

The Mathematical Association of Victoria
Trial Examination 2017
SPECIALIST MATHEMATICS
Written Examination 2

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of Book

<i>Section</i>	<i>Number of Questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 23 pages.
- Formula sheet
- Answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the space provided above on this page.
- Write your **name** on the multiple-choice answer sheet
- Unless otherwise indicated, the diagrams are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$

Question 1

If $z = \sqrt{3} - i$, $w = 4(1 + i)$ and $\text{Arg}\left(\frac{z^k}{w}\right) = \frac{11\pi}{12}$ then a possible value of k is

- A. 2
- B. 3
- C. 6
- D. 8
- E. 10

Question 2

If $2 - i$ is a root of the equation $z^3 + bz^2 + 17z - 15 = 0$ where $b \in R$, then b is

- A. -1
- B. 7
- C. $-\frac{19}{2}$
- D. 1
- E. -7

Question 3

If $(a + i)(1 - ai) = 6 + (b + 2)i$ where $a, b \in R$ then

- A. $ab = 30$
- B. $b + a = 7$
- C. $\frac{b}{a} = 4$
- D. $a + b = -7$
- E. $\frac{b}{a} = -4$

SECTION A – continued

TURN OVER

Question 4

If $\operatorname{cosec}(x) = \frac{4\sqrt{7}}{7}$, $\frac{\pi}{2} < x < \pi$ then $\sec(x) + \tan(x)$ is

- A. $-\frac{4}{3} + \frac{3\sqrt{7}}{16}$
- B. $-\left(\frac{12 - 3\sqrt{7}}{12}\right)$
- C. $-\frac{1}{3}(4 + \sqrt{7})$
- D. $-\frac{4}{3}(1 - \sqrt{7})$
- E. $-\frac{1}{3}(4 - \sqrt{7})$

Question 5

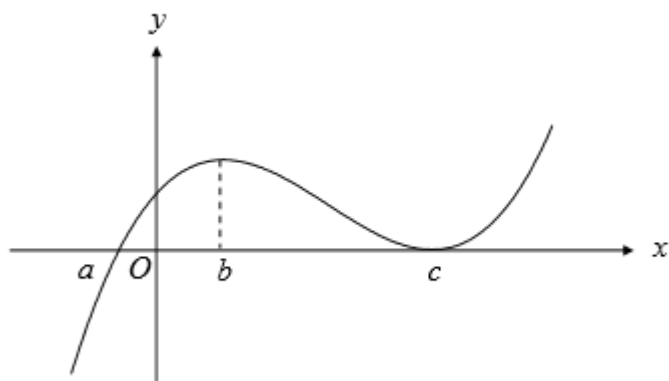
If $3\cos(2\theta) + 5\sin(\theta) - 2 = 0$ then the following are solutions

- A. $\sin(\theta) = 1$ and $\sin(\theta) = -\frac{1}{6}$
- B. $\cos(\theta) = 1$ and $\cos(\theta) = -\frac{1}{6}$
- C. $\sin(\theta) = -1$ and $\sin(\theta) = -\frac{1}{6}$
- D. $\sin(\theta) = -1$ and $\cos(\theta) = \frac{1}{6}$
- E. $\sin(\theta) = 1$ and $\sin(\theta) = \frac{1}{6}$

SECTION A – continue

Question 6

The graph of $y = f(x)$ is shown below.



If $F(x)$ is an antiderivative of $f(x)$, then the graph of $y = F(x)$ will have a

- A. local maximum at $x = b$
- B. local maximum at $x = a$
- C. point of inflexion at $x = a$
- D. zero gradient at $x = b$
- E. point of inflexion at $x = c$

Question 7

If $f'(t) = \ln(2t + 1)$ and $f(0) = 2$ then the value of $f(1)$ can be found by evaluating

- A. $\int_0^1 (\ln(2t + 1) - 2) dt$
- B. $\int_0^2 (\ln(2t + 1)) dt - 1$
- C. $\int_0^1 (2 + \ln(2t + 1)) dt$
- D. $\int_0^1 (2 - \ln(2t + 1)) dt$
- E. $\int_0^1 \ln(2t + 1) dt + 2$

SECTION A - continued

TURN OVER

Question 8

The gradient of the curve $y^2 + 2yx^4 = 33$ when $y = 1$ in the first quadrant is equal to

- A. $-\frac{31}{34}$
- B. $-\frac{32}{17}$
- C. $\frac{31}{34}$
- D. -32
- E. $\frac{31}{17}$

