Neap

Trial Examination 2020

VCE Specialist Mathematics Units 1&2

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

1	A	**B**	C	D	E
2	A	B	**C**	D	E
3	A	B	C	**D**	E
4	A	B	**C**	D	E
5	A	B	C	D	**E**
6	A	B	C	D	**E**
7	A	B	C	**D**	E
8	A	B	**C**	D	E
9	A	**B**	C	D	E
10	A	B	C	**D**	E

11	A	B	C	**D**	E
12	A	B	**C**	D	E
13	A	B	C	D	**E**
14	A	B	**C**	D	E
15	A	B	C	**D**	E
16	**A**	B	C	D	E
17	A	B	**C**	D	E
18	A	B	C	**D**	E
19	**A**	B	C	D	E
20	A	B	**C**	D	E

Question 1 **B**

$$\underset{\sim}{i} + \underset{\sim}{j} = 2(\underset{\sim}{i} - \underset{\sim}{j})$$
$$= -\underset{\sim}{i} + 3\underset{\sim}{j}$$

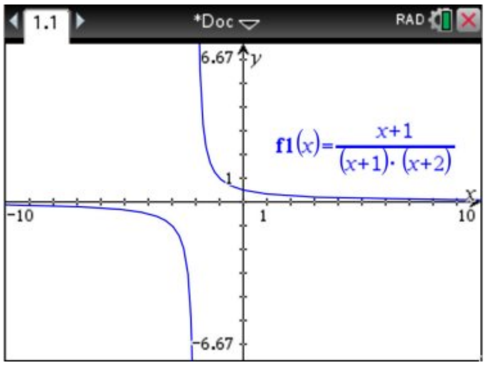
Question 2 **C**

$$r^4 = \frac{9}{1}$$
$$r = \sqrt{3}$$
$$t_6 = r^3 \times t_3$$
$$= 3\sqrt{3} \times 1$$
$$= 3\sqrt{3}$$

Question 3 **D**

$$c^2 = a^2 + b^2 - 2ab\cos(\theta)$$
$$9^2 = 7^2 + 8^2 - 2 \times 7 \times 8\cos(\theta)$$
$$\cos(\theta) = \frac{49 + 64 - 81}{112}$$
$$\theta = \cos^{-1}\left(\frac{32}{112}\right)$$
$$\approx 73°$$

Question 4 **C**



Question 5 **E**

$$\underset{\sim}{a} + \underset{\sim}{b} = (m+4)\underset{\sim}{i} + (2m+4)\underset{\sim}{j}$$

Solving $x = 4 + m$ and $y = 2m + 4$ simultaneously gives infinite solutions for x, y and therefore the solution is vectors $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 9 \\ 12 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$.

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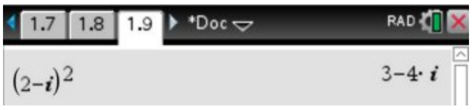
Question 6 **E**



Question 7 **D**

$$\left|m\underset{\sim}{i} - 2m\underset{\sim}{j}\right| = 1$$
$$\sqrt{m^2 + (-2m)^2} = 1$$
$$m^2 + 4m^2 = 1$$
$$m^2 = \frac{1}{5}$$
$$m = \pm\frac{\sqrt{5}}{5}$$

Question 8 **C**

$OB = r,\ OC = 2,\ \angle BOC = 120°$

$$c^2 = a^2 + b^2 - 2ab\cos(\theta)$$
$$BC^2 = r^2 + r^2 - 2r \times r \times \cos(120°)$$
$$= 3r^2$$
$$BC = \sqrt{3}r$$

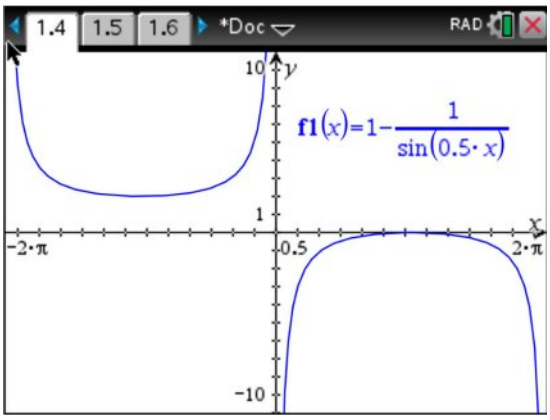
Question 9 **B**

$$9 \times 4 = x(x+9)$$
$$36 = x^2 + 9x$$
$$x^2 + 9x - 36 = 0$$
$$(x+12)(x-3) = 0$$
$$x = 3 \text{ as } x > 0$$

