

Trial Examination 2020

VCE Specialist Mathematics Units 1&2

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of booklet

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
7	7	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 8 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 (7 marks)

- a. A difference equation has the rule $t_{n+1} = 4t_n - 2$, $t_1 = 3$.

Find the value of t_3 .

2 marks

- b. The first, second, and third terms of a geometric sequence are $x - 2$, x and $x + 3$.

- i. Find the value of x .

2 marks

- ii. Find the common ratio between the terms.

1 mark

- iii. Show that the sum of the first n terms in the sequence can be expressed

as $S_n = \frac{3^n}{2^{n-3}} - 8$.

2 marks

Question 2 (7 marks)Let $f(x) = |x - 4| + |x - 6| - 4$.**a.** Find $f(0)$.

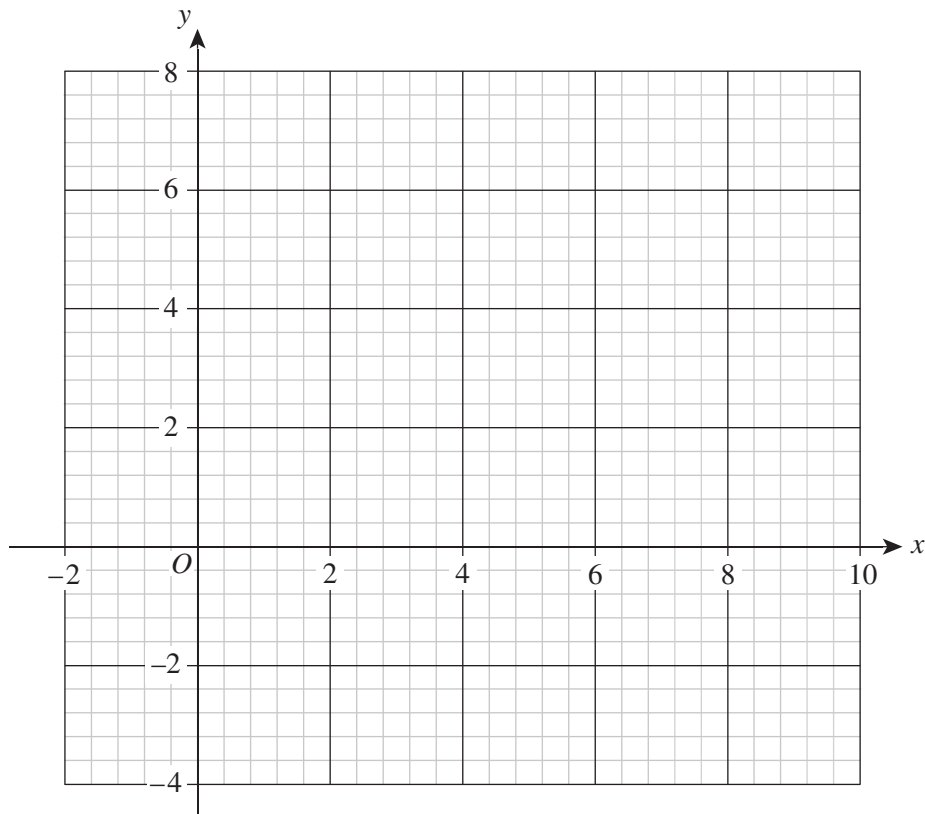
1 mark

b. Solve $f(x) = 0$.

3 marks

c. Sketch the graph of $y = f(x)$ on the set of axes below, showing coordinates of all intercepts and cusp points.

3 marks



Question 3 (6 marks)

A curve is defined by the parametric equations $x = 2^t$ and $y = 2^{t+2} - 5$ for $t \in [0, 3)$.

- a. State the domain and range of the curve.

