

The Mathematical Association of Victoria

Trial Examination 2017

SPECIALIST MATHEMATICS

Written Examination 2 - SOLUTIONS

SECTION A

Question	Answer	Question	Answer
1	D	11	E
2	E	12	D
3	D	13	A
4	C	14	D
5	A	15	C
6	E	16	A
7	E	17	C
8	B	18	D
9	E	19	B
10	B	20	C

Question 1

$$\text{Arg}(z) = -\frac{\pi}{6} \text{ and } \text{Arg}(\bar{w}) = -\frac{\pi}{4}$$

$$\arg\left(\frac{z^k}{\bar{w}}\right) = -\frac{k\pi}{6} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{12}(3-2k)$$

$$\text{When } k=8, \text{ Arg}\left(\frac{z^k}{\bar{w}}\right) = -\frac{13\pi}{12} + 2\pi = \frac{11\pi}{12}$$

Alternatively, students can use substitution to find the values of k

Answer is D

Question 2

The conjugate $2+i$ is also a root as the coefficients of the polynomial are real. If we let a denote the remaining root, the polynomial can be written as

$$(z-2+i)(z-2-i)(z-a) = z^3 - (a+4)z^2 + (4a+5)z - 5a$$

Therefore $4a+5=17$, $a=3$

and $b = -a - 4 = -7$

Answer is E

Question 3

$$\text{If } (a+i)(1-ai) = 6 + (b+2)i$$

$$a+i-a^2i+a = 6 + (b+2)i$$

$$\text{Real part: } 2a = 6 \quad \therefore a = 3$$

$$\text{Imag part: } (1-3^2) = b+2 \quad \therefore b = -10$$

$$\text{Therefore } a+b = -7$$

Answer is D

Question 4

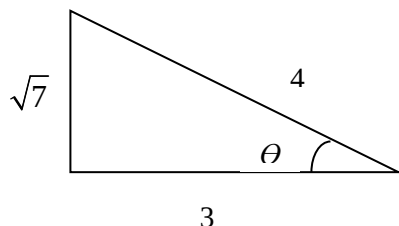
$$\operatorname{cosec}(x) = \frac{4\sqrt{7}}{7} \quad \therefore \sin(x) = \frac{\sqrt{7}}{4}$$

$$\cos(x) = -1\sqrt{\left(1-\frac{7}{4^2}\right)} = -\sqrt{\frac{9}{16}} = -\frac{3}{4} \quad \text{negative as } \frac{\pi}{2} < x < \pi$$

$$\therefore \sec(x) = -\frac{4}{3} \quad \text{and} \quad \tan(x) = \frac{\frac{7}{4}}{-\frac{3}{4}} = -\frac{\sqrt{7}}{3}$$

(or use the right-angled triangle below to see that $\tan(x) = -\tan(\theta) = -\frac{\sqrt{7}}{3}$)

$$\sec(x) + \tan(x) = -\frac{4}{3} - \frac{\sqrt{7}}{3} = -\frac{1}{3}(4 + \sqrt{7})$$



Answer is C

Question 5

$$3\cos(2\theta) + 5\sin(\theta) - 2 = 0$$

$$3(1 - 2\sin^2(\theta)) + 5\sin(\theta) - 2 = 0$$

$$6\sin^2(\theta) - 5\sin(\theta) - 1 = 0$$

$$(6\sin(\theta) + 1)(\sin(\theta) - 1) = 0$$

$$\therefore \sin(\theta) = 1, \sin(\theta) = -\frac{1}{6}$$

Answer is A**Question 6**

At $x = a$ the gradient is zero, and it is negative to the left and positive to the right. Therefore it is a minimum this rules out answers B and C.

At $x = b$ the gradient is positive, not zero which rules out answer D and A.

At $x = c$ the gradient is zero and it is positive to the left and right of this point, therefore it is a point of inflexion.

Answer is E**Question 7**

$$f(b) = f(a) + \int_a^b f'(t) dt$$

$$f(1) = f(0) + \int_0^1 f'(t) dt$$

$$f(1) = 2 + \int_0^1 \ln(2t+1) dt$$

Answer is E**Question 8**

$$y^2 + 2yx^4 = 33$$

$$\therefore 2y \frac{dy}{dx} + 2 \frac{dy}{dx} x^4 + 8yx^3 = 0$$

$$\frac{dy}{dx} = -\frac{8yx^3}{2(y+x^4)} = -\frac{4yx^3}{(y+x^4)}$$

when $y = 1$, $1 + 2x^4 = 33 \therefore x = 2$ (as x is in the first quadrant)

$$\text{thus } \frac{dy}{dx} = -\frac{4 \times 1 \times 2^3}{(1 + 2^4)} = -\frac{32}{17}$$

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$$y^2 + 2 \cdot y \cdot x^4 = 33 \qquad 2 \cdot x^4 \cdot y + y^2 = 33$$

$$\text{solve}(2 \cdot x^4 \cdot y + y^2 = 33, x) | y=1 \text{ and } x>0 \qquad x=2$$

$$\text{impDif}(2 \cdot x^4 \cdot y + y^2 = 33, x, y) \qquad \frac{-4 \cdot x^3 \cdot y}{x^4 + y}$$

$$\frac{-4 \cdot x^3 \cdot y}{x^4 + y} | x=2 \text{ and } y=1 \qquad \frac{-32}{17}$$

