

Thursday 19 November 2020

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 10 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

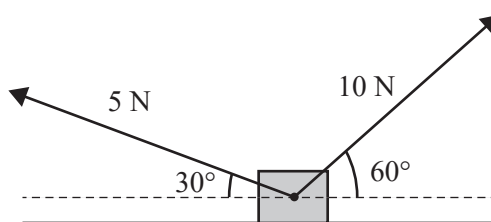
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$

Question 1 (5 marks)

A 2 kg mass is initially at rest on a smooth horizontal surface. The mass is then acted on by two constant forces that cause the mass to move horizontally. One force has magnitude 10 N and acts in a direction 60° upwards from the horizontal, and the other force has magnitude 5 N and acts in a direction 30° upwards from the horizontal, as shown in the diagram below.



- a. Find the normal reaction force, in newtons, that the surface exerts on the mass. 2 marks

- b. Find the acceleration of the mass, in ms^{-2} , after it begins to move. 2 marks

- c. Find how far the mass travels, in metres, during the first four seconds of motion. 1 mark

Question 2 (4 marks)

Evaluate $\int_{-1}^0 \frac{1+x}{\sqrt{1-x}} dx$. Give your answer in the form $a\sqrt{b} + c$, where $a, b, c \in \mathbb{R}$.

Question 3 (3 marks)

Find the cube roots of $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$. Express your answers in polar form using principal values of the argument.

Solve the inequality $3 - x > \frac{1}{|x - 4|}$ for x , expressing your answer in interval notation.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 5 (4 marks)

Let $\underline{a} = 2\underline{i} - 3\underline{j} + \underline{k}$ and $\underline{b} = \underline{i} + m\underline{j} - \underline{k}$, where m is an integer.

The vector resolute of $\underline{a}$ in the direction of $\underline{b}$ is $-\frac{11}{18}(\underline{i} + m\underline{j} - \underline{k})$.

- a.** Find the value of m .

3 marks

- b.** Find the component of $\underline{a}$ that is perpendicular to $\underline{b}$.

1 mark

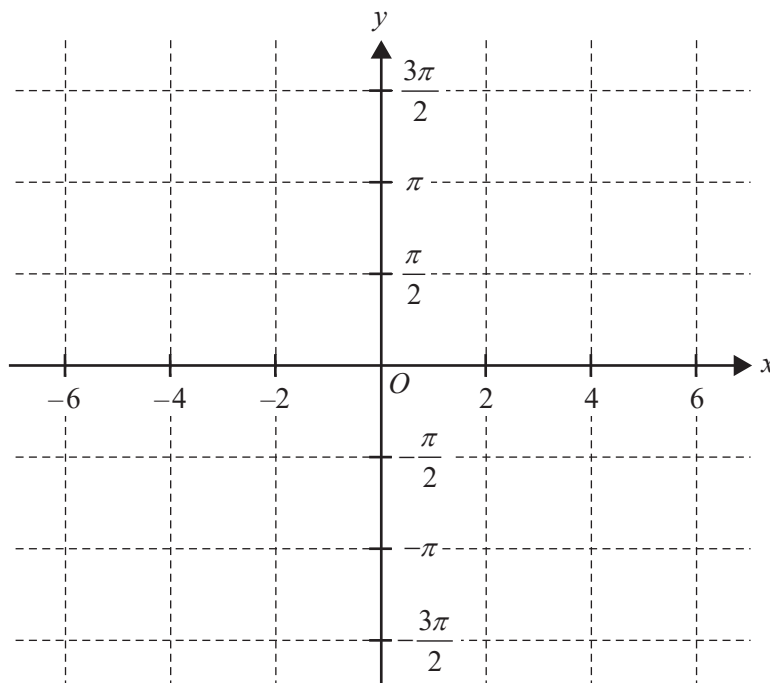
Question 6 (5 marks)

Let $f(x) = \arctan(3x - 6) + \pi$.

- a. Show that $f'(x) = \frac{3}{9x^2 - 36x + 37}$. 1 mark

- b. Hence, show that the graph of f has a point of inflection at $x = 2$. 2 marks

- c. Sketch the graph of $y = f(x)$ on the axes provided below. Label any asymptotes with their equations and the point of inflection with its coordinates. 2 marks



Question 7 (5 marks)

Consider the function defined by

$$f(x) = \begin{cases} mx + n, & x < 1 \\ \frac{4}{1+x^2}, & x \geq 1 \end{cases}$$

where m and n are real numbers.

- a.** Given that $f(x)$ and $f'(x)$ are continuous over R , show that $m = -2$ and $n = 4$. 2 marks

- b.** Find the area enclosed by the graph of the function, the x -axis and the lines $x = 0$ and $x = \sqrt{3}$. 3 marks

Find the volume, V , of the solid of revolution formed when the graph of $y = 2\sqrt{\frac{x^2 + x + 1}{(x+1)(x^2 + 1)}}$ is rotated about the x -axis over the interval $[0, \sqrt{3}]$. Give your answer in the form $V = 2\pi(\log_e(a) + b)$, where $a, b \in \mathbb{R}$.

[illegible]

Question 9 (5 marks)

Consider the curve defined parametrically by

$$x = \arcsin(t)$$

$$y = \log_e(1+t) + \frac{1}{4}\log_e(1-t)$$

where $t \in [0, 1)$.

- a.** $\left(\frac{dy}{dt}\right)^2$ can be written in the form $\frac{1}{a(1+t)^2} + \frac{1}{b(1-t)^2} + \frac{1}{c(1-t)^2}$, where a , b and c are real numbers.

Show that $a = 1$, $b = -2$ and $c = 16$.

2 marks

- b.** Find the arc length, s , of the curve from $t = 0$ to $t = \frac{1}{2}$. Give your answer in the form $s = \log_e(m) + n \log_e(p)$, where $m, n, p \in \mathcal{Q}$.

3 marks

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SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc \sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r \operatorname{cis}(\theta)$	
$ z = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a} \log_e ax+b + c$
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+(f'(x))^2} dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$

Vectors in two and three dimensions

$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$
$ \underline{r} = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$
$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underline{p} = m\underline{v}$
equation of motion	$\underline{R} = m\underline{a}$