

2018 Specialist Mathematics Trial Exam 2 Solutions

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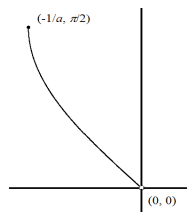
SECTION A – Multiple-choice questions

1	2	3	4	5	6	7	8	9	10
C	D	A	D	A	C	A	E	E	B

11	12	13	14	15	16	17	18	19	20
D	B	A	B	A	C	D	A	E	E

Q1 C

Q2 D

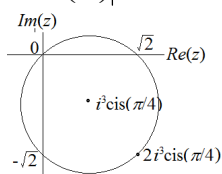


Q3 $x = \sin(\cos^{-1} t)$, $\therefore 0 \leq x \leq 1$, $\sin^{-1} x = \cos^{-1} t = \frac{\pi}{2} - \sin^{-1} t$

$y = \cos(\sin^{-1} t)$, $\therefore \cos^{-1} y = \sin^{-1} t$, $\therefore \cos^{-1} y = \frac{\pi}{2} - \sin^{-1} x$,

$\therefore y = \cos\left(\frac{\pi}{2} - \sin^{-1} x\right) = \sin(\sin^{-1} x) = x$ A

Q4 $\left|z + i \operatorname{cis}\left(\frac{\pi}{4}\right)\right| = \left|z - i^3 \operatorname{cis}\left(\frac{\pi}{4}\right)\right| = 1$ D



Q5 $z = \frac{a}{\sqrt{2}}(-1 + i)$, $z + a\sqrt{2} = \frac{a}{\sqrt{2}}(1 + i)$,

$z - a\sqrt{2}i = -\frac{a}{\sqrt{2}}(1 + i)$, $\therefore \frac{z + a\sqrt{2}}{z - a\sqrt{2}i} = -1$ A

Q6 $\left(n + \frac{3\pi}{22}\right) - \left(n - \frac{4\pi}{11}\right) = \frac{\pi}{2}$, $\therefore O$, z_1 , z_2 and w form a

rectangle. $|w|$ = length of diagonal $= \sqrt{a^2 + b^2}$. C

Q7 $|a| = |b|$, $\therefore a = \pm b$, $\therefore a + b = 0$ or $a - b = 0$

Translate both to the right by $\frac{\alpha}{2}$, $f(x) = \left|a \operatorname{cosec}\left(\frac{x}{2}\right)\right|$ and

$g(x) = \left|b \sec\left(\frac{x - (\alpha - \beta)}{2}\right)\right|$, $f(x) = g(x)$ when $\alpha - \beta = \pm\pi$. A

Q8 Let $\cos^{-1} x - \sin^{-1} y = \frac{\pi}{6}$. Try $\cos^{-1} x = 0$ and $-\sin^{-1} y = \frac{\pi}{6}$.

$\therefore x = 1$ and $y = -\frac{1}{2}$, $\therefore x + y = \frac{1}{2}$. E

Note: For an alternative solution, try $\cos^{-1} x = \frac{2\pi}{3}$ and

$\sin^{-1} y = \frac{\pi}{2}$, $\therefore x = -\frac{1}{2}$ and $y = 1$, $\therefore x + y = \frac{1}{2}$.



Q9 East of the origin, $\tan^{-1} t > 0$ and $\log_e t = 0$, $\therefore t = 1$

$\tilde{v} = \frac{d\tilde{r}}{dt} = \frac{1}{1+t^2} \tilde{i} + \frac{1}{t} \tilde{j} = \frac{1}{2} \tilde{i} + \tilde{j}$, speed $= \sqrt{\left(\frac{1}{2}\right)^2 + 1^2} = \frac{\sqrt{5}}{2}$ E

Q10 $\frac{dy}{dx} = \frac{1}{x} - x$, $y = -0.5$ when $x = 1$

$\therefore y = \log_e |x| - \frac{x^2}{2} = -1$, $x = \alpha = \pm 0.3982, \pm 1.775$

-0.40 is the closest. B

Q11 $-2 = 2u + 2a$, $4 = 4u + 8a$, $a = 2$ and $u = -3$

Let $-3t + t^2 = 0$, $t = 0$ or 3 . D

Q12 $\theta = \cos^{-1} \frac{3+3}{\sqrt{10}\sqrt{11}}$, scalar resolute $= \sqrt{10} \sin \theta \approx 2.6$ B

Q13 $50a = 45g - 50g$, $a = -0.1g \approx -1$, acceleration is downward.