Answer is B

Question 9

$$\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} (\tan^3(x) + \tan(x)) dx = \int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \tan(x)(\tan^2(x) + 1) dx = \int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \tan(x) \sec^2(x) dx$$

$$u = \tan(x), \quad du = \sec^2(x) dx$$

$$x = \frac{\pi}{6}, u = \frac{1}{\sqrt{3}} \quad \text{and} \quad x = \frac{2\pi}{3}, u = -\sqrt{3}$$

$$\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \tan(x) \sec^2(x) dx = \int_{\frac{1}{\sqrt{3}}}^{-\sqrt{3}} u du = - \int_{-\sqrt{3}}^{\frac{1}{\sqrt{3}}} u du$$

Answer is E

Question 10

$$V = \pi \int_2^3 x^2 dy$$

$$y^2 = \frac{16x^4 - 1}{x^4} \text{ rearranging gives } x^2 = \frac{1}{\sqrt{16 - y^2}}$$

$$\therefore V = \pi \int_2^3 \frac{1}{\sqrt{16 - y^2}} dy = \pi \left(\sin^{-1} \left(\frac{3}{4} \right) - \frac{\pi}{6} \right)$$

$$\text{solve}(y^2 = (16x^4 - 1)/x^4, x)$$

$$\left\{ x = \left(\frac{-1}{y^2 - 16} \right)^{\frac{1}{4}} \right\}$$

$$\text{ans}^2$$

$$\left\{ x^2 = \sqrt{\frac{-1}{y^2 - 16}} \right\}$$

□

$$\pi \int_2^3 \frac{1}{(16 - y^2)^{0.5}} dy$$

$$\left(\sin^{-1} \left(\frac{3}{4} \right) - \frac{\pi}{6} \right) \cdot \pi$$

Answer is B

Question 11

For linear dependence we require $\vec{a} = s\vec{b} + t\vec{c}$ where $s, t \in \mathbb{R}$

$$\therefore 8 = sm + t$$

$$-1 = -s + t$$

$$13 = 3s + 2t \quad \text{solve using CAS}$$

$$\begin{cases} 8 = sm + t \\ -1 = -s + t \\ 13 = 3s + 2t \end{cases} \quad s, m, t$$

$$\{m=2, s=3, t=2\}$$

□

Answer is E

Question 12

$\vec{AB} + \vec{BC} = \vec{AC}$ answer A is just a rearrangement of this, therefore A is correct.

The dot product gives $\vec{AB} \cdot \vec{AC} = |\vec{AB}| |\vec{AC}| \cos(A)$, but $|\vec{AB}| = |\vec{BA}|$ therefore B is also correct.

$(\vec{CA} + \vec{AB}) \cdot \vec{CB} = \vec{CB} \cdot \vec{CB} = |\vec{CB}|^2$ therefore C is correct

Angle B is a right angle, therefore $\vec{BA} \cdot \vec{BC} = 0$ and E is correct.

Whereas for a right angled triangle $|\vec{AB}|^2 + |\vec{BC}|^2 = |\vec{AC}|^2$ which is NOT the same as $|\vec{AB}| + |\vec{BC}| = |\vec{AC}|$ therefore D is not correct.

Answer is D

Question 13

$$a = \frac{d}{dx} \left(\frac{1}{2} v^2 \right) = \frac{1}{\sqrt{(9-x^2)}}$$

$$\therefore \frac{v^2}{2} = \int \frac{1}{\sqrt{(9-x^2)}} dx = \sin^{-1} \frac{x}{3} + c \quad x=0, v=2 \therefore c=2$$

$$\therefore \frac{v^2 - 4}{2} = \sin^{-1} \frac{x}{3}$$

$$\therefore x = 3 \sin \left(\frac{v^2 - 4}{2} \right)$$

Answer is A**Question 14**

$$\text{If } \underline{r} = 2\sqrt{t}\underline{i} + (5-t)\underline{j}$$

$$\begin{aligned} \text{distance from the origin} &= \sqrt{((2\sqrt{t})^2 + (5-t)^2)} \\ &= \sqrt{(t^2 - 6t + 25)} \end{aligned}$$

this is a minimum when $2t - 6 = 0$, $t = 3$

Alternative method:

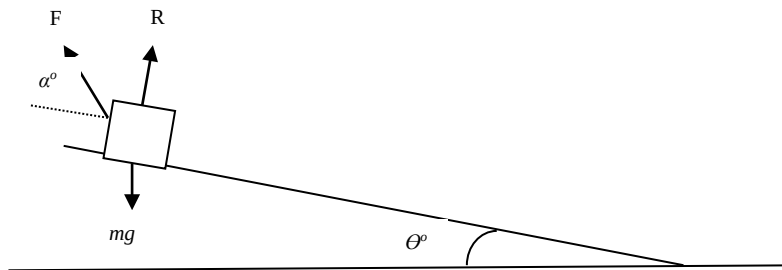
$$\text{If } \underline{r} = 2\sqrt{t}\underline{i} + (5-t)\underline{j} \text{ then } \underline{v} = \frac{1}{\sqrt{t}}\underline{i} - \underline{j}$$

Now the dot product of these two vectors is zero when closest to the origin.

