



Trial Examination 2020

VCE Specialist Mathematics Units 1&2

Written Examination 1

Suggested Solutions

Question 1 (7 marks)

a. $t_2 = 4 \times 3 - 2$ M1
 $= 10$

$t_3 = 4 \times 10 - 2$
 $= 38$ A1

b. i. $\frac{x}{x-2} = \frac{x+3}{x}$ M1

$$x^2 = (x-2)(x+3)$$

$$x^2 = x^2 + x - 6$$

$$0 = x - 6$$

$x = 6$ A1

ii. sequence:

$$x-2, x, x+3$$

$$x = 6 \rightarrow 4, 6, 9$$

$$r = \frac{6}{4} = \frac{9}{6}$$

$$= \frac{3}{2}$$
 A1

iii. $S_n = \frac{a(r^n - 1)}{r - 1}$
 $= \frac{4\left(\left(\frac{3}{2}\right)^n - 1\right)}{\frac{3}{2} - 1}$ M1

$$= \frac{4\left(\frac{3}{2}\right)^n - 4}{\frac{1}{2}}$$

$$= 8\left(\frac{3}{2}\right)^n - 8$$

$$= 2^3 \cdot \frac{3^n}{2^n} - 8$$
 A1

$$= \frac{3^n}{2^{n-3}} - 8$$

Question 2 (7 marks)

a. $f(0) = |0 - 4| + |0 - 6| - 4$
 $= 6$

A1

b. $|x - 4| + |x - 6| - 4 = 0$

Case 1: $x \geq 6$

$$\begin{aligned}(x - 4) + (x - 6) - 4 &= 0 \\ 2x &= 14 \\ x &= 7\end{aligned}$$

Case 2: $4 < x < 6$

$$\begin{aligned}(x - 4) + -(x - 6) - 4 &= 0 \\ -4 + 6 - 4 &= 0 \\ -2 &= 0\end{aligned}$$

 $\therefore$ no solutionCase 3: $x \leq 4$

M1

$$\begin{aligned}-(x - 4) + -(x - 6) - 4 &= 0 \\ -2x + 6 &= 0 \\ x &= 3\end{aligned}$$

Solutions:

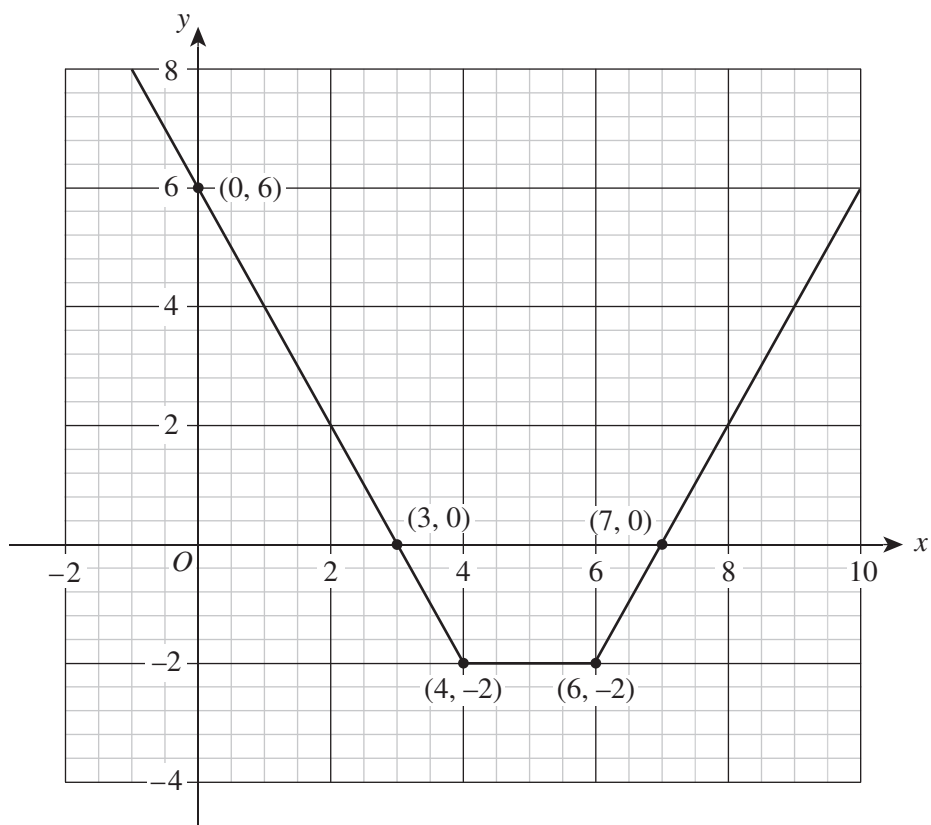
$x = 3$

A1

$x = 7$

A1

c.



