

2018 VCE Specialist Mathematics 2 examination report

General comments

The 2018 Specialist Mathematics examination 2 comprised 20 multiple-choice questions (worth a total of 20 marks) and six extended-answer questions (worth a total of 60 marks).

There were five questions (Questions 2b., 3a., 3ci., 4b. and 5bii.) where students needed to show that a given result was reached. In these cases all steps that led to the given result needed to be clearly and logically set out to obtain full marks.

Answers were generally given in the required forms, except for the following instances where errors in form occurred:

- The domain, range and coordinates of endpoints and the y -intercept in Questions 1a. and 1aiii. were frequently given in approximate form rather than exact form.
- The rate of depth increase in Question 3cii. was frequently given as an exact value rather than to four decimal places as required.
- The point at which the paths of the yachts crossed in Question 4c. was occasionally given in exact form rather than as coordinates correct to three decimal places as required.
- The period of time in Question 4e. was frequently given in hours rather than in minutes.
- Otherwise correct answers to Question 5c. were not always manipulated to the required form.

The examination revealed areas of strength and weakness in student performance.

Areas of strength included:

- setting up of definite integrals and the use of CAS technology to evaluate them
- properties of the inverse tan function
- arc length of a parametrically defined function

Areas of weakness included:

- not carefully reading and responding to all aspects of questions
- not sketching graphs with sufficient care
- identifying the domain of a derivative after giving careful consideration to endpoints and other critical points in Question 1ei.
- complex numbers in general; non-attempts were more common for complex number questions than for other questions

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not necessarily intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section A

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	2	3	4	7	85	0	
2	10	60	23	7	0	0	
3	3	31	5	46	15	0	Option B did not account for common factors and its last term is not irreducible, so should not have Dx in the numerator.
4	49	22	14	12	4	0	
5	13	16	14	41	15	1	
6	4	9	58	10	18	1	
7	4	5	78	12	1	0	
8	14	7	4	4	71	0	
9	2	8	12	69	8	1	
10	65	7	9	6	12	1	
11	4	10	80	5	2	0	
12	36	18	21	9	16	1	Option B would not necessarily satisfy the given statement. Options D and E would not satisfy the given statement. Options A and C would satisfy the given statement, but only A is necessarily true.
13	8	65	11	14	2	0	
14	4	11	75	3	7	0	
15	3	7	17	4	69	0	
16	11	64	13	8	3	1	
17	4	7	19	21	48	1	Option D ignores the initial upwards velocity.
18	6	11	18	62	3	1	
19	4	7	25	7	57	0	
20	13	56	16	10	4	1	

Section B

Question 1a.

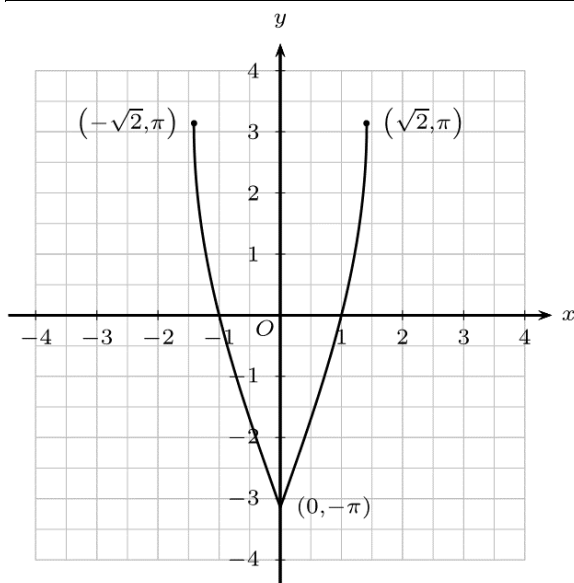
Marks	0	1	2	Average
%	16	16	68	1.5

Domain $[-\sqrt{2}, \sqrt{2}]$, Range $[-\pi, \pi]$

This question was generally handled well. Common errors included: giving open endpoints with round brackets on the intervals, decimal approximations rather than exact values and failing to state the range. Students should read questions carefully and ensure that all required information is supplied in their responses.

