

2019 VCE Specialist Mathematics 2 examination report

General comments

The November 2019 Specialist Mathematics examination 2 comprised 20 multiple-choice questions (worth a total of 20 marks) and six extended-answer questions (worth a total of 60 marks).

There were five questions (Questions 1a., 2ai., 3ai., 3bii., 3biii., 4d. and 5b.) where students needed to show that a given result was reached. In these cases steps that led to the given result needed to be clearly and logically set out to obtain full marks.

Answers were generally given in the required forms but there were indications in Section B that students did not always correctly read and respond to questions:

- some students did not give the cosine of the angle in Question 4a. despite otherwise correct working
- students who were otherwise successful in Question 4d. did not always give the required unit vector
- some students whose work in subsequent parts of Question 5 indicated sufficient understanding did not complete the force diagram in Question 5a.

The examination revealed areas of strength and weakness in student performance.

Areas of strength included:

- determining cartesian forms of parametrically defined curves
- sketching the curve of a given function
- manipulating separable differential equations
- sketching force diagrams.

Areas of weakness included:

- reading and responding to all aspects of questions
- finding the area of a parallelogram in Question 4c.
- relating vector expressions to a spatial arrangement of points
- dealing with motion under constant acceleration over two stages on an inclined plane
- working with linear combinations of sample means.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not necessarily intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section A

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% N/A	Comments
1	3	2	19	4	72	0	There is a local minimum at $x = 2$.
2	4	9	86	1	0	0	
3	2	1	24	65	8	0	
4	18	8	44	17	13	0	$n!$ is a multiple of 4 for $n \geq 4, n \in \mathbb{N}$.
5	8	25	8	38	21	1	Intersection of $y = x - 2, x > 2$ and $y - 1 = -\frac{1}{\sqrt{3}}(x - 5), x < 5$.
6	56	15	8	15	6	1	
7	19	70	5	2	5	0	
8	3	5	4	13	75	0	
9	10	57	12	16	5	0	
10	7	13	58	14	7	1	
11	9	6	13	5	66	0	
12	62	12	12	9	5	1	Option A gives the set of equations that can be used to obtain the values of m , n and p . Explicit solution would result in a null set as it is not possible for a result of a vector to be of greater magnitude than the vector itself.
13	9	16	11	51	13	0	$\vec{F}_1 + \vec{F}_2 = 3(\sqrt{3}\hat{i} + \hat{j}) \Rightarrow \vec{F}_2 = 3\sqrt{3}\hat{i} + \hat{j}$
14	36	27	12	12	12	1	$3g = 7a, a = \frac{3g}{7}, g - T = 1 \times \frac{3g}{7}, T = \frac{4g}{7}$
15	19	17	12	17	35	1	Antidifferentiate to find $r(t)$.
16	49	16	26	6	2	1	$a = v \frac{dv}{dx}$. Note that option C is $\frac{dv}{dx}$.
17	9	8	18	13	50	1	
18	3	9	8	76	4	1	
19	5	6	75	11	3	1	
20	5	70	13	7	4	1	

Section B

Question 1a.

Marks	0	1	2	Average
%	17	7	76	1.6

$$x = \sec(t) + 1 \Rightarrow (x-1)^2 = \sec^2(t), y = \tan(t) \Rightarrow y^2 = \tan^2(t), \tan^2(t) + 1 = \sec^2(t),$$

$$y^2 + 1 = (x-1)^2, y^2 = (x-1)^2 - 1, y^2 = x^2 - 2x + 1 - 1, y^2 = x^2 - 2x, y = \sqrt{x^2 - 2x} \quad (y \geq 0)$$

This question was generally done well. Some students took unnecessarily convoluted approaches with the relationships between the trigonometric expressions. Students were required to work from the parametric forms to reach the cartesian form.

Question 1b.

Marks	0	1	2	Average
%	20	42	38	1.2

$$x \in [2, \infty), y \in [0, \infty)$$

While most students stated the correct range, a significant number gave a domain which did not account for the restriction on t .

