



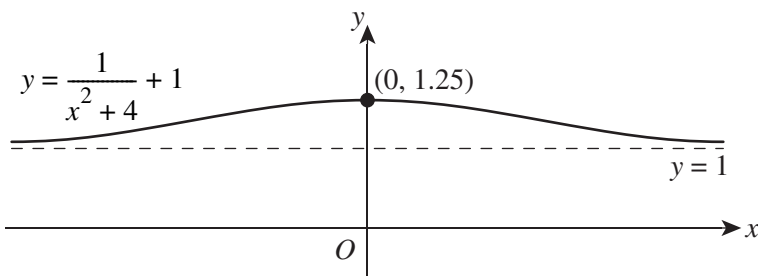
Trial Examination 2018

VCE Specialist Mathematics Units 3&4

Written Examination 1

Suggested Solutions

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Question 1 (3 marks)

correct shape (concavity and asymptotic behaviour) A1
 y-coordinate and stationary point is $(0, 1.25)$ A1
 horizontal asymptote is $y = 1$ A1

Question 2 (3 marks)

The parametric equations are $x = 2 - t^2$ and $y = 4t$.

Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ to obtain $\frac{dy}{dx} = -\frac{2}{t}$. M1

Let the gradient of the normal be m_N .

When $t = 1$, $\frac{dy}{dx} = -2$ and so, $m_N = \frac{1}{2}$. M1

When $t = 1$, $x = 1$ and $y = 4$.

So, $y = \frac{1}{2}x + \frac{7}{2}$. A1

Question 3 (3 marks)

Let X represent the weight of the lemons.

$$X \sim N(58, 9^2)$$

$\bar{X} \sim N\left(58, \frac{9^2}{36}\right)$, that is, $E(\bar{X}) = 58$ and $sd(\bar{X}) = \frac{9}{6} (= 1.5)$ A1

$$\begin{aligned} \Pr(\bar{X} > 61) &= \Pr(\bar{X} > 58 + 2 \times 1.5) & \text{M1} \\ &= 0.025 & \text{A1} \end{aligned}$$

Question 4 (3 marks)

The equations of motion are $T = 3ma$ and $2mg - T = 2ma$. A1

$$3mg - \frac{3T}{2} = 3ma \Rightarrow 3mg - \frac{3T}{2} = T \quad \text{M1}$$

Solving for T gives $T = \frac{6mg}{5}$ (N). A1

Question 5 (5 marks)

It is given that $x^3 + xy^2 - y^3 = 2$.

Using implicit differentiation obtains $3x^2 + y^2 + 2xy\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$.

M1 A1

$$\frac{dy}{dx} = 0 \Rightarrow 3x^2 + y^2 = 0$$

A1

So, $x = y = 0$.

A1

Formal verification (that is the substitution of $x = 0$ and $y = 0$ into $x^3 + xy^2 - y^3 = 2$) shows that $(0, 0)$ is not on C .

M1

Hence, there is no point on C at which $\frac{dy}{dx} = 0$.

Question 6 (5 marks)

The equation to solve is $z^3 = -2 + 2i$.

$$z^3 = 2\sqrt{2}\text{cis}\left(\frac{3\pi}{4} + 2k\pi\right), k = 0, 1, 2$$

M1

$$z = \sqrt{2}\text{cis}\left(\frac{\pi}{4} + \frac{2k\pi}{3}\right), k = 0, 1, 2$$

A1

$$z_1 = \sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$$

A1

Adding or subtracting $\frac{2\pi}{3}$ to obtain the other two solutions:

$$z_2 = \sqrt{2}\text{cis}\left(\frac{11\pi}{12}\right), z_3 = \sqrt{2}\text{cis}\left(-\frac{5\pi}{12}\right)$$

A2

Question 7 (3 marks)

The question asks to prove that $\underline{p} \cdot \underline{q} = |\underline{p}|^2$.

$$\angle OPQ = 90^\circ \text{ and so } \overrightarrow{PO} \cdot \overrightarrow{PQ} = |\overrightarrow{PO}| |\overrightarrow{PQ}| \cos(\angle OPQ) = 0$$

M1

$$\overrightarrow{PO} = -\underline{p} \text{ and } \overrightarrow{PQ} = \underline{q} - \underline{p}, \text{ so } -\underline{p} \cdot (\underline{q} - \underline{p}) = 0.$$

A1

$$-\underline{p} \cdot \underline{q} + \underline{p} \cdot \underline{p} = 0$$

$$\underline{p} \cdot \underline{q} = \underline{p} \cdot \underline{p} \text{ (dot product is distributive)}$$

A1

$$\text{So } \underline{p} \cdot \underline{q} = |\underline{p}|^2.$$

Question 8 (5 marks)

- a. It is given that $a = 2x - \frac{1}{6}x^2$, $0 \leq x \leq 18$.

