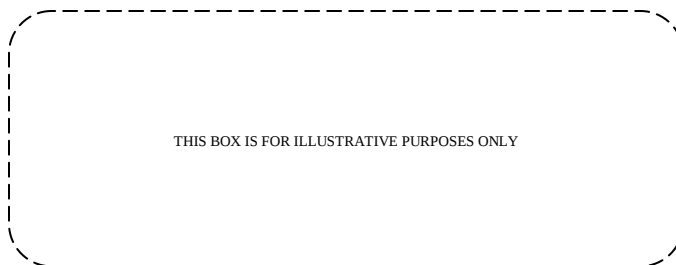




2019 Trial Examination



THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT
NUMBER

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Letter

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SPECIALIST MATHEMATICS

Units 3&4 - Written examination 2

Reading time: 15 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
A	20	20	20
B	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 29 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores zero.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

Given $\cot \theta = \frac{3}{2}$, $\sec \theta =$

- A. $\frac{\sqrt{13}}{3}$ only
- B. $-\frac{\sqrt{13}}{3}$ only
- C. $\frac{\sqrt{13}}{3}$ or $-\frac{\sqrt{13}}{3}$
- D. $-\frac{3}{\sqrt{13}}$ only
- E. $\frac{3}{\sqrt{13}}$ or $-\frac{3}{\sqrt{13}}$

Question 2

$\frac{\cos 2\alpha}{\cos^4 \alpha - \sin^4 \alpha}$ simplifies to

- A. 1
- B. -1
- C. 2
- D. $-\frac{1}{2}$
- E. $\frac{1}{2}$

SECTION A - continued

Question 3

The number of x-intercepts for the function $f(x) = \tan(20x - \pi)$, $x \in (0, 5\pi]$ is:

- A. 4
- B. 19
- C. 20
- D. 99
- E. 100

Question 4

The set of solutions over $\mathbb{C}$ to $z^5 = i$, where $i = \sqrt{-1}$ can be generated from the rule:

- A. $\text{cis}\left(\frac{\pi + 4n\pi}{10}\right)$, where n is an integer
- B. $\text{cis}\left(\frac{\pi + 4n\pi}{5}\right)$, where n is an integer
- C. $\text{cis}\left(\frac{\pi - 4n\pi}{5}\right)$, where n is an integer
- D. $\text{cis}\left(\frac{\pi + 2n\pi}{10}\right)$, where n is an integer
- E. $\text{cis}\left(\frac{\pi + 5n\pi}{10}\right)$, where n is an integer

Question 5

Given $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, a vector parallel to vector $\mathbf{a}$ with a length of 12 units is:

- A. $12\mathbf{a}$
- B. $12\hat{\mathbf{a}}$
- C. $4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$
- D. $-8\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}$
- E. $8\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}$

SECTION A - continued
TURN OVER

Question 6

Consider the following vectors:

$$\overrightarrow{AB} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k} \quad \text{and} \quad \overrightarrow{BC} = 6\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$$

Let M be the midpoint of line AC. The acute angle between line AM and line MB is closest to:

- A. 55°
- B. 57°
- C. 59°
- D. 61°
- E. 63°

Question 7

The cubic polynomial $P(z)$ has coefficients that are **NOT** all real.

Consider the solutions to $P(z) = 0$ over the complex field.

Which statement is true?

- A. $P(z)$ **cannot** have a complex conjugate pair of solutions.
- B. $P(z)$ **cannot** have 3 different complex solutions.
- C. $P(z)$ **must** have exactly 1 real solution.
- D. $P(z)$ **must** have a complex conjugate pair of solutions.
- E. $P(z)$ **could** have two real and one non-real solution.

Question 8

Let $m = 4 + i$ and $n = a + bi$ where $m, n \in \mathbb{C}$ and $a, b \in \mathbb{R}$.

Given $m \times n = -11 + 10i$ then $m + \bar{n} =$

- A. $2 + i$
- B. $2 - 2i$
- C. $1 - 2i$
- D. $2 - i$
- E. $1 + 2i$

SECTION A - continued

Question 9

Consider the two points A and B where point A is on the curve defined by $|z - 2i| = 2$ and point B is on the line $\operatorname{Re}(z) - \operatorname{Im}(z + 4i) = 2$, where $z \in \mathbb{C}$.