3 marks

*intercept points, correct position and coordinate A1**cusp points, correct position and coordinate A1**correct shape A1***Question 3** (6 marks)

a. $t = 0 \rightarrow x = 1, y = -1$

$t = 3 \rightarrow x = 8, y = 28$

domain:

$x \in [1, 8)$

A1

range:

$y \in [-1, 27)$

A1

b. $x = 2^t$ and $y = 2^{t+2} - 5$

$y = 2^2 \times 2^t - 5$

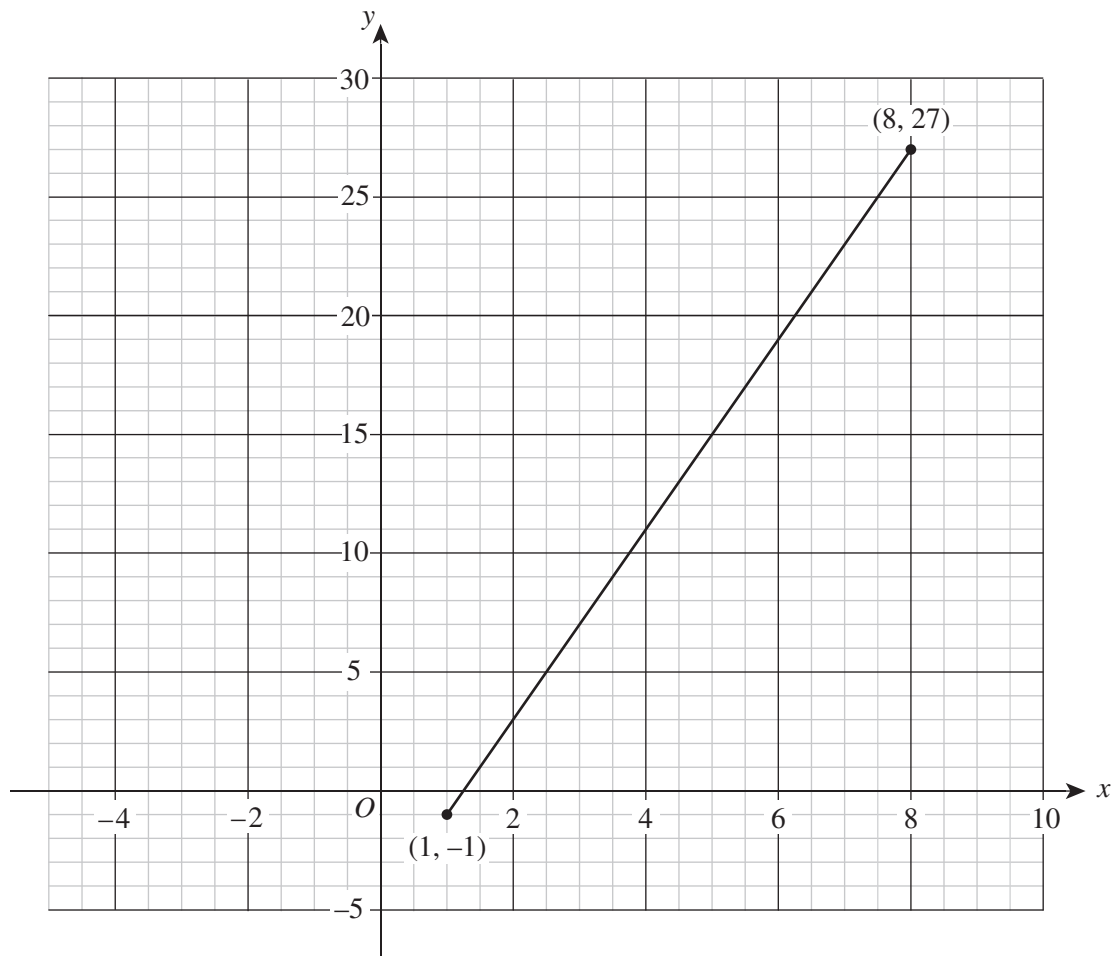
$= 4 \times 2^t - 5$

M1

$= 4x - 5$

A1

c.