Method 1:

$$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) \text{ and so } \frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2x - \frac{1}{6}x^2$$

$$\int \frac{d}{dx}\left(\frac{1}{2}v^2\right) dx = \int \left(2x - \frac{1}{6}x^2\right) dx \quad \text{M1}$$

$$\frac{1}{2}v^2 = x^2 - \frac{1}{18}x^3 + c \quad \text{A1}$$

When $x = 0$, $v = 0$ and so $c = 0$.

$$\text{So } v^2 = 2x^2 - \frac{1}{9}x^3. \quad \text{A1}$$

Method 2:

$$a = v \frac{dv}{dx} \text{ and so } v \frac{dv}{dx} = 2x - \frac{1}{6}x^2$$

$$\int v dv = \int \left(2x - \frac{1}{6}x^2\right) dx \quad \text{M1}$$

$$\frac{1}{2}v^2 = x^2 - \frac{1}{18}x^3 + c \quad \text{A1}$$

When $x = 0$, $v = 0$ and so $c = 0$.

$$\text{So } v^2 = 2x^2 - \frac{1}{9}x^3. \quad \text{A1}$$

- b. $v = 0 \Rightarrow 2x^2 - \frac{1}{9}x^3 = 0 \quad \text{M1}$

$$x^2\left(2 - \frac{1}{9}x\right) = 0$$

$$x = 18 \text{ (m)} \text{ } (x > 0) \quad \text{A1}$$

Question 9 (5 marks)

a. $\sin^2(\theta) = \frac{1}{4}(1 - \cos(2\theta))$

$$\sin^4(\theta) = \frac{1}{4}(1 - \cos(2\theta))^2 \quad \text{M1}$$

$$= \frac{1}{4}(1 - 2\cos(2\theta) + \cos^2(2\theta)) \quad \text{A1}$$

$$= \frac{1}{4}\left(1 - 2\cos(2\theta) + \frac{1 + \cos(4\theta)}{2}\right) \text{ (or equivalent)} \quad \text{A1}$$

Leading to $\sin^4(\theta) = \frac{1}{8}(3 - 4\cos(2\theta) + \cos(4\theta))$.

b. $\int_0^{\frac{\pi}{4}} \sin^4(\theta) d\theta = \int_0^{\frac{\pi}{4}} \frac{1}{8}(3 - 4\cos(2\theta) + \cos(4\theta)) d\theta$ (from **part a.**)

$$= \frac{1}{8} \left[3\theta - 2\sin(2\theta) + \frac{\sin(4\theta)}{4} \right]_0^{\frac{\pi}{4}} \quad \text{M1}$$

$$= \frac{1}{8} \left(\frac{3\pi}{4} - 2\sin\left(\frac{\pi}{2}\right) + \frac{\sin(\pi)}{4} - \left(0 - 2\sin(0) + \frac{\sin(0)}{4}\right) \right)$$

$$= \frac{3\pi - 8}{32} \quad \text{A1}$$

Question 10 (5 marks)

Solve $\arccos(x) + \arccos(2x) = \frac{\pi}{2}$ for x where $x \geq 0$.

Method 1:

Apply $\cos$ to both sides of the equation $\arccos(2x) = \frac{\pi}{2} - \arccos(x)$ to obtain:

$$2x = \cos\left(\frac{\pi}{2} - \arccos(x)\right) \quad \text{A1}$$

Use of $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$ on the RHS to obtain:

$$2x = \cos\left(\frac{\pi}{2}\right)\cos(\arccos(x)) + \sin\left(\frac{\pi}{2}\right)\sin(\arccos(x)) \quad \text{M1}$$

$$\text{So, the equation becomes } 2x = \sqrt{1 - x^2}. \quad \text{A1}$$

$$\text{Squaring both sides gives: } 4x^2 = 1 - x^2. \quad \text{M1}$$

Solving this equation gives:

$$x = \frac{1}{\sqrt{5}} \quad (x \geq 0) \quad \text{A1}$$

Method 2:

Apply $\cos$ to both sides of the equation to obtain:

$$\cos(\arccos(x) + \arccos(2x)) = \cos\left(\frac{\pi}{2}\right) \quad \text{A1}$$

Use of $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ on the LHS to obtain:

$$\begin{aligned} \cos(\arccos(x) + \arccos(2x)) &= \cos(\arccos(x))\cos(\arccos(2x)) - \sin(\arccos(x))\sin(\arccos(2x)) \\ &= 2x^2 - \left(\sqrt{1 - x^2}\right)\left(\sqrt{1 - 4x^2}\right) \end{aligned} \quad \text{M1}$$

$$\text{So, the equation becomes } 2x^2 - \left(\sqrt{1 - x^2}\right)\left(\sqrt{1 - 4x^2}\right) = 0. \quad \text{A1}$$

$$\text{Rearranging and squaring both sides gives: } 4x^4 = 4x^4 - 5x^2 + 1. \quad \text{M1}$$

Solving this equation gives:

$$x = \frac{1}{\sqrt{5}} \quad (x \geq 0) \quad \text{A1}$$