



Fortify Sample Exam A1

SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer booklet of 7 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 (5 marks)

- a. Simplify $\frac{\sqrt{3} - i}{(\sqrt{2} + \sqrt{2}i)^5}$, and express the answer in the form of $a + ib$, where a and b are real numbers. 3 marks

- b. Solve the equation $z^2 + 2z + 5 = 0$ over $z \in \mathbb{C}$. 2 marks

Question 2 (2 marks)

A sample of 49 batteries was found to last an average of 15 hours. If the standard deviation of battery life is known to be 20 minutes, then find the 95% confidence interval for the mean time that the battery will last. Give your answer in the form $a \pm b$ where a and b are real constants.

Question 3 (4 marks)

Find the equation of the normal to the curve given by $\sin(y) = \cos^2(x)$ at the point $\left(\frac{\pi}{4}, \frac{\pi}{6}\right)$.

Question 4 (3 marks)

A particle moves under a force $\vec{F}$ so that its position vector $\vec{r}$ at any time t is given by $\vec{r} = \cos(t)\vec{i} + t^2\vec{j} + e^{2t}\vec{k}$. Distances are measured in metres and time is measured in seconds.

Calculate the magnitude of the acceleration at π seconds.

Question 5 (4 marks)

Find the volume of the solid of revolution when the region bound by the curve $y = \log_e(3 - x)$, the x -axis and the y -axis is rotated about the y -axis.

Question 6 (4 marks)

Consider the vectors $\underline{a} = 2\underline{i} - 2\underline{j} + \underline{k}$ and $\underline{b} = 2\underline{i} - \underline{j} + 3\underline{k}$. Find the vector resolute of b in the direction of a .

Question 7 (4 marks)

The rate of radioactive decay for a particular substance is proportional to the amount Q of the substance at any time t . The differential equation for this situation is $\frac{dQ}{dt} = -kQ$, where k is a constant.

Given that $Q = 20$ when $t = 0$ and $Q = 10$ when $t = 5$, find an equation for Q in terms of t .

Question 8 (4 marks)

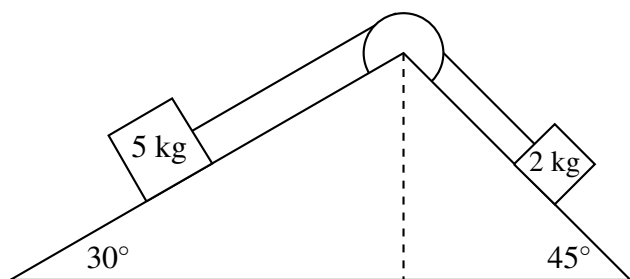
Find the area under the curve $y = \frac{e^x}{e^{2x} - 2e^x + 1}$ from $x = 1$ to $x = 2$. Express your answer as a single fraction.

Question 9 (4 marks)

Let $f(x) = \frac{1}{\arccos(x)}$. Find $f'(x)$ and define the domain of which $f'(x)$ is defined.

Question 10 (6 marks)

Two masses of 5 kg and 2 kg are placed on a smooth inclined plane of 30° and 45° .



- a. Show all forces acting on both masses on the diagram. 2 marks
- b. Find the acceleration of the 5 kg mass down the plane in terms of g . 2 marks

- c. Find the tension in the rope in terms of g . 2 marks



SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Formula Sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a + b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc \sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x + y) = \sin(x) \cos(y) + \cos(x) \sin(y)$	$\sin(x - y) = \sin(x) \cos(y) - \cos(x) \sin(y)$
$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$	$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y)$
$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$	$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$	
$\sin(2x) = 2 \sin(x) \cos(x)$	$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or $\arcsin$	$\cos^{-1}$ or $\arccos$	$\tan^{-1}$ or $\arctan$
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$	
$ z = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $E(\bar{X}) = \mu$ variance $\text{var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a} \log_e ax+b + c$
produce rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+(f'(x))^2} dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$

Vectors in two and three dimensions

$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$
$ \underline{r} = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\underline{i} + \frac{dy}{dt}\underline{j} + \frac{dz}{dt}\underline{k}$
$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underline{p} = m\underline{v}$
equation of motion	$\underline{R} = m\underline{a}$