Hence $2 - (5 - t) = 0$ and so $t = 3$

Answer is D

Question 15



Resolving perpendicular to the plane:

$$R = mg \cos \theta - F \sin \alpha$$

Resolving parallel to the plane:

$$F \cos \alpha - mg \sin \theta = 0$$

Thus, A is wrong because the sign is wrong. B is wrong because the sine and cosine are in the wrong positions. D is wrong because the g is missing. E is wrong because the angles are in the wrong position. The equations above give

$$\frac{F \sin \alpha}{F \cos \alpha} = \frac{mg \cos \theta - R}{mg \sin \theta}$$

$$\tan \alpha = \frac{mg \cos \theta - R}{mg \sin \theta}$$

Answer is C

Question 16

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (2 - t^2 + 5t)\vec{i} + (t - 3 - 1 + 2)\vec{j}$$

$$\text{When } t = 3 \quad \vec{F} = 8\vec{i} + \vec{j}$$

$$\vec{F} = m\vec{a} \quad \therefore \quad \vec{a} = \frac{1}{10}(8\vec{i} + \vec{j})$$

$$|\vec{a}| = \frac{1}{10}\sqrt{(8^2 + 1^2)} = \frac{\sqrt{65}}{10}$$

Answer is A

Question 17

For a 95% confidence interval

$$\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}$$

$$-1.96 \leq \frac{\mu - \bar{x}}{\left(\frac{\sigma}{\sqrt{n}} \right)} \leq 1.96$$

In a two-tailed hypothesis test at the 5% we would reject the null hypothesis if

$$\left| \frac{\mu - \bar{x}}{\frac{\sigma}{\sqrt{n}}} \right| \geq 1.96 \quad \text{this would be the same as saying, reject the null hypothesis if}$$

the confidence interval does not contain μ_0 . This is true for any confidence level.

Answer is C

Question 18

$$\begin{aligned} E(W) &= 2 \times E(X) - E(Y) + 3 \\ &= 2 \times 14.2 - 5.5 + 3 = 25.9 \end{aligned}$$

$$\begin{aligned} SD(W) &= \sqrt{(2^2 \text{Var}(X) + (-1)^2 \text{Var}(Y))} \\ &= \sqrt{(4 \times 2.3 + 1 \times 0.8)} = \sqrt{10} \end{aligned}$$

Answer is D

Question 19

Both A and C are not errors because the correct conclusion is reached. Both D and E are Type I errors because they both lead to a rejection of the null hypothesis in favour of the alternative hypothesis, when the null hypothesis is true.

B is the only Type II error.

Answer is B

Question 20

95% confidence interval is given by

$$800 - 1.96 \times \frac{200}{\sqrt{20}} \leq \bar{x} \leq 800 + 1.96 \times \frac{200}{\sqrt{20}}$$

$$712.346 \leq \bar{x} \leq 887.653$$

$$712 \leq \bar{x} \leq 888 \text{ to the nearest } \mu\text{g/dL}$$

Answer is C

SECTION B

Question 1 (11 marks)

a. $\mathbf{r}(t) = 2\cos(t)\mathbf{i} + (2 + 3\sin(2t))\mathbf{j}, t \geq 0$

$$x = 2\cos(t), \quad y = 2 + 3\sin(2t)$$

$$\cos(t) = \frac{x}{2}, \quad \sin(2t) = \frac{y-2}{3}$$

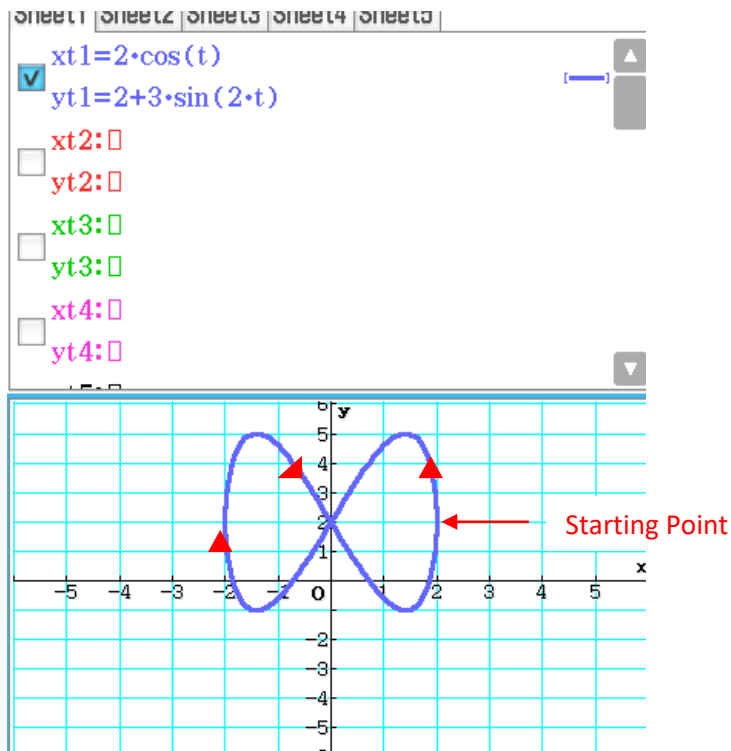
$$\cos(2t) = 2\cos^2(t) - 1 = \frac{x^2}{2} - 1$$

$$\cos^2(2t) + \sin^2(2t) = \frac{(x^2 - 2)^2}{4} + \frac{(y-2)^2}{9} = 1$$

$$\therefore 9(x^2 - 2)^2 + 4(y - 2)^2 = 36$$

[A1]

b. i.