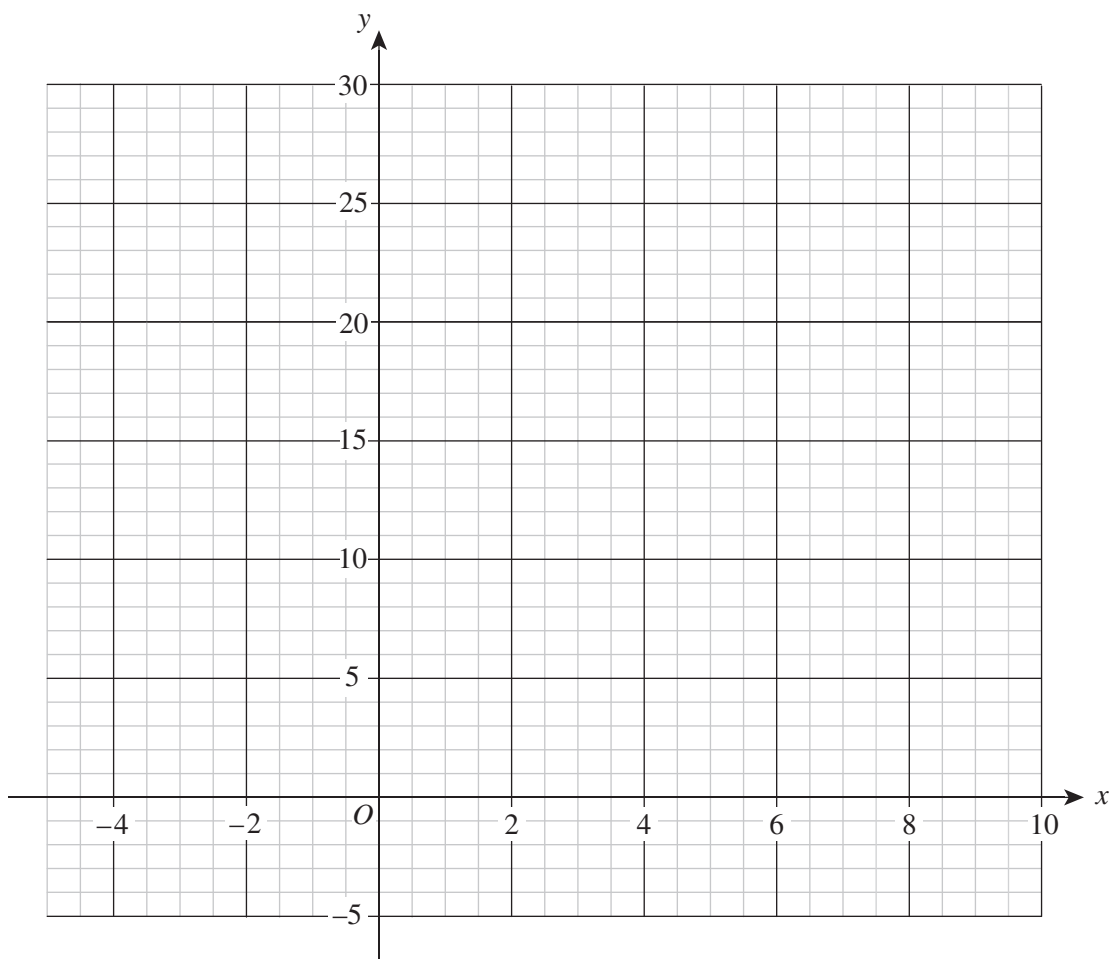
2 marks

- b. Find cartesian equation of the curve.

2 marks

- c. Graph the curve over the required domain on the axes below, showing the coordinates of any end points.

2 marks



Question 4 (5 marks)

- a.** Prove by contradiction that $\sqrt{x} > 2x$, when $0 < x < \frac{1}{4}$. 2 marks

- b.** Prove by mathematical induction that $2^{3n} - 1$ is divisible by 7 for all $n \in \mathbb{N}$. 3 marks

Question 5 (3 marks)

Given two fixed points $A(1, 2)$ and $B(2, 6)$, find the locus of P if the gradient of AP is **half** the gradient of AB .

Question 6 (7 marks)

The position vectors of two points A and B relative to an origin O are $10\hat{i} + 20\hat{j}$ and $25\hat{i} + 5\hat{j}$ respectively.

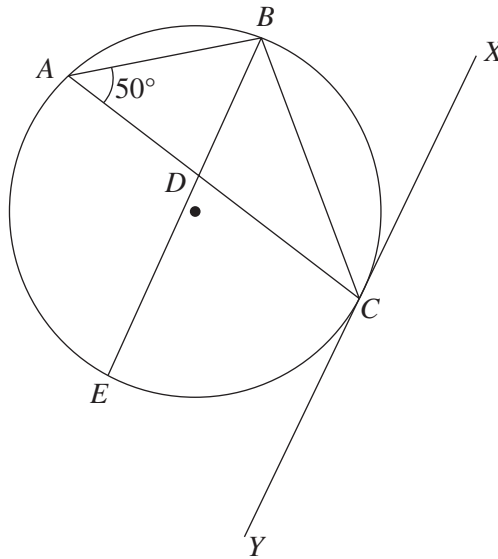
- a. i. Given that $\vec{OC} = \frac{1}{10}\vec{OA}$ and $\vec{OD} = \frac{1}{5}\vec{OB}$, state the position vectors $\vec{OC}$ and $\vec{OD}$. 2 marks

- ii. Hence, find $|\vec{CD}|$. 2 marks

- b. Given that $\vec{OE} = p\vec{OA}$ and $\vec{OF} = p\vec{OB}$, find the value of p such that $\vec{EF}$ is a unit vector. Give your answer in the form $\frac{\sqrt{a}}{b}$, where a and b are integers. 3 marks

Question 7 (5 marks)

In the diagram below, the triangle ABC is inscribed in a circle, and the tangent to the circle at C is parallel to the line BE . The line BE is also the bisector of angle ABC . The point D is the intersection of AC and BE .



- a. Find the magnitude of $\angle BCX$. 1 mark

- b. Find the magnitude of $\angle CBD$. 1 mark

- c. Find the magnitude of $\angle ABC$. 1 mark

- d. If $AD = 4$ cm and $DC = 6$ cm, find the length of ED if it is known that ED is twice as long as BD . 2 marks

END OF QUESTION AND ANSWER BOOKLET

Trial Examination 2020

VCE Specialist Mathematics Units 1&2

Written Examination 2

Formula Sheet

Instructions

This formula sheet is provided for your reference.
A question and answer booklet is provided with this formula sheet.

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SPECIALIST MATHEMATICS FORMULAS**Mensuration**

area of a trapezium	$\frac{1}{2}(a + b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc \sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos C$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2 \sin(x) \cos(x)$	

Vectors in two dimensions

$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$
$ \underline{r} = \sqrt{x^2 + y^2 + z^2} = r$
$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Polar coordinates

$x = r \cos \theta$
$y = r \sin \theta$

END OF FORMULA SHEET