Question 1b.

Marks	0	1	2	3	Average
%	6	13	32	50	2.3



This question required students to label any endpoints and the y -intercept with their coordinates. Not doing this or giving incorrect coordinates frequently caused students to miss out on marks.

Students' graphs were not always precise and accurate as required. For example, many graphs had an obvious turning point at the y -intercept rather than the required shape. Other incorrect responses had endpoints in the incorrect location.

Students are advised to set viewing windows on technology to a scale that closely matches the scale provided on the examination.

Question 1c.

Marks	0	1	Average
%	20	80	0.8

$$f'(x) = \frac{4}{\sqrt{2-x^2}}$$

Parts c. and d. of Question 1 were generally answered well. Some students did not express the derivatives in the required form with a real number in the numerator.

Question 1d.

Marks	0	1	Average
%	19	81	0.8

$$f'(x) = \frac{-4}{\sqrt{2-x^2}}$$

Question 1ei.

Marks	0	1	Average
%	79	21	

$$(-\sqrt{2}, \sqrt{2}) \setminus \{0\}$$

This question was not answered well. The most common error was to include $x=0$ in the domain. Another common error was to include the endpoints $x = \pm\sqrt{2}$.

Question 1eii.

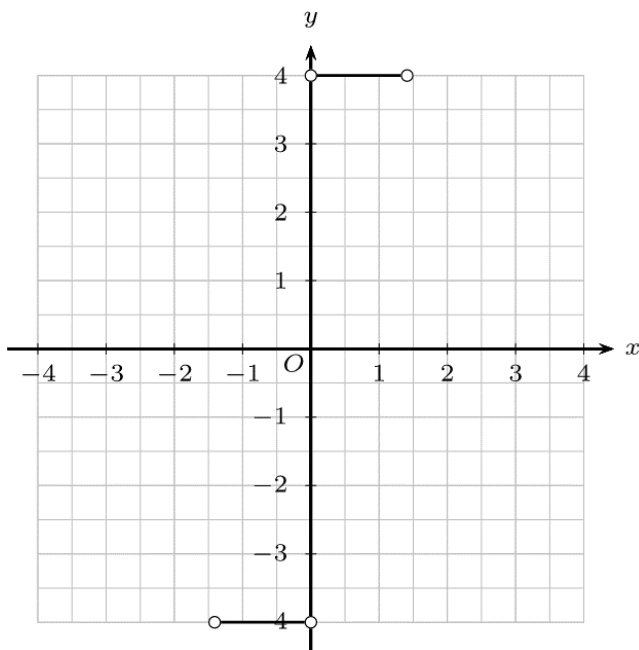
Marks	0	1	Average
%	51	49	

$$g(x) = \begin{cases} 4, & 0 < x < \sqrt{2} \\ -4, & -\sqrt{2} < x < 0 \end{cases}$$

Some graphs were not graphs of functions.

Question 1eiii.

Marks	0	1	2	Average
%	43	18	39	



The majority of students who answered Question 1eii. correctly were also able to sketch a correct graph in this question. Attempts to sketch graphs of $f'(x)$, rather than $g(x)$, were frequently made.

Question 2a.

Marks	0	1	Average
%	30	71	

Centre, $(1,2)$, radius 2

Some students gave only one of the two required parts of the answer. An incorrect radius of $\sqrt{2}$ was occasionally given. Students were not asked to find the expression of the circle at this point but a number did so.

Question 2b.