The lift is either moving upwards but slowing down, or moving downwards and speeding up. A

Q14 $\tilde{v} = 10\tilde{i} - 9.8t\tilde{j}$, $\tan(-45^\circ) = \frac{-9.8t}{10} = -1$, $t \approx 1.0$ B

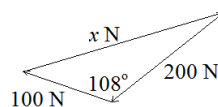
Q15 $\int_{-1}^0 |f(x)| dx \geq 2$ and $\int_0^1 |f(x)| dx \geq \frac{1}{2}$

$\therefore \int_{-1}^1 |f(x)| dx = \int_{-1}^0 |f(x)| dx + \int_0^1 |f(x)| dx \geq \frac{5}{2}$ A

Q16 $\int 2v dv = \int \frac{1}{\sqrt{9-x^2}} dx$, and $v = 0$ at $x = 0$, $\therefore v^2 = \sin^{-1}\left(\frac{x}{3}\right)$

$v = \frac{dx}{dt} = \sqrt{\sin^{-1}\left(\frac{x}{3}\right)}$, $\frac{dt}{dx} = \frac{1}{\sqrt{\sin^{-1}\left(\frac{x}{3}\right)}}$, $\Delta t = \int_1^2 \frac{1}{\sin^{-1}\left(\frac{x}{3}\right)} dx \approx 1.4$ C

Q17 D



$x = \sqrt{100^2 + 200^2 - 2(100)(200)\cos 108^\circ} \approx 250$

Q18 $E(\bar{X}) \approx \mu = 75$, $sd(\bar{X}) = \frac{12}{\sqrt{25}} = 2.4$

$\Pr(\bar{X} = 80) = \Pr(79.5 < \bar{X} < 80.5) \approx 0.01943$ A

Q19 $E(\bar{W}) \approx \mu = 75$, $sd(\bar{W}) = \frac{12}{\sqrt{100}} = 1.2$

p -value $= \Pr(\bar{W} \geq 78 | \mu = 75) \approx 0.0062$ E

Q20 $\Pr(-z < Z < z) = 0.8$, $z = 1.28155$

80% confidence interval for the mean life of the batteries is:

$\left(29.52 - 1.28155 \times \frac{0.45}{\sqrt{36}}, 29.52 + 1.28155 \times \frac{0.45}{\sqrt{36}}\right)$

i.e. $(29.42, 29.62)$ E



SECTION B

Q1a $\overrightarrow{BP} = \frac{1}{2}\overrightarrow{BA} = \frac{1}{2}(\overrightarrow{OA} - \overrightarrow{OB})$,

$\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP} = \overrightarrow{OB} + \frac{1}{2}(\overrightarrow{OA} - \overrightarrow{OB}) = \frac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$

Q1b $\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = \overrightarrow{OM} + \frac{1}{x}\overrightarrow{OM} = \left(\frac{x+1}{x}\right)\overrightarrow{OM}$
 $\therefore \overrightarrow{OM} = \left(\frac{x}{x+1}\right)\overrightarrow{OP} = \left(\frac{x}{x+1}\right)\left(\frac{\overrightarrow{OA} + \overrightarrow{OB}}{2}\right) = \frac{x}{x+1}\left(\frac{2\overrightarrow{OQ} + \overrightarrow{OB}}{2}\right)$