The smallest possible distance between points A and B is:

- A. $4\sqrt{2} - 2$
- B. $4\sqrt{2} + 2$
- C. $4\sqrt{2}$
- D. $2\sqrt{2} + 2$
- E. $2\sqrt{2} - 2$

Question 10

$z^3 + az^2 - iz^2 + bz + iz + c = 0$ where $a, b, c \in \mathbb{R}$ has solutions of $z = 1, z = -i$ and $z = 2i$. The values of a, b and c respectively are:

- A. $a = 1, b = 2, c = -2$
- B. $a = 1, b = -2, c = -2$
- C. $a = -1, b = 2, c = -2$
- D. $a = -1, b = -2, c = 2$
- E. $a = -1, b = 2, c = 2$

Question 11

Given $f(x) = x \tan^{-1} 2x$; $f'\left(\frac{1}{2a}\right)$ for appropriate values of a equals:

- A. $\frac{2a}{a^2+1} + \tan^{-1}\left(\frac{1}{a}\right)$
- B. $\frac{a}{a^2+1} + \tan^{-1}\left(\frac{1}{a}\right)$
- C. $\frac{a}{a^2+1} + \tan^{-1}\left(\frac{1}{2a}\right)$
- D. $\frac{a}{4a^2+1} + \tan^{-1}\left(\frac{2}{a}\right)$
- E. $\frac{2a}{4a^2+1} + \tan^{-1}\left(\frac{2}{a}\right)$

SECTION A - continued
TURN OVER

Question 12

$\int_0^1 \frac{2x}{\sqrt{2x+1}} dx$ can be rewritten as:

- A. $2\int_0^1 u^{1/2} - u^{-1/2} du$ where $u = 2x + 1$
- B. $2\int_0^1 u^{1/2} - u^{-1/2} du$ where $u = \sqrt{2x + 1}$
- C. $\frac{1}{2}\int_1^3 u^{1/2} - u^{-1/2} du$ where $u = \sqrt{2x + 1}$
- D. $\frac{1}{2}\int_1^3 u^{1/2} - u^{-1/2} du$ where $u = 2x + 1$
- E. $\frac{1}{2}\int_1^3 u^{1/2} + u^{-1/2} du$ where $u = 2x + 1$

Question 13

The rule for the volume of a cone of radius r units and height h units can be generated by rotating:

- A. $y = \frac{h}{r}x$ around the x - axis from $x = r$ to $x = h$
- B. $y = \frac{h}{r}x$ around the x - axis from $x = 0$ to $x = r$
- C. $y = \frac{h}{r}x$ around the y - axis from $y = 0$ to $y = r$
- D. $y = \frac{h}{r}x$ around the y - axis from $y = 0$ to $y = h$
- E. $y = \frac{r}{h}x$ around the y - axis from $y = 0$ to $y = r$

Question 14

The area bound by the curves $y = x^3 - 2x^2$ and $y = x^2 - 1$ is closest to:

- A. 1 square unit
- B. 2 square units
- C. 3 square units
- D. 4 square units
- E. 5 square units

Question 15

Which statement is **NOT** true of the curve $x^2 - 6x + y^2 + 8y = 0$?

- A. It is a circle centred at $(3, -4)$
- B. It passes through the origin.
- C. A point on the curve has parametric equations $(5 \cos \theta + 3, 5 \sin \theta - 4)$
- D. The gradient at any point on the curve is given by $\frac{-x+3}{y+4}$
- E. $y = \frac{3}{4}x$ is a tangent to the curve.

Question 16

The maximum negative gradient on $f(x) = x^2 e^x$ occurs when $x =$

- A. $\sqrt{2} + 2$
- B. $\sqrt{2} - 2$
- C. $2 - \sqrt{2}$
- D. $1 - \sqrt{2}$
- E. $\sqrt{2} + 1$

Question 17

Consider the function $a(x) = f(x) \times (g(x))^2$

Given $f(2) = 2$, $g(2) = 0$, $f'(2) = 5$, $g'(2) = 3$, then $a''(2) =$

- A. 36
- B. 24
- C. 18
- D. 12
- E. 0

SECTION A - continued
TURN OVER

Question 18

The lengths of timber pickets are normally distributed with a mean of μ cm and a standard deviation of 1.5 cm.

