

Victorian Certificate of Education
2019

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

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SPECIALIST MATHEMATICS
Written examination 1

Friday 8 November 2019

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 12 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$

Question 1 (4 marks)

Solve the differential equation $\frac{dy}{dx} = \frac{2ye^{2x}}{1+e^{2x}}$ given that $y(0) = \pi$.

Question 2 (3 marks)

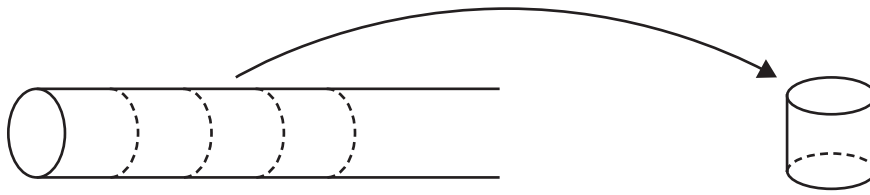
Find all values of x for which $|x - 4| = \frac{x}{2} + 7$.

TURN OVER

Question 3 (3 marks)

A machine produces chocolate in the form of a continuous cylinder of radius 0.5 cm. Smaller cylindrical pieces are cut parallel to its end, as shown in the diagram below.

The lengths of the pieces vary with a mean of 3 cm and a standard deviation of 0.1 cm.



- a. Find the expected volume of a piece of chocolate in cm^3 .

1 mark

- b. Find the variance of the volume of a piece of chocolate in cm^6 .

1 mark

- c. Find the expected surface area of a piece of chocolate in cm^2 .

1 mark

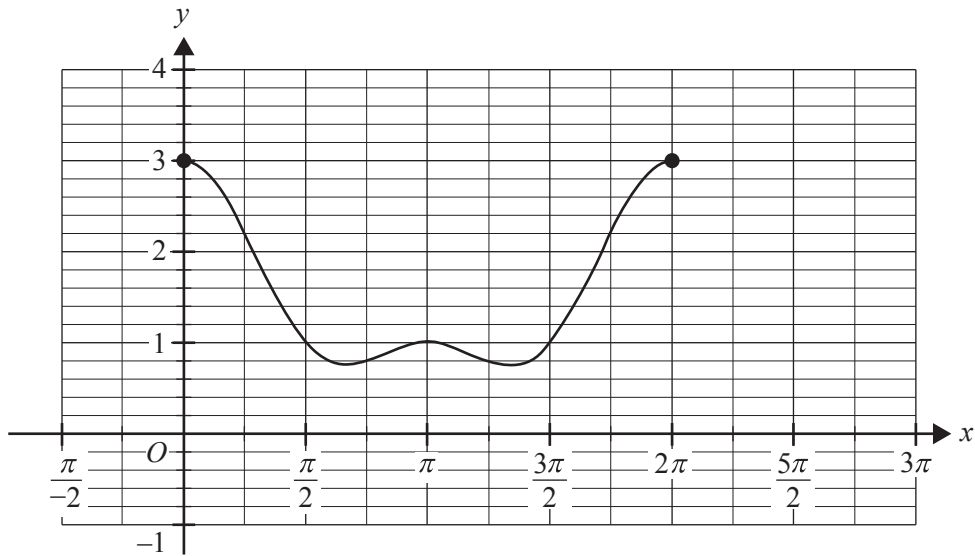
Question 4 (3 marks)

The position vectors of two particles A and B at time t seconds after they have started moving are given by $\mathbf{r}_A(t) = (t^2 - 1)\mathbf{i} + \left(a + \frac{t}{3}\right)\mathbf{j}$ and $\mathbf{r}_B(t) = (t^3 - t)\mathbf{i} + \left(\arccos\left(\frac{t}{2}\right)\right)\mathbf{j}$ respectively, where a is a real constant and $0 \leq t \leq 2$.

Find the value of a if the particles collide after they have started moving.

Question 5 (6 marks)

The graph of $f(x) = \cos^2(x) + \cos(x) + 1$ over the domain $0 \leq x \leq 2\pi$ is shown below.



- a. i. Find $f'(x)$.

1 mark

- ii. Hence, find the coordinates of the turning points of the graph in the interval $(0, 2\pi)$.

2 marks

- b. Sketch the graph of $y = \frac{1}{f(x)}$ on the set of axes above. Clearly label the turning points and endpoints of this graph with their coordinates.

3 marks

Question 6 (3 marks)

Find the value of d for which the vectors $\underline{a} = 2\underline{i} - 3\underline{j} + 4\underline{k}$, $\underline{b} = -2\underline{i} + 4\underline{j} - 8\underline{k}$ and $\underline{c} = -6\underline{i} + 2\underline{j} + d\underline{k}$ are **linearly dependent**.

Question 7 (5 marks)

- a.** Show that $3 - \sqrt{3}i = 2\sqrt{3} \operatorname{cis}\left(-\frac{\pi}{6}\right)$. 1 mark

- b.** Find $(3 - \sqrt{3}i)^3$, expressing your answer in the form $x + iy$, where $x, y \in \mathbb{R}$. 2 marks

- c.** Find the integer values of n for which $(3 - \sqrt{3}i)^n$ is real. 1 mark

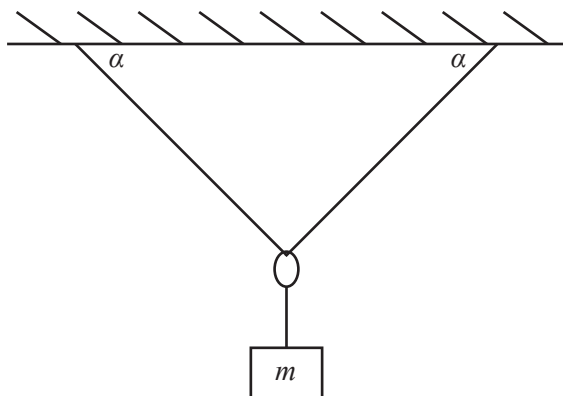
- d.** Find the integer values of n for which $(3 - \sqrt{3}i)^n = ai$, where a is a real number. 1 mark

Question 8 (4 marks)

Find the volume of the solid of revolution formed when the graph of $y = \sqrt{\frac{1+2x}{1+x^2}}$ is rotated about the x -axis over the interval $[0, 1]$.

Question 9 (4 marks)

- a. A light inextensible string is connected at each end to a horizontal ceiling. A mass of m kilograms hangs in equilibrium from a smooth ring on the string, as shown in the diagram below. The string makes an angle α with the ceiling.



Express the tension, T newtons, in the string in terms of m , g and α .

1 mark

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SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc \sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$	$\sin(x-y) = \sin(x) \cos(y) - \cos(x) \sin(y)$
$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$	$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$	
$\sin(2x) = 2 \sin(x) \cos(x)$	$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r \operatorname{cis}(\theta)$	
$ z = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\operatorname{var}(aX + b) = a^2 \operatorname{var}(X)$
for independent random variables X and Y	$\operatorname{var}(aX + bY) = a^2 \operatorname{var}(X) + b^2 \operatorname{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}} \right)$
distribution of sample mean $\bar{X}$	mean $E(\bar{X}) = \mu$ variance $\operatorname{var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a} \log_e ax+b + c$
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+(f'(x))^2} dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$

Vectors in two and three dimensions

$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$
$ \underline{r} = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$
$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underline{p} = m\underline{v}$
equation of motion	$\underline{R} = m\underline{a}$