



Trial Examination 2019

VCE Specialist Mathematics Units 3&4

Written Examination 1

Suggested Solutions

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Question 1 (3 marks)

In the direction of motion we have $2pt + q = ma$.

A1

$$a = \frac{2pt + q}{m} \Rightarrow \frac{dv}{dt} = \frac{2pt + q}{m}$$

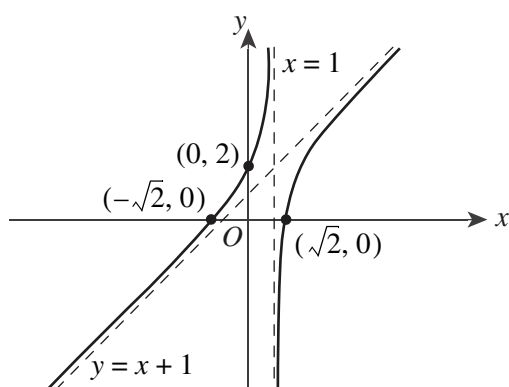
$$v = \frac{pt^2 + qt}{m} + c$$

A1

When $t = 0$, $v = 0$ and so $c = 0$.

$$\text{So } v = \frac{pt^2 + qt}{m}.$$

A1

Question 2 (4 marks)

correct shape (two branches and asymptotic behaviour) A1

correct intercepts with the axes A1

vertical asymptote is $x = 1$ A1

non-vertical asymptote is $y = x + 1$ A1

Question 3 (3 marks)

$$\begin{aligned} \text{a. } E(4X - 3Y) &= 4E(X) - 3E(Y) \\ &= 4 \times 30 - 3 \times 20 \\ &= 60 \end{aligned}$$

A1

$$\begin{aligned} \text{b. } \text{Var}(4X - 3Y) &= 16\text{Var}(X) + 9\text{Var}(Y) \\ &= 16 \times 9 + 9 \times 4 \\ &= 180 \end{aligned}$$

M1

A1

Question 4 (3 marks)

$$\begin{aligned} \vec{AB} &= 3\vec{i} - 2\vec{j} + (m+3)\vec{k} \\ |\vec{OC}| &= 7 \end{aligned}$$

A1

Attempting to solve $3^2 + (-2)^2 + (m+3)^2 = 49$ for m .

M1

$m = -9$ or 3

A1

Question 5 (3 marks)

$$\text{Use of } \tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)} \text{ and } \cot(x) = \frac{1}{\tan(x)} \text{ to obtain } \frac{10 \tan(x)}{1 - \tan^2(x)} = \frac{4}{\tan(x)}. \quad \text{M1}$$

$$\text{Attempting to simplify gives } \tan^2(x) = \frac{2}{7}. \quad \text{M1}$$

$$\tan(x) = \pm \sqrt{\frac{2}{7}} \quad \text{A1}$$

Question 6 (4 marks)

$$\int_0^{\frac{\pi}{6}} \frac{1 + \cos^4(2x)}{\cos^2(2x)} dx = \int_0^{\frac{\pi}{6}} \sec^2(2x) + \cos^2(2x) dx \quad \text{A1}$$

$$\cos(4x) = 2\cos^2(2x) - 1 \Rightarrow \cos^2(2x) = \frac{1}{2} + \frac{1}{2} \cos(4x) \quad \text{M1}$$

$$= \left[\frac{1}{2} \tan(2x) + \frac{x}{2} + \frac{1}{8} \sin(4x) \right]_0^{\frac{\pi}{6}} \quad \text{A1}$$

$$= \frac{\sqrt{3}}{2} + \frac{\pi}{12} + \frac{\sqrt{3}}{16}$$

$$= \frac{27\sqrt{3} + 4\pi}{48} \quad \text{A1}$$

Question 7 (5 marks)

$$\text{a. } \frac{d}{dx} \left(\frac{1}{5} e^{2x} (2 \sin(x) - \cos(x)) \right) = \frac{2}{5} e^{2x} (2 \sin(x) - \cos(x)) + \frac{1}{5} e^{2x} (2 \cos(x) + \sin(x)) \quad \text{M1}$$