Question 1ci.

Marks	0	1	2	Average
%	27	12	61	1.4

$$\frac{1}{\sin(t)}$$

Students who differentiated the parametric functions with respect to t and then applied the chain rule were generally successful. Students who differentiated y in terms of x directly were less successful; some left their answer in terms of x , others had difficulty with the subsequent substitution and simplification.

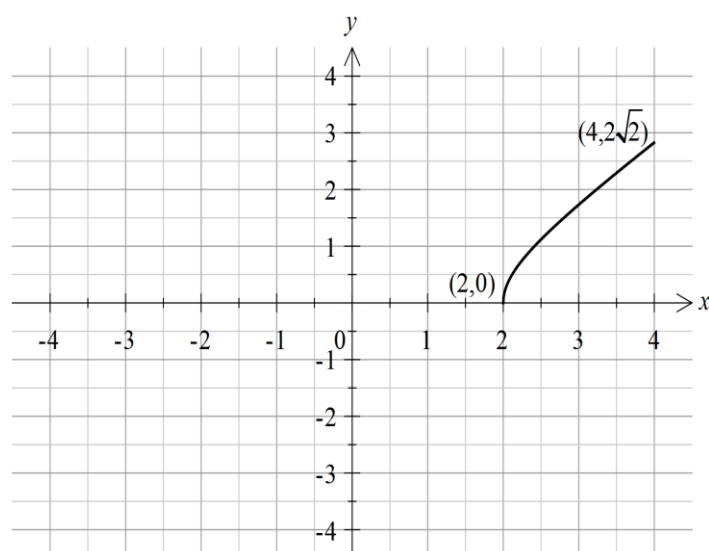
Question 1cii.

Marks	0	1	Average
%	39	61	0.6

1

Question 1d.

Marks	0	1	2	Average
%	4	12	84	1.8



This was generally well done with most students labelling the endpoints with coordinates as required. Students are advised to set viewing windows on technology to a scale that closely matches the scale provided on the examination.

Question 1e.

Marks	0	1	2	Average
%	79	19	3	0.3

$$\int_0^{\arctan 2\sqrt{2}} \pi (\sec t + 1)^2 \sec^2 t \, dt$$

Correct alternatives for the upper terminal of the definite integral include $\arccos\left(\frac{1}{3}\right)$.

Very few students answered this question correctly. The most common incorrect answer was an integral in terms of x . Of those that attempted to give an integral in terms of t , most simply replaced dx with dt .

Question 2ai.

Marks	0	1	Average
%	32	68	0.7

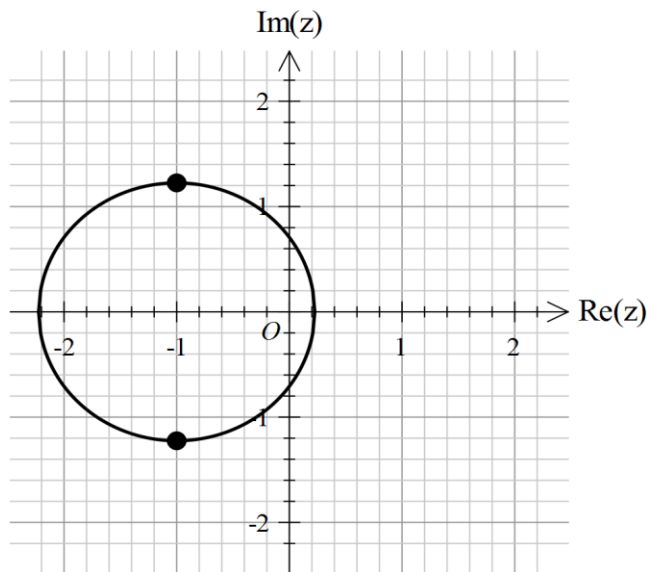
$$z = \frac{-4 \pm \sqrt{16 - 4 \times 2 \times 5}}{2 \times 2} = \frac{-4 \pm \sqrt{-24}}{4} = \frac{-4 \pm i\sqrt{24}}{4} = -1 \pm \frac{2\sqrt{6}i}{4} = -1 \pm \frac{\sqrt{6}i}{2}$$

Most students were able to show that the quadratic equation had the given solutions either by the quadratic formula as above or by completing the square. In a 'show that' question such as this, students are expected to explicitly show that the given information leads to the required conclusion rather than 'verify' that the given values of z are solutions of the equation. The working shown above is an example of a suitable response.