Q1c $\overrightarrow{BQ} = \overrightarrow{BM} + \overrightarrow{MQ} = \overrightarrow{BM} + \frac{1}{y}\overrightarrow{BM} = \left(\frac{y+1}{y}\right)\overrightarrow{BM}$
 $\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM} = \overrightarrow{OB} + \left(\frac{y}{y+1}\right)\overrightarrow{BQ} = \overrightarrow{OB} + \left(\frac{y}{y+1}\right)(\overrightarrow{OQ} - \overrightarrow{OB})$
 $= \overrightarrow{OB} + \left(\frac{y}{y+1}\right)(\overrightarrow{OQ} - \overrightarrow{OB}) = \frac{y\overrightarrow{OQ} + \overrightarrow{OB}}{y+1}$

Q1d Comparing the two expressions for $\overrightarrow{OM}$ in parts b and c,

$\frac{x}{x+1} = \frac{y}{y+1}$ and $\frac{x}{2(x+1)} = \frac{1}{y+1}$

$\therefore x = y = 2$

Q2a $(-i)^{\frac{1}{3}} = (-1)^{\frac{1}{3}} = i$, $\therefore ((-i)^{\frac{1}{3}})^6 = ((-1)^{\frac{1}{3}})^6$, $(-i)^2 = (-1)^3$
 Since $(-i)^4 = (-1)^4$, $\therefore (-i)^2(-i)^4 = (-1)^3(-1)^4$, $(-i)^6 = (-1)^7$
 $\therefore (-i)^{\frac{1}{7}} = (-1)^{\frac{1}{6}}$, $\therefore m = 7$ and $n = 6$ for both values to be higher.

Note: It is possible, for example $(-i)^2 = (-1)^3(-1)^4$,

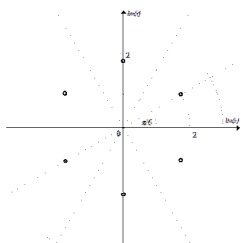
$\therefore (-i)^2 = (-1)^7$, $(-i)^{\frac{1}{7}} = (-1)^{\frac{1}{2}}$, $\therefore m = 7$ and $n = 2$. In this case only one value is higher.

Q2b $(-8i)^{\frac{1}{3}} = (-64)^{\frac{1}{6}}$, $((-8i)^{\frac{1}{3}})^6 = ((-64)^{\frac{1}{6}})^6$,

$\therefore (-8i)^2 = -64$ which is true.

Q2c $z = 2i$

Q2d



Q2e One of the cube roots of $(-8i)^{\frac{1}{3}}$ is $2i$, i.e. $2\text{cis}\left(\frac{\pi}{2}\right)$. The

other two are $2\text{cis}\left(\frac{7\pi}{6}\right)$ and $2\text{cis}\left(\frac{11\pi}{6}\right)$. Multiplying by i will

rotate all by $\frac{\pi}{2}$ anticlockwise. They are $2\text{cis}(\pi)$, $2\text{cis}\left(\frac{5\pi}{3}\right)$ and

$2\text{cis}\left(\frac{\pi}{3}\right)$, i.e. $2\text{cis}\left(-\frac{\pi}{3}\right)$, $2\text{cis}\left(\frac{\pi}{3}\right)$ and $2\text{cis}(\pi)$.

Q3a $\tilde{v} = -10\sin t \tilde{i} + 10\cos t \tilde{j} - 9.8t \tilde{k}$

$\tilde{a} = \frac{d\tilde{v}}{dt} = -10\cos t \tilde{i} - 10\sin t \tilde{j} - 9.8 \tilde{k}$

Magnitude of vertical acceleration = 9.8

Magnitude of horizontal acceleration = $10\sqrt{(-\cos t)^2 + (-\sin t)^2} = 10$

Q3b $\tilde{r} = \int (-10\sin t \tilde{i} + 10\cos t \tilde{j} - 9.8t \tilde{k}) dt$

and $\tilde{r} = 10 \tilde{i} + 44.1 \tilde{k}$ at $t = 0$

$\therefore \tilde{r} = 10\cos t \tilde{i} - 10\sin t \tilde{j} + (44.1 - 4.9t^2) \tilde{k}$

