The Mathematical Association of Victoria

Trial Exam 2019

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers,
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 14 pages
- Formula sheet.
- Working space is provided throughout the book

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the space provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2} where $g = 9.8$.

Question 1 (3 marks)

Let $z^3 + 2iz^2 + bz - i = 0$, $z \in C$, where b is a real constant.

Given that $z = -i$ is a solution to the equation, find all other solutions.

TURN OVER

Question 2 (3 marks)

Find the gradient of the line perpendicular to the graph of

$$x\sin(2y) - y\cos(x) + x = \frac{\pi}{2}$$

at the point where $y = 0$.

Question 3 (4 marks)

The acceleration of a train can vary between -2.4 ms^{-2} and 0.3 ms^{-2}.

The train travels in a straight line from rest at one station to rest at the next station that is 3 km away. Find in seconds the minimum travel time of the train.

TURN OVER

Question 4 (3 marks)

The mass, in grams, of apricots grown on a farm is normally distributed with a mean of 10 and a standard deviation of 2. The farm sells the apricots in paper bags but the bags rip if the total mass of apricots inside them is greater than 120 grams.

Let n be the maximum number of apricots a paper bag can hold without ripping with a probability greater than 0.84. Let Z be the standard normal random variable. Using the fact that, correct to two decimal places, $\Pr(Z < 1) = 0.84$, an equation that can be used to find n is $25n^2 + an + b = 0$ where a and b are integers.

Find the values of a and b.

Question 5 (3 marks)

The altitude of a triangle is decreasing at a rate of 2 cm/min while its area is increasing at a rate of $3\,\text{cm}^2$ /min.

Find, in cm/min, the rate at which the base of the triangle is changing when its altitude is 10 cm and its area is $100\,\text{cm}^2$.

TURN OVER

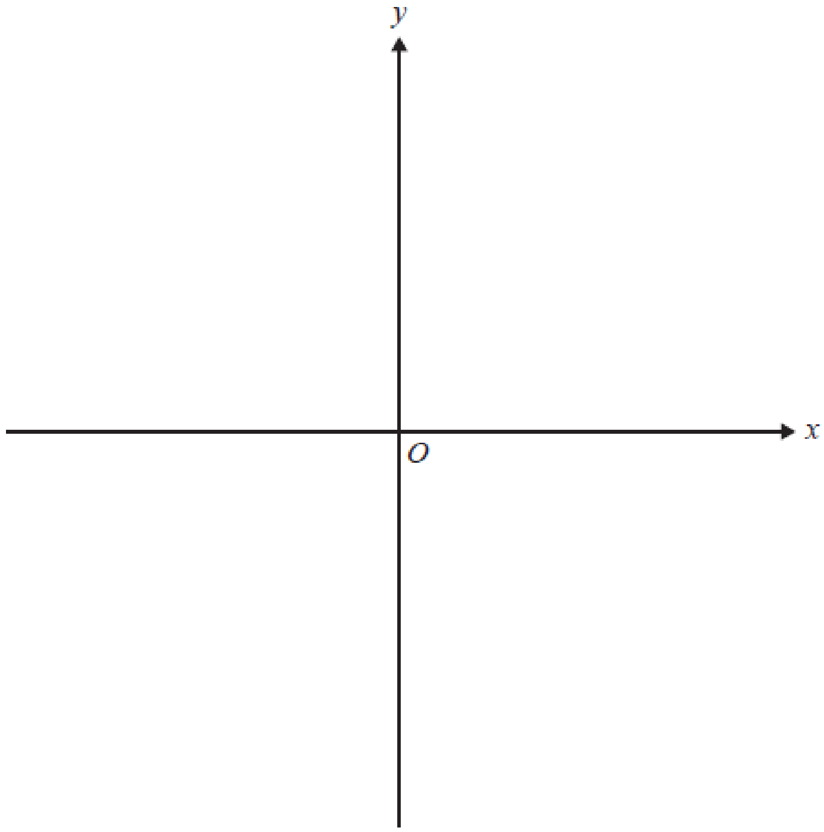
Question 6 (5 marks)

The position vector of an object moving along a curve at time t seconds is given by

$\underset{\sim}{r}(t) = \left(t^2 - 2t\right)\underset{\sim}{i} + (t-1)\underset{\sim}{j},\ 0 \le t \le 2,$

where distances are measured in metres and time is measured in seconds.

a. Sketch the path followed by the object on the axes below, labelling all important features. 3 marks