Question 2aii.

Marks	0	1	Average
%	17	83	0.9

The diagram below shows the solutions to Questions 2aii. (the points) and 2biii. (the circle).



Most students correctly plotted points representing the two solutions. In some cases insufficient care was taken to plot points accurately relative to the supplied gridlines and scale.

Question 2bi.

Marks	0		1	2	Average
%	28		17	55	1.3

$$m=1, n=\frac{\sqrt{6}}{2}$$

$m=-1$ was a common incorrect response.

Question 2bii.

Marks	0	1	Average
%	41	59	0.6

$$(x+1)^2 + y^2 = \frac{3}{2}$$

Incorrect responses included sign errors and algebraic errors resulting from unnecessary attempts to isolate y .

Question 2biii.

Marks	0	1	Average
%	32	68	0.7

Refer to the diagram in Question 2aii.

Question 2c.

Marks	0	1	2	Average
%	70	23	7	0.4

$$-1 \leq d \leq 5$$

Many students did not attempt this question. Of those who did, most abandoned a potentially correct approach before they reached a conclusion. Those who reached a conclusion generally got there via the quadratic formula but unfortunately most of these students found only one end point of the interval.

Question 2d.

Marks	0	1	2	Average
%	72	27	1	0.3

$$p = \frac{b}{2a}, q = \frac{\sqrt{4ac - b^2}}{2|a|}$$

Many students did not attempt this question. Of those who attempted it, most made errors relating to signs; giving the negative of the correct p value or not accounting for the sign of a .

Question 3ai.

Marks	0	1	2	Average
%	28	21	51	1.3

$$k = \frac{1}{a-b} \log_e \left(\frac{r}{s} \right) \quad (\text{given})$$

Students solved the differential equation to find the given expression for k by a variety of correct approaches. Common errors were to neglect a constant of integration or to make mistakes when manipulating logarithmic or exponential terms.

Question 3aii.

Marks	0	1	2	Average
%	60	20	20	0.6

$$a > b \text{ and } r > s \quad \text{or} \quad a < b \text{ and } r < s$$

A significant number of students stated only the first of the above conditions.

Question 3bi.

Marks	0	1	Average
%	25	75	0.8

$$\int e^Q dQ = \int e^t dt$$

Most students answered this correctly using the form above or an alternative such as

$$\int \frac{1}{e^{-Q}} dQ = \int e^t dt.$$

Question 3bii.

Marks	0	1	2	Average
%	27	4	69	1.4

$$\int e^Q dQ = \int e^t dt$$

$$e^Q = e^t + c$$

$$(0,1) \quad e = 1 + c$$

$$c = e - 1$$

$$e^Q = e^t + e - 1$$

$$Q = \log_e(e^t + e - 1)$$

Students handled this well by proceeding from the form of the differential equation given in Question 3bi. to the required solution.

Question 3biii.

Marks	0	1	2	Average
%	38	24	38	1

$$\frac{d^2Q}{dt^2} = \frac{e^t(e-1)}{(e^t + e - 1)^2}, \quad e^t > 0, (e-1) > 0 \text{ and } (e^t + e - 1)^2 > 0 \text{ so } \frac{d^2Q}{dt^2} > 0$$

Most students supplied a correct second derivative but not all of them went on to reasonably justify why the graph does not have a point of inflection.

Question 4a.