Question 10 **D**

$$S_\infty = \frac{a}{1-r}$$
$$= 10a$$
$$10 = \frac{1}{1-r}$$
$$r = \frac{9}{10}$$

Question 11 **D**

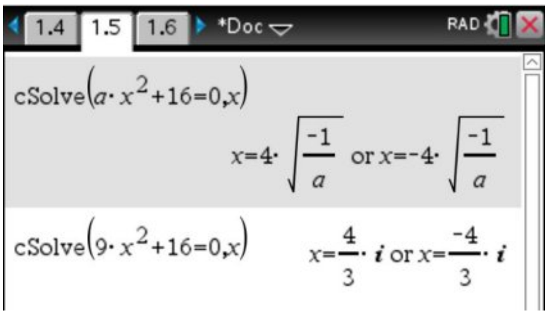


Question 12 **C**

$$z = \frac{3}{2} - 2i$$

$$\text{Im}(z) = -2$$

Question 13 **E**



Question 14 **C**

$$\sqrt{(x-0)^2+(y-6)^2} = \sqrt{(x-2)^2+(y-0)^2}$$

$$x^2 + y^2 - 12y + 36 = x^2 - 4x + 4 + y^2$$

$$12y = 4x + 32$$

$$y = \frac{1}{3}x + \frac{8}{3}$$

Question 15 **D**

$$x^2 + (y-b)^2 - b^2 + c = 0$$

$$x^2 + (y-b)^2 = b^2 - c$$

$$b^2 - c > 0$$

$$c < b^2$$

Question 16 **A**

$$\angle BAC = \frac{1}{2} \times \angle BOC \text{ (subtended angle)}$$

$$\angle BAC = 50°$$

$$\text{obtuse } \angle BAC = 260°$$

$$\angle OBA = 360 - 260 - 50 - 20$$

$$= 30°$$

Question 17 **C**

$$\underset{\sim}{a} . \underset{\sim}{b} = 3m \times m^2 + -6 \times -4$$

$$= 0$$

$$3m^3 = 24$$

$$m^3 = 8$$

$$m = 2$$

Question 18 **D**

$$\cos(\theta) = \frac{\underset{\sim}{a} . \underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}$$

$$= \frac{-2-3}{\sqrt{5} \times \sqrt{10}}$$

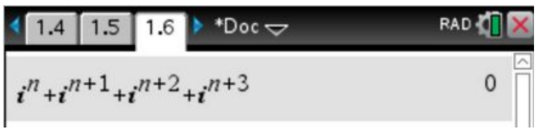
$$= \frac{-5}{\sqrt{50}}$$

$$= -\frac{\sqrt{2}}{2}$$

$$\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$$

$$= \frac{3\pi}{4}$$

Question 19 **A**



Question 20 **C**

length ratio:

$$\frac{AB'}{AB} = \frac{3}{4}$$

area ratio:

$$\left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$\begin{aligned} \text{area } \Delta AB'C' &= \frac{9}{16} \times 40 \\ &= \frac{45}{2} \text{ cm}^2 \end{aligned}$$

SECTION B

Question 1 (8 marks)

a. $r = \dfrac{4}{1 + \sin(\theta)}$

$r + r\sin(\theta) = 4$

Let $y = r\sin(\theta)$ and $x^2 + y^2 = r^2$.

$r + y = 4$

$r = 4 - y$ M1

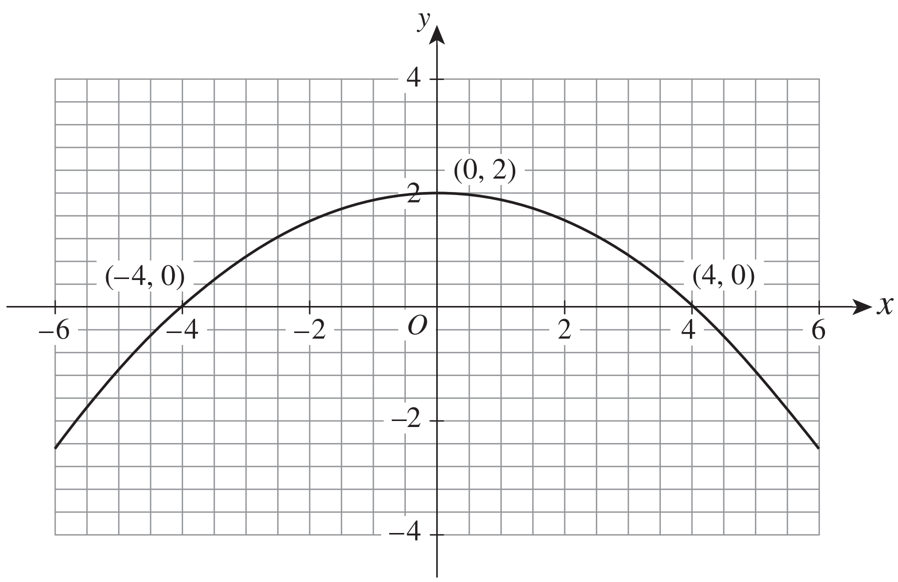
$x^2 + y^2 = (4 - y)^2$

$= 16 - 8y + y^2$

$8y = 16 - x^2$

$y = -\frac{1}{8}x^2 + 2$ A1

b.