What is the smallest sample size required to be at least 99% certain that the sample mean differs by less than 1 cm from the population mean?

- A. 3
- B. 4
- C. 12
- D. 15
- E. 25

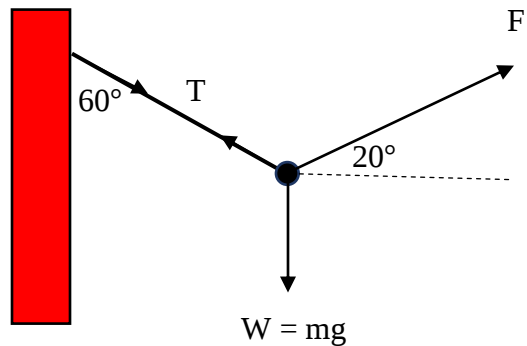
Question 19

A 10 kg mass is just on the point of slipping when placed on a rough plane inclined at 20° to the horizontal. What force, to the nearest newton, needs to be applied directly up the plane for the mass to accelerate at 2 ms^{-2} ?

- A. 93
- B. 87
- C. 81
- D. 54
- E. 20

Question 20

An 8 kg mass attached to a string is pulled away from a vertical wall by force (F) inclined at 20° to the horizontal. Find, correct to the nearest newton, the magnitude of F so that the string maintains an angle of 60°



- A. 89
- B. 96
- C. 100
- D. 104
- E. 107

END OF SECTION A

SECTION B – Extended response questions

Instructions for Section B

Answer **all** questions in the spaces provided.

In **all** questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

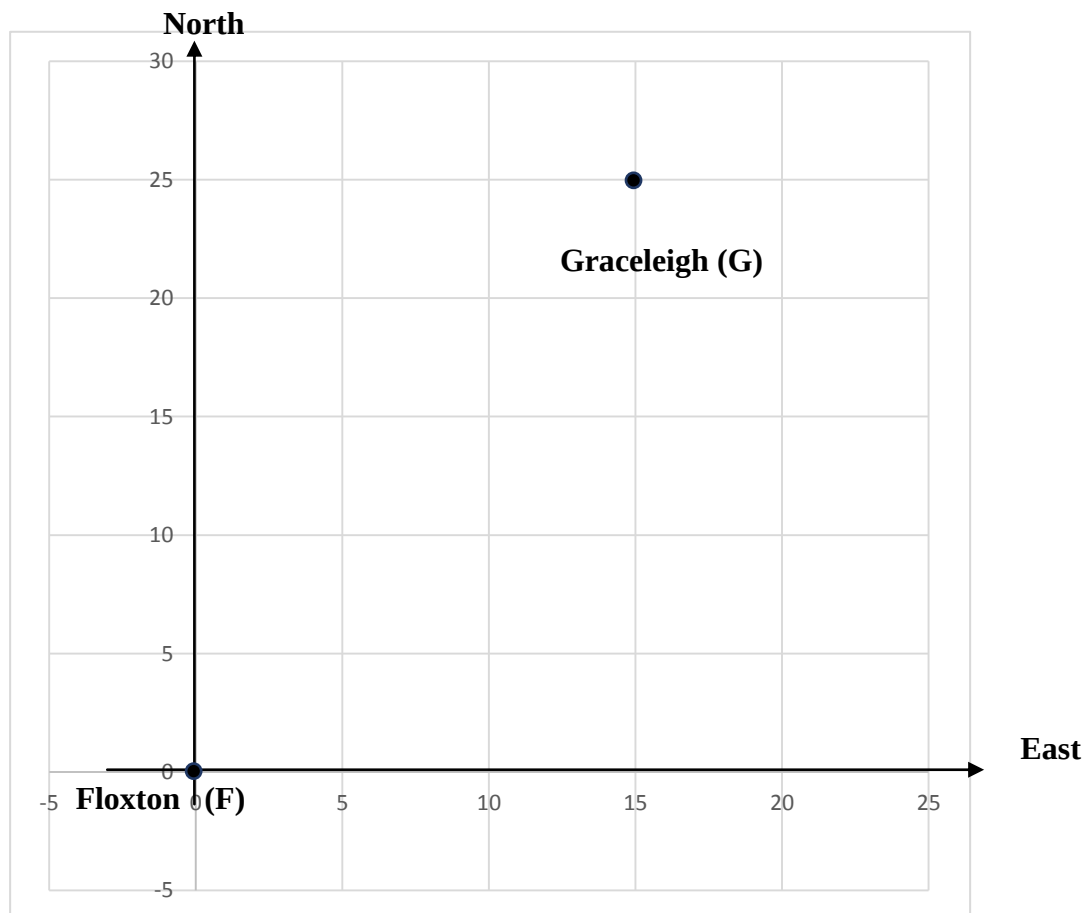
Unless otherwise indicated, the diagrams in this book are **not** to scale.