Marks	0	1	2	Average
%	46	19	35	0.9

$$\overrightarrow{OC} = (4\hat{i} - 2\hat{j} + \hat{k}) + (2\hat{i} + 4\hat{j} - 4\hat{k}) = 6\hat{i} + 2\hat{j} - 3\hat{k}, \quad a = 6, b = 2, c = -3$$

A significant proportion of students did not correctly consider the order of the vertices of the parallelogram and consequently set $\overrightarrow{AB} = \overrightarrow{CD}$. A diagram could assist to avoid this error.

Question 4b.

Marks	0	1	2	Average
%	14	20	65	1.5

$$\cos \theta = \frac{4}{9}$$

Use of the scalar product was generally evident. Some students who would otherwise have been successful did not explicitly answer the question and instead found an approximate value of the angle.

Question 4c.

Marks	0	1	2	Average
%	69	9	22	0.6

$$A = 3 \times 6 \times \sin \theta = 3 \times 6 \times \frac{\sqrt{65}}{9} = 2\sqrt{65}$$

A frequent issue here was the significant proportion of students who multiplied the lengths of two adjacent sides of the parallelogram as if they were finding the area of a rectangle. As for Question 4a., a diagram may help avoid this error.

Question 4d.

Marks	0	1	2	3	Average
%	20	5	26	49	2.1

$$(6\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) \cdot (2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) = 12 - 2 - 10 = 0, \quad (6\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) \cdot (2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}) = 12 + 8 - 20 = 0$$

$$\text{unit vector } \frac{1}{\sqrt{65}}(6\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$$

While most students were able to show that the given vector was perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{AD}$ some of them did not proceed to find the required unit vector.

Question 4e.

Marks	0	1	2	Average
%	96	2	2	0.1

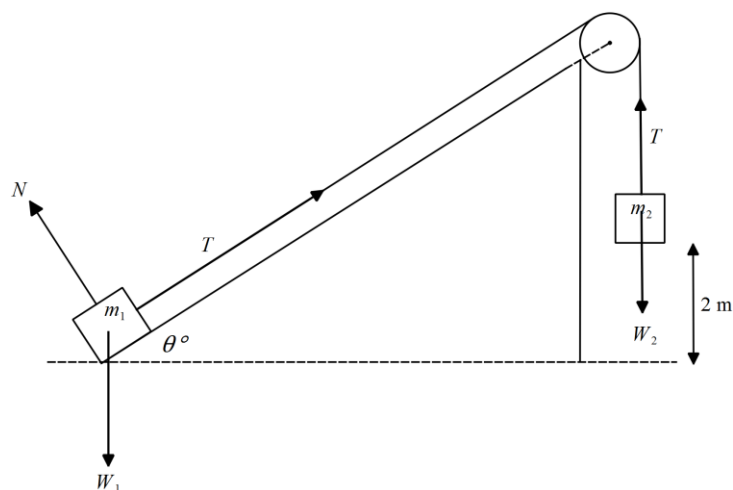
$$\text{Height of pyramid} = (2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) \cdot \frac{(6\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})}{\sqrt{65}} \left(= \frac{36}{\sqrt{65}} \right)$$

$$\text{Volume} = \frac{1}{3} \times (\text{base area}) \times \text{height} = \frac{1}{3} \times 2\sqrt{65} \times \frac{36}{\sqrt{65}} = 24$$

Very few students approached this problem correctly. The majority of those who attempted it made unfounded assumptions about the height or the layout of the pyramid. Of those who used a scalar resolute to find the height of the pyramid, most incorrectly used $\overrightarrow{OP}$ rather than a slant edge of the pyramid such as $\overrightarrow{AP}$.

Question 5a.

Marks	0	1	Average
%	15	85	0.9



Students handled this question well. Common errors included missing one of the forces acting on m_1 or showing one of them in an incorrect direction.

Question 5b.

