

Student's Name.....

Teacher's Name

SPECIALIST MATHEMATICS UNIT 2

EXAMINATION

Paper 1: Short Answer

November 2017

Reading Time: 5 minutes

Writing time 40 minutes

Instructions to students

This exam consists of 12 short-answer questions.

The short-answer questions should be answered in the spaces provided.

All questions should be answered.

There are 40 marks available.

The use of calculators and electronic dictionaries is **NOT** permitted.

Students may **NOT** use any notes or reference material in this examination.

Students are provided with a Formula sheet.

Question 1 (5 marks)

Consider the complex numbers $z_1 = 1 + \sqrt{3}i$, $z_2 = 1 + i$ and $w = \frac{z_1}{z_2}$.

- a) Express z_1 and z_2 in polar form.

2 marks

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- b) Write down

- i) the modulus of w ;

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- ii) the argument of w .

3 marks

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Question 2 (2 marks)

Prove the following identity:

$$\tan^2 x + 1 + \tan x \sec x = \frac{1 + \sin x}{\cos^2 x}.$$

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Question 3 (4 marks)

If $\sin \theta = \frac{\sqrt{5}}{3}$ and $\theta \in \left[0, \frac{\pi}{2}\right]$, find the exact values of:

a) $\sin(2\theta)$

1 mark

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b) $\tan(2\theta)$

2 marks

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c) $\sin\left(\frac{\theta}{2}\right)$

1 mark

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Question 4 (2 marks)

If $\underline{u} = 2\underline{i} + 4\underline{j} - 3\underline{k}$ and $\underline{v} = \underline{i} - \underline{j} - \underline{k}$ find:

a) a unit vector in the direction of $\underline{v}$.

1 mark

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b) a scalar resolute of $\underline{u}$ in the direction of $\underline{v}$.

1 mark

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Question 5 (2 marks)

Express $\frac{4x}{(x+3)(x-1)}$ in partial fraction form.

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Question 6 (3 marks)

Sketch the graph of $y = \frac{1}{x^2 - 6x + 8}$ showing axes intercepts, turning points and asymptotes labelled with coordinates and equations.

Question 7 (2 marks)

A body is moving in a straight line with uniform acceleration and an initial velocity of 12 m/s. If the body stops after 20 metres, find the acceleration of the body.

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Question 8 (7 marks)

A particle's motion is described by the following parametric equations

$$x(t) = 4 \sec(t)$$

$$y(t) = 2 \tan(t)$$

where distance is measured in metres and time, t , in seconds such that $t \in \left[0, \frac{\pi}{2}\right)$.

- a) Show that the Cartesian equation of the path of the particle is $\frac{x^2}{16} - \frac{y^2}{4} = 1$. 2 marks

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- b) Sketch the path of the particle, labelling any asymptotes with their equations. 3 marks

c) Write down the position of the particle at

i) $t = 0$

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ii) $t = \frac{\pi}{4}$

2 marks

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Question 9 (2 marks)

Differentiate $y = \frac{5}{\sqrt{x^3 - 4x}}$.

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Question 10 (3 marks)

The velocity, $v \text{ ms}^{-1}$, at time t , seconds, of a body moving in a straight line is given by $v = 6t^2 + 6t - 12$. Its initial position is 7 metres to the right of O .

Find the body's displacement at $t = 2$.

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Question 11 (4 marks)

The points A , B and C have position vectors $\overrightarrow{OA} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} - \mathbf{j}$ and $\overrightarrow{OC} = \mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$.
Prove that AC bisects OB at right angles.

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Question 12 (4 marks)

An ellipse has equation $x^2 + 4x + 2y^2 = 0$.

- a) Find the coordinates of the centre.

2 marks

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- b) Sketch the ellipse marking axes intercepts.

2 marks

- c) State the length of the minor axis.

1 mark

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END OF EXAMINATION PAPER 1