Question 1 (17 marks)

The following map shows the position of two islands, Floxton (F) and Graceleigh (G).

Floxton is located at $(0, 0)$ and Graceleigh is located at $(15, 25)$

Let 1 km East be $\hat{i}$ and 1 km North be $\hat{j}$



SECTION B – Question 1 - continued
TURN OVER

- a. Write the position of Graceleigh $\overrightarrow{OG}$ in vector notation.

1 mark

A sight-seeing cruise boat leaves Floxton and sails at 20 kmh^{-1} on a bearing of *North* 30° *East*.

- b. Mark the boat's path on the graph provided at the beginning of the question. 1 mark

- c. Prove that the boat passes just to the north of Graceleigh.

2 marks

SECTION B – Question 1 - continued

- d. The position of the boat P, as a function of t where t is the number of hours since the boat has left Floxton is given by $\vec{OP} = \vec{p} = kt(\vec{i} + \sqrt{3}\vec{j})$
Find the value of k .

2 marks

- e. When is the boat due north of Graceleigh?

1 mark

SECTION B – Question 1 - continued
TURN OVER

f. Find $\overrightarrow{PG}$ in terms of t .

2 marks

SECTION B – Question 1 - continued

[illegible]

SECTION B – Question 1 - continued
TURN OVER

A patrol boat has position (B) given by $\vec{OB} = (15 \cos 2t + 15)\tilde{i} + (10 \sin 2t)\tilde{j}$
 where t is the number of hours after the sight-seeing cruise boat leaves Floxton. ~

- h.** How far apart are the two boats when the sight-seeing cruise boat first leaves Floxton?