Q3c $44.1 - 4.9t^2 = 0$, $t = 3$

Q3d When $t = 3$, $\tilde{r} = 10\cos 3 \tilde{i} - 10\sin 3 \tilde{j} + 0 \tilde{k}$

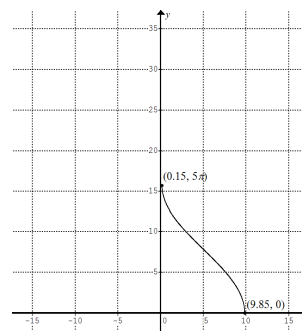
Landing point is $(10\cos 3, -10\sin 3, 0)$, i.e. $(-9.9, -1.4, 0)$ approx.

Q3e When $t = 3$, $\tilde{v} = -10\sin 3 \tilde{i} + 10\cos 3 \tilde{j} - 29.4 \tilde{k}$

Speed = $|\tilde{v}| = \sqrt{(-10\sin 3)^2 + (10\cos 3)^2 + (-29.4)^2} \approx 31.1$

Q3f $\tan \theta \approx \frac{29.4}{10}$, $\theta \approx 71.2^\circ$

Q4a



Q4b Reflect in the x-axis, then translate upwards by 10π ,

$y = 10\pi - 5\cos^{-1}\left(\frac{x-5}{4.85}\right)$

Q4c $V = \int_{5\pi-15}^{5\pi} \pi x^2 dy = \int_{5\pi-15}^{5\pi} \pi \left(4.85\cos\left(\frac{y}{5}\right) + 5\right)^2 dy \approx 1599$

Q4d Flow rate = $\frac{1599}{60} \approx 26.65$

Q4e $\int_0^h \pi \left(4.85\cos\left(\frac{y}{5}\right) + 5\right)^2 dy = \frac{45}{60} \times 1599 = 1199.25$, $h \approx 4.46$

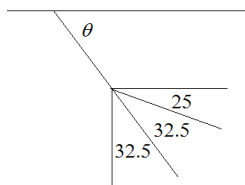


Q5a $a = 0.010 \text{ m s}^{-2}$

Q5b $F_{\text{pull}} - 3g = 3a$, $F_{\text{pull}} = 3g + 3 \times 0.1 = 29.7 \text{ N}$

Q5c $T_2 = T_3 = F_{\text{pull}} = 29.7 \text{ N}$, $T_4 - 1g = 1 \times 0.1$, $T_4 = 9.9 \text{ N}$

Q5d $\theta = 25^\circ + 32.5^\circ = 57.5^\circ$



Q5e $T_1 = T_2 \cos 32.5^\circ + T_3 \cos 32.5^\circ \approx 50.1 \text{ N}$

Q6ai $E(H) = 10 \times \mu = 1700 \text{ cm}$

Q6aii $\text{Var}(H) = 10 \times \text{Var}(X) = 10 \times 20^2 = 4000$

$\text{sd}(H) = \sqrt{\text{Var}(H)} = \sqrt{4000} \approx 63.2 \text{ cm}$

Q6bi $E(\bar{X}) = \mu = 170 \text{ cm}$, $\text{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{10}} \approx 6.3 \text{ cm}$

Q6bii $\Pr(\bar{X} > 175) \approx 0.2$

Q6ci $\bar{x} = 168.9 \text{ cm}$, $s_x = 16.0 \text{ cm}$

Q6cii Approximate 95% confidence interval for μ is

$\left(168.9 - 1.96 \times \frac{16.0}{\sqrt{10}}, 168.9 + 1.96 \times \frac{16.0}{\sqrt{10}} \right) \approx (159.0, 178.8) \text{ cm}$

Q6ciii Let m be the sample size required to reduce the width of the interval by half.

$\frac{1}{\sqrt{m}} = \frac{1}{2} \times \frac{1}{\sqrt{10}}$, $m = 40$

Please inform mathline@itute.com re conceptual and/or mathematical errors