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Specialist Mathematics

2019

Trial Examination 1 (1 hour)

Instructions

Answer **all** questions. Do **not** use calculators.

Unless otherwise specified, an **exact** answer is required to a question.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

In questions where more than one mark is available, show appropriate working or explanation.

Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$

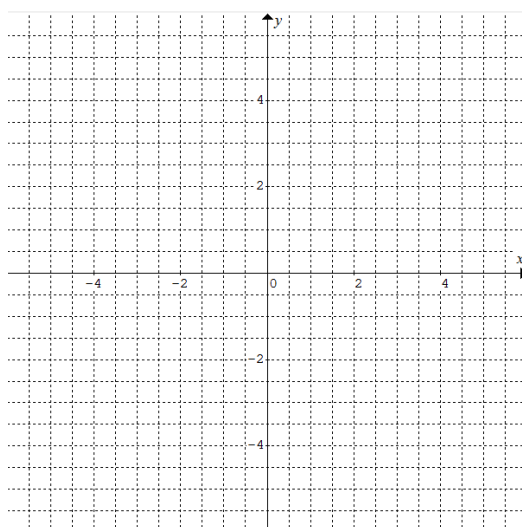
Question 1 Consider $f(x) = \frac{x^2}{4} - \frac{4}{3x^2}$.

- a. Determine the asymptotic behaviour, axis intercepts, nature and coordinates of stationary points, and coordinates of points of inflection of $f(x)$.

4 marks

- b. Sketch the graph of $f(x) = \frac{x^2}{4} - \frac{4}{3x^2}$. Include features found in part a.

2 marks



Question 2 Evaluate $\int_0^1 \left(\frac{xe^{x^2}}{1+e^{x^2}} \right) dx$. Hint: Let $u = 1 + e^{x^2}$.

3 marks

Question 3 $-\frac{\sqrt{6}}{2} + i\frac{\sqrt{2}}{2}$ is a root of $z^6 + n = 0$ where n is a positive integer.

a. Show that $n = 8$.

2 marks

b. Find the other roots of $z^6 + n = 0$.

2 marks

Question 4 $\tilde{p} = \sqrt{2}\tilde{i} - 2\tilde{j} + \sqrt{3}\tilde{k}$ is perpendicular to $\tilde{q} = \alpha\tilde{j} - \beta\tilde{k}$ where $\alpha, \beta \in \mathbb{R}$.

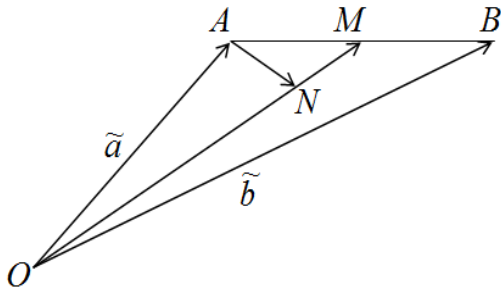
- a. Show that $\frac{\alpha}{\beta} = -\frac{\sqrt{3}}{2}$. 1 mark

- b. The angle between $\tilde{p}$ and $\tilde{j}$ is θ . Evaluate $\cos \theta$. 1 mark

- c. In terms of β only, find a possible $\tilde{r}$ such that $\tilde{p}$, $\tilde{q}$ and $\tilde{r}$ are linearly dependent. 2 marks

Question 5 Solve $\frac{dy}{dx} = -\frac{x(y^2 - 1)}{y(x^2 - 1)}$ for y in terms of x , given that $(-2, 2)$ satisfies the relation. 4 marks

Question 6 In the following diagram, M is the midpoint of line segment AB , and point N divides line segment CM into a ratio of $4:1$. $\overrightarrow{OA} = \tilde{a}$ and $\overrightarrow{OB} = \tilde{b}$.
Express $\overrightarrow{AN}$ in terms of $\tilde{a}$ and $\tilde{b}$. 3 marks



Question 7 Solve for x .

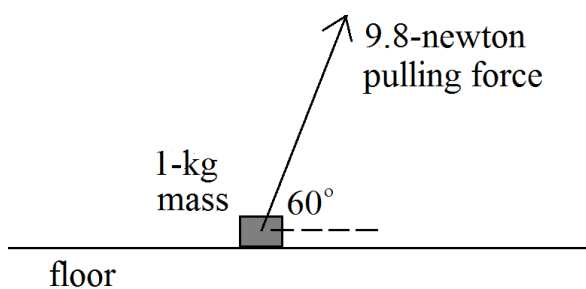
a. $\sin^2\left(-\frac{\pi}{x}\right) - \cos^2\left(\frac{\pi}{x}\right) = \frac{1}{2}$ where $x \in [-1, 1]$ 2 marks

b. $\sin^{-1}\left(\frac{x}{\pi}\right) - \cos^{-1}\left(-\frac{x}{\pi}\right) = \frac{\pi}{2}$ where $x \in [-\pi, \pi]$ 3 marks

Question 8 Show that $y(x^2 - 1) = 2x(y^2 - 1)$ has the same gradient at $(-1, -1)$, $(-1, 1)$, $(1, 1)$ and $(1, -1)$.

3 marks

Question 9 A 1-kg mass is pulled by a 9.8-newton force at 60° angle to the horizontal floor. The mass remains at rest on the floor.



The reaction force of the floor on the mass makes an acute angle of θ° to the horizontal. Determine the exact value of $\tan \theta^\circ$.

3 marks

Question 10 A factory produces nuts and bolts.

Random variables X and Y are nut weight and bolt weight respectively.

Bolts are produced to fit nuts such that $Y = 3(X + 10)$, $\mu_X = 55$ and $\sigma_X = 2$.

Weights are measured in grams.

a. Calculate σ_Y .

1 mark

b. The factory packages its products in bags containing 2 nuts and 2 bolts in each bag.

Let random variable W be the total weight of a bag of 2 nuts and 2 bolts.

Ignore the weight of packaging bag.

Show that $\mu_W = 500$ and $\sigma_W = 16$.

2 marks

100 bags (of 2 nuts and 2 bolts) are sampled randomly. 50 random samples in total are taken.

The distribution of $\bar{W}$ across the 50 samples is approximately normal.

c. Show that 1.6 is the standard deviation of $\bar{W}$ across the samples.

1 mark

d. Estimate $\Pr(484 < \bar{W} < 516)$.

1 mark

End of Exam 1