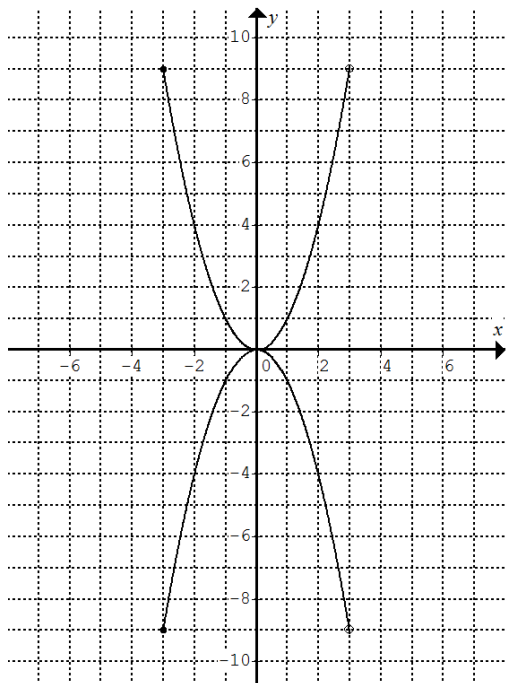




2018 Specialist Mathematics Trial Exam 1 Solutions

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Q1a $|y| = x^2$, $y = \pm x^2$



Q1b $y = \pm x^2$, $\frac{dy}{dx} = \pm 2x$

$$\begin{aligned} \text{Q2 } \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^3 x \, dx &= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x (\sec^2 x - 1) \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\tan x \sec^2 x - \tan x) \, dx \\ &= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x \sec^2 x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{-\sin x}{\cos x} \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x \, d(\tan x) + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\cos x} \, d(\cos x) \\ &= \left[\frac{\tan^2 x}{2} \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} + [\log_e(\cos x)]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = 1 - \frac{1}{2} \log_e 2 \end{aligned}$$

Q3a $\overrightarrow{PQ} = \tilde{b} - \tilde{a}$, $\overrightarrow{AR} = 2\tilde{b} - 2\tilde{a} = 2(\tilde{b} - \tilde{a})$, $\overrightarrow{AR} = 2\overrightarrow{PQ}$
 $\therefore AR \parallel PQ$ and $AR = 2PQ$

Q3b From part a, $PQ \parallel SR$, $\therefore \overrightarrow{PQ} = n\overrightarrow{SR}$.

Since R is the midpoint of CQ , and $\overrightarrow{SR} = \tilde{d} - \tilde{c}$, $\therefore \overrightarrow{PQ} = 2(\tilde{d} - \tilde{c})$

Q3c From parts a and b, $\overrightarrow{AR} = 2\overrightarrow{PQ}$ and $\overrightarrow{PQ} = 2\overrightarrow{SR}$, $\therefore \overrightarrow{AR} = 4\overrightarrow{SR}$,

$AR = 4SR$, $\therefore AS = 3SR$, hence $AS : SR = \frac{AS}{SR} = 3$

Q4a A vector dependent of $\tilde{p}$ is $m\tilde{p} = m(\tilde{i} - 2\tilde{j} + 3\tilde{k})$ where $m \neq 0$.
 By choosing $m = 1.5$, a possible vector is $1.5\tilde{i} - 3\tilde{j} + 4.5\tilde{k}$.

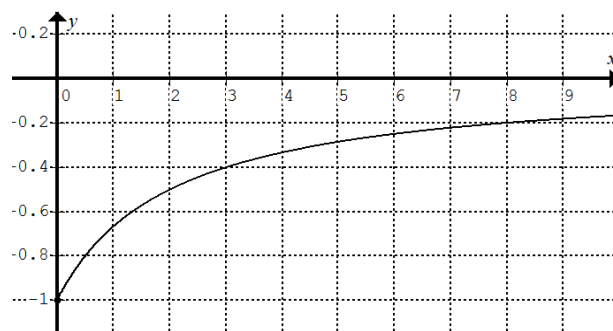
Q4b Let $x\tilde{i} + y\tilde{j} + z\tilde{k}$ be a vector independent of $\tilde{p}$.

$$\therefore (x\tilde{i} + y\tilde{j} + z\tilde{k}) \cdot (\tilde{i} - 2\tilde{j} + 3\tilde{k}) = 0, \quad x - 2y + 3z = 0$$

Choose $y = 1$ and $z = 1$, $\therefore x = -1$, and the vector is $-\tilde{i} + \tilde{j} + \tilde{k}$.