[A1]

ii. Starting point is (2,2). It starts moving away from this point in an anticlockwise direction. [A1]

- c. The period of $\cos(t)$ is 2π and the period of $\sin(2t)$ is π , therefore it takes 2π seconds to complete one circuit [A1]

d.

$$\begin{aligned}\dot{\mathbf{r}}(t) &= -2\sin(t)\mathbf{i} + 6\cos(2t)\mathbf{j} \\ \therefore |\dot{\mathbf{r}}(t)| &= \sqrt{(4\sin^2(t) + 36\cos^2(2t))}\end{aligned}\quad [\text{A1}]$$

$$4\sin^2(t) = 2 - 2\cos(2t)$$

$$\begin{aligned}\therefore |\dot{\mathbf{r}}(t)| &= \sqrt{(2 - 2\cos(2t) + 36\cos^2(2t))} \\ &= \sqrt{2(1 - \cos(2t) + 18\cos^2(2t))}\end{aligned}$$

- e. To find the maximum and minimum points solve

$$\begin{aligned}\frac{d}{dt}(1 - \cos(2t) + 18\cos^2(2t)) &= 0 \\ \therefore 2\sin(2t) - 72\cos(2t)\sin(2t) &= 0 \\ 2\sin(2t)(1 - 36\cos(2t)) &= 0 \\ \sin(2t) = 0, \quad \cos(2t) &= \frac{1}{36}\end{aligned}\quad [\text{M1}]$$

$$\sin(2t) = 0 \quad t = \frac{\pi}{2}, \pi$$

$$\left|\dot{\mathbf{r}}\left(\frac{\pi}{2}\right)\right| = \sqrt{(2 + 2 + 36)} = \sqrt{40} = 2\sqrt{10}$$

$$|\dot{\mathbf{r}}(\pi)| = \sqrt{(2 - 2 + 36)} = \sqrt{36} = 6$$

$$\cos(2t) = \frac{1}{36} \quad |\dot{\mathbf{r}}(t)| = \sqrt{(2 - 2 \times \frac{1}{36} + 36 \times \frac{1}{36^2})} = \frac{\sqrt{71}}{6}\quad [\text{A1}]$$

Therefore the maximum is $2\sqrt{10} \text{ ms}^{-1}$ and the minimum is $\frac{\sqrt{71}}{6} \text{ ms}^{-1}$

f. i.

$$\begin{aligned}
 \text{Length travelled } L &= \int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \\
 &= \int_0^2 \sqrt{(-2\sin(t))^2 + (6\cos(2t))^2} dt \\
 &= \int_0^2 \sqrt{2(1 - \cos(2t) + 18\cos^2(2t))} dt
 \end{aligned}$$

[A1]

ii. $L = 8.97$ [A1]

$$\int_0^2 (2(1 - \cos(2x) + 18(\cos(2x))^2))^{0.5} dx = 8.968044233$$

g. Need to solve $\frac{dy}{dx} = 3$

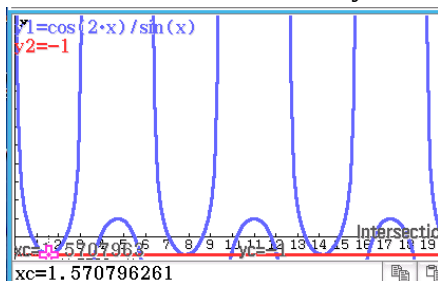
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6\cos(2t)}{-2\sin(t)} =$$

[M1]

$$\therefore \frac{\cos(2t)}{\sin(t)} = -1$$

$t = 1.57$ secs [A1]

The value of t can be found graphically, or by solving with CAS. If using CAS solve, students need to ensure they select the correct root as there are multiple solutions.



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solve(cos(2x)/sin(x)=-1,x)
{x=6.283185307*constn(1)-2.61799387}

