Neap:

Trial Examination 2019

VCE Specialist Mathematics Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (3 marks)

In the direction of motion we have $2pt + q = ma$. A1

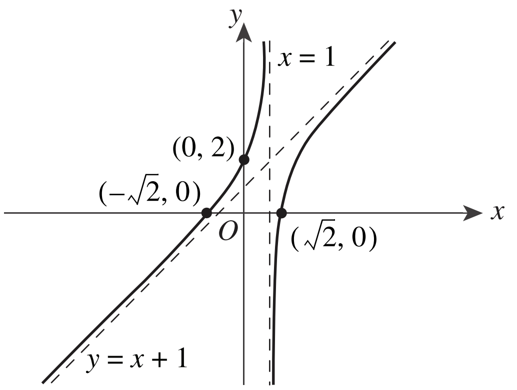
$$a = \frac{2pt+q}{m} \Rightarrow \frac{dv}{dt} = \frac{2pt+q}{m}$$

$$v = \frac{pt^2+qt}{m} + c$$ A1

When $t = 0$, $v = 0$ and so $c = 0$.

So $v = \dfrac{pt^2+qt}{m}$. A1

Question 2 (4 marks)



correct shape (two branches and asymptotic behaviour) A1
correct intercepts with the axes A1
vertical asymptote is x = 1 A1
non-vertical asymptote is y = x + 1 A1

Question 3 (3 marks)

a. $$\begin{aligned} \mathrm{E}(4X-3Y) &= 4\mathrm{E}(X) - 3\mathrm{E}(Y) \\ &= 4\times 30 - 3\times 20 \\ &= 60 \end{aligned}$$ A1

b. $$\mathrm{Var}(4X-3Y) = 16\mathrm{Var}(X) + 9\mathrm{Var}(Y)$$ M1
$$= 16\times 9 + 9\times 4$$
$$= 180$$ A1

Question 4 (3 marks)

$\overrightarrow{AB} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + (m+3)\underset{\sim}{k}$

$\left|\overrightarrow{OC}\right| = 7$ A1

Attempting to solve $3^2 + (-2)^2 + (m+3)^2 = 49$ for m. M1

$m = -9$ or 3 A1

Question 5 (3 marks)

Use of $\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$ and $\cot(x) = \dfrac{1}{\tan(x)}$ to obtain $\dfrac{10\tan(x)}{1-\tan^2(x)} = \dfrac{4}{\tan(x)}$. M1

Attempting to simplify gives $\tan^2(x) = \dfrac{2}{7}$. M1

$$\tan(x) = \pm\sqrt{\frac{2}{7}}$$ A1

Question 6 (4 marks)

$$\int_0^{\frac{\pi}{6}} \frac{1+\cos^4(2x)}{\cos^2(2x)}dx = \int_0^{\frac{\pi}{6}} \sec^2(2x) + \cos^2(2x)dx$$ A1

$$\cos(4x) = 2\cos^2(2x) - 1 \Rightarrow \cos^2(2x) = \frac{1}{2} + \frac{1}{2}\cos(4x)$$ M1

$$= \left[\frac{1}{2}\tan(2x) + \frac{x}{2} + \frac{1}{8}\sin(4x)\right]_0^{\frac{\pi}{6}}$$ A1

$$= \frac{\sqrt{3}}{2} + \frac{\pi}{12} + \frac{\sqrt{3}}{16}$$

$$= \frac{27\sqrt{3}+4\pi}{48}$$ A1

Question 7 (5 marks)

a. $$\frac{d}{dx}\left(\frac{1}{5}e^{2x}(2\sin(x) - \cos(x))\right) = \frac{2}{5}e^{2x}(2\sin(x) - \cos(x)) + \frac{1}{5}e^{2x}(2\cos(x) + \sin(x))$$ M1

$$= \frac{4}{5}e^{2x}\sin(x) + \frac{1}{5}e^{2x}\sin(x) - \frac{2}{5}e^{2x}\cos(x) + \frac{2}{5}e^{2x}\cos(x)$$ A1

So $\dfrac{d}{dx}\left(\dfrac{1}{5}e^{2x}(2\sin(x) - \cos(x))\right) = e^{2x}\sin(x)$.

b. $$\int \frac{dy}{\sqrt{1-y^2}} = \int e^{2x}\sin(x)dx$$ A1

$$\sin^{-1}(y) = \frac{1}{5}e^{2x}(2\sin(x) - \cos(x)) + c$$ M1

When $x = 0$, $y = 0$ and so $c = \dfrac{1}{5}$.

$$y = \sin\left(\frac{1}{5}e^{2x}(2\sin(x) - \cos(x)) + \frac{1}{5}\right)$$ A1

Question 8 (5 marks)

$(a + bi)^2 = a^2 - b^2 + 2abi$ and so $a^2 - b^2 = 7$ and $2ab = -6\sqrt{2}$. M1 A1

Attempting to eliminate a variable, for example b: $b = -\dfrac{3\sqrt{2}}{a}$ to form $a^4 - 7a^2 - 18 = 0$. M1

(Alternatively, eliminating variable a forms $b^4 + 7b^2 - 18 = 0$.)

Solving gives $a = 3$, $b = -\sqrt{2}$; that is, $z = 3 - \sqrt{2}i$. A1

And $a = -3$, $b = \sqrt{2}$; that is, $z = -3 + \sqrt{2}i$. A1

Question 9 (5 marks)

Attempting to differentiate implicitly to obtain $e^{-y} - xe^{-y}\dfrac{dy}{dx} + e^{y}\dfrac{dy}{dx} - 1 = 0$. M1

At the point $(k, \log_e(k))$, $\dfrac{1}{k} - \dfrac{dy}{dx} + k\dfrac{dy}{dx} - 1 = 0$. M1

$\dfrac{dy}{dx} = \dfrac{1}{k} \Rightarrow m_N = -k$, where m_N is the gradient of the normal. A1

The equation of the normal is $y - \log_e(k) = -k(x - k)$.

When $x = 0$, $y = \dfrac{2k^2 + 1}{2}$ and so $\dfrac{2k^2 + 1}{2} - \log_e(k) = k^2$. M1

So $k = \sqrt{e}$. A1

Question 10 (5 marks)

a. $1 - x^2 \geq 0$

$-1 \leq x \leq 1$ but $x \neq 0$

The maximal domain of f is $-1 \leq x \leq 1,\ x \neq 0$. A1

b. $$f'(x) = \left(\frac{1}{1 + \frac{1 - x^2}{x^2}}\right)\left(\frac{-x^2(1 - x^2)^{-\frac{1}{2}} - (1 - x^2)^{\frac{1}{2}}}{x^2}\right)$$ M1 A1

$$= \left(\frac{x^2}{x^2 + 1 - x^2}\right)\left(\frac{-x^2(1 - x^2)^{-\frac{1}{2}} - (1 - x^2)^{\frac{1}{2}}}{x^2}\right)$$

$$= x^2\left(\frac{\frac{-x^2}{\sqrt{1 - x^2}} - \sqrt{1 - x^2}}{x^2}\right)$$

$$= \frac{-x^2}{\sqrt{1 - x^2}} - \sqrt{1 - x^2}$$ M1

$$= \frac{-x^2 - (1 - x^2)}{\sqrt{1 - x^2}} \text{ and so } f'(x) = -\frac{1}{\sqrt{1 - x^2}}$$ A1