$\therefore$ a unit vector independent of $\tilde{p}$ is $\frac{1}{\sqrt{3}}(-\tilde{i} + \tilde{j} + \tilde{k})$. There are infinitely many other possible unit vectors.

Q5a $x = 2t$, $y = -\frac{1}{t+1}$, eliminating t , $y = -\frac{2}{x+2}$ for $x \geq 0$



x -intercept $(0, -1)$, asymptote $y = 0$

Q5b Length $= \int_5^{10} \sqrt{(x')^2 + (y')^2} \, dt = \int_5^{10} \sqrt{4 + \frac{1}{(t+1)^4}} \, dt \approx \int_5^{10} 2 \, dt = 10$

Note: $\frac{1}{(t+1)^4} \ll 4$ for large t .

Q6a $\frac{d}{dx} \left(\frac{1}{2} v^2 \right) = \frac{4+v^2}{2}$, $\frac{dx}{d(v^2)} = \frac{1}{4+v^2}$, $x = \log_e(4+v^2) + c$

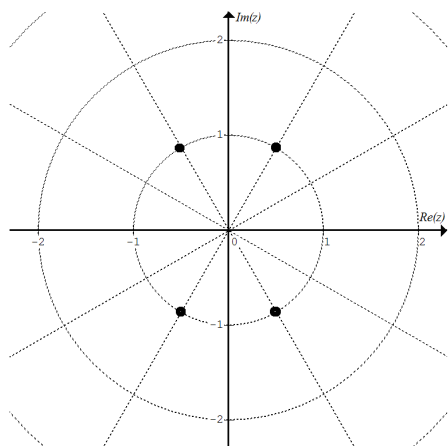
$v = 0$ when $x = 0$, $x = \log_e \left(\frac{4+v^2}{4} \right)$, $v = \pm 2\sqrt{e^x - 1}$, $|v| = 2\sqrt{e^x - 1}$

Q6b $e^x - 1 \geq 0$, i.e. $x \geq 0$. The UFO travels from $x > 0$ towards $x = 0$ with decreasing speed. It stops momentarily at $x = 0$, reverses its direction and travels away from $x = 0$ with increasing speed.



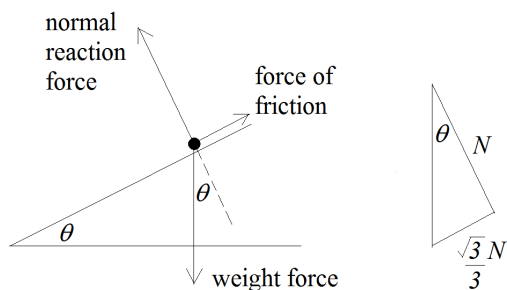
Q7a $f(z) = z^4 + z^2 + 1 = z^4 + 2z^2 + 1 - z^2 = (z^2 + 1)^2 - z^2$
 $= (z^2 + z + 1)(z^2 - z + 1) = 0 \therefore z = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i, z = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

Q7b $|z| = 1, \theta = \pm \frac{\pi}{3}, \pm \frac{2\pi}{3}$



Q8 Net force is zero at constant speed and direction.

Refer to the following diagrams, $\tan \theta^\circ = \frac{\sqrt{3}}{3}, \theta = 30$



Q9a $E(X + 2Y) = E(X + X - 1) = E(2X - 1) = 2E(X) - 1 = 1.5$

Q9b $\text{Var}(X + 2Y) = \text{Var}(2X - 1) = 2^2 \times \text{Var}(X) = 2$

Q10a $\mu = 32, \sigma = 15, E(\bar{X}) = \mu = 32, \text{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{15}{10} = 1.5$

Q10b The distribution of $\bar{X}$ is approximately normal, and
 $(29, 35) = (E(\bar{X}) - 2\text{sd}(\bar{X}), E(\bar{X}) + 2\text{sd}(\bar{X}))$

$\therefore \Pr(29 < \bar{X} < 35) \approx 0.95$

Please inform mathline@itute.com re conceptual and/or mathematical errors