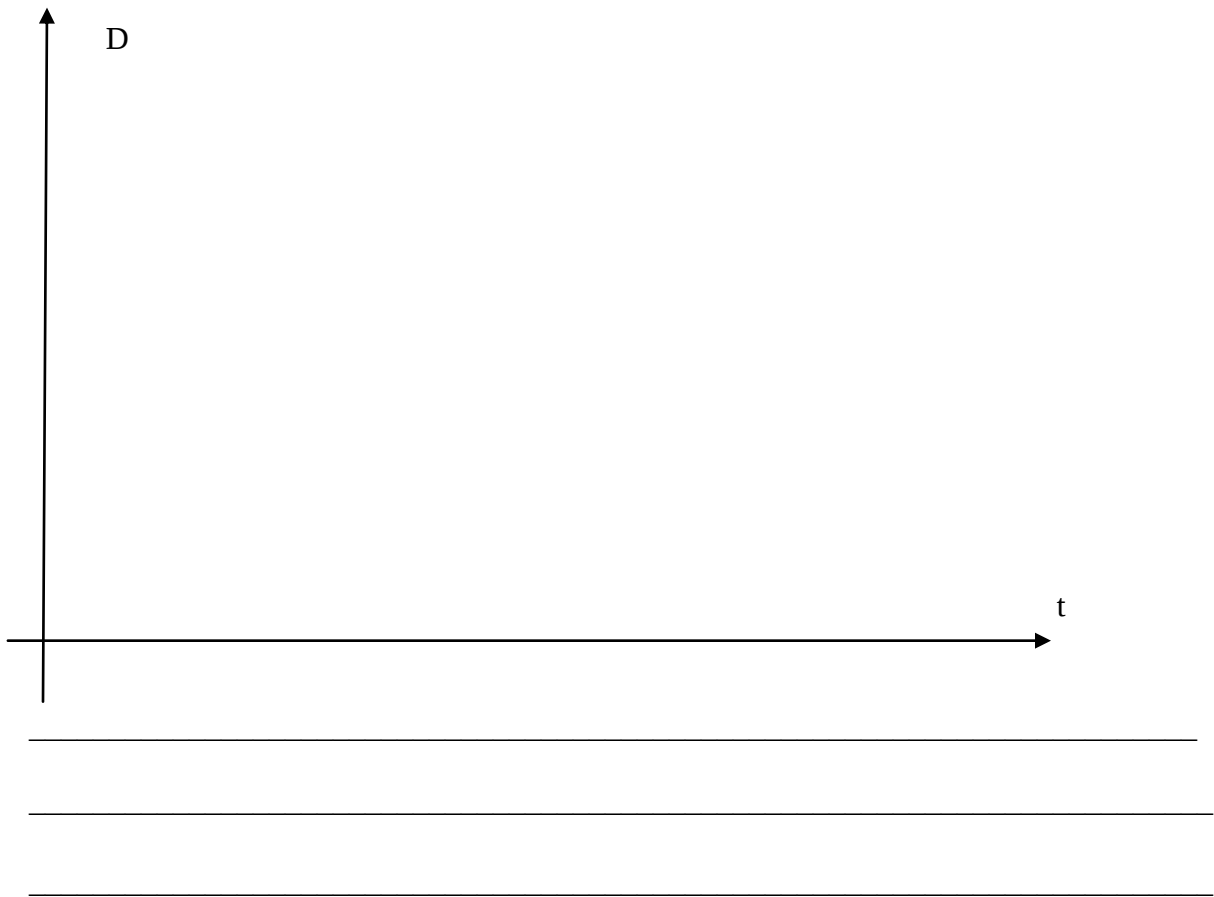
2 marks

- i.** Find the Cartesian equation of the patrol boat.

2 marks

SECTION B – Question 1 - continued

- j. Sketch and label the graph of the distance (D) between the two boats for $t \in [0,2]$
Interpret the local minimum.