Question 9

$\int_{\pi/6}^{2\pi/3} (\tan^3(x) + \tan(x))dx$ is equivalent to

- A. $\int_{-\sqrt{3}}^{1/\sqrt{3}} u du$
- B. $\int_{1/\sqrt{3}}^{-\sqrt{3}} (u^3 + u) du$
- C. $\int_{\pi/6}^{\pi/3} (u^3 + u) du$
- D. $\int_{1/\sqrt{3}}^{-\sqrt{3}} u^3 du$
- E. $-\int_{-\sqrt{3}}^{1/\sqrt{3}} u du$

SECTION A – continued

Question 10

The graph of the function $y = \sqrt{\frac{16x^4 - 1}{x^4}}$, between the lines $y = 2$ and $y = 3$ is rotated about the y -axis to form a solid of revolution. The volume formed is given by

- A. $\cos^{-1}\left(\frac{3}{4}\right) - \frac{\pi}{3}$
- B. $\pi \sin^{-1}\left(\frac{3}{4}\right) - \frac{\pi^2}{6}$
- C. $\frac{\ln 7}{8} - \frac{\ln 3}{8}$
- D. $\pi \sin^{-1}\left(\frac{3}{4}\right) + \frac{\pi^2}{6}$
- E. $\frac{\pi}{4} \left(\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{1}{2}\right) \right)$

Question 11

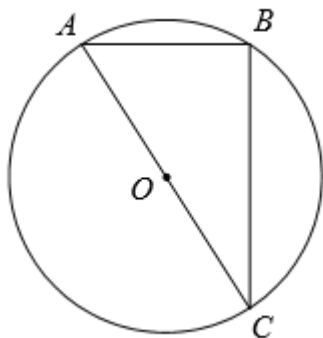
$\mathbf{a} = 8\mathbf{i} - \mathbf{j} + 13\mathbf{k}$, $\mathbf{b} = m\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{c} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ are linearly dependent when

- A. $m = \frac{1}{3}$
- B. $m = -\frac{2}{3}$
- C. $m = -2$
- D. $m = -\frac{1}{3}$
- E. $m = 2$

SECTION A – continued

TURN OVER

Question 12



If the points A, B and C lie on the circumference of the circle whose centre is O, which of the following statements is **not** true?

- A. $\overrightarrow{AB} = \overrightarrow{AC} - \overrightarrow{BC}$
- B. $\overrightarrow{AB} \cdot \overrightarrow{AC} = |\overrightarrow{BA}| |\overrightarrow{AC}| \cos(A)$
- C. $(\overrightarrow{CA} + \overrightarrow{AB}) \cdot \overrightarrow{CB} = |\overrightarrow{CB}|^2$
- D. $|\overrightarrow{AB}| + |\overrightarrow{BC}| = |\overrightarrow{AC}|$
- E. $\overrightarrow{BA} \cdot \overrightarrow{BC} = 0$

Question 13

The acceleration $a \text{ ms}^{-2}$ of a body moving in a straight line when it is x m from the origin is given by

$a = \frac{1}{\sqrt{9-x^2}}$. If the velocity, v , of the body is 2 ms^{-1} when it passes through the origin, then the function for x is given by

- A. $x = 3 \sin\left(\frac{v^2 - 4}{2}\right)$
- B. $x = 2 \sin\left(\frac{2 - v^2}{3}\right)$
- C. $x = 3 \cos\left(\frac{v^2 - 2}{2}\right)$
- D. $x = 3 \sin(v^2 - 2)$
- E. $x = 2 \sin^{-1}\left(\frac{v}{3}\right)$

SECTION A - continued

Question 14

The position vector of a particle at time t , $t \geq 0$, is given by $\underline{r} = 2\sqrt{t}\underline{i} + (5-t)\underline{j}$. The particle is closest to the origin when

- A. $t = 0$
- B. $t = 1$
- C. $t = \frac{3}{2}$
- D. $t = 3$
- E. $t = 5$

Question 15

A smooth inclined plane makes an angle of θ° to the horizontal. An object of mass m kg is held at rest on this plane by a force F newtons inclined at α° upwards to the plane. If the normal reaction of the mass on the plane is R newtons, which of the following statements is true?

