

The Mathematical Association of Victoria

Trial Exam 2020

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers,
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 17 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Note: This examination was written for the Adjusted 2020 VCE Mathematics Study Design and accordingly does not include the Specialist Mathematics Area of Study 6 (Probability and Statistics).

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Instructions

Answer **all** questions in the space provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$ where $g = 9.8$.

Question 1 (2 marks)

Find $\frac{dy}{dx}$ at the point where $x = 0$ for the curve defined by the relation $y^3 - xy + e^{-x} = -7$.

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c. Express g as a piecewise (hybrid) function.

1 mark

TURN OVER

Question 3 (5 marks)

a. Express $\frac{4}{4-x^2}$ in partial fraction form.

1 mark

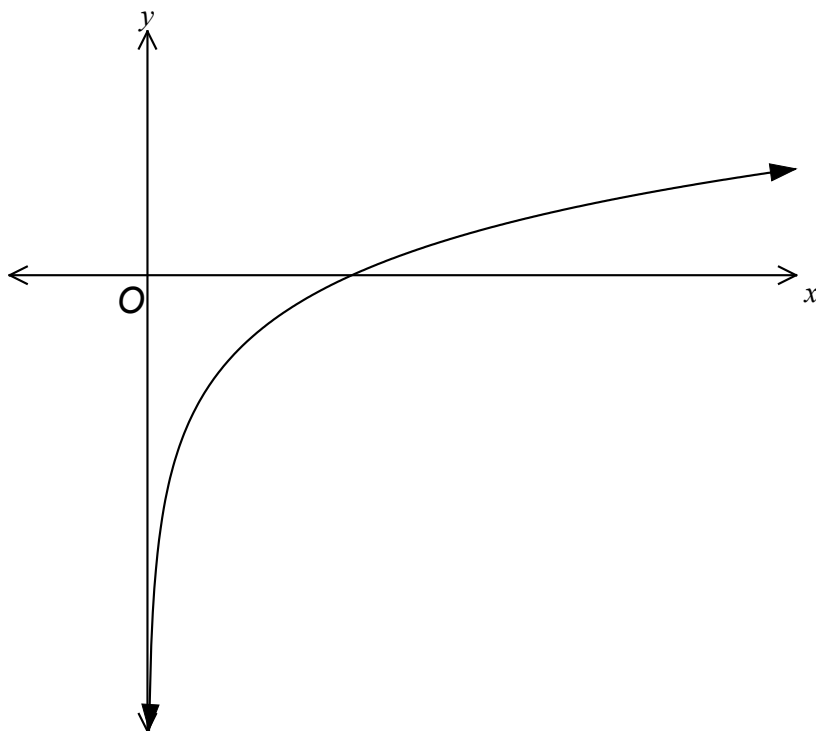
- 4 marks

[illegible]

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Question 4 (3 marks)

Part of the graph of $y = \log_2(x)$ is shown below.



Sketch the graph of $y = \frac{1}{\log_2(x)}$ on the set of axes above, given that it has a point of inflection at $x = \frac{1}{e^2}$. Clearly label its asymptotes with their equation and any endpoints and points of intersection with $y = \log_2(x)$ with their coordinates.

Working space

Question 6 (5 marks)

After being given an initial push, an object of mass 4 kg slides down a rough plane inclined at 30° to the horizontal with an acceleration of 3 ms^{-2} .

After four seconds the velocity of the object is 14 ms^{-1} .

- a. Find, in newtons, the size of the friction force acting on the object.

2 marks

Working space

Question 7 (5 marks)

- a. Write the function $f(x) = \frac{x}{\sin\left(2 \arcsin\left(\sqrt{x}\right)\right)}$, $0 < x < 1$, in the form $f(x) = \frac{a\sqrt{x}}{\sqrt{b-x}}$ where

where a and b are positive real constants.

2 marks

- 3 marks

[illegible]

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Question 8 (6 marks)

Let $z = r\text{cis}(\theta)$ be a solution to $z^n - 3i\bar{z} = 0$, $z \in \mathbb{C}$, where $n \in \mathbb{Z}^+$ and $n \geq 2$.

- a. Find all possible values of $|z|$.

2 marks

- b. Show that either $\theta = \frac{(1+4k)}{2(n+1)}\pi$, $k \in \mathbb{Z}$, or θ is undefined.

3 marks

c. State **with justification** the number of solutions to $z^5 - 3i\bar{z} = 0$.

1 mark

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