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Students Name:………………………………………

SPECIALIST MATHEMATICS UNITS 3 & 4

TRIAL EXAMINATION 1

2019

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
An exact answer is required to a question unless otherwise specified.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Formula sheets can be found on pages 12 - 14 of this exam.

Question 1 (3 marks)

Find the equation of the tangent to the curve $3y^2 + 2xy = 7$ at the point (2, 1).

Question 2 (4 marks)

A 40 kg trolley sits on the floor of a lift.

a. The lift accelerates downwards at the rate of 1.8 ms^{-2}. Find the reaction of the lift floor on the trolley in newtons. 2 marks

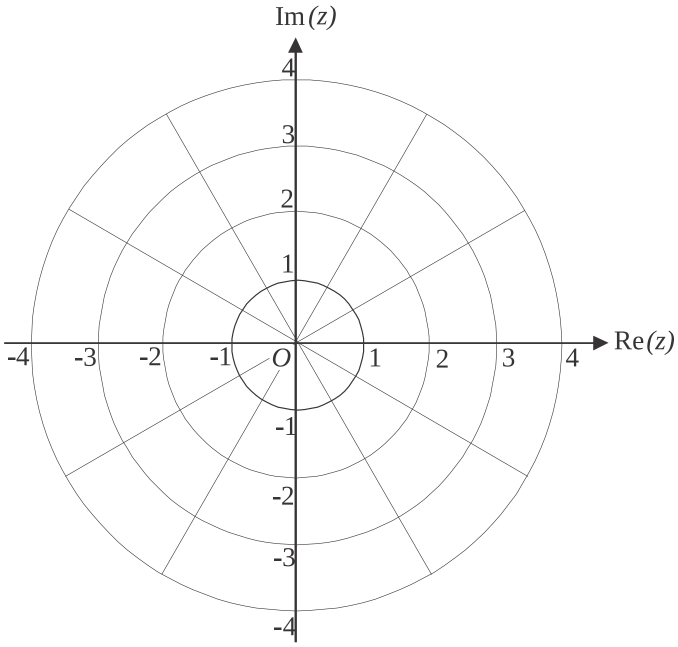
b. The lift stops and then accelerates upwards so that the reaction of the lift floor on the trolley is 448 newtons. Find the acceleration of the lift upwards in ms^{-2}. 2 marks

Question 3 (4 marks)

The equation $z^3 - 3z^2 + 12z + 16 = 0, \quad z \in C$, has one root given by $z = 4\text{cis}\left(\frac{\pi}{3}\right)$.

a. Find the other two roots of the equation in the form $a + bi$ where $a, b \in R$. 3 marks

b. Plot the roots of the equation on the Argand diagram below. 1 mark



Question 4 (3 marks)

The mass, in grams, of mussels farmed in a bay, are normally distributed with a variance of 9.
The mussels are sold locally in bags of 100.
One such bag has a mass of 2400 grams.
Use this information, together with an integer multiple of the standard deviation, to calculate an approximate 95% confidence interval for the mean mass of mussels farmed in the bay.

Question 5 (4 marks)

The points M, N and P have position vectors, relative to a fixed origin, given respectively by

$$\underset{\sim}{m} = 2\underset{\sim}{i} + a\underset{\sim}{j}, \quad \underset{\sim}{n} = \underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k} \text{ and } \underset{\sim}{p} = \underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}, \quad \text{where } a \text{ is a real constant.}$$

The magnitude of angle MNP is $\frac{\pi}{4}$. Find the value of a. Give your answer in the form $\frac{b + c\sqrt{d}}{f}$, where b, c, d and f are integers.

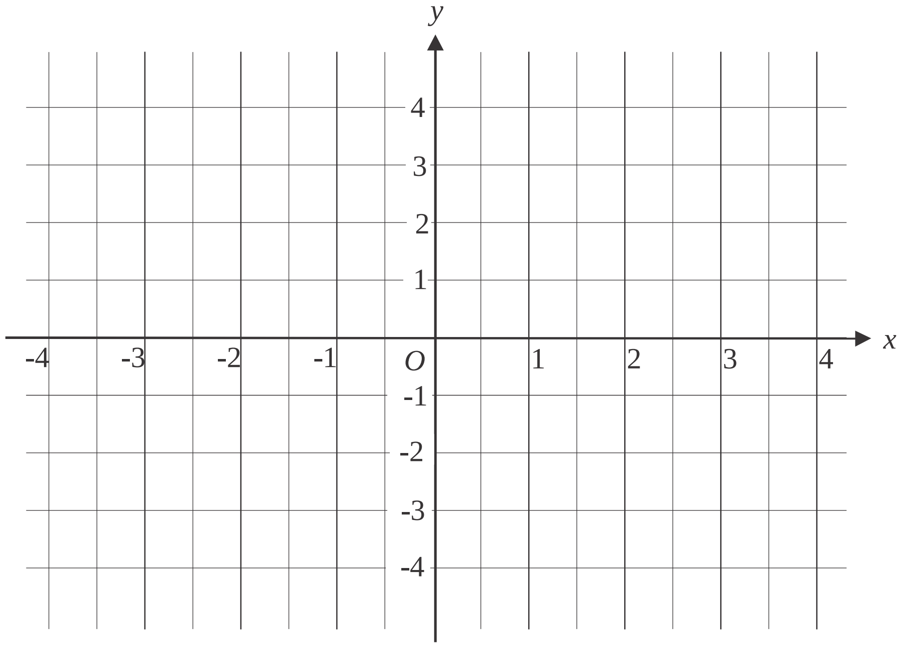
Question 6 (4 marks)

Evaluate $\int_0^{\sqrt{3}} \frac{3+x}{x^2+3}\,dx$.

Question 7 (4 marks)

Sketch the graph of $y = \dfrac{1-x}{x^2-2x}$ on the set of axes below.

Label any asymptotes with their equations and any intercepts with their coordinates.



Question 8 (3 marks)

Find $\sec(x)$ given that $x = \arcsin\left(\frac{4}{5}\right) - \arctan\left(\frac{5}{12}\right)$.

Question 9 (4 marks)

Solve the differential equation $(1+x^2)\frac{dy}{dx}-\frac{1}{x}=0$ for y, given that $x>0$ and $y(1)=2$.

Question 10 (7 marks)

Let $f(x) = \arcsin\left(\frac{x+1}{2}\right)$.

a. Find $f'(x)$. Express your answer in the form $\frac{a}{\sqrt{-(x+b)(x-a)}}$ where a and b are positive integers. 2 marks

b. Show that the rule of the inverse function of f, f^{-1}, is given by $f^{-1}(x) = 2\sin(x) - 1$. 1 mark

c. Let S be the region enclosed by the graph of f^{-1} and the x and y-axes.
Find the volume of the solid of revolution that is generated when the region S is rotated about the x-axis. 4 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$	
$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\text{E}(aX + b) = a\text{E}(x) + b$ $\text{E}(aX + bY) = a\text{E}(x) + b\text{E}(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\,\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $\text{E}(\bar{X}) = \mu$ variance $\text{var}\left(\bar{X}\right) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}\,dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos\,(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}\,dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1}\,dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+(f'(x))^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r}=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert=\sqrt{x^2+y^2+z^2}=r$
$\dot{\underset{\sim}{r}}=\frac{d\underset{\sim}{r}}{dt}=\frac{dx}{dt}\underset{\sim}{i}+\frac{dy}{dt}\underset{\sim}{j}+\frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1.\underset{\sim}{r}_2=r_1r_2\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$

Mechanics

momentum	$\underset{\sim}{p}=m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R}=m\underset{\sim}{a}$