- A. $R = mg \cos \theta + F \sin \alpha$
- B. $F \sin \alpha - mg \cos \theta = 0$
- C. $\tan \alpha = \frac{mg \cos \theta - R}{mg \sin \theta}$
- D. $F = \frac{m \sin \theta}{\cos \alpha}$
- E. $\tan \alpha = \frac{F \cos \theta}{R - F \sin \theta}$

Question 16

A body of mass 10 kg is acted upon by three forces $\underline{F}_1$, $\underline{F}_2$ and $\underline{F}_3$ all measured in Newtons. If, at time t seconds, $\underline{F}_1 = 2\underline{i} + (t-3)\underline{j}$, $\underline{F}_2 = -t^2\underline{i} - \underline{j}$ and $\underline{F}_3 = 5t\underline{i} + 2\underline{j}$ then the magnitude of the acceleration of the body in ms^{-2} when $t = 3$ is

- A. $\frac{\sqrt{65}}{10}$
- B. $\frac{73}{10}$
- C. 65
- D. 9
- E. $\frac{\sqrt{65}}{5}$

SECTION A – continued
TURN OVER

Question 17

A confidence interval estimate for a population mean can be used to test a null hypothesis about the population mean only if

- A. The distribution is symmetrical
- B. A one-tailed test is used
- C. A two-tailed test is used
- D. All of the above statements are true
- E. None of the above statements are true

Question 18

X is a random variable with $E(X) = 14.2$ and $\text{Var}(X) = 2.3$. Y is another random variable with $E(Y) = 5.5$ and $\text{Var}(Y) = 0.8$. If X and Y are independent variables and $W = 2X - Y + 3$ then

- A. $E(W) = 22.9$ and $\text{SD}(W) = \sqrt{13}$
- B. $E(W) = 22.9$ and $\text{SD}(W) = \sqrt{10}$
- C. $E(W) = 33.9$ and $\text{SD}(W) = 10$
- D. $E(W) = 25.9$ and $\text{SD}(W) = \sqrt{10}$
- E. $E(W) = 25.9$ and $\text{SD}(W) = \sqrt{13}$

Question 19

A Type II error is made when the

- A. alternative hypothesis is accepted when it is true
- B. null hypothesis is accepted when it is false
- C. null hypothesis is true and we accept it
- D. null hypothesis is rejected when it is true
- E. alternative hypothesis is accepted when it is false

Question 20

The distribution of serum vitamin E in a certain population is approximately normal with mean $800\mu\text{g/dL}$ and standard deviation $200\mu\text{g/dL}$, ($\mu\text{g/dL}$ stands for micrograms per decilitre). In a random sample of 20 people, the mean vitamin E level would lie within which range approximately 95% of the time

- A. Between $780\mu\text{g/dL}$ and $820\mu\text{g/dL}$
- B. Between $408\mu\text{g/dL}$ and $1192\mu\text{g/dL}$
- C. Between $712\mu\text{g/dL}$ and $888\mu\text{g/dL}$
- D. Between $760\mu\text{g/dL}$ and $839\mu\text{g/dL}$
- E. Between $625\mu\text{g/dL}$ and $975\mu\text{g/dL}$

END OF SECTION A

SECTION B**Instructions for Section B**

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$

Question 1 (11 marks)

A particle travels in a closed circuit. At time t secs it has a position vector, measured in metres, given by

$$\underline{r}(t) = 2\cos(t)\underline{i} + (2 + 3\sin(2t))\underline{j}, t \geq 0$$

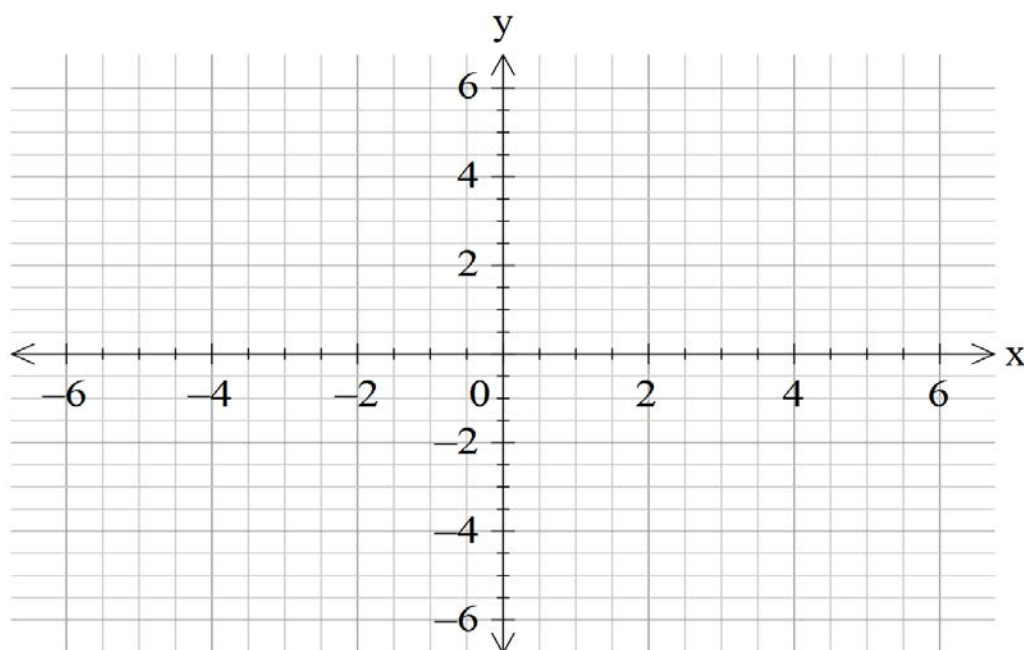
- a. Show that the Cartesian equation of the path of the particle can be written as

