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Students Name:.....

SPECIALIST MATHEMATICS UNITS 3 & 4

TRIAL EXAMINATION 1

2018

Reading Time: 15 minutes

Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
An exact answer is required to a question unless otherwise specified.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude $g \text{ m/s}^2$ where $g = 9.8$
Formula sheets can be found on pages 11 – 13 of this exam.

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Question 1 (2 marks)

Evaluate $\int_0^{\frac{\pi}{6}} \cos^2(3x) \, dx$.

Question 2 (3 marks)

Given the relation $5y - 2x^2y + x = 7$, find the equation of the tangent to the graph of this relation at the point $(1, 2)$.

Question 3 (3 marks)

The weight of eggs produced at a free range farm varies normally with a mean of 68 g and a standard deviation of 4 g.

These eggs are sold in boxes which contain 16 eggs.

Find, correct to four decimal places, the approximate probability that the mean weight of an egg in a randomly selected box is less than 65 g.

Question 4 (3 marks)

Find the value of a given that $z = 2 - i$ is a solution to the equation

$z^3 - 7z^2 + (a^2 + 1)z - (4a - 1) = 0$, where a is a real constant.

Question 5 (5 marks)

Consider the three vectors $\underline{a} = \underline{i} + 2\underline{j} + 2\underline{k}$, $\underline{b} = 2\underline{i} + 3\underline{j} + d\underline{k}$ and $\underline{c} = \underline{i} - 2\underline{j} + 2\underline{k}$ where d is a real constant.

- a.** Find a unit vector in the direction of $\underline{c}$. 1 mark

- b.** Find the vector resolute of $\underline{a}$ perpendicular to $\underline{c}$. 2 marks

- c.** Find the value of d if the three vectors are **linearly dependent**. 2 marks

Question 6 (4 marks)

Solve the differential equation

$$-\frac{1}{x} \frac{dy}{dx} = \sqrt{\frac{4-y^2}{4-x^2}}, \text{ given that } y(2) = \sqrt{3}.$$

Express your answer in the form $y = a \cos\left(b - \sqrt{c - x^2}\right)$ where a, b and $c \in \mathbb{R}$.

Question 7 (3 marks)

The velocity v , in ms^{-1} , of a 5 kg mass is given by $v = 4 \arccos(2x^2 - 1)$ where x , in metres, is its displacement from the origin and $x > 0$.

Find the net force F , in newtons, acting on the mass in terms of x .

Question 8 (5 marks)

Let $f: (-\infty, 4] \rightarrow \mathbb{R}$, $f(x) = (x-1)\sqrt{4-x}$.

S represents the region enclosed by the graph of f and the x -axis.

- a.** Find the area of S . 3 marks

[illegible]

- b.** The region S is rotated around the x -axis to form a solid of revolution. Find the volume of this solid of revolution. 2 marks

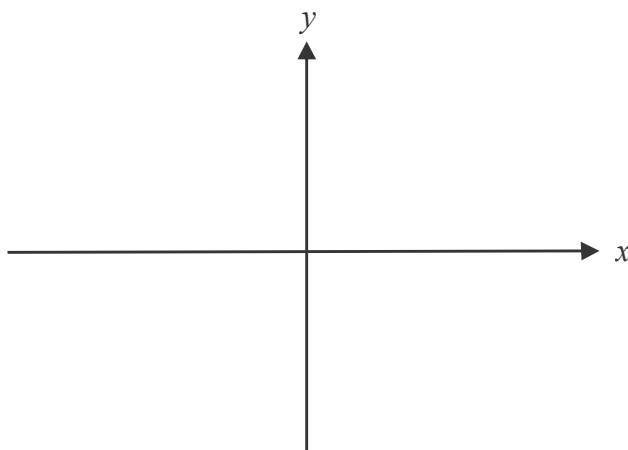
Question 9 (4 marks)

A curve is defined parametrically by $x = 2\sqrt{4-t}$ and $y = 2\sqrt{t+4}$.
Find the length of this curve from $t=2$ to $t=4$.

Question 10 (8 marks)

- a. Sketch the graph of $f(x) = 2\arctan(3x)$ on the set of axes below.
Label any asymptotes with their equations.

2 marks



- b. Find the rule and the domain of the inverse function f^{-1} .

2 marks

- c. Find $\int \tan\left(\frac{x}{2}\right) dx$. 1 mark

- d. Hence find the area enclosed by the graph of f , the x -axis and the line $x = \frac{1}{3}$. 3 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc \sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

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Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$	
$ z = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(x) + b$ $E(aX + bY) = aE(x) + bE(Y)$ $\text{var}(aX + b) = a^2 \text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2 \text{var}(X) + b^2 \text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}} \right)$
distribution of sample mean $\bar{X}$	mean $E(\bar{X}) = \mu$ variance $\text{var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a} \log_e ax+b + c$
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+(f'(x))^2} dx \quad \text{or} \quad \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$

Vectors in two and three dimensions

$\underline{r} = x \underline{i} + y \underline{j} + z \underline{k}$
$ \underline{r} = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$
$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underline{p} = m \underline{v}$
equation of motion	$\underline{R} = m \underline{a}$