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solve(cos(2x)/sin(x)=-1,x)
{x=6.283185307*constn(3)+1.570796327}

```

Question 2

a.

$$\overrightarrow{AB} = \underline{b} - \underline{a} = -3\underline{i} + (2-m)\underline{j}$$

$$\overrightarrow{AD} = \underline{d} - \underline{a} = 5\underline{i} + (6-m)\underline{j} \quad [A1]$$

b. For a rhombus the opposite sides must be identical vectors

$$\overrightarrow{AB} = \overrightarrow{DC}$$

$$-3\underline{i} + (2-m)\underline{j} = (n-6)\underline{i} - 4\underline{j} \quad [M1]$$

$$\therefore n = 3 \text{ and } m = 6 \quad [A1]$$

$$\begin{aligned} \text{OR use } |\overrightarrow{AB}|^2 &= |\overrightarrow{AD}|^2 \\ \therefore 9 + (2-m)^2 &= 25 + (6-m)^2 \\ m &= 6 \text{ etc.} \end{aligned}$$

c.

$$\overrightarrow{AE} = (\overrightarrow{AB} \cdot \widehat{AD}) \widehat{AD} \quad [M1]$$

$$\overrightarrow{AD} = 5\underline{i} \quad \therefore \widehat{AD} = \underline{i} \quad [A1]$$

$$\overrightarrow{AE} = ((-3\underline{i} - 4\underline{j}) \cdot \underline{i}) \underline{i} = -3\underline{i}$$

d.

$$\text{Area of triangle ABE} = \frac{1}{2} |\overrightarrow{AE}| |\overrightarrow{EB}| \quad [M1]$$

$$|\overrightarrow{AE}| = 3, \quad \overrightarrow{EB} = \overrightarrow{AB} - \overrightarrow{AE} = -4\underline{j}, \quad |\overrightarrow{EB}| = 4$$

$$\text{Area} = \frac{1}{2} \times 3 \times 4 = 6 \text{ units}^2 \quad [A1]$$

e.

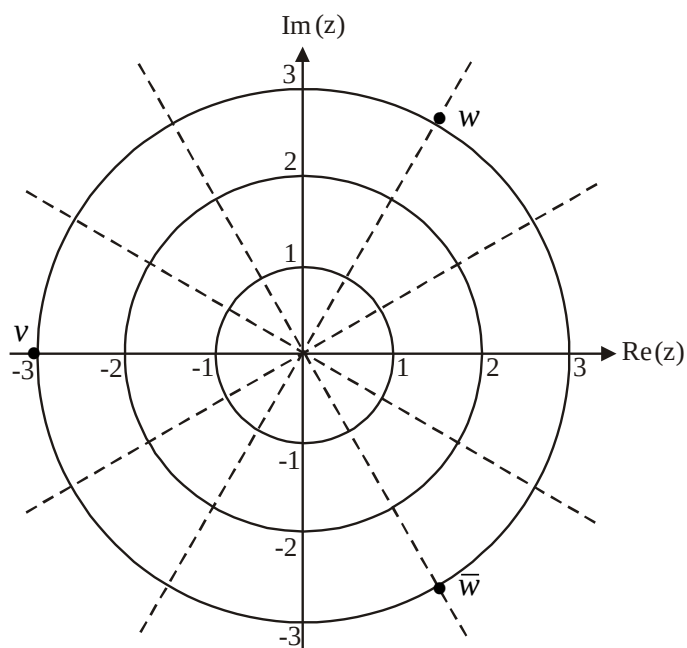
$$\theta = \cos^{-1} \left(\frac{\overrightarrow{BD} \cdot \overrightarrow{BF}}{|\overrightarrow{BD}| |\overrightarrow{BF}|} \right)$$

$$\overrightarrow{BD} = 8\underline{i} + 4\underline{j} \quad \text{and} \quad \overrightarrow{BF} = \overrightarrow{BC} + \frac{2}{3} \overrightarrow{CD} = 5\underline{i} + \frac{2}{3} (3\underline{i} + 4\underline{j}) = \frac{21}{3} \underline{i} + \frac{8}{3} \underline{j} \quad [M1]$$

$$\theta = \cos^{-1} \left(\frac{\frac{1}{3} (21 \times 8 + 8 \times 4)}{\frac{1}{3} \sqrt{(21^2 + 8^2)} \sqrt{(8^2 + 4^2)}} \right) = 5.71^\circ \quad [A1]$$

Question 3

a.



All roots correctly plotted and labelled. Equally spaced by $\frac{2\pi}{3}$ around a circle radius 3 [A1]

b.

$$k = w^3 = 3^3 \operatorname{cis}\left(3 \times \frac{\pi}{3}\right) = 27 \operatorname{cis}(\pi) \quad [\text{A1}]$$

$$k = -27$$

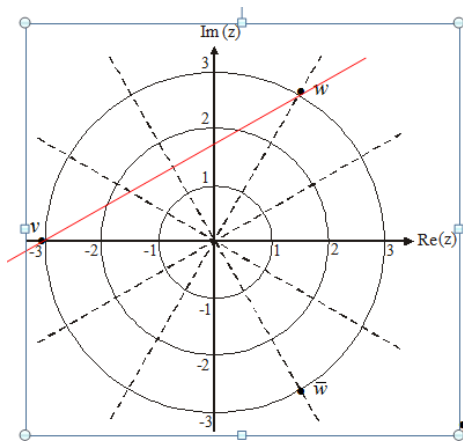
c.

$$\operatorname{Im}(w) = 3 \sin\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{2}, \quad \operatorname{Re}(w) = 3 \cos\left(\frac{\pi}{3}\right) = \frac{3}{2} \quad [\text{M1}]$$

