



2018 Trial Examination

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT
NUMBER

								Letter
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SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION & ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 12 pages.

Instructions

- Print your student number in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

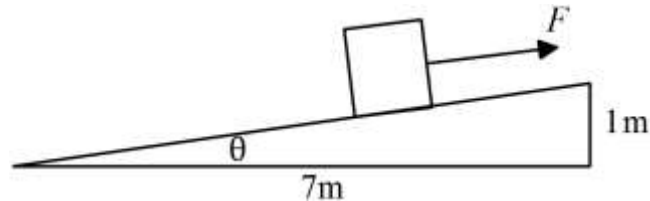
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagram in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 (3 marks)

A constant force of F newtons is applied to a body of mass 10kg , parallel to a smooth inclined plane with angle of elevation θ as shown on the diagram below.



- a. Show that $\sin \theta = \frac{\sqrt{2}}{10}$.

1 mark

- b. Find the force F required to give the body an acceleration of $\sqrt{2} \text{ m/s}$ up the plane.
Write your answer in the form $a(b + g)$, where $a, b \in R$ and g is acceleration due to gravity.

2 marks

TURN OVER

Question 2 (3 marks)

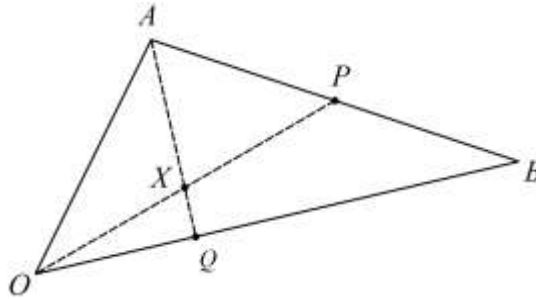
Find all possible values for the gradient of the curve $\sin(x) \sin(y) = \frac{1}{2}$ at the points where $x = \frac{\pi}{4}$, $0 < y < 2\pi$.

Question 3 (3 marks)

Solve the equation $z^3 = (1 - i)^6$ for z , writing the solutions in Cartesian form.

Question 4 (5 marks)

In triangle OAB , let P be the midpoint of AB and Q be the point on OB such that $\overrightarrow{QB} = 2\overrightarrow{OQ}$. Let X be the point of intersection of AQ and OP .



- a. Given that $\underline{\underline{a}} = \overrightarrow{OA}$ and $\underline{\underline{b}} = \overrightarrow{OB}$, express $\overrightarrow{AQ}$ in terms of $\underline{\underline{a}}$ and $\underline{\underline{b}}$.

1 mark

- b. Express $\overrightarrow{OP}$ in terms of $\underline{\underline{a}}$ and $\underline{\underline{b}}$.

1 mark

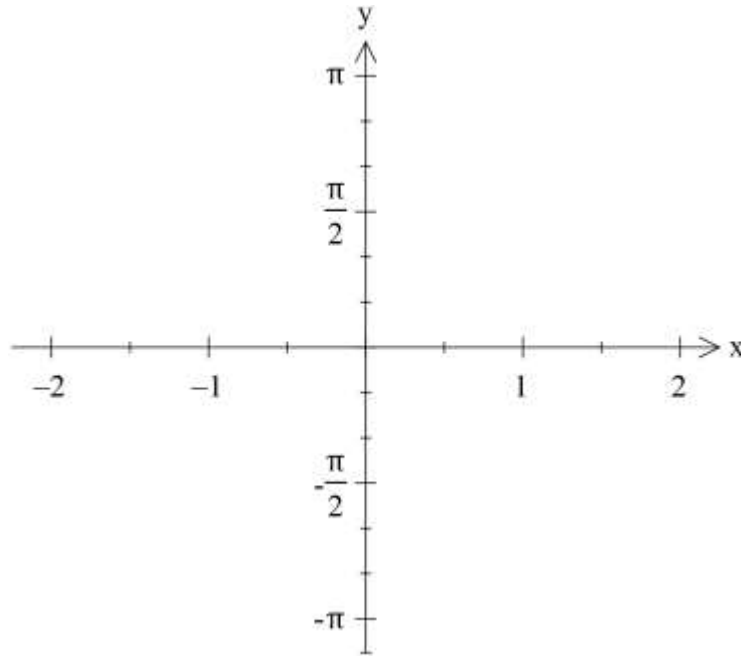
- c. Let k be the rational number such that $\overrightarrow{AX} = k\overrightarrow{AQ}$. Find k .

3 marks

TURN OVER

Question 5 (5 marks)

- a. Sketch the graph of $y = \arcsin(\frac{1}{2} - x)$ on the axis below.
Label endpoints in coordinate form.



2 marks

- b. Find the exact volume of the solid obtained when the section of the graph of $y = \arcsin(\frac{1}{2} - x)$ where $x \geq 0$ is rotated around the y axis.

3 marks

Question 6 (4 marks)

Two teams of soccer players are timed running 100m sprints. Team A's times are normally distributed with mean 14.7 seconds and standard deviation 0.8 seconds. Team B's times are normally distributed with mean 13.7 seconds and standard deviation 0.6 seconds.

- a. Using the normal random variable $D = A - B$ to represent the difference in times, find the probability that a randomly selected player from Team A would be faster than a randomly selected player from Team B. Correct your answer to two decimal places.

2 marks

- b. If a random sample of four players is taken from each team, find the probability that the average time for the four Team A players would be faster than the average time for the four Team B players. Correct your answer to three decimal places.
You may assume that each selection is independent and normally distributed.

2 marks

TURN OVER

Question 7 (5 marks)

Solve the differential equation $\frac{dy}{dx} = \frac{y-2}{1+4x^2}$, given that $y\left(-\frac{1}{2}\right) = 1$.

Give the solution for y in terms of x .

[illegible]

Question 8 (6 marks)

Consider the function $f: R^+ \setminus \{2\} \rightarrow R, f(x) = \frac{\sqrt{x}}{x-2}$

- a.** Find the x values for any stationary points of f , or show that none exist.

2 marks

TURN OVER

- b.** Find the x values for any points of inflection of the function f , or show that none exist.

[illegible]

4 marks

Question 9 (6 marks)

- a. Find an anti-derivative for the function g , where $g(x) = \frac{\log_e|x-1|}{x-1}$.

1 mark

- b. Find x intercepts for the function g .

1 mark

- c. Find the area bound by the graph of g , the x axis and the line $x = -2$.

1 mark

TURN OVER

- d. Solve the equation below to find k in terms of a , where $a < 1$ and $k > 1$.

$$\int_0^a \frac{\log_e |x-1|}{x-1} dx = \int_2^k \frac{\log_e |x-1|}{x-1} dx$$

3 marks

END OF QUESTION AND ANSWER BOOK