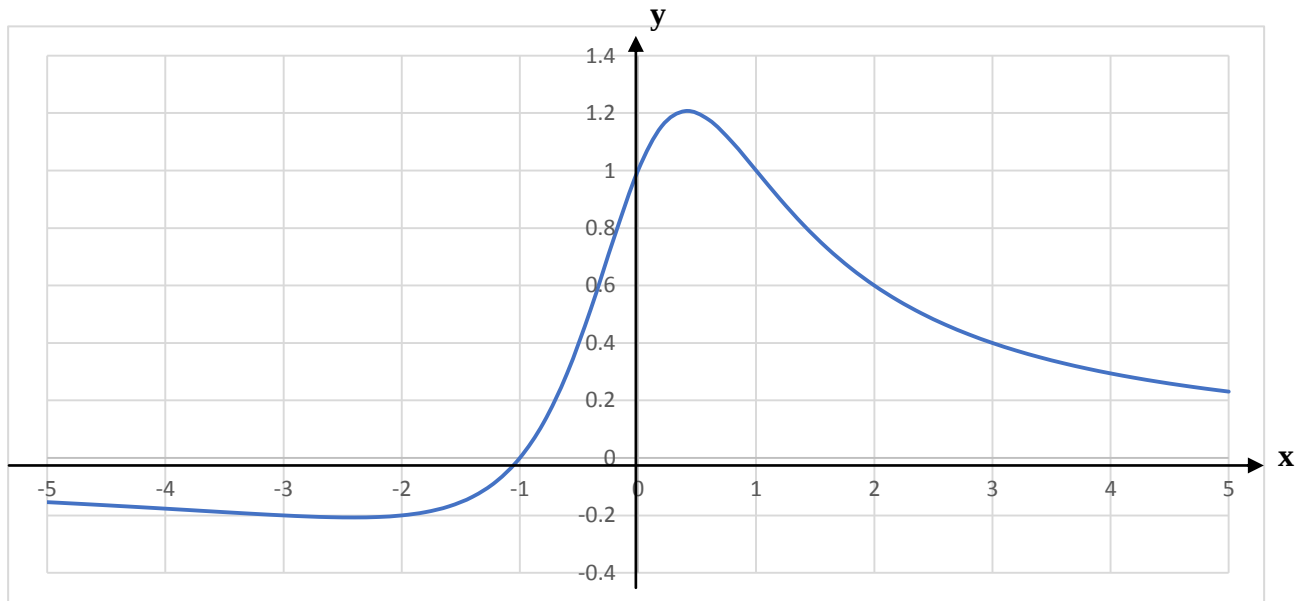
3 marks

$1 + 1 + 2 + 2 + 1 + 2 + 3 + 1 + 1 + 3 = 17$ marks

SECTION B – continued
TURN OVER

Question 2 (14 marks)

The following graph shows the function $f: [-5, 5] \rightarrow \mathbb{R}$ where $f(x) = \frac{x+1}{x^2+1}$



- a.** Find the derivative of $f: [-5, 5] \rightarrow \mathbb{R}$ where $f(x) = \frac{x+1}{x^2+1}$

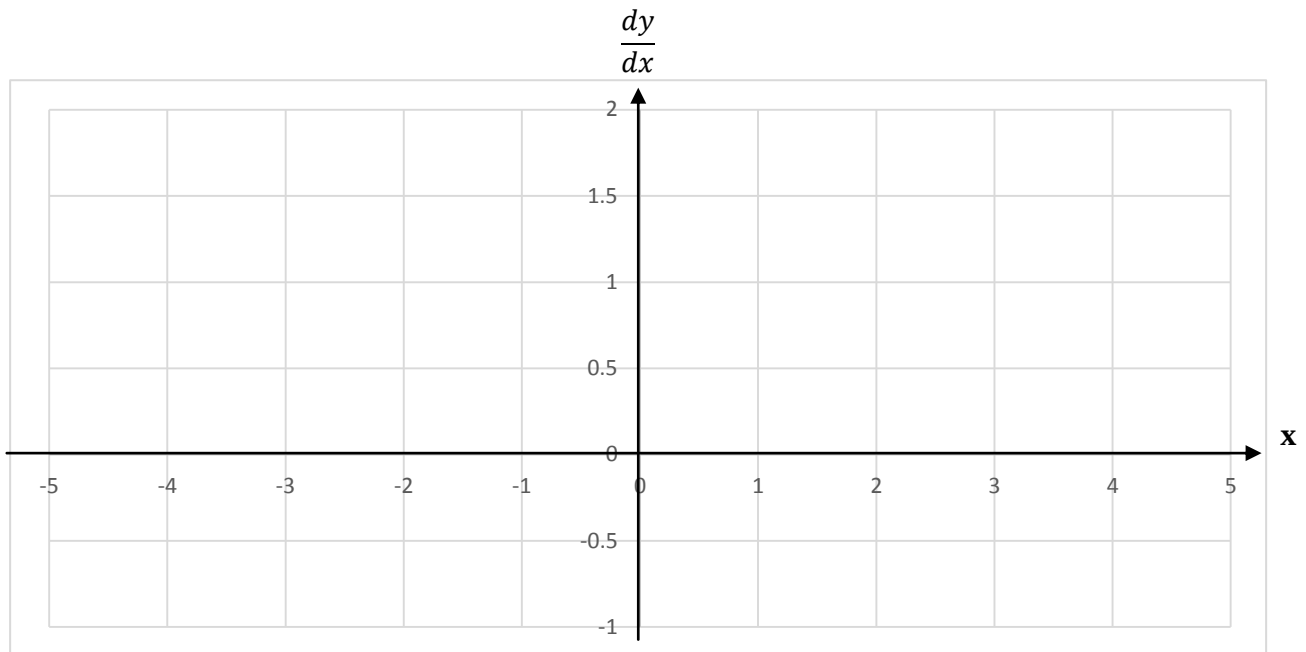
1 mark

SECTION B – Question 2 - continued

- b. Find the exact co-ordinates of the stationary points to $f: [-5, 5] \rightarrow R$ where $f(x) = \frac{x+1}{x^2+1}$
Mark these on the graph above.