Working space

b. Find the cartesian coordinates of the point on the path where the velocity and acceleration of the object are perpendicular. 2 marks

TURN OVER

Question 7 (5 marks)

a. Prove the identity $2\sec(\theta) = \dfrac{\sec^2\left(\dfrac{\theta}{2}+\dfrac{\pi}{4}\right)}{\tan\left(\dfrac{\theta}{2}+\dfrac{\pi}{4}\right)}$, $\theta \in \left[0, \dfrac{\pi}{2}\right)$ 2 marks

b. Hence or otherwise find the arc length of the curve $y = \log_e\left(\cos(x)\right)$ from $x = 0$ to $x = \dfrac{\pi}{6}$. 3 marks

Question 8 (10 marks)

a. Find the derivative of $\sin^{-1}\left(\frac{2}{x-1}\right)$.

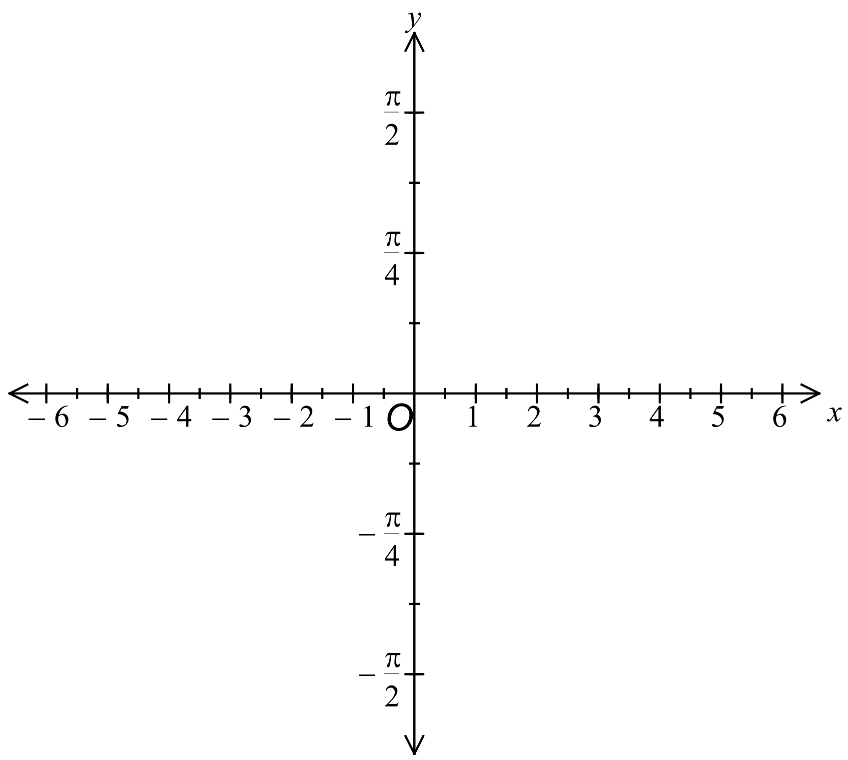
Give your answer in the form $\frac{a}{k(x)\sqrt{(x+b)(x+c)}}$ where a, b, c are integers and $k(x)$ is a function. 3 marks

TURN OVER

Consider the function $f : D \to R$, where $f(x) = \sin^{-1}\left(\frac{2}{x-1}\right)$ and D is the maximal domain of f.

b. Determine the maximal domain D and range of f. 3 marks

c. Sketch the graph of $y = f(x)$ on the axes provided below, labelling any asymptotes with their equation and any endpoints with their coordinates. 3 marks



d. Consider the differential equation $\frac{dy}{dx} = \sin^{-1}\left(\frac{2}{x-1}\right)$ and $y(4) = -2$.

Euler's method with a step size of 0.1 is used to find an approximate value of y when $x = 5$.

Explain whether this approximate value will be an overestimate or an underestimate of the exact value of y. Do **not** attempt to use Euler's method or to solve the differential equation. 1 mark

TURN OVER

Question 9 (4 marks)

Let $I = \displaystyle\int \frac{e^{\frac{x}{2}}}{\sqrt{3e^{-x} - e^{x} + 4}}\, dx$

a. By making the substitution $u = e^{x}$, show that $I = \displaystyle\int \frac{1}{\sqrt{3 + 4u - u^{2}}}\, du$. 2 marks

b. Hence evaluate I in terms of x. 2 marks

END OF QUESTION AND ANSWER BOOK