Marks	0	1	2	Average
%	16	31	54	

$$|x + iy + 1| = \sqrt{2}|x + iy - i| \Rightarrow (x+1)^2 + y^2 = 2(x^2 + (y-1)^2) \Rightarrow x^2 + 2x + 1 + y^2 = 2x^2 + 2y^2 - 4y + 2$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 4y = 0 \Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 = 4 \Rightarrow (x-1)^2 + (y-2)^2 = 4$$

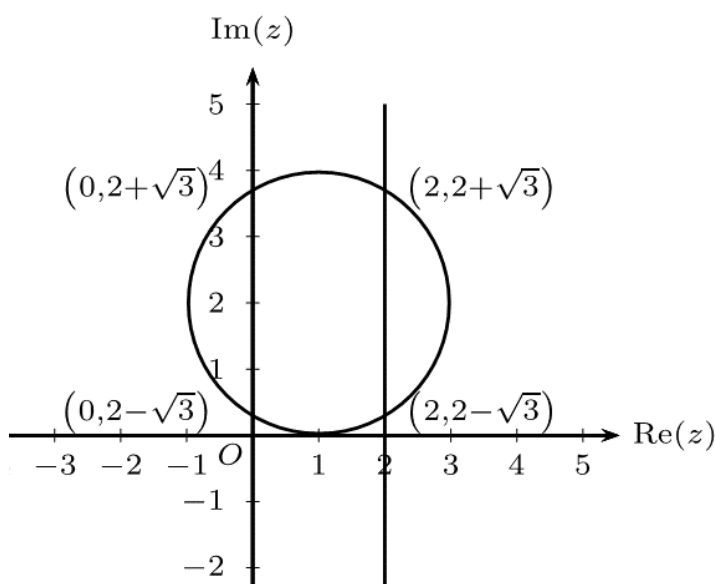
Circle, centre, $(1,2)$, radius 2

Most students were able to correctly find an expression that did not involve i . In a 'show that' question such as this, students are expected to explicitly show that the given relation leads to the required conclusion. The working shown above is an example of a suitable response.

Question 2c.

Marks	0	1	2	Average
%	23	19	58	

The diagram below shows the solutions to Questions 2c. (the circle) and 2d. (the line).



The circle was generally drawn correctly. Students did not always supply the coordinates of the y-intercepts as required by the question. Some coordinates were incorrectly given as imaginary numbers.

Question 2d.

Marks	0	1	2	Average
%	27	18	55	1.3

Refer to the diagram in Question 2c.

While the vertical line was usually sketched correctly, coordinates of the points of intersection with the circle were not always shown. Coordinates were sometimes presented as decimal approximations.

Question 2e.

Marks	0	1	2	3	Average
%	50	15	4	31	1.2

$$A = \frac{1}{2} \times 2^2 \times \left(\frac{2\pi}{3} - \sin\left(\frac{2\pi}{3}\right) \right) = \frac{4\pi}{3} - \sqrt{3}$$

Students who used standard formulas to find the segment area were generally more successful than those who took a definite integral approach. Many students did not start the problem with a correct sector angle.

Question 3a.

Marks	0	1	2	Average
%	40	27	32	0.9

$$V = \int_0^h \frac{\pi}{4} (4y^2 + 1) dy = \frac{\pi}{4} \left[\frac{4}{3} y^3 + y \right]_0^h = \frac{\pi}{4} \left(\frac{4}{3} h^3 + h \right)$$

Approximately half of the students were able to either set up an appropriate definite integral or find an antiderivative and attempt to evaluate the constant of integration. Of these, many did not explicitly show that the first part of their response yielded the required volume.

Question 3b.

Marks	0	1	2	Average
%	28	17	56	1.3

$$V\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi\sqrt{3}}{4}, \quad \frac{\pi\sqrt{3}}{8} = \frac{\pi}{4} \left(\frac{4}{3} h^3 + h \right) \Rightarrow h = 0.59$$

While the approach above was the most common, other correct approaches were used. A common error was to fail to halve the volume.

Question 3ci.

Marks	0	1	2	Average
%	34	30	36	1

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{4}{\pi(4h^2+1)} \times (0.04 - 0.05\sqrt{h}) = \frac{4}{\pi(4h^2+1)} \times \frac{4-5\sqrt{h}}{100} = \frac{4-5\sqrt{h}}{25\pi(4h^2+1)}$$

Most students were able to correctly state $\frac{dV}{dt}$ and find $\frac{dV}{dh}$ and then use this to find $\frac{dh}{dV}$ before proceeding. A few students did not understand the importance of brackets when multiplying the derivative expressions.