2 marks

parabolic shape A1

labelled intercepts A1

c. $r + r\sin(\theta) = \frac{4}{n}$

Let $y = r\sin(\theta)$ and $x^2 + y^2 = r^2$. M1

$r + y = 4$

$r = 4 - y$

$$x^2 + y^2 = \left(\frac{4}{n} - y\right)^2$$

$$= \frac{16}{n^2} - \frac{8}{n}y + y^2 \quad \text{M1}$$

$$\frac{8}{n}y = \frac{16}{n^2} - x^2$$

$$y = -\frac{n}{8}x^2 + \frac{2}{n}$$

y-intercept:

$\left(0, \frac{2}{n}\right)$ A1

x-intercept:

Let $y = 0$.

$$-\frac{n}{8}x^2 + \frac{2}{n} = 0$$

$$x = \pm\frac{4}{n}$$

The x-intercepts are therefore $\left(-\frac{4}{n}, 0\right)$ and $\left(\frac{4}{n}, 0\right)$. A1

Note: Alternatively, students may use dilation of factor of $\frac{1}{n}$ as their method mark.

Question 2 (10 marks)

a. Let $z_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$.

$$r^2 = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$r = 1$$ A1

$$\theta = \tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right)$$

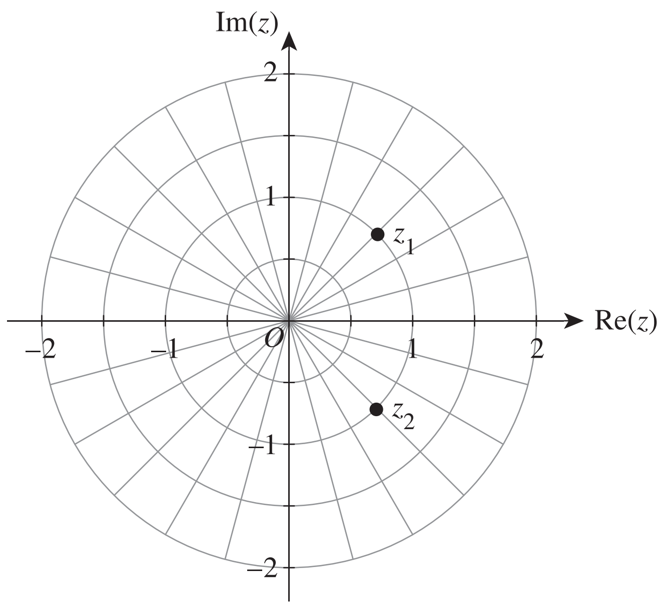
$$= \frac{\pi}{4}$$ A1

$$z_1 = \text{cis}\left(\frac{\pi}{4}\right)$$

b. **i.** $z_2 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$ A1

ii. $z_2 = \text{cis}\left(-\frac{\pi}{4}\right)$ A1

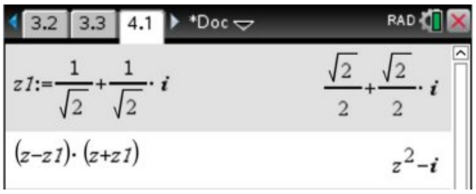
c.



correct z_1 A1
correct z_2 A1

d. **i.** $(z - z_1)(z + z_1) = \left(z - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)\right)\left(z + \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)\right)$ M1

$(z - z_1)(z + z_1) = z^2 - i$ A1



ii.

$$z^2 = i$$

$$z^2 - i = 0$$

$(z - z_1)(z + z_1) = z^2 - i$ M1

$$= 0$$

$z = z_1$ or $z = -z_1$

$z = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ or $z = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$ A1

Question 3 (9 marks)

a. $\angle PAR$ corresponding to $\angle QRC$

$\angle RPA$ corresponding to $\angle CQR$ M1

$\therefore \Delta RAP$ is similar to ΔCRQ (two angles equal in a triangle) A1

b. $PB = d - x$ A1

c. $|RQ| = |PB|$

$$= d - x$$

$\frac{|RQ|}{|AP|} = \frac{d - x}{x}$ M1

area ratio $= \frac{(d - x)^2}{x^2}$ M1

$$= \frac{d^2 - 2dx + x^2}{x^2}$$

$$= \frac{d^2}{x^2} - \frac{2d}{x} + 1$$

d. ΔABC is right-angled (inscribed in semi-circle).