3 marks

- c. Sketch $y = f'(x)$ on the axes below. Label intercepts and stationary points in exact form.



3 marks

SECTION B – Question 2 - continued
TURN OVER

- d. Let $g(x) = f'(x)$

Write down the exact range of $y = g(x)$.

1 mark

- e. Explain in words what this range of values represents, in terms of the original function $y = f(x)$.

1 mark

SECTION B – Question 2 - continued

[illegible]

Page 19 of 29

- g. Find the volume generated when the region enclosed by the x -axis, the y -axis and $y = f(x)$ is rotated around the x -axis.

2 marks

1 + 3 + 3 + 1 + 1 + 3 + 2 = 14 marks

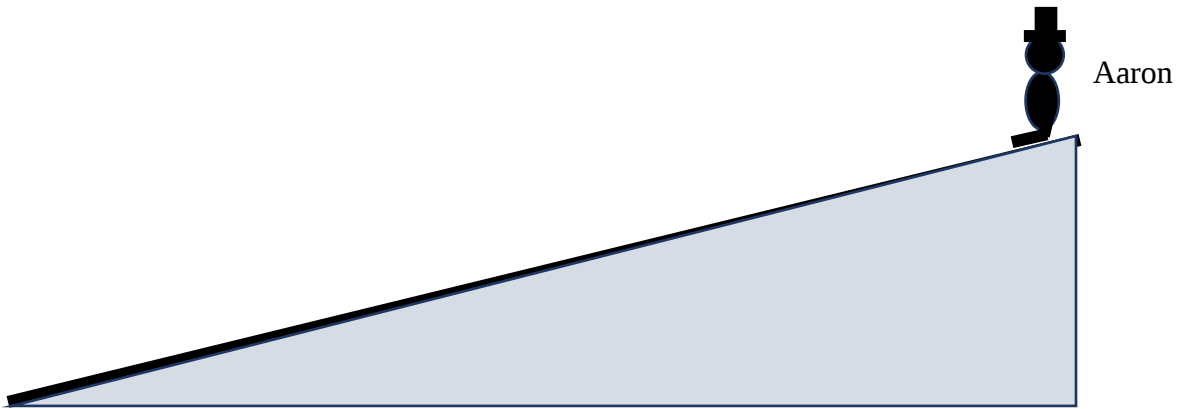
SECTION B – continued

Question 3 (12 marks)

A 5-metre playground slide is inclined at 30° to the horizontal. A 40 kg boy (Aaron) sits at the very top of the slide.

Initially assume that the contact between the boy and the surface of the slide is smooth.

- a.** Show all the forces acting on Aaron on the diagram below.



1 mark

- b.** How long will it take Aaron to reach the bottom of the slide?
Give your answer correct to one decimal place.

1 mark

SECTION B – Question 3 - continued
TURN OVER

- c. Find Aaron's speed at the bottom of the slide

1 mark

In fact, the contact between Aaron and the surface of the slide is not smooth. A resistive force of F_R newtons (due to friction) acts directly up the plane.

- d. If Aaron's speed at the bottom of the slide is 5 m s^{-1} find F_R

2 marks

SECTION B – Question 3 - continued

Aaron wants to have a faster ride. He asks his sister to give him a push from the top of the slide. She provides a pushing force (P newtons) directly down the slide where:

$$P = 960t, \quad 0 \leq t \leq 0.5$$

$$P = 0 \text{ otherwise}$$

- e. Show that the Aaron's speed 0.5 *seconds* after the start of being pushed is 4.25 ms^{-1}

2 marks

- f. Find the distance that Aaron has travelled down the slide 0.5 *seconds* after the start of being pushed.