$$9(x^2 - 2)^2 + 4(y - 2)^2 = 36$$

1 mark

- b. i Sketch the circuit of the particle on the set of axes below. There is no need to find the x -intercepts.

1 mark



SECTION B – Question 1 – continued
TURN OVER

ii. Mark the starting point and the directions of motion of the particle on the diagram above. 1 mark

c. How long does it take the particle to complete one full circuit? 1 mark

d. Find an expression for the speed of the particle at time t secs. 1 mark

e. For $0 < t \leq 2\pi$, what are the maximum and minimum speeds of the particle? 2 marks

SECTION B – Question 1 – continued

f.

- i.** Write down a definite integral for the length that the particle has travelled in the first 2 seconds.

1 mark

- ii.** Find this length. Give your answer to two decimal places.

1mark

- g.** Find the first time at which the particle will be travelling in a direction parallel to the line $y = 3x$. Give your answer correct to two decimal places.

2 marks

SECTION B — continued
TURN OVER

Question 2 (9 marks)

Points A, B, C and D have position vectors $\underline{a} = \underline{i} + m\underline{j}$, $\underline{b} = -2\underline{i} + 2\underline{j}$, $\underline{c} = n\underline{i} + 2\underline{j}$ and $\underline{d} = 6\underline{i} + 6\underline{j}$ relative to the origin O, where m and n are real numbers.

- a. Find $\overrightarrow{AB}$ and $\overrightarrow{AD}$ in terms of m . 1 mark

- b. If ABCD is a rhombus, find the values of m and n . 2 marks

- c. Find $\overrightarrow{AE}$ the vector resolute of $\overrightarrow{AB}$ parallel to $\overrightarrow{AD}$. 2 marks

- d. Find the area of triangle ABE. 2 marks

SECTION B – Question 2 – continued

- e. If F divides $\overrightarrow{CD}$ in the ratio 2:1, find the angle θ° between vector $\overrightarrow{BD}$ and $\overrightarrow{BF}$. Give your answer correct to two decimal places. 2 marks

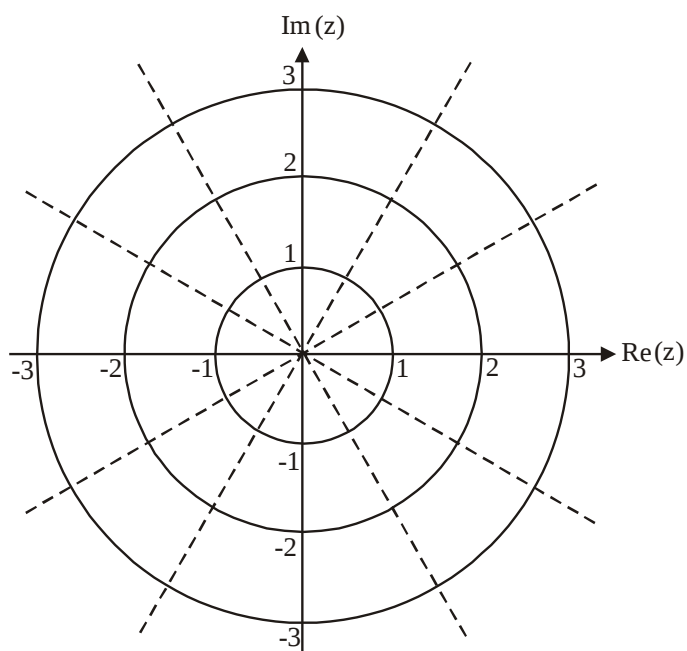
SECTION B — continued
TURN OVER

Question 3 (11 marks)

$w, \bar{w}$ and v are the solutions of the equation $\{z : z^3 = k, z \in \mathbb{C}\}$ where $w = 3\text{cis}\left(\frac{\pi}{3}\right)$

- a. Plot $w, \bar{w}$ and v on the Argand diagram below.

1 mark



- b. Find the value of k .