$$\sqrt{3} \operatorname{Im}(w) - \operatorname{Re}(w) = \frac{3 \times 3}{2} - \frac{3}{2} = \frac{6}{2} = 3 \quad [\text{A1}]$$

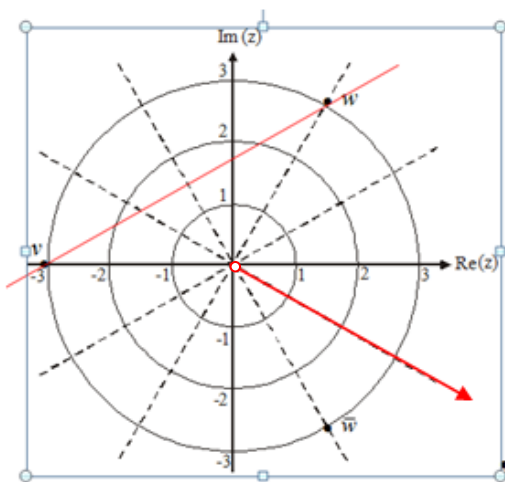
d.

- i. The line should go through the intercepts $x = -3$ and $y = \sqrt{3}$ and it should have an angle of $\frac{\pi}{6}$ to the horizontal (it goes through w and v , if they have been plotted correctly) [A1]



ii. $\text{Arg}(\bar{w}) = -\frac{\pi}{3} \therefore \text{Arg}(\bar{w}^{\frac{1}{2}}) = -\frac{\pi}{6}$

Sketch $\{z : \text{Arg}(z) = -\frac{\pi}{6}\}$ straight line extended from the origin, on an angle of $-\frac{\pi}{6}$ to the horizontal. (The origin should be excluded).



[A1]

e. i.

$$\frac{4w}{u} = \frac{6(1+\sqrt{3}i)}{(1+i)} = 3(\sqrt{3}+1+(\sqrt{3}-1)i)$$

Edit Action Interactive

0.5 1 2 $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{9}$ $\frac{1}{10}$ $\frac{1}{11}$ $\frac{1}{12}$ $\frac{1}{13}$ $\frac{1}{14}$ $\frac{1}{15}$ $\frac{1}{16}$ $\frac{1}{17}$ $\frac{1}{18}$ $\frac{1}{19}$ $\frac{1}{20}$ $\frac{1}{21}$ $\frac{1}{22}$ $\frac{1}{23}$ $\frac{1}{24}$ $\frac{1}{25}$ $\frac{1}{26}$ $\frac{1}{27}$ $\frac{1}{28}$ $\frac{1}{29}$ $\frac{1}{30}$ $\frac{1}{31}$ $\frac{1}{32}$ $\frac{1}{33}$ $\frac{1}{34}$ $\frac{1}{35}$ $\frac{1}{36}$ $\frac{1}{37}$ $\frac{1}{38}$ $\frac{1}{39}$ $\frac{1}{40}$ $\frac{1}{41}$ $\frac{1}{42}$ $\frac{1}{43}$ $\frac{1}{44}$ $\frac{1}{45}$ $\frac{1}{46}$ $\frac{1}{47}$ $\frac{1}{48}$ $\frac{1}{49}$ $\frac{1}{50}$ $\frac{1}{51}$ $\frac{1}{52}$ $\frac{1}{53}$ $\frac{1}{54}$ $\frac{1}{55}$ $\frac{1}{56}$ $\frac{1}{57}$ $\frac{1}{58}$ $\frac{1}{59}$ $\frac{1}{60}$ $\frac{1}{61}$ $\frac{1}{62}$ $\frac{1}{63}$ $\frac{1}{64}$ $\frac{1}{65}$ $\frac{1}{66}$ $\frac{1}{67}$ $\frac{1}{68}$ $\frac{1}{69}$ $\frac{1}{70}$ $\frac{1}{71}$ $\frac{1}{72}$ $\frac{1}{73}$ $\frac{1}{74}$ $\frac{1}{75}$ $\frac{1}{76}$ $\frac{1}{77}$ $\frac{1}{78}$ $\frac{1}{79}$ $\frac{1}{80}$ $\frac{1}{81}$ $\frac{1}{82}$ $\frac{1}{83}$ $\frac{1}{84}$ $\frac{1}{85}$ $\frac{1}{86}$ $\frac{1}{87}$ $\frac{1}{88}$ $\frac{1}{89}$ $\frac{1}{90}$ $\frac{1}{91}$ $\frac{1}{92}$ $\frac{1}{93}$ $\frac{1}{94}$ $\frac{1}{95}$ $\frac{1}{96}$ $\frac{1}{97}$ $\frac{1}{98}$ $\frac{1}{99}$ $\frac{1}{100}$

cExpand(6(1+3^0.5i)÷(1+i))

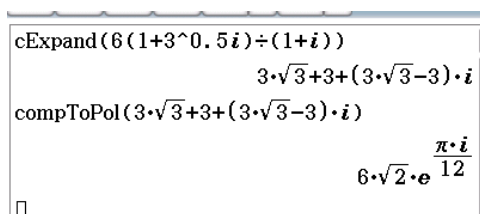
$3\sqrt{3}+3+(3\sqrt{3}-3)\cdot i$

[A1]

ii.

$$w = 3\text{cis}\left(\frac{\pi}{3}\right), \quad u = \sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$$

$$\frac{4w}{u} = \frac{4 \times 3}{\sqrt{2}} \text{cis}\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = 6\sqrt{2}\text{cis}\left(\frac{\pi}{12}\right)$$