Marks	0	1	Average
%	20	80	0.8

$$m_2 g - m_1 g \sin(\theta) = 0, m_1 g \sin(\theta) = m_2 g, \sin(\theta) = \frac{m_2}{m_1} \quad (\text{given})$$

By either considering the masses in isolation and solving simultaneous equations or by considering the whole system, most students were able to show the required result.

Question 5ci.

Marks	0	1	Average
%	61	39	0.4

$$\theta < \sin^{-1}\left(\frac{m_2}{m_1}\right)$$

Question 5cii.

Marks	0	1	2	Average
%	33	13	54	1.2

$$m_2 g - m_1 g \sin(\theta) = (m_1 + m_2) a, \quad a = \frac{m_2 g - m_1 g \sin(\theta)}{m_1 + m_2}$$

Most students were able to set up and attempt to solve suitable equations of motion.

Question 5d.

Marks	0	1	2	3	4	5	Average
%	54	26	4	6	2	9	1

$$\text{Stage 1: } a = \frac{m_2 g - 2m_2 g \times \frac{1}{4}}{2m_2 + m_2} = \frac{g}{6}, v = \sqrt{\frac{2g}{3}}, \text{ Stage 2: } a = -g \sin(\theta) = -\frac{g}{4}, s = \frac{4}{3}$$

$$\text{Distance: } 2 + \frac{4}{3} = \frac{10}{3} \text{ (m)}$$

Relatively few students accounted for the two stages. Most students did not successfully proceed beyond finding the initial acceleration of the system. Of those who did proceed to considering beyond the first stage, some erred in the direction of acceleration in the second stage.

Most students used constant acceleration formulas; some calculus approaches were seen.

A number of students used the displacement in the first stage (2 m) to find the time taken and then used that time to find the displacement (2 m), without demonstrating insight that this process was unfruitful.

Question 6a.

Marks	0	1	2	Average
%	40	31	29	0.9

$$\bar{X} \sim N\left(375, \left(\frac{15}{\sqrt{50}}\right)^2\right), \Pr(370 < \bar{X} < 375) = 0.4908$$

$$\Pr(\text{at least one sample mean between 370 and 375}) = 0.741$$

Of the students who found the standard deviation of the sample mean, about half successfully used a binomial distribution to answer the question.

Question 6b.

Marks	0	1	2	3	Average
%	67	15	2	15	0.7

$$\text{Var}(\bar{X}_1 - \bar{X}_2) = \left(\frac{15}{\sqrt{50}}\right)^2 + \left(\frac{15}{\sqrt{50}}\right)^2 = 9, \Pr(-2 < \bar{X}_1 - \bar{X}_2 < 2) = 0.495$$

Many students did not make a reasonable start to this question, or they were unable to correctly find the variance for the combined distributions. Very few students indicated an understanding that the difference between the samples could be negative and found $\Pr(\bar{X}_1 - \bar{X}_2 < 2)$ rather than correctly finding $\Pr(-2 < \bar{X}_1 - \bar{X}_2 < 2)$ or $\Pr(|\bar{X}_1 - \bar{X}_2| < 2)$.

Question 6c.

Marks	0	1	Average
%	34	66	0.7

$$H_0: \mu = 375, H_1: \mu \neq 375$$

While most students correctly stated the null and alternative hypotheses for a two-tailed test, answers indicating a one-tailed test were relatively frequent.

Question 6d.

Marks	0	1	Average
%	41	59	0.6

$$p = 0.046$$

Students who gave correct hypotheses in Question 6c. were usually successful here.

Question 6e.

Marks	0	1	Average
%	41	59	0.6

As $p < 0.05$ reject H_0 . The sample suggests the machine is not working properly.

Question 6f.

Marks	0	1	Average
%	64	36	0.4

$$\Pr(\bar{X} < x_c) = 0.025 \Rightarrow x_c = 372.1$$

This question was often not attempted. Most students who did attempt it answered correctly. The most frequent incorrect response was 372.5, resulting from $\Pr(\bar{X} < x_c) = 0.05$.