Many students moved directly from the product of the derivatives to the required expression, without explicitly showing that their product led to the final (given) answer.

Question 3cii.

Marks	0	1	Average
%	26	74	0.8

0.0153

The majority of students were able to use the supplied derivative to find the required rate for the given depth. Some students did not give the answer in the required decimal form.

Question 3d.

Marks	0	1	2	Average
%	34	10	56	1.2

$$t = \int_0^{0.25} \frac{25\pi(4h^2+1)}{4-5\sqrt{h}} dh = 9.8$$

Most students were able to set up a correct definite integral. Transcription errors were occasionally present in the integrand.

Question 3e.

Marks	0	1	2	Average
%	64	10	25	0.6

$$h = 0.4 + \frac{4-5\sqrt{0.4}}{25\pi(4 \times 0.4^2 + 1)} \times 5 = 0.43$$

Many students did not explicitly demonstrate their use of Euler's method. Of those who did, a number incorrectly substituted into their expression.

Question 3f.

Marks	0	1	2	Average
%	61	15	24	0.7

$$\frac{dh}{dt} = 0 \Rightarrow h = \frac{16}{25}, \quad \frac{\sqrt{3}}{2} - \frac{16}{25} = 0.23$$

Most students who attempted this question understood that they need to solve $\frac{dh}{dt} = 0$ to find the limiting water level. Many students who correctly found h did not subtract their value from the height of the top of the fountain.

Question 4a.

Marks	0	1	2	Average
%	11	8	81	1.7

A: $y = x^2 - 1$, B: $y = x + 3$

A variety of less simplified, but correct, forms were given and accepted.

Question 4b.

Marks	0	1	2	Average
%	19	8	72	1.6

$$t+1=t^2 \Rightarrow t = \frac{1+\sqrt{5}}{2}, \quad t^2+2t=t^2+3 \Rightarrow t = \frac{3}{2}, \quad \frac{1+\sqrt{5}}{2} \neq \frac{3}{2}$$

The approach shown is one of several of correct ways to show that the yachts do not collide.

Question 4c.

Marks	0	1	2	Average
%	25	41	34	1.1

(2.562, 5.562)

Missing the condition that $t \geq 0$, led, incorrectly, to two points being provided. Many otherwise correct responses were not expressed in the required form, as coordinates correct to three decimal places.

Question 4d.

Marks	0	1	2	Average
%	43	32	25	0.8

$$\sqrt{(2t+2)^2+1} > \sqrt{8t^2} \Rightarrow 0 \leq t < \frac{5}{2}$$

Many students approached this question incorrectly. Some set up inequalities using magnitudes of the position vectors, others attempted to compare the velocities of the yachts rather than correctly set up an equation or inequation involving the speeds. Of those who approached the question correctly, a number ignored the domain of t , giving answers that included negative values.

Question 4e.

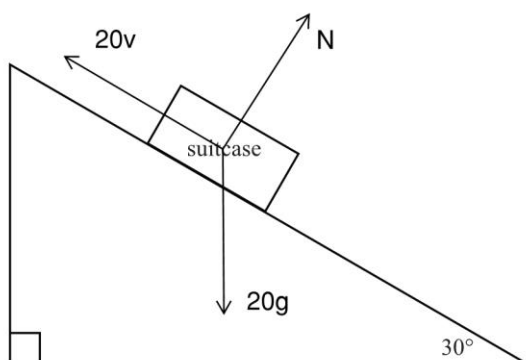
Marks	0	1	2	Average
%	54	18	28	0.8

$$(t^2 - t - 1)^2 + (3 - 2t)^2 < 0.2^2 \Rightarrow 1.529 < t < 1.597, \text{ Period in minutes, } 4.1 \text{ (91.7 to 95.8)}$$

Many students did not attempt Question 4e. A common misconception was evident when students attempted to solve $|r_B| - |r_A| < 0.2$, rather than the correct $|r_B - r_A| < 0.2$. Responses in terms of hours, rather than minutes, were given by a number of students who did not respond to the specifics of the question.