1 mark

- c. Show that $w \in \{z : \sqrt{3} \text{Im}(z) - \text{Re}(z) = 3\}$.

2 marks

- d. On the Argand diagram given in part a.

- i. Sketch $\{z : \sqrt{3} \text{Im}(z) - \text{Re}(z) = 3\}$

1 mark

SECTION B – Question 3 – continued

- ii. Sketch the ray given by $\{z : \text{Arg}(z) = \text{Arg}(\bar{w}^{\frac{1}{2}})\}$

2 marks

- e. If $u = 1 + i$

- i. Express $\frac{4w}{u}$ in Cartesian form

1 mark

- ii. Express $\frac{4w}{u}$ in Polar form

1 mark

- iii. Hence, express $\tan\left(\frac{\pi}{12}\right)$ in the form $a - \sqrt{b}$ where a and b are positive integers.

2 marks

SECTION B – continued
TURN OVER

Question 4 (9 marks)

A tank initially contains 100 litres of water with 5kg of salt dissolved uniformly into it. A salt solution with a concentration of 0.1kg/litre flows into the tank at a rate of 2litres/min. The mixture is kept uniform by stirring and flows out at the rate of 2litres/min. If x kilograms is the amount of salt in the tank after t minutes

- a. Show that the differential equation for x in terms of t is

$$\frac{dx}{dt} = \frac{10 - x}{50}$$

1 mark

- b. Solve this differential equation to give x as a function of t .

2 marks

- c. Calculate the amount of salt in the tank after 2 minutes. Give your answer correct to 2 decimal places.

1 mark

- d. Sketch a graph of x against t

1 mark

**SECTION B – Question 4** – continued

e. The outflow is now adjusted to 3litres/min instead of 2litres/min

i. Set up a new differential equation for x in terms of t . Do not attempt to solve it.

1 mark

ii. Use Euler's method with increments of 1 minute to predict the amount of salt in the tank after 2 minutes. Give your answer correct to 3 decimal places.

2 marks

iii. Verify that the following is a solution for this differential equation

$$x = \frac{1}{10}(100 - t) - \frac{5}{1000000}(100 - t)^3$$

2 marks

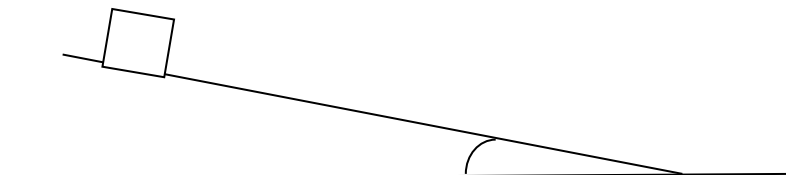
TURN OVER

Question 5 (11 marks)

A baggage ramp in an airport is 6 metres in length and is inclined at 25 degrees to the horizontal. A suitcase of mass 15kg is initially at rest at the top of the ramp. The suitcase slides down the ramp under the force of gravity with a constant frictional force of 40N acting upon it.

- a. Mark in all of the forces acting on the suitcase.

1 mark



- b. Find the acceleration of the suitcase down the ramp. Give your answer correct to 2 decimal places.

2 marks

- c. Find the time taken for the suitcase to reach the bottom of the ramp. Give your answer correct to 2 decimal places.

2 marks

- d. An identical suitcase, also initially at rest, is pushed down the ramp with a force of $30 - 50t$ for the first 0.6 seconds. Show that the acceleration of this suitcase at time t where $0 < t \leq 0.6$ is approximately

$$3.47 - \frac{10}{3}t$$

1 mark

- e. Using this approximate acceleration, find the speed of this second suitcase when $t = 0.6$. Give your answer correct to 2 decimal places.

2 marks

- f. How long does it take this suitcase to reach the bottom of the ramp? Give your answer correct to 2 decimal places.

3 marks

TURN OVER

Question 6 (9 marks)

The monthly turnover of a retail shop is normally distributed with an average of \$40500 per month and a standard deviation of \$4500

- a. What is the probability that in a three monthly period of time the mean turnover per month would be more than \$42000? Give your answer correct to three decimal places.

2 marks

- b. What is the probability that the shop will turnover more than half a million dollars in a year? Give your answer correct to three decimal places.

2 marks

- c. What is the probability of them turning over more than half a million dollars in a year, given that they are guaranteed to turn over at least \$480000? Give your answer correct to three decimal places.

2 marks

SECTION B – Question 6 – continued

- d. Following an advertising campaign the next three month period produced an average monthly turnover of \$46500. At the 5% level, test whether the advertising campaign has lead to a significant increase in the monthly turnover.

3 marks

END OF QUESTION AND ANSWER BOOK