$$= \frac{4}{5} e^{2x} \sin(x) + \frac{1}{5} e^{2x} \sin(x) - \frac{2}{5} e^{2x} \cos(x) + \frac{2}{5} e^{2x} \cos(x) \quad \text{A1}$$

$$\text{So } \frac{d}{dx} \left(\frac{1}{5} e^{2x} (2 \sin(x) - \cos(x)) \right) = e^{2x} \sin(x).$$

$$\text{b. } \int \frac{dy}{\sqrt{1-y^2}} = \int e^{2x} \sin(x) dx \quad \text{A1}$$

$$\sin^{-1}(y) = \frac{1}{5} e^{2x} (2 \sin(x) - \cos(x)) + c \quad \text{M1}$$

$$\text{When } x = 0, y = 0 \text{ and so } c = \frac{1}{5}.$$

$$y = \sin \left(\frac{1}{5} e^{2x} (2 \sin(x) - \cos(x)) + \frac{1}{5} \right) \quad \text{A1}$$

Question 8 (5 marks)

$$(a + bi)^2 = a^2 - b^2 + 2abi \text{ and so } a^2 - b^2 = 7 \text{ and } 2ab = -6\sqrt{2}. \quad \text{M1 A1}$$

$$\text{Attempting to eliminate a variable, for example } b: b = -\frac{3\sqrt{2}}{a} \text{ to form } a^4 - 7a^2 - 18 = 0. \quad \text{M1}$$

$$(\text{Alternatively, eliminating variable } a \text{ forms } b^4 + 7b^2 - 18 = 0.)$$

$$\text{Solving gives } a = 3, b = -\sqrt{2}; \text{ that is, } z = 3 - \sqrt{2}i. \quad \text{A1}$$

$$\text{And } a = -3, b = \sqrt{2}; \text{ that is, } z = -3 + \sqrt{2}i. \quad \text{A1}$$

Question 9 (5 marks)

$$\text{Attempting to differentiate implicitly to obtain } e^{-y} - xe^{-y} \frac{dy}{dx} + e^y \frac{dy}{dx} - 1 = 0. \quad \text{M1}$$

$$\text{At the point } (k, \log_e(k)), \frac{1}{k} - \frac{dy}{dx} + k \frac{dy}{dx} - 1 = 0. \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{1}{k} \Rightarrow m_N = -k, \text{ where } m_N \text{ is the gradient of the normal.} \quad \text{A1}$$

$$\text{The equation of the normal is } y - \log_e(k) = -k(x - k).$$

$$\text{When } x = 0, y = \frac{2k^2 + 1}{2} \text{ and so } \frac{2k^2 + 1}{2} - \log_e(k) = k^2. \quad \text{M1}$$

$$\text{So } k = \sqrt{e}. \quad \text{A1}$$

Question 10 (5 marks)

a. $1 - x^2 \geq 0$

$-1 \leq x \leq 1$ but $x \neq 0$

The maximal domain of f is $-1 \leq x \leq 1, x \neq 0$.

A1

b.
$$f'(x) = \left(\frac{1}{1 + \frac{1-x^2}{x^2}} \right) \left(\frac{-x^2(1-x^2)^{-\frac{1}{2}} - (1-x^2)^{\frac{1}{2}}}{x^2} \right)$$

M1 A1

$$= \left(\frac{x^2}{x^2 + 1 - x^2} \right) \left(\frac{-x^2(1-x^2)^{-\frac{1}{2}} - (1-x^2)^{\frac{1}{2}}}{x^2} \right)$$

$$= x^2 \left(\frac{\frac{-x^2}{\sqrt{1-x^2}} - \sqrt{1-x^2}}{x^2} \right)$$

$$= \frac{-x^2}{\sqrt{1-x^2}} - \sqrt{1-x^2}$$

M1

$$= \frac{-x^2 - (1-x^2)}{\sqrt{1-x^2}} \text{ and so } f'(x) = -\frac{1}{\sqrt{1-x^2}}$$

A1