1 mark

SECTION B – Question 3 - continued

TURN OVER

- g.** Find Aaron's speed at the bottom of the slide now.

2 marks

- h.** A horizontal rubber mat at the bottom of the slide slows Aaron down by providing a resistive force of $100x$ newtons where x is the horizontal distance from the end of the slide in metres.

Find, to the nearest cm, how far Aaron comes to rest from the base of the slide.

2 marks

$1 + 1 + 1 + 2 + 2 + 1 + 2 + 2 = 12$ marks

SECTION B – continued

Question 4 (17 marks)

Mr Braggs, a Specialist Mathematics teacher has found an old exam question relating to the solutions to $z^3 = -64i$. He wants to compare how long his class takes to answer the question compared to the published data.

- a.** Show by substitution that $z = 4i$ is a solution to $z^3 = -64i$

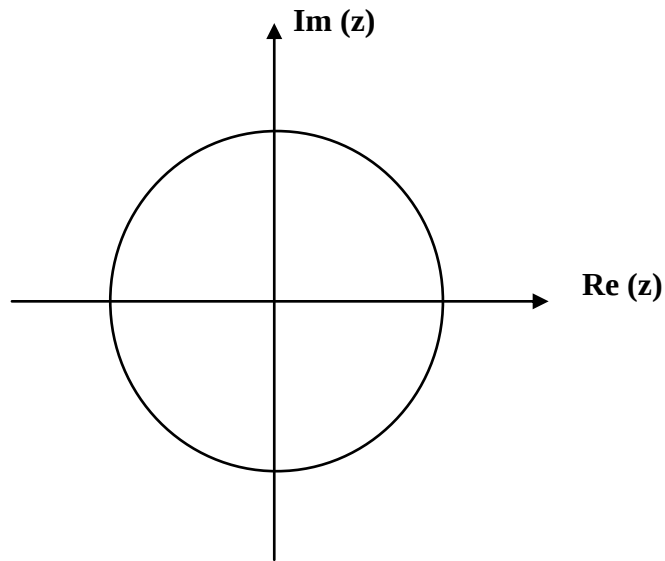
1 mark

- b.** Write $4i$ in $r \operatorname{cis} \theta$ form. (Call this z_1).

1 mark

SECTION B – Question 4 - continued
TURN OVER

- c. Plot z_1 on the axes below then plot z_2 and z_3 the other two solutions to $z^3 = -64i$



2 marks

- d. Write z_2 and z_3 in both polar and rectangular form.

2 marks

SECTION B – Question 4 - continued

- e. Solve $z^2 = -64i$, writing your answers in polar form.

2 marks

- f. Using one of your roots from each of $z^3 = -64i$ and $z^2 = -64i$, find one solution to $z^6 = -64i$ in rectangular form.

2 marks

SECTION B – Question 4 - continued
TURN OVER

Parts **a.** to **f.** of Question 4 were originally from an old Applied Mathematics exam paper.
 Time (t minutes) to complete the question from the state-wide population of students is well modelled by a continuous random variable with a pdf given by:

$$m(t) = at(t - 20); \quad 0 \leq t \leq 20$$

$$= 0, \text{ otherwise}$$

g. Find the mean and standard deviation of m .

3 marks

- h.** Mr Braggs is worried that his students take too long to answer application questions. To test this hypothesis, what mean time (correct to 2 decimal places) would Mr Bragg's class of 23 students need to record so that this time is higher than the state-wide mean at the 0.01 level of significance?

Assume that Mr Bragg's class is an independent sample from the entire population of students, who have attempted the question.

2 marks

- i.** In fact, Mr Braggs has a total of 25 students in his class. The two students who were absent on the day of the test, attempt the question the following day. What mean time (correct to 1 decimal place) would these two students need to record so that the entire class of 25 would achieve the same level of significance as the original 23 students?

2 marks

1 + 1 + 2 + 2 + 2 + 2 + 3 + 2 + 2 = 17 marks

END OF QUESTION AND ANSWER BOOK