$\therefore \angle RCQ = \angle ARP$

$= 90°$

$AR^2 + RP^2 = AP^2$

$1^2 + RP^2 = x^2$

$RP = \sqrt{x^2 - 1}$ M1

area of $\Delta ARP = \frac{1}{2}bh$

$= \frac{1}{2} \times 1 \times \sqrt{x^2 - 1}$

$= \frac{\sqrt{x^2 - 1}}{2}$ M1

area ratio $= \frac{(d-x)^2}{x^2}$

$d = 4 \rightarrow$ area ratio $= \frac{(4-x)^2}{x^2}$ M1

area of $\Delta CRQ = \frac{\sqrt{x^2 - 1}(4-x)^2}{2x^2}$ A1

Question 4 (11 marks)

a. **i.** $\overrightarrow{OA} = 10\underset{\sim}{i} + 18\underset{\sim}{j}$ and $\overrightarrow{OB} = 20\underset{\sim}{i} + 6\underset{\sim}{j}$. A1

ii. $\overrightarrow{AB} = \overrightarrow{OA} - \overrightarrow{OB}$

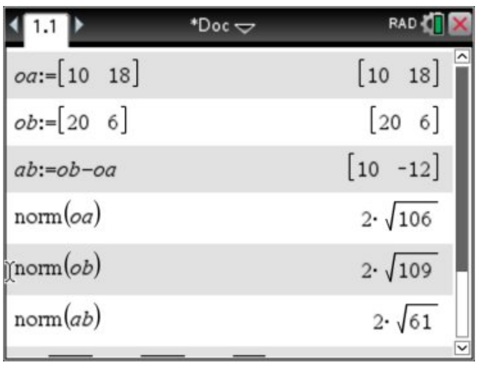
$= (20 - 10)\underset{\sim}{i} + (6 - 18)\underset{\sim}{j}$ M1

$= 10\underset{\sim}{i} - 12\underset{\sim}{j}$ as required

b. $|\overrightarrow{OA}| = 2\sqrt{106}$ $\quad |\overrightarrow{OB}| = 2\sqrt{109}$ $\quad |\overrightarrow{AB}| = 2\sqrt{61}$ A1

distance $= |\overrightarrow{OA}| + |\overrightarrow{OB}| + |\overrightarrow{AB}|$

$= 57$ km A1

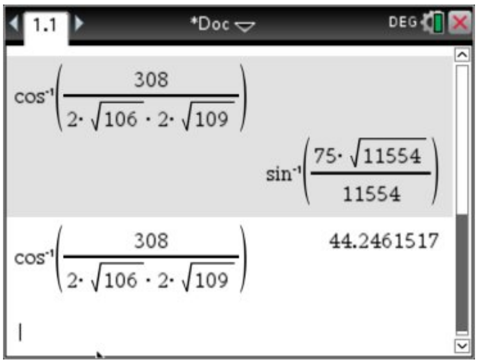


c. **i.** $\overrightarrow{OA} \cdot \overrightarrow{OB} = 308$ A1

ii. $\theta = \cos^{-1}\left(\dfrac{\overrightarrow{OA} \cdot \overrightarrow{OB}}{\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|}\right)$

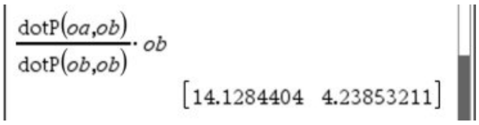
$= \cos^{-1}\left(\dfrac{308}{2\sqrt{106} \times 2\sqrt{109}}\right)$ M1

$\approx 44°$ A1



d. $\underset{\sim}{u} = \dfrac{\underset{\sim}{a}.\underset{\sim}{b}}{\underset{\sim}{b}.\underset{\sim}{b}}\underset{\sim}{b}$ M1

$= 14.1\underset{\sim}{i} + 4.2\underset{\sim}{j}$ A1



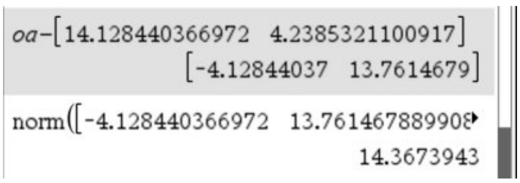
e. $\underset{\sim}{v} = \overrightarrow{OA} - \underset{\sim}{u}$