Handwritten derivation for part ii:

$$\frac{4w}{u} = \frac{4 \times 3}{\sqrt{2}} \text{cis}\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = 6\sqrt{2}\text{cis}\left(\frac{\pi}{12}\right)$$

[A1]

iii.

$$\tan\left(\frac{\pi}{12}\right) = \frac{3\sqrt{3}-3}{3\sqrt{3}+3} \quad \text{[M1]}$$

$$= \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$\tan\left(\frac{\pi}{12}\right) = 2-\sqrt{3} \quad \text{[A1]}$$

Question 4 (9 marks)

a. $\frac{dx}{dt} = \text{Inflow} - \text{Outflow}$

Inflow $= 2 \times 0.1 = 0.2$

Outflow $= \frac{2x}{100}$

$\therefore \frac{dx}{dt} = 0.2 - \frac{2x}{100} = \frac{20 - 2x}{100} = \frac{10 - x}{50}$

[A1]

b.

$\int \frac{1}{10 - x} dx = \int \frac{1}{50} dt$

[A1]

$-\log_e |10 - x| = \frac{t}{50} + C$

$t = 0, x = 5 \quad \therefore C = -\log_e 5$

$\log_e(10 - x) - \log_e 5 = -\frac{t}{50}$

$\frac{10 - x}{5} = e^{-t/50}$

$x = 10 - 5e^{-t/50}$

[M1]

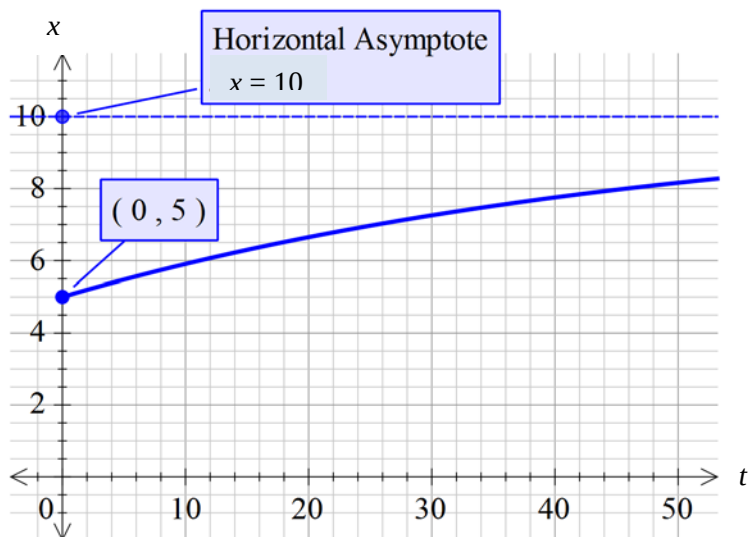
c.

$t = 2, x = 10 - 5e^{-0.02 \times 2} = 5.196$

[A1]

$x = 5.20\text{kg}$

d. Sketch over domain $t \geq 0$, y intercept is 5 and there is an asymptote at $y = 10$



[A1]

e.

i.

$$\text{Inflow} = 2 \times 0.1 = 0.2 \text{ (same)}$$

$$\text{Outflow} = \frac{3x}{100 + 2t - 3t}$$

$$\frac{dx}{dt} = 0.2 - \frac{3x}{100 - t}$$

[A1]

ii. Using Euler's method with $h = 1$ when $t = 2, x = 5.097$

t	x	$\frac{dx}{dt}$
0	5	0.05
1	$5 + 1 \times 0.05$ $= 5.05$	0.04696
2	$5.05 + 1 \times 0.04696$ $= 5.097$	

[M1]

[A1]

iii. Differential equation is

$$\frac{dx}{dt} = 0.2 - \frac{3x}{100 - t}$$

$$\frac{dx}{dt} + \frac{3x}{100 - t} = 0.2$$

$$\text{If } x = \frac{1}{10}(100 - t) - \frac{5}{1000000}(100 - t)^3$$

$$\frac{dx}{dt} = -\frac{1}{10} + \frac{15}{1000000}(100 - t)^2$$

[M1]

$$\text{L.H.S} = \frac{dx}{dt} + \frac{3x}{100 - t}$$

$$= -\frac{1}{10} + \frac{15}{1000000}(100 - t)^2 + \frac{3}{(100 - t)} \left(\frac{1}{10}(100 - t) - \frac{5}{1000000}(100 - t)^3 \right)$$

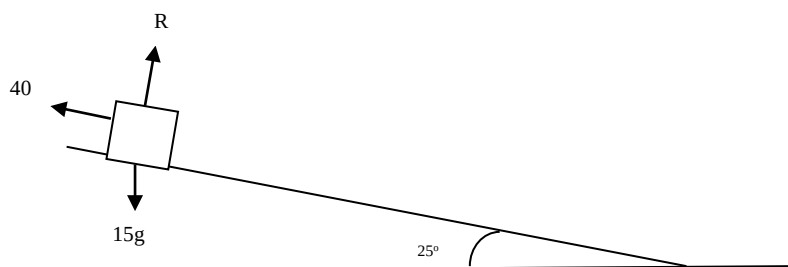
$$= -\frac{1}{10} + \frac{15}{1000000}(100 - t)^2 + \frac{3}{10} - \frac{15}{1000000}(100 - t)^2$$

$$= \frac{2}{10} = 0.2 = \text{R.H.S}$$

[A1]

Question 5

a.



[A1]

b.

$$15a = 15g \sin 25^\circ - 40$$

[M1]

$$a = g \sin 25^\circ - \frac{8}{3}$$

$$= 1.47 \text{ ms}^{-2}$$

[A1]

c.

$$u = 0 \text{ ms}^{-1}, a = 1.47499 \text{ ms}^{-2}, s = 6 \text{ m}$$

[M1]

$$s = ut + \frac{1}{2}at^2$$

$$\therefore 6 = \frac{1}{2} \times 1.47499 \times t^2$$

$$t = \sqrt{\frac{12}{1.47499}} = 2.85 \text{ secs}$$

