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Specialist Mathematics

2018

Trial Examination 1 (1 hour)

Instructions

Answer **all** questions. Do **not** use calculators.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working or explanation **must** be shown.

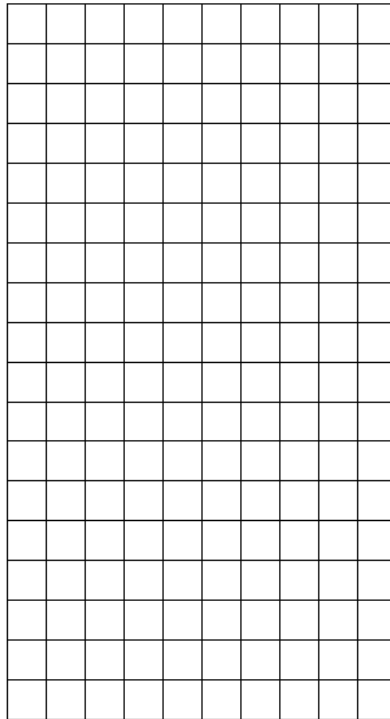
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$

Question 1 Consider relation $T = \{(x, y) : |y| = x^2, -3 \leq x < 3\}$.

a. Sketch the graph of relation T .

2 marks



b. Differentiate relation T with respect to x . Express your answer in terms of x in simplest form.

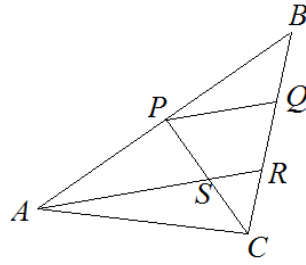
1 mark

Question 2 Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^3 x \, dx$.

3 marks

Question 3 ABC is a triangle. P is the midpoint of side AB . Q and R are points of trisection of side BC . Lines AR and CP intersect at S .

Let vectors $\overrightarrow{BP}$, $\overrightarrow{BQ}$, $\overrightarrow{CS}$ and $\overrightarrow{CR}$ be $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ and $\tilde{d}$ respectively.



- a. By expressing $\overrightarrow{PQ}$ and $\overrightarrow{AR}$ in terms of $\tilde{a}$ and $\tilde{b}$, show that line PQ is parallel to line AR , and $AR = 2PQ$. 2 marks

- b. Hence express $\overrightarrow{PQ}$ and $\overrightarrow{SR}$ in terms of $\tilde{c}$ and $\tilde{d}$. 1 mark

- c. Hence find the value of the ratio $AS : SR$. 2 marks

Question 4 Consider $\tilde{p} = \tilde{i} - 2\tilde{j} + 3\tilde{k}$.

- a. Write down a vector dependent of $\tilde{p}$. 1 mark

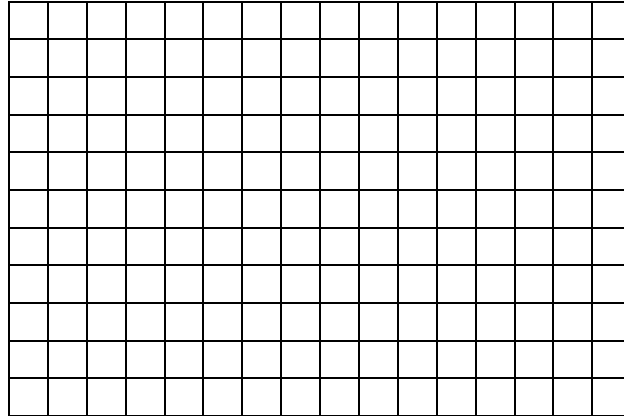
- b. Find a 3-dimensional unit vector independent of $\tilde{p}$. 3 marks

Question 5 The position of a particle from a fixed origin at time t seconds, $t \geq 0$, is given by

$$\tilde{r} = 2t \tilde{i} - \left(\frac{1}{t+1} \right) \tilde{j} \text{ where } |\tilde{r}| \text{ is in metres.}$$

- a. Sketch the path of the particle. Show intercept(s) and asymptote(s).

3 marks



- b. Determine the approximate length (m) of the path of the particle during $5 \leq t \leq 10$.

2 marks

Question 6 The relationship between the acceleration and velocity v of an unidentified flying object (UFO) moving in a straight line (the x -axis) is given by $a = \frac{4+v^2}{2}$. The particle stops momentarily at $x = 0$.

- a. Find $|v|$ in terms of x .

3 marks

- b. Describe the motion of the UFO in terms of position, speed and direction of motion.

2 marks

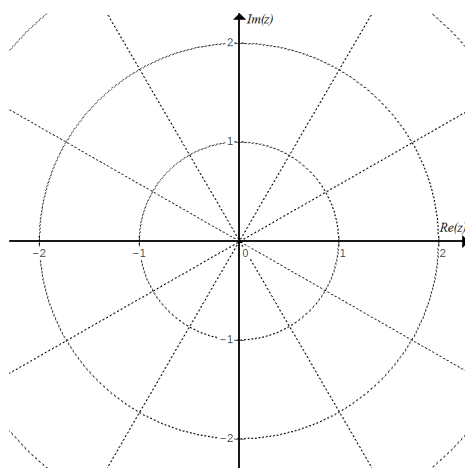
Question 7 Consider $f(z) = z^4 + z^2 + 1$

a. Solve $f(z) = 0$ for z in $x + yi$ form.

3 marks

b. Plot the solutions to $f(z) = 0$ on the Argand diagram below.

2 marks



Question 8 A particle slides down a plane at constant speed. The plane inclines at θ° with a horizontal surface.

The force of friction between the particle and the plane is $\frac{\sqrt{3}}{3}$ of the normal reaction force of the plane on the particle. Draw a force diagram and determine the value of θ .

4 marks

Question 9 Consider random variables X and Y where $Y = \frac{X-1}{2}$, $E(X) = 1.25$ and $\text{Var}(X) = 0.5$.

a. Find $E(X + 2Y)$. 1 mark

b. Find $\text{Var}(X + 2Y)$. 1 mark

Question 10 The age X of a large population has a mean of 32 and standard deviation of 15.
A random sample of 100 people is taken from the population. The mean age of the sample is $\bar{X}$.

a. Determine the expectation and standard deviation of $\bar{X}$. 2 marks

b. Hence find an approximate value for $\Pr(29 < \bar{X} < 35)$. Give a reason to justify your approximation. 2 marks

End of Exam 1