$= 10\underset{\sim}{i} + 18\underset{\sim}{j} - (14.128\ldots\underset{\sim}{i} + 4.238\ldots\underset{\sim}{j})$

$= -4.128\ldots\underset{\sim}{i} + 13.76\ldots\underset{\sim}{j}$ M1

minimum distance $= \left|\underset{\sim}{v}\right|$

$\left|\underset{\sim}{v}\right| = 14.4$ km A1



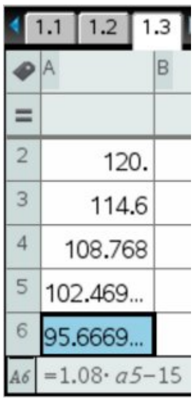
Question 5 (12 marks)

a. **i.** $125 - 4 \times 15 = 65$ A1

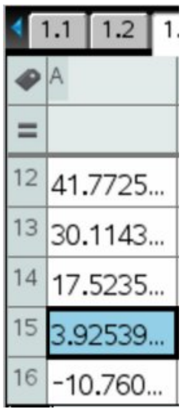
ii. $\dfrac{125}{15} = 8.33\ldots$

$\therefore 8$ years A1

b. **i.** $C_5 = 96$ A1



ii. 14 years A1



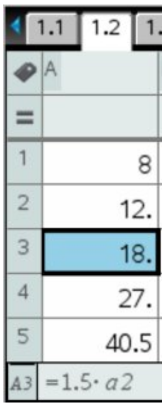
c. $m = 125 \times 0.08$

$= 10$ A1

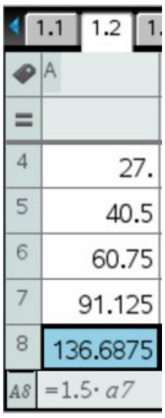
$n = 1 + \dfrac{15}{125}$

$= 1.12$ A1

d. **i.** 18 ducks after 2 years A1



ii. 7 years A1



e. 8×1.5^n A1

f. $D_{n+1} = 1.5D_n,\ D_0 = 8$ A1

g. after 5 years: $C_5 = 96$ and $D_5 = 73$

after 6 years: $C_5 = 88$ and $D_5 = 106$ M1

Therefore there will be more ducks than chickens after 6 years. A1

Question 6 (10 marks)

a. $y = \frac{x^2}{4a}$ and $x = 2at$

$y = \frac{(2at)^2}{4a} = \frac{4a^2t^2}{4a} = at^2$ as required A1

b. **i.** $y - y_1 = m(x - x_1)$

$y - \frac{a}{t^2} = t\left(x - -\frac{2a}{t}\right)$ M1

$= tx + 2a$

$y = tx + 2a + \frac{a}{t^2}$ A1

ii. $m_{RP} = -\frac{1}{m_{QR}}$

$= -\frac{1}{t}$

$y - y_1 = m(x - x_1)$

$y - at^2 = -\frac{1}{t}(x - 2at)$ M1

$y = -\frac{1}{t}x + 2a + at^2$ A1

iii. Let $y_{QR} = y_{RP} \to tx + 2a + \frac{a}{t^2} = -\frac{1}{t}x + 2a + at^2$

$(t^2 + 1)x = at^3 - \frac{a}{t}$

$x = \dfrac{\frac{at^4 - a}{t}}{t^2 + 1}$

$= \frac{a(t^4 - 1)}{t(t^2 + 1)}$

$= \frac{a(t^2 - 1)(t^2 + 1)}{t(t^2 + 1)}$ M1

$= a\left(t - \frac{1}{t}\right)$

$y = -\frac{1}{t}x + 2a + at^2$

$= ta\left(t - \frac{1}{t}\right) + 2a + \frac{a}{t^2}$ M1

$= at^2 - a + 2a + \frac{a}{t^2}$

$x = a\left(t - \frac{1}{t}\right),\ y = a\left(t^2 + 1 + \frac{1}{t^2}\right)$ A1

c. $x = a\left(t - \dfrac{1}{t}\right)$

$x^2 = a^2\left(t^2 - 2 + \dfrac{1}{t^2}\right)$ M1

$= a^2\left(t^2 + 1 + \dfrac{1}{t^2} - 3\right)$

$= a.a\left(t^2 + 1 + \dfrac{1}{t^2}\right) - 3a^2$

$= ay - 3a^2$

$ay = x^2 + 3a^2$

$y = \dfrac{x^2}{a} + 3a$ A1