2 marks

*correct endpoints A1**correct shape A1***Question 4** (5 marks)

- a. Assume the contradictory position that $\sqrt{x} \leq 2x$, for $x > 0$.

$$x \leq 4x^2$$

$$1 \leq 4x$$

M1

$$x \geq \frac{1}{4}$$

The original assumption is incorrect and therefore $0 < x < \frac{1}{4}$.

A1

b. Assume $2^{3n} - 1 = 7k$ where $n, k \in \mathbb{Z}^+$. M1

$$\text{For } n = 1 \rightarrow \text{LHS} = 2^{3 \times 1} - 1 = 7 \rightarrow k = 1$$

$\therefore$ true for $n = 1$

$$\text{For } n + 1 \rightarrow \text{LHS} = 2^{3(n+1)} - 1 \quad \text{M1}$$

$$\text{LHS} = 2^{3n+3} - 1$$

$$= 8 \times 2^{3n} - 1$$

$$= 8(2^{3n} - 1) + 7$$

$$= 8(7k) + 7$$

$$= 7(8k + 1)$$

As $8k + 1$ is an integer, $7(8k + 1)$ is divisible by 7. A1

As the rule is true for $n = 1$ and consecutive cases, the rule is true for all n for $n \in \mathbb{Z}^+$.

Question 5 (3 marks)

$$m_{AB} = \frac{6-2}{2-1}$$

$$= 4$$

$$\rightarrow m_{AP} = 0.5 \times 4$$

$$= 2$$

A1

Let the point $P(x, y)$.

$$\rightarrow m_{AP} = \frac{y-2}{x-1}$$

M1

$$\frac{y-2}{x-1} = 2$$

$$y - 2 = 2(x - 1)$$

$$y = 2x$$

A1

Question 6 (7 marks)

a. i. $\overrightarrow{OC} = i + 2j$ A1

$$\overrightarrow{OD} = 5i + j \quad \text{A1}$$

ii. $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$ M1

$$= 5i + j - (i + 2j)$$

$$|\overrightarrow{CD}| = \sqrt{4^2 + (-1)^2}$$

$$= \sqrt{17} \quad \text{A1}$$

b. $\vec{EF} = \vec{OF} - \vec{OE}$

$$= p\vec{OB} - p\vec{OA}$$

$$= p(25\hat{i} + 5\hat{j} - 10\hat{i} - 20\hat{j})$$

$$= p(15\hat{i} - 15\hat{j}) \quad \text{M1}$$

$$|\vec{EF}| = \sqrt{p^2(15)^2 + (-15)^2} \quad \text{M1}$$

$$= p\sqrt{450}$$

$$= p\sqrt{9 \times 5 \times 5 \times 2}$$

$$= 15p\sqrt{2}$$

unit vector $\frac{|\vec{EF}|}{|\vec{EF}|} = 1$

$$15p\sqrt{2} = 1$$

$$p = \frac{\sqrt{2}}{30} \quad \text{A1}$$

Question 7 (5 marks)

- a. Using the alternate segment theorem:
- $$\angle BCX = 50^\circ \quad \text{A1}$$
- b. Using alternate angles on parallel lines:
- $$\angle CBD = 50^\circ \quad \text{A1}$$
- c. As $\angle ABC$ is twice the bisected angle,
- $$\angle ABC = 2 \times 50^\circ = 100^\circ. \quad \text{A1}$$
- d. Let $ED = x \rightarrow DB = 2x$ M1
- $$ED \times DB = AD \times DC$$
- $$x \times 2x = 6 \times 4$$
- $$2x^2 = 24$$
- $$x = \sqrt{12} \quad \text{A1}$$
- $$= 2\sqrt{3} \text{ cm}$$