Question 5a.

Marks	0	1	Average
%	56	44	0.5



Although the particular values of the forces were not required at this point, labels here should have been correct if they were not general (e.g. W for the weight force was acceptable). Many students labelled the resistance force incorrectly as v , rather than $20v$.

Question 5bi.

Marks	0	1	Average
%	41	59	0.6

$$20g \sin(30^\circ) - 20v = 20a$$

Many students who incorrectly labelled the resistance force in Question 1a. corrected themselves here. Some students gave an expression for the net force on the suitcase rather than an equation of motion involving acceleration. Various correct, simplified forms of the equation above were accepted.

Question 5bii.

Marks	0	1	Average
%	34	66	0.7

$$20g \sin(30^\circ) - 20v = 20a \Rightarrow 10g - 20v = 20a, \quad a = \frac{10g - 20v}{20} = \frac{g - 2v}{2}$$

Question 5c.

Marks	0	1	2	Average
%	28	49	23	1

$$v \frac{dv}{dx} = \frac{g-2v}{2} \Rightarrow x = -v + 4.9 \log_e \left(\frac{4.9}{4.9-v} \right)$$

Most students set up a differential equation using the appropriate derivative form of acceleration. Many students who successfully solved the equation, by a variety of suitable means, did not give the solution in the required form.

Question 5d.

Marks	0	1	Average
%	74	26	0.3

4.81

Difficulty with Question 5c. led to many students not answering this question successfully.

Question 5ei.

Marks	0	1	Average
%	59	41	0.4

$$\int_0^{4.5} \frac{2}{9.8-2v} dv$$

An incorrect integrand, typically the reciprocal of the correct integrand, was common.

Question 5eii.

Marks	0	1	Average
%	59	41	0.4

2.51

Correct definite integrals in the previous question generally resulted in correct responses for this question.

Question 6a.

Marks	0	1	Average
%	30	70	0.7

$$H_0: \mu = 150, \quad H_1: \mu < 150$$

The question was answered well. Common errors included: poor notation such as $H_0 = 150$ or similar, and not understanding the nature of a one-tailed test, evidenced by answers such as $H_1: \mu \neq 150$.

Question 6b.

Marks	0	1	Average
%	16	84	0.9

$$\frac{15}{\sqrt{50}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

This question was generally well answered. A variety of correct, exact forms were accepted.

Question 6c.

Marks	0	1	2	Average
%	22	18	60	1.4

$$p = \Pr(\bar{X} < 145 \mid \mu = 150) = 0.0092$$

Most students obtained the correct value of p . A small number of these did not write p to the required four decimal places. Some students inappropriately used calculator syntax in place of correct working or notation. Students must take care with notation as some responses incorrectly stated that $p = \Pr(X < 145 \mid \mu = 150)$.

Question 6d.

Marks	0	1	Average
%	24	76	0.8

As $p(=0.0092) < 0.05$, reject H_0 .

Most students were able to draw the appropriate conclusion. Some students did not supply a reason for their conclusion as required by the question. Occasional errors caused some students to miss out on the mark. For example, some responses incorrectly stated that $0.0092 > 0.05$.

Question 6e.

Marks	0	1	Average
%	52	48	0.5

146.51, 146.52

In this instance, both values above were accepted.

Question 6f.

Marks	0	1	Average
%	89	11	0.1

0.24

Only a small number of students attempted this question.

Question 6g.

Marks	0	1	Average
%	48	52	0.5

(139.5, 150.5)

While many students answered this correctly and concisely, arithmetic errors caused some students to miss out on the mark. Some students appeared to use a 95% confidence interval rather than the required 99% confidence interval. It is important for students to read questions carefully.