



## Victorian Certificate of Education – Free Trial Examinations

STUDENT NUMBER

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# SPECIALIST MATHEMATICS

## Free Trial Written Examination 1

Reading time: 15 minutes

Writing time: 1 hour

### QUESTION AND ANSWER BOOK

#### Structure of book

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| 10                         | 10                                        | 40                     |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

#### Materials supplied

- Question and answer book of 9 pages
- Formula sheet
- Working space is provided throughout the book.

#### Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

#### At the end of the examination

- You may keep the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**Instructions**

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

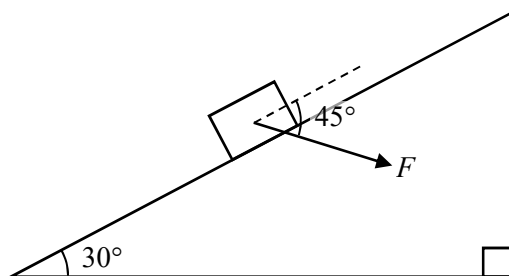
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude  $g \text{ ms}^{-2}$ , where  $g = 9.8$ .

**Question 1** (3 marks)

A 12 kg mass on a smooth plane inclined at  $30^\circ$  is held in equilibrium by a force of  $F$  newtons, acting  $45^\circ$  to the inclined plane, as shown below.



- a. On the diagram above, show all other forces acting on the mass and label them. 1 mark
- b. Find  $F$ , in newtons, expressing your answer in the form  $a\sqrt{b}g$ , where  $a, b \in \mathbb{N}$ . 2 marks

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**Question 2** (3 marks)

Given that  $\cos(2\theta) = \frac{3}{8}$ , where  $\theta \in \left(\frac{3\pi}{4}, \pi\right)$ , find  $\text{cis}(\theta)$  in cartesian form.

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**Question 3** (4 marks)

Find the equation of the line perpendicular to the curve given by  $e^{xy} = y^2 - 3$  at the point  $(0, 2)$ .

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**Question 4** (3 marks)

The masses of pears grown on a particular farm vary normally with a mean of 200 grams and a standard deviation of 15 grams. The pears are sold to supermarkets in boxes of 25 pears.

Given that  $\Pr(-2 < Z < 2) = 0.955$ , correct to three decimal places, where  $Z \sim N(0, 1)$ , calculate the probability that the mean mass per pear in a randomly selected 25-pear box exceeds 206 grams. Give your answer correct to **three** decimal places.

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**Question 5** (4 marks)

Let  $g: \mathbb{C} \rightarrow \mathbb{C}$ ,  $g(z) = z^4 - 2z^3 + 8z - 16$ .

Find all solutions of  $g(z) = 0$  for  $z$  in cartesian form given that  $z = 2 \operatorname{cis}\left(\frac{\pi}{3}\right)$  is a solution.

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**Question 6 (4 marks)**

Find  $\int_1^2 \frac{4}{x^2(x+2)} dx$ , expressing your answer in the form  $a + \log_e\left(\frac{b}{c}\right)$ , where  $a, b, c \in \mathbb{N}$ .

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

### Question 7 (4 marks)

Relative to a fixed origin  $O$ , the points  $A$  and  $B$  are defined by the position vectors  $\underline{a} = \underline{i} + \underline{j} + \underline{k}$  and  $\underline{b} = \underline{i} - \underline{j} + m\underline{k}$  respectively, where  $m \in \mathbb{R}$ .

Given that the cosine of the angle  $AOB$  is  $\frac{-1}{\sqrt{6}}$ , find the area of the triangle  $AOB$ .

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

**Question 8** (5 marks)

a. Show that  $\frac{d}{dx} \left[ \log_e \left( x + \sqrt{x^2 + k} \right) \right] = \frac{1}{\sqrt{x^2 + k}}.$

1 mark

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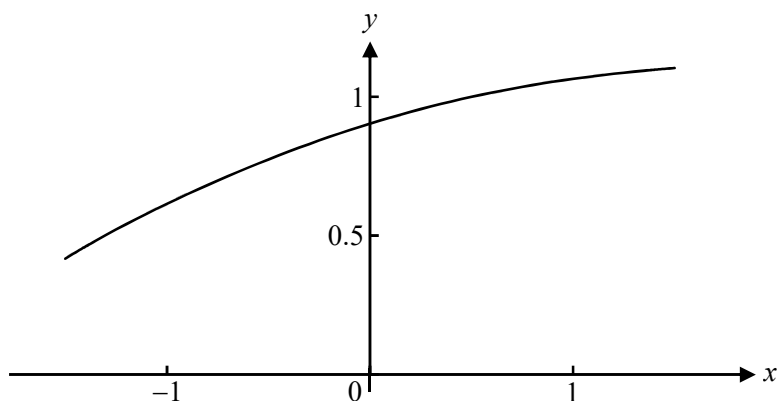


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Let  $h(x) = \frac{\sqrt{x+2}}{\sqrt[4]{x^2+6}}$ . Part of the graph of  $h$  is shown below.



- b. Find the volume generated when the region bounded by the graph of  $h$ , the  $x$ -axis, and the lines  $x = -\sqrt{2}$  and  $x = \sqrt{2}$ , is rotated about the  $x$ -axis. Express your answer in the form  $\pi \log_e(b)$ , where  $b > 1$ .

4 marks

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**Question 9** (6 marks)

The velocity vector, in  $\text{ms}^{-1}$ , of a 5 kg body moving relative to an origin  $O$  at time  $t$  seconds is given by  $\dot{\mathbf{r}}(t) = 2\sin^2(t)\mathbf{i} + 4\cos^2(t)\mathbf{j}$ , where  $t \in [0, \pi]$ .

- a. i.** Express  $\dot{\mathbf{r}}(t)$  in terms of  $\cos(2t)$ . 1 mark

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- ii.** Hence, find  $\mathbf{r}(t)$  given that  $\mathbf{r}(0) = \mathbf{i} + 2\mathbf{j}$ . 2 marks

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- b.** Find the value(s) of  $t$  for which the magnitude of the net force acting on the body is  $5\sqrt{5}$  N. 3 marks

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**Question 10** (4 marks)

Find the arc length of the curve given by  $y = \frac{e^x + e^{-x}}{2}$  from  $x = -1$  to  $x = 1$ .

**END OF QUESTION AND ANSWER BOOK**