[A1]

Note: accept 2.85 if student has used the rounded answer from part b.

d.

$$15a = 30 - 50t + 15 \times 1.47$$

[A1]

$$\text{the acceleration is approximately } a = 3.47 - \frac{10t}{3}$$

e.

$$\text{speed } v = \int_0^{0.6} \left(3.47 - \frac{10t}{3} \right) dt$$

[M1]

$$= 1.48 \text{ ms}^{-1}$$

[A1]

$$\int_0^{0.6} (3.47 - (10/3)x) dx$$

1.482

f.

1st part of journey:

$$v = 3.47t - \frac{10}{6}t^2$$

[M1]

$$\therefore \text{distance travelled in the first 0.6 secs} = \int_0^{0.6} \left(3.47t - \frac{10}{6}t^2 \right) dt = 0.5046$$

$$\int_0^{0.6} (3.47x - (10/6)x^2) dx$$

0.5046

2nd part of journey:

$$u = 1.48 \text{ ms}^{-1}, a = 1.47 \text{ ms}^{-2}, s = 6 - 0.5046 = 5.4954 \text{ m}$$

[M1]

$$5.4954 = 1.48t + \frac{1}{2} \times 1.47t^2 \quad \therefore t = 1.9070$$

$$\text{solve}(5.4954 = 1.48x + 0.5 \times 1.47x^2, x)$$

{x=-3.920629486, x=1.907024043}

$$\text{Total time} = 0.6 + 1.9070 = 2.51 \text{ secs}$$

[A1]

Question 6 (9 marks)

a. $X \sim N(40500, 4500^2)$

For a sample size of 3 $\bar{X} \sim N(40500, \frac{4500^2}{3})$

[M1]

Therefore $\Pr(\bar{X} \geq 42000) = 0.282$

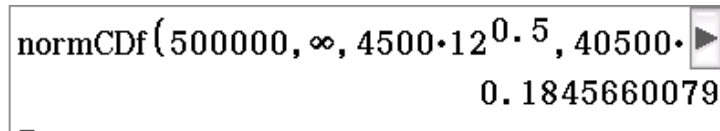
[A1]

b.

$$\text{Let } Y = X_1 + X_2 + \dots + X_{12} \quad [\text{M1}]$$

$$Y \sim N(12 \times 40500, 12 \times 4500^2)$$

$$\Pr(Y \geq 500000) = 0.185 \quad [\text{A1}]$$



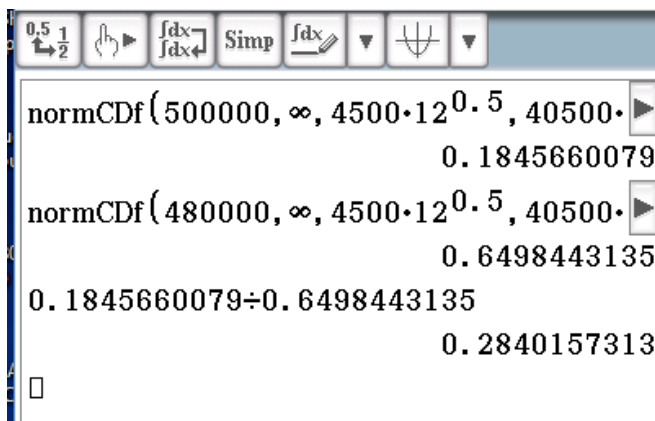
normCDF(500000, ∞, 4500·12^{0.5}, 40500·) = 0.1845660079

c.

$$\Pr(Y \geq 480000) = 0.6498$$

$$\Pr(Y \geq 500000 / Y \geq 480000) = \frac{\Pr(Y \geq 500000)}{\Pr(Y \geq 480000)} \quad [\text{M1}]$$

$$\Pr(Y \geq 500000 / \geq 480000) = \frac{0.184566}{0.649844} = 0.284 \quad [\text{A1}]$$



normCDF(500000, ∞, 4500·12^{0.5}, 40500·) = 0.1845660079
 normCDF(480000, ∞, 4500·12^{0.5}, 40500·) = 0.6498443135
 0.1845660079 ÷ 0.6498443135 = 0.2840157313

d. As we are interested in whether there has been a significant increase we use a one-sided test

$$H_0 : \mu = 40500 \quad [\text{M1}]$$

$$H_1 : \mu > 40500$$

$$\text{For a sample size of 3} \quad \bar{X} \sim N(40500, \frac{4500^2}{3})$$

$$\Pr(\bar{X} \geq 46500) = 0.01046 \quad [\text{A1}]$$

$$p\text{-value} = 0.01046$$

$p < 0.05$ there is significant evidence to reject the Null Hypothesis

There is evidence to suggest that the advertising campaign has lead to a significant increase in the monthly turnover [A1]

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normCdf(46500, ∞,  $\frac{4500}{3^{0.5}}$ , 40500)  
0.01046066767
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□